Two-dimensional tunable magnetic photonic crystals

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We have calculated the photonic band structure for a cubic block of magnetically saturated ferrite material having a triangular array of circular holes that is under the influence of an external static magnetic field $\mathbf{H}_{\rm ex}=H_{\rm ex}\mathbf{z}$ applied in the hole direction. In this two-dimensional magnetic photonic crystal, the saturated magnetism can affect the transverse electric (TE) mode whose electric field lies in the z axis, but not the transverse magnetic mode. The photonic band gaps of the TE mode shift to lower frequencies and their widths become narrower compared to the case when the relative magnetic permeability $\mu=1.0$. Both the position and the width of the photonic band gap increase in frequency as $H_{\rm ex}$ increases and the gap never becomes larger than that for $\mu=1.0$.

Recently, there has been much attention paid to periodic dielectric composites (photonic crystals) that possess photonic band gaps (PBG's), in which electromagnetic waves cannot propagate in any direction.[1–7] It is well known that the periodic variation of dielectric constant or refractive index can give rise to PBG's. However, we have recently demonstrated that the wave impedance plays the essential role in the formation of PBG's rather than the dielectric constant or the refractive index.[8] The effects of magnetic permeability on PBG's, investigated by Sigalas *et al.*,[9] can be well explained by the role of wave impedance in the formation of PBG's.

Up to now, magnetic materials have not attracted much attention for photonic crystals, since the relative permeability μ of magnetic material is equal to 1.0 in the optical range. However, most ferrites have values of μ quite different from 1 in the microwave range and thus can be exploited for microwave PBG's. When ferrites are employed in microwave devices, they are usually operated in the saturated state since they can be very lossy below saturation in the microwave range. The μ value of magnetically saturated ferrites in the microwave range depends on the saturation magnetization, the microwave frequency, and the external static magnetic field $\mathbf{H}_{\rm ex}$. Therefore, ferrites make tunable PBG's possible with applied magnetic fields. Recently, a report was made on one-dimensional tunable metallic photonic crystals using an external magnetic field.[10] Also, a tunable optical PBG can be realized by controlling the orientational order in a nematic liquid crystal infiltrated into the void regions of PBG materials.[11]

In this report, we investigate theoretically the properties of two-dimensional (2D) ferrite photonic crystals for two independent polarizations. Circular holes in a triangular array are assumed to be drilled along the z axis of a cubic ferrite block perpendicular to the propagation direction of the microwaves. The external static magnetic field is assumed to be applied so that $\mathbf{H}_{\rm ex}=H_{\rm ex}\mathbf{z}$.

The magnetic field of the transverse magnetic (TM) mode, which is parallel to $\mathbf{H}_{\rm ex}$, does not interact with the dipole moments of the saturated ferrite medium, and thus μ of the ferrrite is 1.0.[12] Therefore, for the TM mode, the characteristics of a 2D z-biased ferrite photonic crystal are the same as those of a 2D dielectric photonic crystal with the dielectric constant of the ferrite medium. Since the dielectric losses of ferrites can be neglected in the ferrite operating frequency range and the dielectric constants of typical ferrite materials are in the range of 12.0–16.0,[13] we can expect rather wide PBG's in this TM mode.

In the transverse electric (TE) mode, the magnetic field of the mode is perpendicular to $\mathbf{H}_{\rm ex}$ and thus it induces a precession of magnetic dipoles around the external field at the mode frequency. Thus, for the TE mode, μ of the ferrite is a function of frequency ω, the saturation magnetization of ferrite, M_s, and the strength of the applied field $H_{\rm ex}$ as

$$\mu=\frac{(\omega_{\rm ex}+\omega_m)^2-\omega^2}{\omega_{\rm ex}(\omega_{\rm ex}+\omega_m)-\omega^2}, \qquad (1)$$

where $\omega_{\rm ex}=\gamma H_{\rm ex}$ and $\omega_m=4\pi\gamma M_s$.[12] γ is the ratio of the spin magnetic moment to the spin angular momentum, the gyromagnetic ratio. The TE mode in a 2D ferrite photonic crystal satisfies

$$\sum_{\mathbf{K}'} M_{\mathbf{K},\mathbf{K}'}E_{\mathbf{K}'}=\frac{\omega^2}{c^2}E_{\mathbf{K}}, \qquad (2)$$

where c is the velocity of light in vacuum and

$$M_{\mathbf{K},\mathbf{K}'}=\sum_{\mathbf{G}''} \mu^{-1}_{\mathbf{G}''}\epsilon^{-1}_{\mathbf{K}-\mathbf{K}'-\mathbf{G}''}(\mathbf{K}'+\mathbf{G}'')\cdot\mathbf{K}', \qquad (3)$$

with $\mathbf{K}=\mathbf{k}+\mathbf{G}$, $\mathbf{K}'=\mathbf{k}+\mathbf{G}'$. Here, $\mathbf{k}$ is a wave vector in the Brillouin zone, $\mathbf{G},\mathbf{G}'$, and $\mathbf{G}''$ are reciprocal lattice vectors, and $\mu^{-1}_{\mathbf{G}''}$ and $\epsilon^{-1}_{\mathbf{K}-\mathbf{K}'-\mathbf{G}''}$ are the Fourier transforms of $\mu(\mathbf{r})$ and $\epsilon(\mathbf{r})$, respectively. Equation (2) cannot be solved by the standard matrix diagonalization method[2] because $\mu^{-1}_{\mathbf{G}''}$ depends on ω. However, we can calculate the matrix solutions of Eq. (2) by means of the effective way to treat frequency-

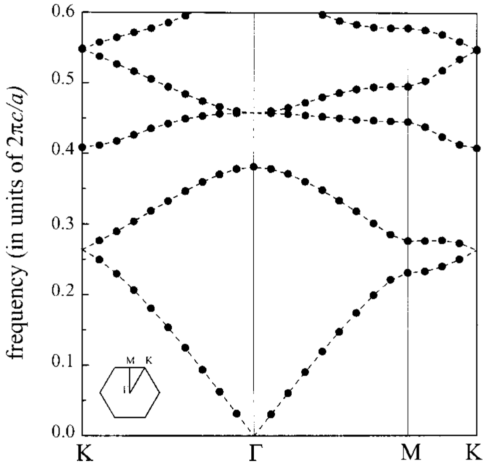

FIG. 1. The lowest five photonic bands for the TE mode of a triangular lattice of air rods in a ferrite medium with magnetic permeability 1.0. The frequency is normalized to $2\pi c/a$, where a is the lattice constant and c the velocity of photons in vacuum. The radius of the air rod is taken as $0.34a$ and the dielectric constant of the ferrite medium as 12.25. The dashed lines are the photonic band structure obtained from the standard matrix diagonalization method. The inset denotes the first Brillouin zones of a two-dimensional triangular lattice.

dependent media already reported:[14,15] we took 1000 mesh points for $0<\omega a/2\pi c\leqslant 1$ for a given $\mathbf{k}$, where a is the lattice constant. At these mesh points, we calculated the determinants of the matrix equation and found the region where the determinants of the neighboring mesh points change sign.

Figure 1 shows the lowest five photonic bands of a triangular lattice of air rods in a saturated ferrite medium for the TE mode when the permeability μ and the dielectric constant of the ferrite medium, ϵ, are assumed to be 1.0 and 12.25, respectively. The radius of the air rod is $0.4233a$ and the frequency is normalized to $2\pi c/a$. These photonic bands can also be obtained from a standard matrix diagonalization method,[9] the results of which are indicated by the dashed lines. We used 253 plane waves in both cases. The two are in excellent agreement with each other, as expected.

Figure 2 represents the lowest five photonic bands of a ferrite triangular photonic crystal for the TE mode and the dependence of μ on $\omega a/2\pi c$ when $\omega_{\mathrm{ex}}a/2\pi c=1.0$ and $\omega_m a/2\pi c=0.5$. The band structure obtained for $\mu=1.0$ and $\epsilon=12.25$ is also drawn as dashed lines for comparison. One can see that the band frequencies are suppressed, especially at high frequencies, and the magnitude of the PBG is decreased. Recently, we have demonstrated that the wave impedance $\sqrt{\mu/\epsilon}$ plays a major role in determining the magnitude of PBG's while the refractive index $\sqrt{\mu\epsilon}$ is related to the position of PBG's.[16] Note that the refractive index of ferrite is larger than that of air but the wave impedance of the ferrite medium is smaller than that of the air. Therefore, as the relative permeability μ of the ferrite increases with ω, so does the contrast of refractive index between the ferrite and air, while the contrast of wave impedance between the two decreases. Thus the suppression of band frequencies and the reduction of PBG magnitude compared with the case of $\mu=1.0$ are evidently due to the increased ratio of refractive index and the decreased ratio of wave impedance, respectively, with the increase of frequency. Moreover, we can also

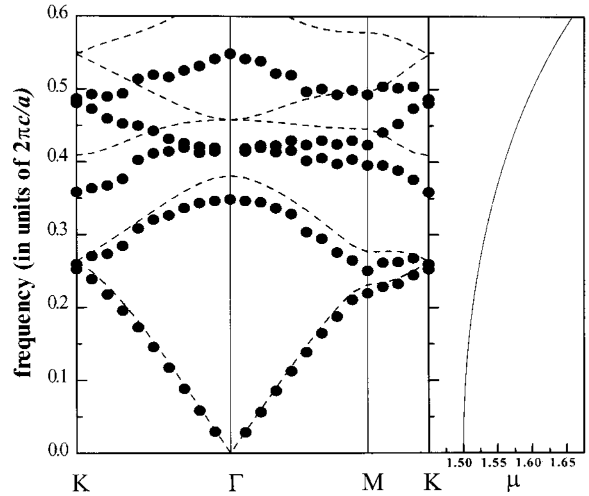

FIG. 2. The lowest five photonic bands of a ferrite triangular photonic crystal for the TE mode and the dependence of μ on $\omega a/2\pi c$ when $\omega_{\mathrm{ex}}a/2\pi c=1.0$ and $\omega_m a/2\pi c=0.5$. Other calculation parameters are the same as those in Fig. 1. The dashed lines are the photonic bands when $\mu=1.0$ and $\epsilon=12.25$.

safely say that this behavior would be a general property of 2D saturated ferrite photonic crytals for the TE mode.

The relative permeability of a saturated ferrite medium is always larger than 1.0 and becomes close to 1.0 as ω_{ex} increases in the low frequency range. Thus, the increase of H_{ex} makes the position of the PBG shift to higher frequency and the width of the PBG larger, but the width of the PBG will never be larger than that of a PBG with $\mu=1.0$. In Fig. 3, the dotted points represent the lowest five photonic bands of a ferrite triangular photonic crystal for the TE mode when $\omega_{\mathrm{ex}}a/2\pi c=2.5$ and $\omega_m a/2\pi c=0.5$. The dashed lines are for the case of $\mu=1.0$ and $\epsilon=12.25$. One can observe that the position of the PBG shifts to higher frequency and the width of the PBG is larger than that of the PBG in Fig. 2. This well displays the dependence of the position and the width of the PBG on H_{ex}. In typical ferrite materials, $4\pi M_s$ is 1700–3000 G, so that $\omega_m/2\pi$ is 4.8–8.4 GHz. For example, in

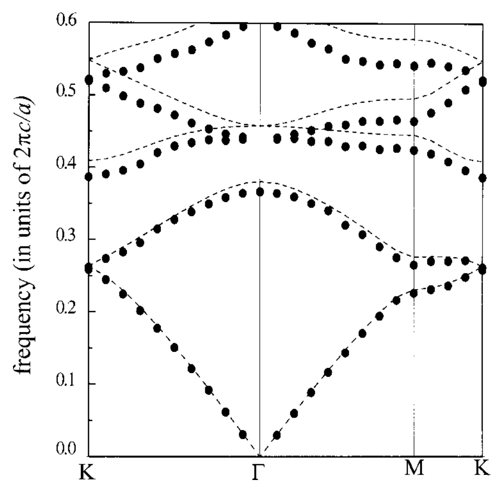

FIG. 3. The lowest five photonic bands of a ferrite triangular photonic crystal for the TE mode when $\omega_{\mathrm{ex}}a/2\pi c=2.5$ and $\omega_m a/2\pi c=0.5$. The dashed lines are the photonic band structure when $\mu=1.0$ and $\epsilon=12.25$.

magnesium ferrite (TT1-105) where $4\pi M_s = 1750$ G and the dielectric constant is 12.2,[13] the PBG for the geometry employed is expected to lie between 3.59 and 3.78 GHz when $H_{ex} = 8750$ Oe.

In Fig. 4, the edge frequencies of the PBG shown in Fig. 3 are plotted as a function of $\omega_{ex} a/2\pi c$. The solid circles and open circles denote the top and bottom frequencies of the PBG, respectively. One can see that they increase with increase of ω_{ex} and apparently this is due to the fact that the permeability of the ferrite decreases as H_{ex} increases. When H_{ex} is very large, their values will be saturated at those of $\mu = 1.0$. Ferrite photonic crystals can thus be tuned in such a way that their PBG's shift to higher frequencies and their widths become larger as H_{ex} increases.

In conclusion, we have investigated the general properties of saturated ferrite 2D photonic crystals under an externally applied static magnetic field. The properties of a saturated 2D ferrite photonic crystal for the TM mode are the same as those of a 2D dielectric photonic crystal with the dielectric constant of the ferrite medium. Since μ of a ferrite medium increases as ω increases and is always larger than 1.0, the contrast of the refractive index between the ferrite medium and the air increases but that of the wave impedance decreases in the TE mode. Thus, the positions of the bands shift toward lower frequencies and the widths of the PBG's are smaller than those for $\mu = 1.0$. We have also shown that the position and the width of a PBG can be tuned by applying an external magnetic field to photonic crystals constructed with ferrites.

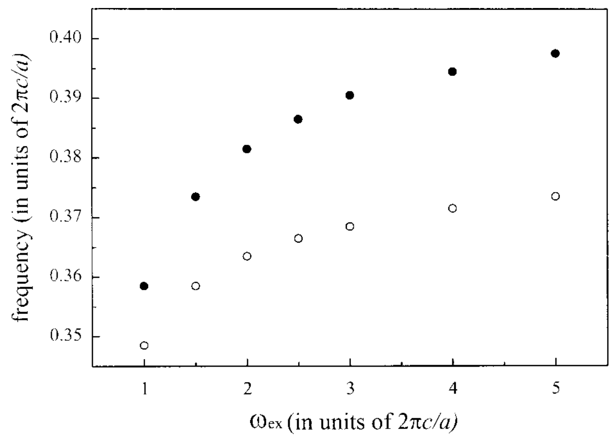

FIG. 4. The edge frequencies of the PBG shown in Fig. 3 as a function of $\omega_{ex} a/2\pi c$ when $\omega_m a/2\pi c = 0.5$. The solid circles and open circles denote the top and bottom frequencies of the PBG, respectively.

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