Kondo effect in crossed Luttinger liquids

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We present results for the Kondo effect in two *crossed* Luttinger liquids by using boundary conformal field theory. We predict two types of critical behaviors: either a two-channel Kondo fixed point with a nonuniversal Wilson ratio or a theory with an anomalous reponse that is identical to that found by Furusaki and Nagaosa for the Kondo effect in a single Luttinger liquid. Moreover, we discuss the relevance of perturbations like channel anisotropy in restoring a Fermi-liquid-like Kondo fixed point, and compare our results with the Kondo effect in a two-band Hubbard system modeled by a channel-dependent Luttinger Hamiltonian. The suppression of backscattering off the impurity produces a model similar to the four-channel Kondo theory. Consequences are discussed.

I. INTRODUCTION

The one dimensional (1D) conductors differ fundamentally from those in three dimensions, where the low-energy properties can be described very well by Landau's Fermi liquid theory. In 1D, the resulting state is often of the Luttinger liquid (LL) type.[1,2] The physics of such low-dimensional systems has received much attention lately, mainly due to advances in nanofabrication[3] and the discovery of novel 1D materials such as carbon nanotubes.[4] The study of magnetic impurities in 1D unconventional correlated hosts has attracted great interest in the last few years. The Kondo effect in a LL yields two possible fixed points[5–7]. *Either* the system behaves rather like a Fermi liquid [with a nonuniversal Wilson ratio and an $SU(2)_{k=2}$ spin symmetry[7]] *or* it indeed has the non-Fermi-liquid properties predicted by Furusaki and Nagaosa.[8]

In this paper, we study the Kondo effect in two *crossed* Luttinger liquids,[9] i.e., two correlated 1D metals coupled in a pointlike manner via a magnetic impurity. An important question is examined: are the two fixed points cited above stable when several conducting channels interact through a pointlike Kondo coupling? The geometry of our system is shown in Fig. 1. The authors of Ref. 10 have studied the Kondo effect in a two-band Hubbard chain modeled by a channel-dependent Luttinger Hamiltonian. On the other hand, for the most general two-band problem investigated in Ref. 11, a prominent repulsive Hubbard interaction normally destroys the LL phase producing a metallic spin-gapped phase with a leading d-wave order parameter. The resulting Kondo problem becomes very difficult to handle.

In our case, the two Luttinger liquids are supposed to be *noninteracting* [except at the impurity site]. In particular, we do not include an electron-electron interaction for two particles that belong to different conducting channels. Further experiments on magnetic impurities implanted in 1D quantum wires or carbon nanotubes[9] could provide impetus for studying this model.

II. MODEL

As long as the angle Γ [that is depicted in Fig. 1] does not tend to zero, we can separate the two degenerate Luttinger liquids and can neglect the electron-electron interaction between channels with different i $(i=1,2)$. As in Ref. 9, we consider that x measures deviations from the magnetic impurity in both conducting channels. In such sense, we have only one coordinate left.

The Hamiltonian

$$\mathcal{H}=\mathcal{H}_o+\mathcal{H}_U+\mathcal{H}_K \tag{1}$$

for this two-channel Kondo model [with left (L) and right (R) moving electrons per channel] consists of the term for free electrons

$$\mathcal{H}_o=v_F\left\{\psi^{\dagger}_{iR\sigma}i\frac{d}{dx}\psi_{iR\sigma}-\psi^{\dagger}_{iL\sigma}i\frac{d}{dx}\psi_{iL\sigma}\right\}, \tag{2}$$

with v_F being the Fermi velocity and $i=1,2$ channel index; an electron-electron (e-e) interaction term

$$\mathcal{H}_U=Uj^i_p j^i_{p'},\; j^i_{L(R)}=:\psi^{\dagger}_{iL(R)\alpha}\psi_{iL(R)\alpha}:, \tag{3}$$

with $U>0$;[12] and forward and backward scatterings off the impurity:

$$\mathcal{H}_K=\lambda_F\psi^{\dagger}_{iL(R)\alpha}(0)\boldsymbol{\sigma}_{\alpha\beta}\psi_{iL(R)\beta}(0)\cdot\mathbf{S} + \lambda_B\psi^{\dagger}_{iL(R)\alpha}(0)\boldsymbol{\sigma}_{\alpha\beta}\psi_{iR(L)\beta}(0)\cdot\mathbf{S}, \tag{4}$$

where $\boldsymbol{\sigma}$ are the usual spin-1/2 matrices. For the physically relevant case, we have $\lambda_F=\lambda_B=\lambda_K$ (*the usual Kondo interaction*).

Conduction electrons of one liquid respond to a spin flip of the impurity caused by the interactions with electrons of

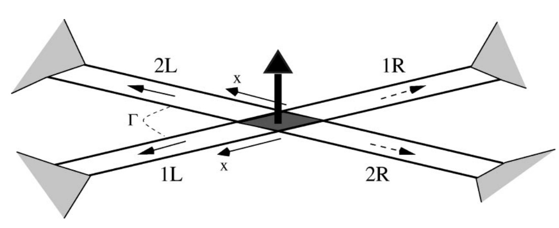

FIG. 1. Two Luttinger liquids coupled only at $x=0$ via the Kondo effect. The angle Γ is assumed to remain finite.

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the other liquid. In this way, there is an *induced* interaction between the liquids. We could also include another interaction of the form:

$$\lambda_m \epsilon_{ij}[\psi^{\dagger}_{iL(R)\alpha}\boldsymbol{\sigma}_{\alpha\beta}\psi_{jL(R)\beta}+\psi^{\dagger}_{iL(R)\alpha}\boldsymbol{\sigma}_{\alpha\beta}\psi_{jR(L)\beta}]\mathbf{S}, \quad (5)$$

where $\epsilon_{ij}=1$ for $i=1$, $j=2$ and zero otherwise. First, we neglect the term in Eq. (5).

We subsequently study this problem by using boundary conformal field theory (BCFT). The heart of the method, pioneered by Affleck and Ludwig,[13,14] is to replace the impurity by a scale invariant boundary condition. It was successfully applied to study the low-temperature properties of a spin-1/2 magnetic impurity coupled to a LL,[5–7] and to solve the Kondo effect in the particular two-band Hubbard chain of Ref. 10.

Below, we shall precisely discuss how the geometry of Fig. 1 influences the two fixed points found in Ref. 10.

III. ONLY FORWARD SCATTERING OFF THE IMPURITY

We first study the case of only *forward* scattering off the impurity, i.e. $\lambda_B=0$. Let us start with a *free* electron gas where $U=0$.

A. Four-channel Kondo model with free electrons

To solve this case using BCFT, it is convenient to define right and left movers on the half-plane $x\geqslant 0$ (see Fig. 1), so that

$$\psi_{iR\alpha}(t,x)\equiv\psi_{iL\alpha}(t,-x), \quad (6)$$

with $i=1,2$, and to confine the system to the finite interval $x\in[-l,l]$. Fields are *left* movers only and it is useful to rename $\chi_{1\alpha}(x)=\psi_{1L\alpha}(x)$, $\chi_{2\alpha}(x)=\psi_{1L\alpha}(-x)$, $\chi_{3\alpha}(x)=\psi_{2L\alpha}(x)$, and $\chi_{4\alpha}(x)=\psi_{2L\alpha}(-x)$. Keeping only $\lambda_F\neq 0$ (forward scattering off the impurity) in Eq. (3), it follows:

$$\mathcal{H}_F=\lambda_F\mathbf{J}(0)\cdot\mathbf{S}. \quad (7)$$

Here, $\mathbf{J}$ is the electron spin current density: $\mathbf{J}(x)=\sum_{i=1}^{k}\chi^{\dagger}_{i\alpha}(x)\boldsymbol{\sigma}_{\alpha\beta}\chi_{i\beta}(x)$ and $k=4$. Note that the information about the number of channels is contained in the commutation rules satisfied by these currents,[14] indicating that $J^a(x)$ form an $SU(2)_k$ Kac-Moody algebra.

Generally, we must also introduce,

$$J(x)=\sum_{i=1}^{k}\chi^{\dagger}_{i\alpha}(x)\chi_{i\alpha}(x), J^A(x)=\sum_{ij\alpha}\chi^{\dagger}_{i\alpha}(x)\mathbf{T}^A_{ij}\chi_{j\alpha}(x), \quad (8)$$

where $\mathbf{T}^A_{ij}$ are the generators of the $SU(k)$ group. Thus, the free Hamiltonian $\mathcal{H}_o$ can be rewritten in a suitable Sugawara form,

$$\mathcal{H}_o=\frac{v_F}{2\pi}\int dx\frac{J(x)J(x)}{4k}+\frac{\mathbf{J}(x)\mathbf{J}(x)}{k+2}+\frac{J^A(x)J^A(x)}{k+2}\cdot \quad (9)$$

This allows one to formulate the problem entirely in terms of the electron spin current, $\mathbf{J}(x)$. It leads to an effective four-channel (left-handed) Kondo theory.[14] Briefly, we summarize the arguments below.

The unperturbed problem organizes into a product of *three* conformal towers labeled by the quantum numbers (Q,j,j_f), respectively the charge, the spin, and the flavor of the system. Starting with an even number of particles the high-temperature physics is described by the set $(Q=0,j=0,\text{flavor singlet})$. For the special value:

$$\lambda_F^*=\frac{v_F}{k+2}, \quad (10)$$

[the unique solvable point in the isotropic region, which is commonly identified as the fixed point of the model[15]], we can absorb the impurity spin by redefining the spin current as that of electrons and impurity:

$$\mathbf{J}(x)\rightarrow\mathbf{J}(x)+2\pi\mathbf{S}\delta(x). \quad (11)$$

For the overscreening Kondo effect, the absorption of the impurity spin takes place in the weak-coupling limit and then the groundstate degeneracy g is not exactly 1 as in the completely screened situation,[14] but it takes a *noninteger* value smaller than 2 (the groundstate degeneracy at high temperatures). Then, some extra *nonmagnetic* degrees of freedom occur at the impurity site.

Near the fixed point, the Hamiltonian can be written as the fixed point Hamiltonian plus possible perturbations

$$\mathcal{H}=\mathcal{H}_F+\sum_i \gamma_i\mathcal{O}_i(0). \quad (12)$$

We can classify all the possible perturbations $\mathcal{O}_i$ in the physical problem according to the representation theory of the underlying Kac-Moody algebra at the fixed point.

For the overscreening case, nontrivial boundary operators may appear that do not occur in the bulk theory. The triplet operator $\boldsymbol{\Phi}$ always occurs.[14] This selection rule describes a new content of boundary scaling operators. The low-temperature properties are now governed by the *leading-correction-to-scaling boundary operator* (LCBO). This must preserve all the symmetries of $\mathcal{H}_o+\mathcal{H}_F^*$. We obtain a *unique* LCBO: $\mathbf{J}^{-1}\cdot\boldsymbol{\Phi}\delta(x)$, which has the scaling dimension $\Delta_S=1+2/(k+2)$ for a left-handed theory. Then, adding

$$\delta H=\gamma_1\mathbf{J}^{-1}\cdot\boldsymbol{\Phi}(0), \quad (13)$$

to the total Hamiltonian, the leading contribution to low-temperature thermodynamics is second order in γ_1. For $k=4$, we have[14]

$$C_{imp}\sim T^{2/3}+\cdots,\chi_{imp}\sim T^{-1/3}+\cdots T\rightarrow 0. \quad (14)$$

As pointed out by Fabrizio and Gogolin, the same conclusion holds at a particular anisotropic Kondo limit, namely the so-called Toulouse point.[16]

B. Role of repulsive interactions in each channel

When $U\neq 0$, the bulk Hamiltonian $\mathcal{H}_{TL}$ can also be written on a Sugawara form, using the redefinitions[5]

$$J^i_{L(R)}(x)=\cosh\eta{:}\psi^{\dagger}_{iL(R)\alpha}(x)\psi_{iL(R)\alpha}(x){:} + \sinh\eta{:}\psi^{\dagger}_{iR(L)\alpha}(x)\psi_{iR(L)\alpha}(x){:} \quad (15)$$

$$\mathbf{J}^i_{L(R)}(x) = :\psi^\dagger_{iL(R)\alpha}(x)\boldsymbol{\sigma}_{\alpha\beta}\psi_{iL(R)\beta}(x):,$$

where the currents $J^i_p(x)$ and $\mathbf{J}^i_p(x)$ (where $i=1,2$ and $p=L,R$) satisfy the $U(1)$ and (level-1) $SU(2)_1$ Kac-Moody algebras, respectively and

$$\tanh 2\eta = U/(v_F+U). \tag{16}$$

They generate the critical Luttinger bulk Hamiltonian

$$\mathcal{H}_{TL} = \int_0^l dx \frac{v_c}{8\pi}:J^i_p(x)J^i_p(x): + \frac{v_F}{8\pi}:\mathbf{J}^i_p(x)\mathbf{J}^i_p(x):. \tag{17}$$

Note that $\mathcal{H}_{TL}$ is invariant under the chiral symmetry $\mathcal{G} = \{U(1)_L \times U(1)_R \times SU(2)_{1,L} \times SU(2)_{1,R}\}^2$. The model yields separation of spin and charge and the velocity for charge zero sound modes is given by

$$v_c = v_F\sqrt{1+2U/v_F} = v_F K^{-1}. \tag{18}$$

The parameter $K=e^{-2\eta}$ can be identified as the usual Luttinger exponent. At high temperatures, the spin quasiparticles, from the SU(2), level-1 Wess-Zumino-Witten conformal field theory, are the usual spin-1/2 doublets namely *spinons* [which bring fractional spins].

By analytic continuation, the theory in Eq. (17) is equivalent to a chiral (left-handed) theory on $[-l,l]$. As the four currents are coupled via **S**, the forward Kondo exchange breaks $\{SU(2)_{1,L} \times SU(2)_{1,L}\}^2$ of $\mathcal{G}$ down to the diagonal level-4 subalgebra $SU(2)_4$. For conformal theories with an $SU(2)_k$ symmetry, the free energy is proportional to the ''central charge'' defined as[17]

$$C = \frac{3k}{k+2}\cdot \tag{19}$$

Thus, we can decompose a $4\times SU(2)_1$ Sugawara Hamiltonian (with $C=4$) onto an $SU(2)_4$ one (with $C=2$) and a remainder describing the flavor sector [here, an $SU(2)_4$ critical theory with $C=2$, as well]. This analysis can be routed via the so-called *coset* construction[18]. Then, since only the spin sector $SU(2)_4$ is coupled to the impurity, we predict the same *unique* LCBO as for the case without electron-electron interaction [a boundary operator coming from the charge sector or the flavor one (only) is characterized by a coupling constant which goes to zero when the ultraviolet cutoff goes to infinity]. Using the general formula of Ref. 7, we obtain a Wilson ratio

$$R_W = \frac{\chi_{imp}C}{\chi C_{imp}} = 4(1+K), \tag{20}$$

where, C and χ are the bulk quantities. It should be noted that R_W is universal only for a perfect isotropic Kondo exchange[19] and in the limit $U\to 0$: it takes the value $R_W = 8$.[14]

To conclude, the presence of the electron-electron interaction makes the Kondo crossover highly nonuniversal. The impurity screening leads to a new symmetry for the bulk Hamiltonian, and then to new *N*-body excitations in the infrared limit [coming from the $SU(2)_4 \times SU(2)_4$ (flavor-spin) sectors]. However, note that charge quasiparticles with charges $Q=+e$, are still the usual ''holons'' of the LL.

On the other hand, the low-temperature thermodynamics due to the impurity screening is still the same as the one found in the noninteracting case, because the impurity spin couples only to *individual* electrons.

IV. BACKSCATTERING EFFECTS

Let us now include $\lambda_B = \lambda_F \neq 0$. First, to confirm that the presence of backward scattering off the impurity leads to a *new* fixed point, we start with $U=0$. With no e-e interaction, it is convenient to use the so-called Weyl basis[10]

$$\psi^i_\pm(x) = [\psi^i_{L,\sigma}(x) \pm \psi^i_{R,\sigma}(-x)]/\sqrt{2}. \tag{21}$$

Then, $(\mathcal{H}_o + \mathcal{H}_K)$ transforms into a four-channel Kondo theory, but with the impurity coupled to the electrons in only the two positive parity channels, namely ψ^1_+ and ψ^2_+. Thus, we obtain an effective two-channel[20] (left-handed) Kondo Hamiltonian.[10]

Here, it is well-known that the forward Kondo scattering term breaks the $SU(2)_1 \times SU(2)_1$ subgroup of $\mathcal{H}_o$ down to $SU(2)_2 \times \mathcal{Z}_2$, where $\mathcal{Z}_2$ is a critical theory with a central charge C=1/2 equivalent to an *Ising* model.[5,7] The model renormalizes to a marginal non-Fermi liquid with *logarithmic* corrections. It can be simply obtained by taking γ_1 as the unique LCBO (note that $\Delta_S = 3/2$ for $k=2$). The low-temperature thermodynamics at the impurity site is given by[14]

$$C_{imp} \propto T\ln\left(\frac{T_K}{T}\right) + \cdots, \chi_{imp} \propto \ln\left(\frac{T_K}{T}\right) + \cdots T\to 0\,. \tag{22}$$

When $U\neq 0$, the e-e interaction mixes left- and right-moving fields, and hence becomes highly *nonlocal* in the Weyl-basis. Although efforts have been made to handle consistently the non-local terms appearing from the interaction,[21] in our problem it is very difficult to describe the Kondo fixed point in the (ψ^1_+, ψ^2_+) basis.

However, demanding that any associated LCBO must correctly reproduce the noninteracting limit as $U\to 0$, the possible critical theories can be deduced:

(a) From the spin sector only LCBO with the scaling dimension $\Delta_S = 3/2$ can occur. The only contribution from the $SU(2)_4$ sector is then the identity and its descendants. *This implies a recombination of conformal towers in the spin sector*.

(b) A LCBO including a charge or a flavor field unambiguously must be characterized by a scaling dimension $\Delta_T \to 1$ as $U\to 0$ [producing no boundary correction in the noninteracting limit $U\to 0$].

A. Two-channel Kondo physics when $U\neq 0$

To guess the precise symmetry of the Hamiltonian in the critical region, we can use the following points.

First, we can exploit the expectation that the full Kondo interaction $\mathcal{H}_K$ can be described as a *renormalized* boundary condition (selection rule) on $\mathcal{H}_{TL}$, analogous to the forward interaction obtained for $U=0$. In particular, λ_F should scale towards the *solvable* point $\lambda^*_F = v_F/4$ (with k=2) although

λ_B goes to strong couplings when $U\neq 0$ or $K\neq 1$ [see below]. Second, the full Hamiltonian must also contain an Ising sector.

As an important consequence, when $U\neq 0$ we must write the fixed-point Hamiltonian $\mathcal{H}_0+\mathcal{H}_F^*$ as a C=2 critical theory with L and R movers having an $SU(2)_{2,L}\times SU(2)_{2,R}\times\mathcal{Z}_2$ symmetry. The presence of backward scattering off the impurity breaks $SU(2)_{2,L}\times SU(2)_{2,R}$ down to $SU(2)_2$.

Let us now precisely describe the content of scaling boundary operators. If $\mathbf{J}^1_{L(R)}$ and $\mathbf{J}^2_{L(R)}$ [given by Eq. (15)] satisfy the level-1 Kac-Moody algebra, then the diagonal currents given by

$$\mathbf{J}_{L(R)}=\mathbf{J}^1_{L(R)}+\mathbf{J}^2_{L(R)}, \tag{23}$$

satisfy the level-2 one. Thus, we can write the Hamiltonian as a sum of an $SU(2)_{2,L}\times SU(2)_{2,R}$ Sugawara Hamiltonian and an Ising model. Such procedure, for example, has been successfully applied to treat the two-leg spin ladder problem.[22] We can easily complete the ''square'' at the solvable point $\lambda_F^*=v_F/4$ via the use of the transformation: $\mathbf{J}_{L(R)}(x)\to\mathbf{J}_{L(R)}(x)+2\pi\mathbf{S}\delta(x)$. The Kac-Moody algebras for channels L and R are no longer independent. As for the noninteracting U-limit, $\lambda_F^*=v_F/4$ will be identified as the true fixed point of the model. However, it should be noted that the recursion law for λ_F

$$\frac{d\lambda_F}{d\ln L}=\frac{\lambda_F^2}{2\pi v_F}+\frac{\lambda_B^2}{2\pi v_F}-\frac{k}{2}\frac{\lambda_F^3}{(2\pi v_F)^2}+\cdots, \tag{24}$$

does not allow to find the precise forward Kondo exchange infrared value, namely λ_F^*. We can only assume that, as for $U=0$, the presence of the last term that occurs with a *minus* sign should prevent λ_F to flow to strong couplings.

The eigenstates in the $SU(2)_{2,L}\times SU(2)_{2,R}$ sector appear in conformal towers labeled by the spin quantum numbers $j=0,1/2,1$. The corresponding primary fields are the identity $\mathbf{1}$, the fundamental field g, and the triplet operator (a 3×3 matrix) $\mathbf{\Phi}=\Sigma_{i,j}\Phi_{Li}\Phi_{Rj}$. They have the scaling dimensions, $\Delta_S=\frac{1}{2}j(j+1)$. Similarly, there are three primary fields in the Ising sector given by $\phi=\mathbf{1},\sigma,\epsilon$ with scaling dimensions Δ_I given by $0,\frac{1}{8},1$, respectively.

In respect to the noninteracting case $U\to 0$, the absorption of the impurity must give for forward scattering $(j,\phi)=(0 \text{ or } 1,\mathbf{1})$.[14] Simply, through the examination of spin singlets from $SU(2)_{2,L}\times SU(2)_{2,R}$, one obtains the following LCBO:

$$\delta\mathcal{H}=\gamma_1\{\mathbf{J}_L^{-1}\mathbf{\Phi}_L(0)+\mathbf{J}_R^{-1}\mathbf{\Phi}_R(0)\}. \tag{25}$$

By construction, $\mathbf{\Phi}_L$ and $\mathbf{\Phi}_R$ have the halved dimension 1/2. Thus, we can easily check that $\delta\mathcal{H}$ produces a two-channel-like Kondo fixed point with transport properties given in Eq. (22). We like to point out the following remark. Although $\mathbf{J}_L$ and $\mathbf{J}_R$ are coupled through the impurity screening, the symmetry $SU(2)_{2,L}\times SU(2)_{2,R}$ of the total Hamiltonian cannot be broken at the fixed point because a descendant field $\mathbf{J}_p^{-1}$ with $p=L,R$ acts only on a primary field from the p sector. The same fixed point has been found for the overscreened Kondo effect in a two-band Hubbard chain,[10] *meaning that the geometry of the system does not affect spin properties in the infrared limit*. We can check that the Wilson ratio is universal only when $U\to 0$; it takes the value 8/3.[14]

B. Generalized tunneling process à la Furusaki-Nagaosa

Neglecting the λ_m term, for *each* LL the charge eigenstates organize into a product of two $U(1)$ conformal towers, labeled by the two quantum numbers $(Q_i,\Delta Q_i)$, the sum and difference of net charge in the left and right channels. Introducing the usual charge variables $(i=1,2)$,

$$(J_L^i+J_R^i)=\frac{1}{\sqrt{2\pi K}}\partial_x\phi_{ci},\quad (J_L^i-J_R^i)=\sqrt{\frac{K}{2\pi}}\Pi_{ci}, \tag{26}$$

the charge part of the free Hamiltonian can be identified as two independent Luttinger models.[1,2]

Now, we must carefully treat backward scattering off the impurity. Indeed, the corresponding term in Eq. (4) breaks the chiral $U(1)$ invariance of $\mathcal{H}_{TL}$. The selection rule for combining the two $U(1)$ conformal towers may change. Thus, ΔQ_i is no longer restricted to zero, and the charge sector should make nontrivial contributions to the content of scaling operators leading to another possible fixed point in the critical region.

The backscattering term (4) is usually expressed in the so-called spinon basis as[6,7]

$$\mathcal{H}_B=\lambda_B\sum_{i=1,2}\{\mathrm{Tr}(g_i\boldsymbol{\sigma})\cos\sqrt{2\pi}\phi_{ci}(0)\}\cdot\mathbf{S}, \tag{27}$$

and the spin operators $g_i\in SU(2)_{1,L}\times SU(2)_{1,R}$. Using simple scaling arguments (with $L\equiv 1/T$)

$$\frac{d\lambda_B}{d\ln L}=\frac{1}{2}(1-K)\lambda_B+\mathcal{O}\left(\frac{\lambda_B\lambda_F}{\pi v_F}\right), \tag{28}$$

we find that prominent backscattering off the impurity supports a Kondo effect for ferromagnetic as well as antiferromagnetic Kondo exchanges.[23] The Kondo temperature yields the same power-law dependence on the exchange coupling $T_K\propto\lambda_B^{2/(1-K)}$ as for the single LL case.[8,24] To summarize, when $K\neq 1$ the flow of $\lambda_B\neq 0$ goes to infinity whereas the forward Kondo scattering exchange scales to the precise intermediate value given by Eq. (8), with k=2.

When $T\ll T_K$, we have the formation of a bound state (with spin S=0) between any electron near the Fermi level and the impurity spin. However, a nonmagnetic extra degree of freedom remains at the impurity site because λ_F^* is not too strong [let us remind that only the forward Kondo exchange can really absorb or screen the impurity spin]. Precisely, for $k=2$, the groundstate degeneracy is exactly $g=\sqrt{2}$,[14] and it can be interpreted as a residual Majorana fermion at the origin.[25]

On the other hand, the fact that $\lambda_B\to+\infty$ can be interpreted as follows. In the infra-red region, the cosine terms of Eq. (27) become pinned at the origin and $\langle\cos\sqrt{2\pi}\phi_{ci}(0)\rangle$ = constant or $\phi_{ci}(0)=\sqrt{\pi/2}$.[7] Simply, it means that the charge quasiparticles (*holons*) move completely away from the origin [despite the relatively weak value of the forward Kondo exchange at the fixed point], due to the concrete spin-

TABLE I. Different fixed points and physical behaviors reported in this paper, for the Kondo effect in crossed Luttinger liquids. Notations are explained in the text and * is from Ref. 14.

Impurity	Susceptibility	Specific heat	Conductance/Resistivity	Fixed point
$\lambda_F \neq 0$ and $\lambda_B = 0$	$T^{-1/3}$	$T^{2/3}$	$\rho \propto T^{1/3}$*	4-channel-like
$\lambda_F = \lambda_B \neq 0$ **1-**	const.$+T^{1/K}$	$T^{(1/K)-1}$	$G \propto T^{(1/K)-1}$	Furusaki-Nagaosa
2-	$\ln T$	$T \ln T$	$\rho \propto \sqrt{T}$*	2-channel-like, Or
If $\delta, \lambda_m \neq 0$	const.*	T*	$\rho \propto T^2$*	1-channel-like

charge separation occurring in a 1D metallic wire for $K \neq 1$: only spin degrees of freedom couple to the impurity in the infrared region. Finally, since a bound state between an electron of the Fermi sea and the impurity spin acts as a strong nonmagnetic barrier at $x=0$ and since $\lambda_B \to +\infty$, exotic tunneling phenomena can take place. In the infrared limit, we must decompose the backscattering term λ_B (written via g_1 and g_2) in the (Ising)$\otimes g$ basis (which has been used to absorb the impurity spin). After some complicated algebra, the result is[22]

$$\mathrm{Tr}(g_1 \boldsymbol{\sigma}) + \mathrm{Tr}(g_2 \boldsymbol{\sigma}) = \sqrt{2}\,\mathrm{Tr}(g \boldsymbol{\sigma}) \cdot \sigma. \tag{29}$$

The lowest dimension operator with $\Delta Q_i \neq 0$ allowed by the forward selection rule is obtained from $(Q_i, \Delta Q_i, j, \phi) = (0, \pm 2, 0, \mathbf{1})$, has the scaling dimension $1/2K$ and can be written as: $\cos \sqrt{2\pi}\tilde{\phi}_{ci}(0)$. Then, possible couplings of $SU(2)_2$ and Ising towers to the $U(1)$ towers yield the following candidate LCBO:

$$\delta\mathcal{H} = \gamma_2 \mathrm{Tr}(g \boldsymbol{\sigma}) \cdot \sigma \sum_{i=1,2} \cos \sqrt{2\pi}\tilde{\phi}_{ci}(0), \tag{30}$$

and $\gamma_2 \propto 1/\lambda_B$. Such term describes a collective tunneling process of two electrons (one in each LL), which breaks the spin singlet at the impurity site.

Since there is no Hubbard coupling between channels 1 and 2, a tunneling phenomenon including a renormalized (channel-dependent) LL charge parameter cannot occur. This is the main difference with the Kondo effect in a two-band Hubbard chain.[10] Here, physical properties exhibit an exact duality between high- and low-temperature fixed points, replacing $K \to 1/K$.[7] We can check that such an operator with scaling dimension $\Delta_T = \frac{1}{2}(1/K+1)$ [which goes to 1 as U goes to zero] shows the same anomalous scaling in temperature as the one predicted by Furusaki and Nagaosa for the Kondo effect in a LL.[8] Thus, the impurity specific heat and the conductance also exhibit the same anomalous temperature dependence with a leading term (at $T \to 0$)

$$G_{imp}(T) \propto T^{(1/K)-1}, C_{imp}(T) \propto T^{(1/K)-1}, \tag{31}$$

which vanishes when $K \to 1$,[5,7,8] i.e., for the noninteracting case. The current-voltage curve associated with this tunneling process obeys

$$G(V) \equiv \frac{dI}{dV} \propto |V|^{(1/K)-1}, \tag{32}$$

[thermal energy has been replaced by electric energy]. When $K=1$ a linear I-V curve is predicted, consistent with expectations for non-interacting electrons, which are partially transmitted through a nonmagnetic barrier. For $K \neq 1$, we obtain

$$I \propto |V|^{1/K}, \tag{33}$$

and then the linear conductance is strictly zero. This is a simple reflection of the suppressed density of states in a LL.

V. CONCLUSIONS AND DISCUSSIONS ON RELEVANT PERTURBATIONS

Summarizing, we have studied the low-temperature properties of a spin-1/2 magnetic impurity coupled to two crossed conducting channels, each described by a Luttinger model. Using boundary conformal field theory, we have reached the important conclusion that the problem still admits two possible fixed points: either the theory remains a marginal non-Fermi liquid with logarithmic corrections in the presence of electron-electron interactions, or electron correlations drive the system to another non-Fermi liquid fixed point obtained originally by Furusaki and Nagaosa for the Kondo effect in a LL.

However, as in the case without e-e interaction,[26] the previous marginal non-Fermi liquid is unstable in presence of a *small* channel anisotropy $\delta = (\lambda_F^1 - \lambda_F^2)$. Adding the corresponding term

$$\mathcal{H}_A = \delta(\mathbf{J}_L^1 - \mathbf{J}_L^2 + \mathbf{J}_R^1 - \mathbf{J}_R^2)\mathbf{S} = \delta(\epsilon_L \boldsymbol{\Phi}_L + \epsilon_R \boldsymbol{\Phi}_R)\mathbf{S}, \tag{34}$$

to the Hamiltonian destabilizes the symmetric forward scattering fixed point. As in the Kondo effect in a LL (Ref. 7) or the famous two-impurity model in a three dimensional Fermi-liquid environment,[27] the LCBO $\mathbf{J}_p^{-1}\boldsymbol{\Phi}_p$ $(p=L,R)$ is excluded by parity conservation. We have used the notations: $\epsilon = \epsilon_L \epsilon_R$. Here, ϵ_p enters as an allowed boundary operator of scaling dimension $\Delta_I = 1$, producing a one-channel (Fermi-liquid-like) fixed point, ruled by the new selection rule $\delta^* \to +\infty$. There are now three irrelevant leading operators of dimension 2, namely $\mathbf{J}_L^1\mathbf{J}_L^1$, $\mathbf{J}_R^1\mathbf{J}_R^1$, $\mathbf{J}_L^1\mathbf{J}_R^1$.[7] To conclude, either Fermi-liquid-like [with an $SU(2)_{k=2}$ spin symmetry] or non-Fermi liquid *à la* Furusaki-Nagaosa could be still realized experimentally in multichannel 1D quantum wires or carbon nanotubes satisfying the geometry presented here.

Note also that the suppression of backscattering off the impurity produces a low-energy physics identical to that of the four-channel Kondo model.

Finally, $\lambda_m = \lambda_F = \lambda_B \neq 0$ seems also to be a relevant [but not very realistic] perturbation. Indeed, passing to an odd-even parity basis, $(a,b) = 1/\sqrt{2}(\psi_1 \pm \psi_2)$ [when $U \to 0$] the impurity couples only to the fermionic channel a. This also leads to a Fermi-liquid-like fixed point or to the Furusaki-Nagaosa non-Fermi-liquid one.

A summary of various physical behaviors is given in Table I.

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