Photon modes in photonic crystals undergoing rigid vibrations and rotations

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We explore the nature of photon modes associated with photonic crystals undergoing rigid time-dependent spatial displacements in a noninertial frame of reference and prove that under certain conditions these modes retain many of the spatial symmetries allowed in a static photonic crystal. Moreover, it is proved quite generally that such noninertial modes possess a temporal Bloch-like symmetry. Conserved ‘‘quantum numbers’’ are identified and a convenient scheme for labeling noninertial modes is presented.

The idea of using periodic dielectric materials (photonic crystals) to alter the dispersion relation of photons[1–3] has received widespread interest and consideration because of numerous potential applications.[4] It has been shown by several authors[4–7] that passive elements such as waveguide bends, channel drop filters, mirror surfaces, etc. can be substantially improved if constructed on the basis of photonic crystals. Recently, a strong interest has developed for the incorporation of nonlinear materials into photonic crystals. Investigations in the framework of field-dependent dielectric media have led to several suggestions[8–10] on the possibility of constructing active elements such as optical switches, and on the realization of dynamical effects such as second-harmonic generation and induced interband transitions in photonic crystals. In all of these studies, the photonic crystals are constrained to be in a static, or inertial, frame of reference. Nevertheless, it should be possible to develop active photonic crystal elements even with *linear* materials by working with *nonstationary* photonic crystals.[14] Before one can begin to explore this possibility it is necessary to have a fundamental framework of understanding of the nature of the photonic states in such noninertial systems.

In this Brief Report, we explore the properties of photon modes associated with photonic crystals undergoing rigid time-dependent spatial vibrations and rotations in the nonrelativistic limit. Several fundamental theorems about the form and the symmetries of the resulting electromagnetic modes in a noninertial frame are presented. Specifically, we prove that photonic crystals undergoing such rigid displacements can exhibit solutions that, under certain circumstances, exhibit Bloch-like spatial and temporal translational symmetries and/or point-subgroup rotational symmetries. To our knowledge, this is the first time photonic crystals have been investigated in a noninertial frame of reference.

We begin by considering the case of a photonic crystal rigidly vibrating with an amplitude Δ and a driving frequency Ω. In this case Maxwell's equations take the form

$$\nabla\times H(\vec{r},t)=\epsilon(\vec{r}-\vec{\Delta}(t))\frac{\partial E(\vec{r},t)}{c\,\partial t}+\frac{\partial\epsilon(\vec{r}-\vec{\Delta}(t))}{c\,\partial t}E(\vec{r},t),$$

$$\nabla\times E(\vec{r},t)=-\frac{\partial H(\vec{r},t)}{c\,\partial t},\qquad(1)$$

where $\epsilon(\vec{r},t)$ is a spatially periodic time dependent function such that there exists $\vec{R}$ so that for all $\vec{r}$: $\epsilon(\vec{r}+\vec{R}-\vec{\Delta}(t))=\epsilon(\vec{r}-\vec{\Delta}(t))$ and $\vec{\Delta}(t)=\vec{\Delta}(t+2\pi/\Omega)$. We now prove that a Bloch-like theorem still holds in the noninertial case.

To prove the theorem we search for a solution to the noninertial problem in a complete plain wave basis

$$\begin{pmatrix}H(\vec{r},t)\\E(\vec{r},t)\end{pmatrix}=\int d\vec{k}d\omega\begin{pmatrix}H(\vec{k},\omega)\\E(\vec{k},\omega)\end{pmatrix}|\vec{k},\omega\rangle,\qquad(2)$$

where we define $|\vec{k},\omega\rangle=[1/(2\pi)^2]\exp(i\vec{k}\cdot\vec{r}-i\omega t)$ and $\langle\vec{k}_0,\omega_0|\vec{k},\omega\rangle=\delta(\vec{k}_0-\vec{k})\,\delta(\omega_0-\omega)$.

Substituting Eq. (2) into Maxwell's equations Eq. (1) and multiplying both sides by $(1/i)\langle\vec{k}_0,\omega_0|$ we obtain

$$0=H(\vec{k}_0,\omega_0)\times\vec{k}_0-\int d\vec{k}d\omega\frac{\omega}{c}E(\vec{k},\omega)$$
$$\times\langle\vec{k}_0,\omega_0|\epsilon(\vec{r}-\vec{\Delta}(t))|\vec{k},\omega\rangle-i\int d\vec{k}d\omega E(\vec{k},\omega)$$
$$\times\left\langle\vec{k}_0,\omega_0\left|\frac{\partial\epsilon(\vec{r}-\vec{\Delta}(t))}{c\,\partial t}\right|\vec{k},\omega\right\rangle,\qquad(3)$$
$$0=E(\vec{k}_0,\omega_0)\times\vec{k}_0+\frac{\omega_0}{c}H(\vec{k}_0,\omega_0).$$

We now proceed to evaluate the integrals over the wave vector space in Eq. (3). Since $\epsilon(\vec{r}+\vec{R})=\epsilon(\vec{r})$ we can represent a dielectric function in the reciprocal vector space as $\epsilon(\vec{r})=\Sigma_{\vec{G}}\epsilon_{\vec{G}}\exp(i\vec{G}\cdot\vec{r})$. Substituting this representation of $\epsilon(\vec{r})$ into Eq. (3) the first integral can be written

$$\int d\vec{k}d\omega\frac{\omega}{c}E(\vec{k},\omega)\langle\vec{k}_0,\omega_0|\epsilon(\vec{r}-\vec{\Delta}(t))|\vec{k},\omega\rangle$$
$$=\int d\vec{k}d\omega\frac{\omega}{c}E(\vec{k},\omega)\sum_{\vec{G}}\epsilon_{\vec{G}}$$
$$\times\langle k_0,\omega_0|\exp(i\vec{G}\cdot\vec{r}-i\vec{G}\cdot\vec{\Delta}(t))|\vec{k},\omega\rangle.\qquad(4)$$

Now the spatial part of the matrix element can be trivially calculated to give

$$\langle \vec{k}_0,\omega_0|\exp(i\vec{G}\cdot\vec{r}-i\vec{G}\cdot\vec{\Delta}(t))|\vec{k},\omega\rangle = \delta(\vec{k}+\vec{G}-\vec{k}_0)\frac{1}{2\pi}\int dt \exp(i(\omega_0-\omega)t-i\vec{G}\cdot\vec{\Delta}(t)). \tag{5}$$

Since $\vec{\Delta}(t)$ is a periodic function of time with period $2\pi/\Omega$, the integral over time can be rewritten in the following way:

$$\frac{1}{2\pi}\int dt \exp(i(\omega_0-\omega)t-i\vec{G}\cdot\vec{\Delta}(t)) = \left[\sum_{l=-\infty}^{+\infty}\exp\left(i(\omega_0-\omega)\frac{2\pi}{\Omega}l\right)\right]\frac{1}{2\pi}\int_0^{2\pi/\Omega}dt \times\exp(i(\omega_0-\omega)t-i\vec{G}\cdot\vec{\Delta}(t)). \tag{6}$$

Using the identity $\Sigma_{l=-\infty}^{+\infty}\exp(i(\omega_0-\omega)(2\pi/\Omega)l) = \Omega\Sigma_{l=-\infty}^{+\infty}\delta(\omega-(\omega_0+l\Omega))$ one finally arrives at

$$\int d\vec{k}d\omega\frac{\omega}{c}E(\vec{k},\omega)\langle k_0,\omega_0|\epsilon(\vec{r}-\vec{\Delta}(t))|\vec{k},\omega\rangle = \frac{1}{c}\sum_{\vec{G}}\epsilon_{\vec{G}}\sum_{l=-\infty}^{+\infty}E(\vec{k}_0-\vec{G},\omega_0+l\Omega)(\omega_0+l\Omega) \times\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt \exp(-i\vec{G}\cdot\vec{\Delta}(t)-il\Omega t). \tag{7}$$

Proceeding in exactly the same fashion, the second integral in Eq. (3) can be manipulated to give

$$i\int d\vec{k}d\omega E(\vec{k},\omega)\left\langle \vec{k}_0,\omega_0\left|\frac{\partial\epsilon(\vec{r}-\vec{\Delta}(t))}{c\partial t}\right|\vec{k},\omega\right\rangle = -\frac{1}{c}\sum_{\vec{G}}\epsilon_{\vec{G}}\sum_{n=-\infty}^{+\infty}E(\vec{k}_0-\vec{G},\omega_0+l\Omega) \times l\Omega\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt \exp(-i\vec{G}\cdot\vec{\Delta}(t)-il\Omega t). \tag{8}$$

Combining the results above, we arrive at the following form of Maxwell's equations in the wave vector representation:

$$0=H(\vec{k}_0,\omega_0)\times\vec{k}_0-\frac{\omega_0}{c}\sum_{\vec{G}}\epsilon_{\vec{G}}\sum_{l=-\infty}^{+\infty}E(\vec{k}_0-\vec{G},\omega_0 +l\Omega)\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt \exp(-i\vec{G}\cdot\vec{\Delta}(t)-il\Omega t),$$

$$0=E(\vec{k}_0,\omega_0)\times\vec{k}_0+\frac{\omega_0}{c}H(\vec{k}_0,\omega_0). \tag{9}$$

There are three immediate conclusions that can be drawn from the form of Eq. (9). First, modes with different $\vec{k}_0$ within the conventional Brillouin zone do not mix so that it is still possible to define a ''good'' quantum number $\vec{k}_0$ for a vibrating photonic crystal, regardless of the direction of vibration.[11] Secondly, for a given mode with a native band frequency ω_0 and amplitude

$$\begin{pmatrix} H(\vec{k}_0,\omega_0) \\ E(\vec{k}_0,\omega_0) \end{pmatrix},$$

the harmonics with the satellite frequencies $\omega_0+l\Omega$ and amplitudes

$$\begin{pmatrix} H(\vec{k}_0,\omega_0+l\Omega) \\ E(\vec{k}_0,\omega_0+l\Omega) \end{pmatrix}$$

are also present. And finally, for a given $\vec{k}_0$ there will be a discrete set of $\omega_{0,n}$'s which satisfy Eq. (9). These $\omega_{0,n}$'s are analogous to the photon bands of the static photonic crystal.

In general therefore, any time dependent solution

$$\begin{pmatrix} H_\Omega(\vec{r},t) \\ E_\Omega(\vec{r},t) \end{pmatrix}$$

of Eq. (1) can be expressed in a basis set of noninertial modes each satisfying Eq. (1) and characterized by a set of ''good'' quantum numbers $\vec{k}$ and ω_n so that

$$\begin{pmatrix} H_{\vec{k},\Omega,\omega_n}(\vec{r},t) \\ E_{\vec{k},\Omega,\omega_n}(\vec{r},t) \end{pmatrix} = \sum_{\vec{G}}\sum_{l=-\infty}^{+\infty}\begin{pmatrix} H(\vec{k}-\vec{G},\omega_n+l\Omega) \\ E(\vec{k}-\vec{G},\omega_n+l\Omega) \end{pmatrix} \times\exp(i(\vec{k}-\vec{G})\cdot\vec{r}-i(\omega_n+l\Omega)t). \tag{10}$$

It is straightforward to see that such modes possess a spatial and temporal Bloch symmetry

$$\begin{pmatrix} H_{\vec{k},\Omega,\omega_n}\left(\vec{r}+\vec{R},t+\frac{2\pi}{\Omega}\right) \\ E_{\vec{k},\Omega,\omega_n}\left(\vec{r}+\vec{R},t+\frac{2\pi}{\Omega}\right) \end{pmatrix} = \exp\left(i\vec{k}\cdot\vec{R}-i\omega_n\frac{2\pi}{\Omega}\right) \times\begin{pmatrix} H_{\vec{k},\Omega,\omega_n}(\vec{r},t) \\ E_{\vec{k},\Omega,\omega_n}(\vec{r},t) \end{pmatrix}. \tag{11}$$

Further symmetries are possible by considering a point group operator $\hat{O}$ such that the vector of vibrations $\vec{\Delta}$ is left invariant under $\hat{O}\vec{\Delta}=\vec{\Delta}$. In this case substituting $\vec{k}$ by $\hat{O}\vec{k}$ in Eq. (9) and acting with $\hat{O}^{-1}$ from the left and remembering that $\epsilon_{\hat{O}\vec{G}}=\epsilon_{\vec{G}}$ for any reciprocal vector $\vec{G}$, one can derive

$$0=\hat{O}^{-1}H(\hat{O}\vec{k}_0,\omega_0)\times\vec{k}_0-\frac{\omega_0}{c}\sum_{\vec{G}}\epsilon_{\vec{G}} \times\sum_{l=-\infty}^{+\infty}\hat{O}^{-1}E(\hat{O}(\vec{k}_0-\vec{G}),\omega_0+l\Omega)\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt \times\exp(-i\vec{G}\cdot\vec{\Delta}(t)-il\Omega t),$$

$$0=\hat{O}^{-1}E(\hat{O}\vec{k}_0,\omega_0)\times\vec{k}_0+\frac{\omega_0}{c}\hat{O}^{-1}H(\hat{O}\vec{k}_0,\omega_0). \tag{12}$$

This equation has exactly the same form as Eq. (9) thus implying that for any vector $\vec{k}$, $\hat{O}^{-1}H(\hat{O}\vec{k},\omega)=H(\vec{k},\omega)$ and $\omega_n(\hat{O}\vec{k})=\omega_n(\vec{k})$.

From this, it immediately follows that under the conditions $\hat{O}\vec{\Delta}=\vec{\Delta}$ the noninertial electromagnetic modes of a photonic crystal will possess the additional symmetry

$$\hat{O}^{-1}\begin{pmatrix} H_{\hat{O}\vec{k},\Omega,\omega_n}(\vec{r},t) \\ E_{\hat{O}\vec{k},\Omega,\omega_n}(\vec{r},t) \end{pmatrix}=\begin{pmatrix} H_{\vec{k},\Omega,\omega_n}(\hat{O}^{-1}\vec{r},t) \\ E_{\vec{k},\Omega,\omega_n}(\hat{O}^{-1}\vec{r},t) \end{pmatrix}. \tag{13}$$

If in addition to $\hat{O}\vec{\Delta}=\vec{\Delta}$, the wave vector $\vec{k}$ satisfies $\hat{O}\vec{k}=\vec{k}+\vec{G}$ then the discrete set $\omega_n(\vec{k})$ can be designated in terms of irreducible representations of a subgroup $\{\hat{O}\}$ of the original small point group of $\vec{k}$ in the standard way.

Let us now turn to the case of a photonic crystal rigidly rotating with an angular frequency $\vec{\Omega}=\vec{e}_\Omega\Omega$. In this case Maxwell's equations take the form

$$\nabla\times H(\vec{r},t)=\epsilon(\vec{r}(\vec{\Omega},t))\frac{\partial E(\vec{r},t)}{c\,\partial t}+\frac{\partial\epsilon(\vec{r}(\vec{\Omega},t))}{c\,\partial t}E(\vec{r},t),$$

$$\nabla\times E(\vec{r},t)=-\frac{\partial H(\vec{r},t)}{c\,\partial t}, \tag{14}$$

where

$$\vec{r}(\vec{\Omega},t)=\vec{e}_\Omega\cdot(\vec{e}_\Omega\cdot\vec{r})+(\vec{e}_\Omega\times\vec{r})\sin(\Omega t)+(\vec{e}_\Omega\times\vec{r})\times\vec{e}_\Omega\cos(\Omega t).$$

Rewriting the modes in the plain wave basis set and employing the same techniques as in the previous section we arrive at

$$0=H(\vec{k}_0,\omega_0)\times\vec{k_0}-\frac{\omega_0}{c}\sum_{\vec{G}}\epsilon_{\vec{G}}\sum_{l=-\infty}^{+\infty}\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt\, E(\vec{k}_0-(\vec{e}_\Omega\cdot\vec{G})\cdot\vec{e}_\Omega+\vec{V}_{rot}(t),\omega_0+l\Omega)\exp(-il\Omega t),$$

$$0=E(\vec{k}_0,\omega_0)\times\vec{k_0}+\frac{\omega_0}{c}H(\vec{k}_0,\omega_0), \tag{15}$$

where we define $\vec{V}_{rot}(t)=(\vec{e}_\Omega\times\vec{G})\sin(\Omega t)-(\vec{e}_\Omega\times\vec{G})\times\vec{e}_\Omega\cos(\Omega t)$.

The general solution to Eq. (15) does not possess Bloch-like character. To obtain a Bloch form we must restrict $\vec{e}_\Omega$ to lie along one of the real space lattice vectors. Under this constraint, the set $(\vec{e}_\Omega\cdot\vec{G})\cdot\vec{e}_\Omega$ represents a set $\vec{g}_\Omega$ of reciprocal lattice vectors associated with an effective one-dimensional periodic structure. Thus from Eq. (15) one can easily deduce that $\vec{k}_0$ will only couple to wave vectors of the form $\vec{k}=(\vec{k}_0\cdot\vec{e}_\Omega)\cdot\vec{e}_\Omega+\vec{g}_\Omega+\vec{S}_{orth}$ where $\vec{S}_{orth}$ is any vector in the space orthogonal to $\vec{e}_\Omega$.

The electromagnetic fields in real space can then be expanded[12] as

$$\begin{pmatrix} H_{\omega_0,\Omega}(\vec{r},t) \\ E_{\omega_0,\Omega}(\vec{r},t) \end{pmatrix}=\sum_{\vec{G}_\Omega}\sum_{l=-\infty}^{+\infty}\int d\vec{S}_{orth}\begin{pmatrix} H((\vec{k}_0\cdot\vec{e}_\Omega)\cdot\vec{e}_\Omega-\vec{g}_\Omega+\vec{S}_{orth},\omega_0+l\Omega) \\ E((\vec{k}_0\cdot\vec{e}_\Omega)\cdot\vec{e}_\Omega-\vec{g}_\Omega+\vec{Sp}_{orth},\omega_0+l\Omega) \end{pmatrix}\times\exp(i[(\vec{k}_0\cdot\vec{e}_\Omega)\cdot\vec{e}_\Omega-\vec{g}_\Omega+\vec{S}_{orth}]\cdot\vec{r}-i(\omega_0+l\Omega)t). \tag{16}$$

Since, $\vec{e}_\Omega$ was chosen in such a way that $\{\vec{g}_\Omega\}$ is a reciprocal space for some effective one-dimensional periodic structure with a period $\vec{R}_\Omega\equiv R_\Omega\vec{e}_\Omega$, it follows immediately that Eq. (16) can be written in Bloch form. In particular, for such a mode there will exist a "good" quantum number $(\vec{k}\cdot\vec{e}_\Omega)\cdot\vec{e}_\Omega\equiv k_\Omega\vec{e}_\Omega$ and a set of continuous intervals of ω's[13] so that

$$\begin{pmatrix} H_{k_\Omega,\Omega,\omega}\left(\vec{r}+\vec{R}_\Omega,t+\frac{2\pi}{\Omega}\right) \\ E_{k_\Omega,\Omega,\omega}\left(\vec{r}+\vec{R}_\Omega,t+\frac{2\pi}{\Omega}\right) \end{pmatrix}=\exp\left(ik_\Omega R_\Omega-i\omega\frac{2\pi}{\Omega}\right)\begin{pmatrix} H_{k_\Omega,\Omega,\omega}(\vec{r},t) \\ E_{k_\Omega,\Omega,\omega}(\vec{r},t) \end{pmatrix}. \tag{17}$$

Further symmetries can be deduced if we also restrict $\vec{e}_\Omega$ such that $\hat{O}\vec{e}_\Omega=\vec{e}_\Omega$. In this case, substituting $\vec{k}_0$ by $\hat{O}\vec{k}_0$ in Eq. (15), acting with $\hat{O}^{-1}$ from the left, and remembering that $\epsilon_{\hat{O}\vec{G}}=\epsilon_{\vec{G}}$ for any reciprocal vector $\vec{G}$, gives

$$0=\hat{O}^{-1}H(\hat{O}\vec{k}_0,\omega_0)\times\vec{k_0}-\frac{\omega_0}{c}\sum_{\vec{G}}\epsilon_{\vec{G}}\sum_{l=-\infty}^{+\infty}\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}dt\,\hat{O}^{-1}E(\hat{O}[\vec{k}_0-(\vec{e}_\Omega\cdot\vec{G})\cdot\vec{e}_\Omega+\vec{V}_{rot}(t)],\omega_0+l\Omega)\exp(-il\Omega t),$$

$$0=\hat{O}^{-1}E(\hat{O}\vec{k}_0,\omega_0)\times\vec{k_0}+\frac{\omega_0}{c}\hat{O}^{-1}H(\hat{O}\vec{k}_0,\omega_0). \tag{18}$$

By comparing Eq. (18) with Eq. (15) it follows that for any vector $\vec{k}$, $\hat{O}^{-1}H(\hat{O}\vec{k},\omega)=H(\vec{k},\omega)$. Substitution of this result into Eq. (16) leads to the following additional symmetry properties for the solution:

$$\hat{O}^{-1}\begin{pmatrix} H_{k_\Omega,\Omega,\omega}(\hat{O}\vec{r},t) \\ E_{k_\Omega,\Omega,\omega}(\hat{O}\vec{r},t) \end{pmatrix} = \begin{pmatrix} H_{k_\Omega,\Omega,\omega}(\vec{r},t) \\ E_{k_\Omega,\Omega,\omega}(\vec{r},t) \end{pmatrix}. \qquad (19)$$

For both the rotation and vibration scenarios the $\omega_n(\Omega)$ spectrum will generally be rather complex. To illustrate what this spectrum will typically consist of consider the following argument. Since changing $\omega_n(\Omega)$ to $\omega_n(\Omega)+l\Omega$ for any integer l leads to the same state [see Eqs. (10) and (16)], all the labels $\omega_n(\Omega)$ can be mapped trivially to the interval $[-\Omega/2,\Omega/2]$. For any proper choice of wave vector, each corresponding $\omega_n(\Omega)$ will be a band of modes as sketched in Fig. 1. Since plotting a complete band structure is very involved, it is instructive to illustrate a simple case where we only have two bands and where coupling between the modes is very weak (that corresponds to $\Delta\Omega/c\rightarrow 0$ for vibrations and $L\Omega/c\rightarrow 0$ for rotations). Under these conditions the frequencies of the bands folded into the interval $[-\Omega/2,\Omega/2]$, $\omega_n(\Omega)$, will correspond approximately to $\omega_n(\Omega)\simeq\omega_n(0)+l\Omega$ over the whole range of a driving frequency Ω. For a special set of driving frequencies $\Omega=(\omega_2-\omega_1)/l$, bands of the same symmetry will exhibit a near crossing (inset on Fig. 1). The amplitude of this splitting will be proportional to the coupling parameter and will become vanishingly small as the value of l increases. In practice, therefore the major splitting will occur at the primary interband resonant frequency $\Omega\sim\omega_2-\omega_1$. The approach described above can, of course, be readily generalized to include more bands. Finally, we conclude with the observation that in the weak coupling limit it can be shown[14] that a possibility exists of using vibrations and rotations to induce interband transitions between the photon crystal modes in a novel and controlled fashion without the necessity of employing nonlinear materials.

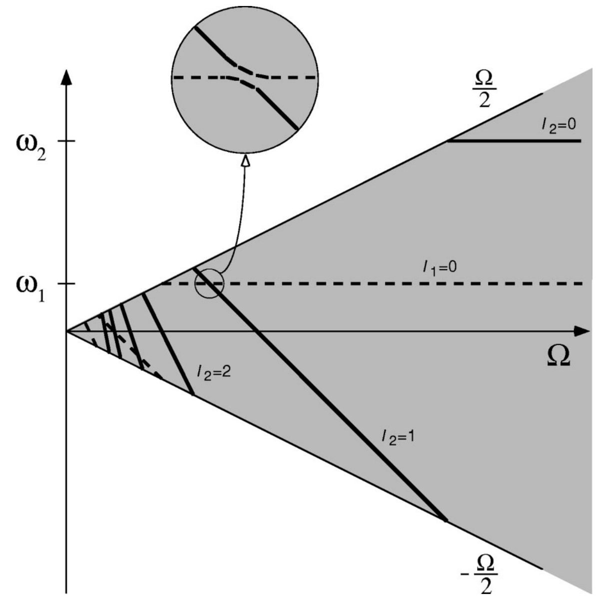

FIG. 1. Noninertial band structure as a function of driving frequency Ω is presented for a case of two nonstationary modes. For each value of Ω the frequencies of the bands $\omega_{1,2}(\Omega)\approx\omega_{1,2}(0)+l_{1,2}\Omega$ are mapped into the interval $[-\Omega/2,\Omega/2]$. For a set of driving frequencies $\Omega=(\omega_2-\omega_1)/l$ bands will exhibit a near crossing as shown in the inset.

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[1] E. Yablonovich, Phys. Rev. Lett. **58**, 1059 (1987).

[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).

[3] J.D. Joannopoulos, R.D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, NJ, 1995).

[4] See, e.g., IEEE Trans. Microwave Theory Tech. (to be published).

[5] A. Mekis, Shanhui Fan, and J. D. Joannopoulos, Phys. Rev. B **58**, 4809 (1998).

[6] S. Fan, Pierre R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **80**, 960 (1998).

[7] J. N. Winn, Yoel Fink, Shanhui Fan, and J. D. Joannopoulos, Opt. Lett. **23**, 1573 (1998).

[8] P. Tran, Phys. Rev. Lett. **21**, 1138 (1996).

[9] J. Martorell, R. Vilaseca, and R. Corbalan, Appl. Phys. Lett. **70**, 702 (1997).

[10] J. N. Winn, Shanhui Fan, and John D. Joannopoulos, Phys. Rev. B **59**, 1551 (1999).

[11] Note that for a 1D or 2D periodic photonic crystal, if one chooses a vector of vibration $\vec{\Delta}$ perpendicular to the reciprocal vector space of the structure, only the $l=0$ term survives in Eq. (9) and the problem reduces (in nonrelativistic limit) to that of the static case.

[12] Note that in practice the characteristic velocities of a rotating photonic crystal must be much smaller then the speed of light. Thus, if L is the size of a photonic crystal in a crossection perpendicular to the axis of revolution, then $L\ll c/\Omega$. This effectively limits the integration range in Eq. (16) to $|\vec{S}_{\rm orth}|\in(2\pi/L,+\infty)$, thus defining a small coupling parameter $L\Omega/c$.

[13] This set of continuous intervals is analogous to the continuum of frequencies that arise from the projection of a three-dimensional band structure $\omega_n(\vec{k})$ onto a particular $\vec{k}$ direction.

[14] M. Skorobogatiy and J.D. Joannopoulos, Phys. Rev. B **61**, 5293 (2000).