Persistent current in a one-dimensional correlated disordered ring

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Persistent currents and charge stiffness in one-dimensional (1D) correlated disordered rings are investigated by a transfer-matrix method. It is found that, under certain conditions the electronic states do not feel the existence of the disorder and the ring recovers the ordered case. If the occupied energy is just at the unscattered state energy level, the persistent current is not reduced and shows the behavior of sudden jumps regardless of the disorder, contrary to the general theorem where the current is obviously reduced and becomes a smooth function of flux, passing through zero. For other occupied energy, the persistent current is scattered and decreased as expected. The results indicate that a quantum interference effect can be observed in 1D correlated disordered rings. Further, the results provide evidence that the realization of delocalized states in the experiments is easily obtained by observing only the persistent current of finite length rings, in contrast to the 1D disordered chain where the infinite length is needed.

I. INTRODUCTION

The problem of persistent currents in one-dimensional (1D) metal or semiconductor mesoscopic rings threaded by a magnetic flux has attracted much attention in the last decade. In isolated mesoscopic metallic rings, Büttiker *et al.* have predicted that static magnetic fields should induce persistent current.[1] The paper has initiated a renewal of interest in the persistent current topic.[2–16] The persistent currents are a consequence of the boundary conditions imposed by the field, and are periodic functions of the magnetic flux ϕ threading the ring, with a fundamental period equal to the quantum of magnetic flux $\phi_0 = hc/e$. It is believed that the disorder and interaction are two important factors that affect the amplitude of the persistent current in 1D mesoscopic rings. In close analogy to the behavior of 1D localization, it has been theoretically predicted that all forms of disorder are found to round the steps in the $I-\phi$ characteristic and to reduce the amplitude of persistent currents.[3,5–7,13–16] Such a localization dependence has generally been discussed by using the concept of single electronic localization,[17] in which all electronic states in 1D disordered systems are always localized. On the other hand, models that include the electron-electron interactions have been investigated theoretically. Some results have indicated that both long-range and short-range electron-electron interactions suppress the persistent current.[18–20] Conversely, Wang, Wang, and Zhu have found that the persistent current is enhanced by the electron-electron interactions in a disordered ring by means of the world-line quantum Monte-Carlo simulations.[21] They do not directly compare their result with the experimental observations. But, their result is more close to the direction of the experimental results. Meantime, their result is also in agreement with that of other theoretical calculations.[22,23]

However, there are some examples of 1D disordered systems where the extended states have been found. It has been shown that the existence of extended states is a consequence of introducing short-range correlations on the disordered distribution. In a 1D random dimer model,[24,25] Dunlap *et al.* have shown that when one of the site energies is assigned at random to pairs of lattices (that is, two sites in succession), $\sqrt{N}$ of the electronic states are delocalized, which is consistent with other theoretical results.[26,27] This is a tight-binding model with site-diagonal disorder and is constructed by randomly inserting a number of identical dimers into a purely periodic chain. The random dimer model has been generalized to include more complex arrangements of symmetrical defects[28] and other models of paired correlations,[29] without suppressing the existence of many extended states. We have also shown that the extended states appear in other 1D random models, for example, the 1D site-diagonal disordered tight-binding model with random period[30] and 1D site-diagonal tight-binding random cluster model.[31] The random dimer model is also in this class with two atoms in each inserted cluster. In these 1D correlated disordered models, the competition between the long-range disorder and the short-range correlation causes the appearance of delocalization and long-range transport.[32,33] More recently, the delocalization has also been found in 1D tight-binding model with long-range correlated disorder.[34] It has been indicated that the presence of mobility edges arises from the long-range correlations in the disordered distribution. It is natural to ask whether there exist fixed persistent currents regardless of disorder or how the persistent currents are influenced by the disorder in the correlated disordered rings if the rings are constructed by the 1D random segment chains. To the best of our knowledge, so far the effect of the disorder on the persistent currents in 1D correlated disordered rings remains unexplored. The study of the persistent currents in 1D correlated disordered rings may provide a profound understanding of the 1D disordered localization. Therefore, it is worthwhile to explore the properties of persistent currents in mesoscopic

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correlated disordered rings. In this paper, we focus on a class of 1D tight-binding correlated disordered rings, the random segment rings, which can be constructed by randomly inserting a number of identical segments into a purely periodic chain. Using the transfer-matrix method, we show rigorously that there exist fixed persistent currents regardless of disorder at some selected energies. From a numerical scheme, we illustrate the energy spectrum, the persistent currents and the charge stiffness of a sample ring with different choices of parameters. The existence of the fixed persistent currents in 1D correlated disordered rings is further confirmed, contrary to the general theorem on the decrease of persistent currents in 1D disordered rings. The existence of the fixed persistent currents is equivalent to the appearance of delocalized states in the 1D disordered chain. Therefore, the realization of delocalized states in the experiments is easily obtained by observing only finite length rings, in contrast to the 1D disordered chain where the infinite length is needed. In addition, the effect of the electron-electron interaction on the persistent current will also be discussed appropriately.

II. THEORY

In the tight-binding approximation, the 1D disordered ring can be described by Anderson model with site-diagonal disorder. The effect of the magnetic field is simply to change the boundary condition along the ring. Thus, the Hamiltonian for the Anderson model can be written as follows[3,15]

$$H=\sum_{i=1}^{N}\epsilon_i\hat{a}_i^+\hat{a}_i+\sum_{i=1}^{N-1}t(\hat{a}_{i+1}^+\hat{a}_i+\hat{a}_i^+\hat{a}_{i+1})+te^{i(2\pi\phi/\phi_0)}\hat{a}_N^+\hat{a}_1+te^{-i(2\pi\phi/\phi_0)}\hat{a}_1^+\hat{a}_N, \tag{1}$$

where $\hat{a}_i^+(\hat{a}_i)$ creates (annihilates) an electron in the Wannier state at the ith lattice site, ϵ_i is the on-site energy and t is the hopping-matrix element connecting neighboring site. The lattice constant is taken to be 1 and the lattice sum runs from $j=1$ to N.

The ring is made up of alternating connections of segments of two species A and B. Thus, ϵ_i takes the value ϵ_A or ϵ_B depending on the species of the ith site. We consider a random distribution in which the lengths of A segments are random and the lengths of B segments are fixed. This means that the length of a particular A segment is a random variable. The ring arrangement is

$$\underbrace{\ldots AAA\ldots}_{L_i}\underbrace{BBB\ldots}_{m}\underbrace{AAA\ldots}_{L_{i+1}}\underbrace{BBB\ldots}_{m}\underbrace{AAA\ldots}_{L_{i+2}}\underbrace{BBB\ldots}_{m},$$

where L_i is the length of segment A and m is the length of segment B. We denote $P_A(L_i)$ as the stochastic function describing the length distributions of A site segments. The form and the extent of the disorder are completely controlled by the function. Here, we consider the distribution as follows

$$P_A(L_i)=\sum_j p_j\delta(L_i-j), \tag{2}$$

where

$$\delta(l)=\begin{cases}1 & l=0\\ 0 & l\neq 0\end{cases}$$

and p_j is the probability of finding a segment of A having j sites. The whole lattice is formed by sequential connections of A and B segments.

The tight-binding equation may be rewritten in a transfer matrix form,

$$\Psi(l+1)=\hat{M}(l)\Psi(l), \tag{3}$$

where $\Psi(l)$ is a column vector $\binom{\psi_l}{\psi_{l-1}}$ and $\hat{M}(l)$ is a transfer matrix $\begin{pmatrix}E-\epsilon_l/t & -1\\ 1 & 0\end{pmatrix}$. Repeated application of $\hat{M}(l)$ to the disordered ring gives

$$\begin{pmatrix}\psi_{N+1}\\ \psi_N\end{pmatrix}=\prod_{i=1}^{N}\hat{M}(i)\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}=e^{i(2\pi\phi/\phi_0)}\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}. \tag{4}$$

For the disordered ring, the product of matrices in Eq. (4) becomes

$$\ldots\hat{M}_A^{L_i}\hat{M}_B^{m}\hat{M}_A^{L_{i+1}}\hat{M}_B^{m}\ldots, \tag{5}$$

where $\hat{M}_A$ or $\hat{M}_B$ is the matrix $\hat{M}(i)$ with ϵ_i equal to ϵ_A or ϵ_B.

From the theory of matrices, the mth power of the 2×2 unimodular matrix $\hat{M}_B$ can be written as[35]

$$\hat{M}_B^m=u_{m-1}(x)\hat{M}_B-u_{m-2}(x)\hat{I}, \tag{6}$$

where $x=\frac{1}{2}\,\mathrm{Tr}(\hat{M}_B)$, $\hat{I}$ is a unit matrix, $u_m(x)$ is the mth Chebyshev polynomial of the second kind, which obeys the recurrence relation

$$u_{m+1}(x)=2xu_m(x)-u_{m-1}(x),\quad m\geqslant 0 \tag{7}$$

with $u_{-1}=0$ and $u_0=1$. If $|x|\leqslant 1$, $u_m(x)$ has the form

$$u_{m-1}(x)=\frac{\sin(m\theta)}{\sin\theta}, \tag{8}$$

where $\theta=\cos^{-1}(x)$. If

$$x=x_l=\cos\left(\frac{l\pi}{m}\right),\quad l=1,2,\ldots,m-1,\quad \text{for}\quad m\geqslant 2, \tag{9}$$

we have $u_{m-1}(x_l)=0$ and $u_{m-2}(x_l)=(-1)^{l+1}$. From Eq. (6), one has

$$\hat{M}_B^m=(-1)^l\hat{I}, \tag{10}$$

where l represents the lth unscattered electronic states. Based on these formulas, the energy of the unscattered state corresponding to x_l in Eq. (9) is $E_l=\epsilon_B+2t\cos(l\pi/m)$, ($l=1,2,\ldots,m-1$ for $m\geqslant 2$). From Eq. (10), for the state with eigenenergy E_l, the matrix string of Eq. (5) is only composed of matrices $\hat{M}_A$ and $(-1)^l\hat{I}$. Since matrix $(-1)^l\hat{I}$ has no influence on the amplitude of wave function, this matrix string is equivalent to that of periodic ring, which only consists of matrix $\hat{M}_A$. Therefore, if the Fermi level is just equal to E_l, the persistent current will recover to that of the ordered ring.

III. RESULTS AND DISCUSSIONS

In order to understand the effect of disorder on the properties of the ring threaded by a magnetic flux, we numerically calculate the flux-dependent energy levels of electronic states by using Eq. (5). Once the flux-dependent energy levels are obtained, the persistent current contributed from the nth level is

$$I_n(\phi) = -c\frac{\partial E_n(\phi)}{\partial \phi}. \tag{11}$$

If the electrons occupy the highest energy level N_e, the total persistent current of the system is

$$I(\phi) = \sum_{n=1}^{N_e} I_n(\phi). \tag{12}$$

We discuss first the effect of disorder on the energy levels. Figure 1(a) shows the energy spectrum of a 1D disordered ring as a function of magnetic flux ϕ. For the sake of comparison, the energies are also exhibited for the 1D ordered ring in Fig. 1(b). In disordered case, every energy curve changes smoothly with flux. It is found that the presence of the disorder opens gaps at the points $\phi/\phi_0 = 0$ and ± 0.5 of the intersection of the corresponding energy bands of ordered ring. The behavior is similar to that of the general 1D disordered ring, in which impurities and disorder lead to the presence of gaps in energy spectrum at the point $\phi/\phi_0 = 0$ and ± 0.5.[3,13] However, we will show that there are substantial changes in the persistent currents for 1D rings with different types of disordered distributions.

It is easily found that for 1D ring with random segments, the resonance condition is satisfied when $E_l = \epsilon_B + 2t\cos(l\pi/m)(l=1,2,\ldots,m-1)$. In these energy levels, the electronic states do not feel the existence of the random segments. The ring recovers the ordered ring, only built by A atoms. If the occupied energy is just at the resonance energy level, the persistent current is not reduced and shows the behavior of delocalized states. At other occupied energies, the electronic states feel the existence of disordered scattering. Instead of sudden jumps, the current is obviously reduced and becomes a continuous function of flux, passing through zero. In Fig. 2, we show the current of the 1D disordered ring at both occupied energies (resonance and off-resonance) with different electronic occupied numbers $N_e = 22$ and $N_e = 28$, corresponding to (a) and (b), respectively. For the sake of comparison, the persistent current in 1D ordered ring is exhibited with $N_e = 28$ in Fig. 2(c). Here, the maximum persistent current of the periodic tight-binding model $I_0 = (4\pi c/N\phi_0)\sin(N_e\pi/N)$ is taken to be the current unit. We can see that when the energy of the unscattered state coincides with the Fermi level of the ring, the persistent current is equivalent to that of a ring without disorder and shows the behavior of delocalized states, which is not reduced and appears sudden jumps; while in other energy the persistent current is reduced and becomes a smooth function of flux. In Fig. 2(b), we also show the result for an odd N_e case with dashed curve. It is found that the current-flux curve shifts $\phi/\phi_0 = 1/2$ along the flux axis when compared with that (solid curve) for even N_e in Fig. 2(b). The parity effect of the electronic filling with odd and even numbers is similar to that of the periodic system.[3,36] In the sample of our calculation, when filling with odd number of electrons we cannot find the unscattered persistent current because the Fermi level of the ring does not coincide with the energy of the delocalized state. If one changes the parameters of the disordered ring, such as, ring length, random-segment length or atomic site energy etc., the unscattered persistent current can still be found and only shifts $\phi/\phi_0 = 1/2$ along the flux axis when compared with that for even N_e in Fig. 2(a). It is expected to have a high conductivity in the delocalized energy. This indicates that the properties of the disordered ring at the unscattered states are independent of the disorder. The conclusion is not in agreement with that of general disordered rings, in which all electronic states are scattered, and the amplitude of persistent current should be reduced and becomes a smooth function of magnetic flux ϕ.[3,5–7,13,14] However, the result is the evidence of the existence of delocalized states in 1D correlated disordered system. Therefore, the ex-

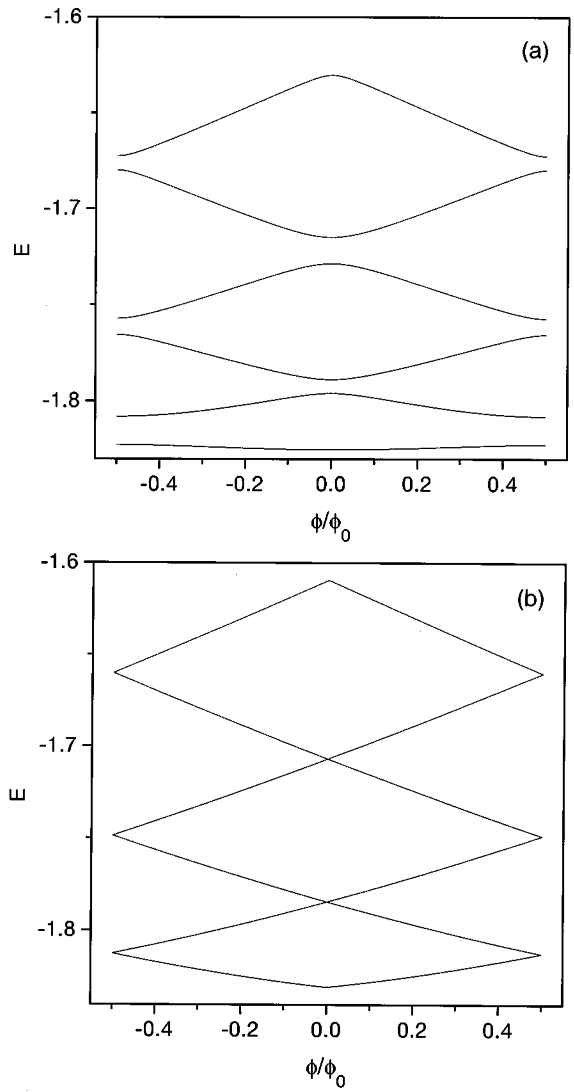

FIG. 1. Several lower electronic energy levels as a function of the flux ϕ/ϕ_0 in 1D rings with $N=80$, $m=3$, $\epsilon_A=0.0$, and $\epsilon_B=-0.5$. The parameters in Eq. (3) are (a) disordered ring, $p_j=\frac{3}{5}$ for $j=3$, $p_j=\frac{1}{5}$ for $2\leqslant j\leqslant 4$, $j\neq 3$, and $p_j=0$ otherwise, and (b) periodic ring, $p_j=1$ for $j=3$ and $p_j=0$ otherwise. All energies are in unit of t.

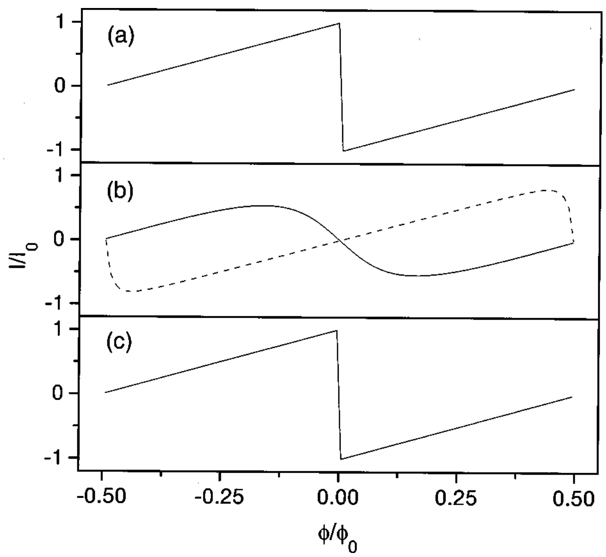

FIG. 2. The persistent current I vs ϕ/ϕ_0 for 1D rings with $N=80$, $m=3$, $\epsilon_A=0.0$, and $\epsilon_B=-0.54$. The disordered ring with the highest occupied energy level (a) $N_e=22$ and (b) $N_e=28$ (solid curve) and $N_e=21$ (dashed curve), and other parameters are the same as Fig. 1(a); the periodic ring (c) the highest occupied energy level $N_e=28$ and other parameters are the same as Fig. 1(b).

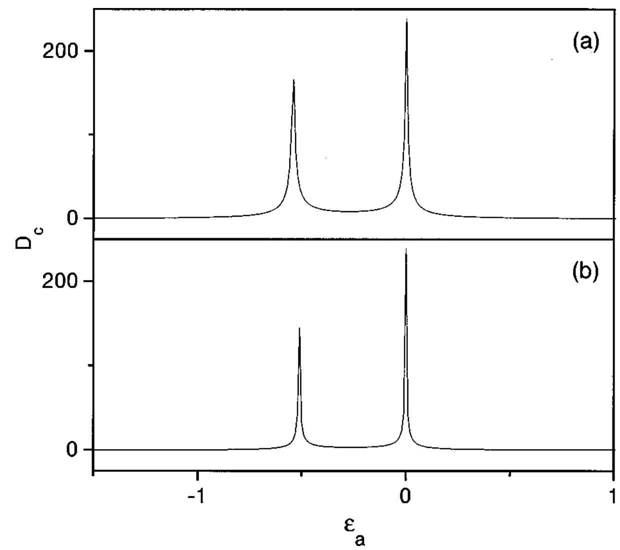

FIG. 3. Charge stiffness D_c vs on-site energy ϵ_B of random segment with $N=80$, $N_e=22$, $\epsilon_A=0.0$, and $m=3$. The parameters in Eq. (3) are (a) $p_j=\frac{4}{10}$ for $j=3$, $p_j=\frac{3}{10}$ for $2\leq j\leq 4$, $j\neq 3$, and $p_j=0$ otherwise; (b) $p_j=\frac{6}{20}$ for $j=3$, $p_j=\frac{6}{20}$ for $2\leq j\leq 4$, $j\neq 3$, $p_j=\frac{1}{20}$ for $j=1$ and $j=5$, and $p_j=0$ otherwise.

istence of delocalized states is easily observed by using the 1D disordered ring with finite length.

Another important physical magnitude is the charge stiffness, which represents the response of the persistent current to the applied flux. Kohn has shown that the localization characteristics of electronic state is closely tied to the charge stiffness,[37]

$$D_c=\frac{1}{2}\frac{d^2(E_0/N)}{d(\phi/N)^2}\bigg|_{\phi=0^+}, \tag{13}$$

where E_0 is the ground-state energy of a ring in a field. The definition of D_c assumes a ring-shaped geometry of length N threaded by a flux ϕ. Kohn noted that for an insulating system D_c decays exponentially to zero, while it remains finite for a metal. In terms of the ground state energy E_0, changes in the magnetic flux have a profoundly different effect in a metal than an insulator. Thus, the charge stiffness, which characterizes the ground state of the system can be used as a measure of a transition from the localized regime (insulator) to the ballistic one (perfect metal).

We calculate D_c for several systems with different fillings and different random segment lengths, as a function of the site energy of identical random segments. At a certain electronic filling, when the site energy ϵ_B is changed so that the resonance energy $E_l=\epsilon_B+2t\cos(l\pi/m)$ coincides with the Fermi level of the disordered ring, we expect a high conductivity, and therefore a maximum in the charge stiffness. Numerical results of the charge stiffness are shown in Fig. 3 as a function of ϵ_B for the disordered rings with two different degrees of randomnesses, where the total length N of ring and the length m of random segment are taken to be 80 and 3, respectively. Several facts are worth noticing in the Fig. 3: (i) The charge stiffness shows two peaks, one at $\epsilon_B=0$ (corresponding to a system built only by A atoms and without disorder because of $\epsilon_B=\epsilon_A=0$), and the other in the region of the resonance. (ii) The height of the peaks converges to a finite nonzero value, indicating that the systems are metallic at those energies. (iii) D_c takes a finite nonzero value at the energy of resonance state, and the position of the peak is independent of the degree of randomness. For example, the peaks have the same positions for different disordered degrees and the fixed segment length m in Figs. 3(a) and 3(b), corresponding to the parameters in Eq. (3) to be $p_j=\frac{4}{10}$ for $j=3$, $p_j=\frac{3}{10}$ for $2\leq j\leq 4$, $j\neq 3$ and $p_j=0$ otherwise, and $p_j=\frac{6}{20}$ for $j=3$, $p_j=\frac{6}{20}$ for $2\leq j\leq 4$, $j\neq 3$, $p_j=\frac{1}{20}$ for $j=1$ and $j=5$ and $p_j=0$ otherwise, respectively. Furthermore, if we consider the disordered rings with different random segment m, it can be found that the peaks of charge stiffness have different locations, satisfying Eq. (10), in which the electronic states do not feel the existence of random segments. There are $m-1$ finite nonzero peaks (expect for that of $\epsilon_B=0$), in which the electrons freely pass through the disordered ring, regardless of disorder. Another worth noticing fact is that the charge stiffness peaks appear when the energy levels of delocalized states in the disordered ring correspond to the Fermi energy, the highest occupied energy level. Similar behaviors can also be found for odd-number electrons, but with the flux being shifted by $\phi_0/2$. We think that if one want to observe the delocalized state in 1D system, the system must be infinite or at least much longer than the localized length. If the system is a finite ring, the localization theory is easily confirmed by observing the current because the persistent current is very sensitive to the configurations of rings. Experiment requires only a very short finite ring. Our results provide also a new electronic interference phenomenon, regardless of disorder, in the correlated disordered rings. In our model calculation, we consider onsite energies distributed in such a way that the impurity always appears in finite segments of fixed size. Delocalized states arise from resonant modes, which present no scattering through these finite structures. Such states form a discrete set of energy values. We can observe the unscattered persistent current and the effect of high conductivity at particular delocalized energies. If a long-range correlation is introduced in the disordered distribution, one can find the existence of an Anderson transition with mobility edges separating delocalized and localized states in the 1D disordered system.[34] The delocalized states exist in a finite range of energy values. Therefore, if the ring is constructed by the 1D chain with long-range correlated disorder, it can be predicted that the unscattered persistent current and the high conductivity will appear in a finite range of energy values. The behavior in this system needs further exactly study.

Further, we discuss appropriately the effect of the electron-electron interactions on the persistent current in the 1D correlated disordered ring. The discussion is based on the results we have obtained in 1D correlated disordered system with electron-electron interactions in the Hartree-Fock approximation.[38] With a certain electronic filling, in the Hartree-Fock approximation, it has been found that the occupied number of electrons is different in the atomic sites of the random segments. So, the renormalized energies of the sites in the random segments are different. This means that the matrix product of in Eq. (4) can not be expressed as the product of matrices $\hat{M}_A^{L_i}$ and $\hat{M}_B^{m}$. In this disordered matrix product, the 1D correlated disordered system becomes a general 1D disordered model. The extended states will disappear in a rigorous sense. The result is contrary to the noninteracting case, in which the extended states are not affected by the disorder. Therefore, in the 1D correlated disordered ring with the electron-electron interactions, one can predict that the persistent current may be suppressed by the interactions in the Hartree-Fock approximation. This conclusion is in agreement with that of the general 1D disordered ring with the interactions.[18–20] However, with a more exact method, one may obtain much more interesting results, for example, by means of the world-line Monte Carlo simulations[21,39] and the exact diagonalization calculations[22] etc. The exact results in the 1D correlated disordered ring with the electron-electron interactions are beyond our present model calculations. It remains an open question whether the novel persistent current still exists in a 1D correlated disordered ring when the electron-electron interactions are treated exactly.

IV. CONCLUSIONS

We have studied the persistent currents and charge stiffness in 1D disordered ring, which is constructed by randomly inserting a number of identical segments into a purely periodic ring. Using the transfer-matrix method, we have exactly shown that under certain conditions some electronic states do not feel the existence of the disorder and the ring recovers the ordered case. If the occupied energy is just at the energy level of the unscattered state, the persistent current is not reduced and shows the behavior of sudden jumps regardless of disorder, contrary to the general theorem where the current is obviously reduced and becomes a smooth function of flux, passing through zero; while for other occupied energy, the persistent current is scattered and decreased as expected. At a certain electronic filling, when the site energy of identical random segments is changed so that the unscattered state energies coincide with the Fermi level of the disordered ring, it is expected to have a high conductivity, and therefore a charge stiffness maximum. It is also found that if the identical segment length is m, there are $m-1$ finite nonzero peaks in the charge stiffness. The results indicate that a quantum interference effect can be observed in 1D correlated disordered mesoscopic rings. Furthermore, the results provide evidence that the realization of delocalized states in the experiments is easily obtained by observing only the fixed persistent current of finite length rings, in contrast to the 1D disordered chain where the infinite length is needed.

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