

## Irreversibility fields of superconducting niobium alloys

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The irreversibility line of superconductors is most usually established from magnetization curves. However, many low- $T_c$  materials show extremely reversible magnetization curves, while still having a finite critical current. Confirmation of a reversibility line requires other measurements. We have made measurements of dc magnetization, ac susceptibility, and magnetoresistivity as a function of applied field and temperature on Nb alloy samples in order to investigate the irreversibility line in low- $T_c$  superconductors. The results show that there exists an observable field region below the mean-field critical field  $B_{c2}$ , where the magnetization is reversible during a cycle of increasing and decreasing field, which is in agreement with a previous report by Suenaga *et al.* In addition to dc magnetization, ac susceptibility and magnetoresistivity measurements were also carried out on the same sample as alternative techniques to probe the irreversibility line to determine the best way of distinguishing a genuine thermally activated reversibility from a finite, but low, critical current density. The results showed that the collapse of the dc magnetic hysteresis, the onset of the diamagnetic ac susceptibility (or the peak of the ac loss) and the zero resistance occur at nearly the same field, namely, the irreversibility field  $B_{irr}$ . These experimental observations indicate that the irreversibility line is not unique to high- $T_c$  oxides but also exists in conventional superconducting metallic alloys although much closer to  $B_{c2}$ . However, it is difficult to reconcile these results with measurements on other low- $T_c$  materials which show zero resistance up to the surface critical field  $B_{c3}$ .

### I. INTRODUCTION

The unusual behavior of high- $T_c$  superconductors in the mixed state attracts considerable attention. One of the most important phenomena is the existence of the so-called irreversibility line in the field-temperature ( $B$ - $T$ ) plane. This line separates the  $B$ - $T$  plane into two regions: a reversible one (corresponding to high temperature and field) and an irreversible one (low  $T$  and  $B$ ). A number of models have been proposed to explain the existence of the irreversibility line.<sup>1-7</sup> Although the mechanism for the observed easy flux line movement in high- $T_c$  oxides is yet to be understood completely, it is generally agreed that the phenomenon results from a complex scenario including very weak electronic coupling between the superconducting  $\text{CuO}_2$  layers (which leads to a very large anisotropy), a short coherence length, and high working temperatures. Flux lattice melting and decoupling of layers are the main contenders, although it now seems that these are not as different as at first thought.

The characteristics of the irreversibility line are a reversible diamagnetic magnetization and a broadening of the resistive transition at low currents by a field, corresponding to a linear resistivity. Neither result on its own establishes an irreversibility line. Magnet inhomogeneity combined with sample movement makes most magnetometers much less sensitive to hysteresis than to magnetization. Resistive

broadening can be due to inhomogeneity in the sample which may have a much wider range of  $B_{c2}$  than  $T_c$  and these measurements are further complicated by surface currents. All techniques can only set a lower limit to  $J_c$ , although since  $J_c$  usually varies as a power law if this limit is well below a factor of about  $(1 - B/B_{c2})^2$  of the low-field value we have pretty convincing evidence of a true irreversibility line.

Relevant early experiments were those of Wade<sup>8</sup> and Campbell, Evetts, and Dew-Hughes.<sup>9</sup> Wade used a Clarke slug to measure the  $V$ - $I$  characteristics of Pb-In at voltages down to  $10^{-12}$  V, looking for the exponential curve characteristic of flux creep. This only appeared within 100 G of  $B_{c2}$ ; at lower fields a sharp jump in voltage was observed, and no measurable linear resistivity was reported at any field. Campbell, Evetts, and Dew-Hughes<sup>9</sup> measured the critical current of Pb-Bi near the upper critical field and found a finite critical current at all fields up to  $B_{c2}$  in plated samples. These results suggest that the reversible region in low- $T_c$  materials is too small to be observed. Clem, Kerchner, and Sekula<sup>10</sup> reported measurements on NbTa which had all the characteristics of a reversibility line, except that presence of harmonics in the susceptibility indicated a low  $J_c$  rather than a linear resistivity. In all these experiments the value of  $B_{c2}$  was determined from the dc magnetization curve.

More recently resistive broadening has been seen in some

specially prepared quasi-two-dimensional films such as In/InO,<sup>11</sup> amorphous Mo-Ge,<sup>12</sup> and Nb-Ge.<sup>13</sup> Also Suenaga *et al.*<sup>14</sup> have investigated the irreversibility line in low-temperature superconducting NbTi and Nb<sub>3</sub>Sn multifilamentary wires using dc magnetization measured in a superconducting quantum interference device (SQUID) magnetometer. They found that these materials have a clear irreversibility line which is substantially lower than the superconducting  $B_{c2}(T)$  phase line. They also found that their  $B_{\text{irr}}$  data fitted well to a flux lattice melting line formulated by Houghton, Pelcovits, and Sudbø,<sup>4</sup> and thus they interpreted the observed irreversibility line as a melting line of the flux lattice. Schmidt, Israeloff, and Goldman<sup>15</sup> investigated the irreversibility line of pure Nb film samples following a similar procedure to that used by Suenaga *et al.*<sup>14</sup> According to their data and analysis, they also concluded that the irreversibility line in Nb is the flux lattice melting line. The authors showed that their flux creep data in the region near the irreversibility line can neither be explained by the flux-line depinning theory<sup>1</sup> nor the vortex glass phase-transition theory.<sup>5</sup>

Besides the dc magnetometry technique, a number of different techniques have been employed to measure the irreversibility line in high- $T_c$  oxides. Magnetoresistivity and ac susceptibility measurements are the most commonly used. It has been demonstrated<sup>16–19</sup> that data from magnetic measurements, particularly the observed dissipation peak in the ac susceptibility, can be explained by the electromagnetic skin effect using resistivity data. It is expected therefore that both resistivity and ac susceptibility measurements would show transitions in accordance with the disappearance of dc magnetic hysteresis at the irreversibility field or temperature in low- $T_c$  materials just as in the high- $T_c$  oxides. Drulis *et al.*<sup>20</sup> have investigated the irreversibility line in Nb film and NbSe<sub>2</sub> single-crystal samples using a vibrating reed (which is equivalent to the ac susceptibility method<sup>21</sup>) and magnetoresistivity methods. The authors also concluded that there exists an extended region of magnetic reversibility in these low- $T_c$  samples. However, no data of dc magnetic measurement and  $B_{c2}$  were presented in their work. To clarify the situation we have measured dc magnetization, ac susceptibility, and resistivity as a function of applied field and temperature on Nb alloy samples and the results of these measurements are reported in this paper.

## II. EXPERIMENT

The samples used in this study were NbTi alloys in the form of a multifilamentary wires and a bulk cylindrical rod and NbTa in a bulk rod. The multifilamentary NbTi wire consists of individual NbTi filaments embedded in a continuous copper matrix. Most measurements were conducted on a vibrating sample magnetometer (VSM 3001, Oxford Instruments) although some dc magnetization measurements were performed on a SQUID magnetometer (Quantum Design MPMS) with a scan length of 2 cm. The variation of the magnetic field is less than 0.01% at this scan length.

Since the reversible region in low- $T_c$  superconductors is expected to be small, measurements need to be carried out carefully if  $B_{\text{irr}}$  and  $B_{c2}$  are to be distinguished from each other. Thus it is crucial to maintain good temperature stabil-

ity during isothermal magnetization and other measurements. For this reason the magnetometer probe was adapted so that resistive and susceptibility measurements could be done at the same time as the dc magnetic measurements without removing the sample. A calibrated carbon in glass resistance thermometer was placed at the sample position to check the temperature stability inside the VSM. It was found that the temperature variation and drift  $\Delta T$ , in the range  $4.2 < T < 20$  K, was less than 0.05 K during a time period of 1 h, which corresponds approximately to the time scale of individual measurements.

In order to perform the dc magnetization measurements, the NbTi wire was cut into segments of length  $\sim 3$  mm and several lengths were sealed tightly together inside a short piece of heat shrink plastic sleeving. Measurements were taken with this bundle of wire segments placed perpendicular to the applied field (i.e., horizontally) to avoid the effect of surface currents.<sup>22</sup> Measurements were also made with the sample placed vertically at selected temperatures. The resistivity was measured using a standard four-probe method with a low frequency (77 Hz) ac driving current. To achieve a high normal-state voltage the outer layer of copper on a short segment of the multifilamentary wire sample was etched away with concentrated nitric acid, leaving only the current contact area. Electrical leads were connected to the sample using silver paint. The sample was placed horizontally adjacent to the bundle of short wire segments which was used for magnetization measurements. The dc magnetization and resistivity were measured virtually simultaneously with this geometry.

The ac susceptibility was measured inductively by placing the specimen inside a sample holder on which a copper drive coil was wound. A small ac field was generated by this solenoid and the induced signal was picked up by the sense coil of the VSM and measured by a lock-in amplifier. This allows the ac susceptibility to be compared with the dc magnetization in the same apparatus at almost the same time. Similarly, the resistivity measurements were made in a modified probe which allowed the dc magnetization to be measured without changing anything. This is important since  $B_{\text{irr}}$  is rather close to  $B_{c2}$  in the low- $T_c$  samples so any small difference in temperature or magnetic-field measurement will make the comparison of data between different techniques difficult. All the measurements were carried out at fixed temperature.

### Comparison of techniques

The central problem is to distinguish between a low, but finite, critical current supported by pinning centers, and a true reversible region with a linear resistivity as is seen in high- $T_c$  superconductors. The results are complicated by surface currents. Different techniques differ in their sensitivity, and the sample shape is also relevant. We can set order of magnitude limits to the detectable  $J_c$  in samples of individual radius  $R$  and total volume  $V$  as follows:

- (i) If the magnetometer sensitivity is  $M J_c < 1.5 \text{ M/RV}$ . This is the smallest detectable magnetization.
- (ii) Normally a higher limit is set if the field inhomogeneity is greater than the penetration field.  $J_c < \Delta B / \mu_0 R$ .
- (iii) For a resistive measurement  $J_c < V / \rho L$  where  $V$  is the

TABLE I. The minimum detectable  $J_c$  for each sample.

	2-mm Nb Ta	1.8-mm NbTi	10 $\mu$ -NbTi
Magnetometer limit	0.38 (0.01)	0.38 (0.01)	38 (0.01)
Magnet inhomogeneity	2 (0.2)	26 (2)	5000 (2)
ac susceptibility	0.01 ( $10^{-3}$ )	0.01 ( $10^{-3}$ )	2 ( $10^{-3}$ )
Resistive	0.1 (0.01)	0.1 (0.009)	0.1 ( $5 \times 10^{-5}$ )

voltage sensitivity,  $\rho$  the resistivity (approximately the flux flow resistivity which is near the normal state value), and  $L$  the length.

(iv) For a susceptibility measurement the situation is more complicated and discussed in more detail below. However, one limit is if the amplitude is much greater than the penetration field.  $J_c < h_{ac}/R$  where  $h_{ac}$  is the lowest useful amplitude of the ac drive field.

Similar limits can be obtained for a critical surface current.

Table I shows minimum critical current density detectable on the assumptions above close to  $B_{c2}$  in A/cm<sup>2</sup> with the minimum surface current in amps/cm in parentheses following. From Table I we can draw a number of qualitative conclusions. Perhaps the first is that in early experiments the real irreversibility of many samples was obscured by the inhomogeneity of the magnets in the magnetometers in use at the time. In particular filamentary NbTi could have a  $J_c$  of 5000 A/cm<sup>2</sup> and still appear reversible. Only integrating magnetometers avoided movement of the sample, and these suffered from drift of the integrator which made them inaccurate for high- $\kappa$  materials. Although magnetometers are less sensitive than other methods in detecting a critical current, they work at very low frequency so that if there is a low linear resistivity as opposed to a low  $J_c$ , they may be the most sensitive detection method.

### III. ac INDUCTIVE TRANSITION

It was argued above that the detection limit of the ac susceptibility technique was when the penetration field was less than the amplitude, but this assumes that the amplitude is sufficient to generate a critical state and this is rarely the case. In a field  $B$  with an ac amplitude  $b$  and a radius  $R$  the flux-line movement is  $bR/B$  and at the lowest amplitudes used this was about one thousandth of the vortex spacing. We are therefore in the linear regime. There are three approaches to describing the inductive transition in the linear regime,<sup>23–26</sup> but they give similar results at low frequencies. We give here a simple derivation for the low-frequency linear limit based on adding the vortex displacement due to elastic movement to the plastic deformation due to creep. We assume harmonic oscillations and that the flux flow resistivity can be neglected at low frequencies.

For the elastic deformation  $y_{el} \mathbf{B} \times \mathbf{J} = \alpha \mathbf{y}_{el}$  and for the plastic deformation  $\mathbf{B} \times \mathbf{J} = -\eta j \omega \mathbf{y}_{pl}$  where  $\alpha$  is the Labusch parameter and  $\eta$  the viscosity due to flux creep, which is related to the measured resistivity by  $\eta = \rho/B^2$ .

The total displacement is

$$\mathbf{y} = \mathbf{y}_{pl} + \mathbf{y}_{el} = -\mathbf{B} \times \mathbf{J} (1/\alpha + j\omega/\eta).$$

Now

$$\mu_0 \mathbf{J} = -\nabla^2 \cdot \mathbf{A} = -\nabla^2 \cdot \mathbf{E}/j\omega = -B \nabla^2 \cdot \mathbf{y}.$$

Putting  $\alpha = B^2/\mu_0 \lambda'^2$  and  $\delta^2 = 2\rho/\mu_0 \omega$  the equation of motion is  $\nabla^2 \cdot \mathbf{y} = k^2 \mathbf{y}$  where

$$k^2 = (\lambda'^2 + \delta^2/2j)^{-1}.$$

We thus get a skin depth equation, or London equation, with a complex penetration depth. The solutions are well known, for a cylinder of radius  $a$  parallel to the field the susceptibility is

$$\chi = 2I_1(ka)/I_0(ka).$$

If  $\lambda'$  is large we can use the first term in the power series expansion:

$$\chi = (ka)^2 = a^2 (\lambda'^2 + \delta^2/2j)^{-1}.$$

As the temperature is reduced both  $\lambda'$  and  $\delta$  reduce from infinity to very small values. We expect  $\lambda'^2$  to be proportional to the depth of the energy wells, and therefore obey a power law in  $(1 - B/B_{c2})$  while  $\delta$  depends on the flux creep resistivity and goes to zero exponentially at the irreversibility line. Since this is much more rapid than the change in  $\lambda'$  we can assume  $\lambda'$  is constant over the transition. The loss peak occurs when  $\delta = \sqrt{2}\lambda'$  and the height is exactly half the susceptibility at zero skin depth.

There are two possible scenarios. First, if the irreversibility line is very close to  $B_{c2}$  there will be a very small loss peak at this point, but diamagnetism will not appear until a lower temperature when  $\lambda'$  becomes comparable to the sample size. The second possibility is that the irreversibility line is far enough below  $B_{c2}$  for  $\lambda'$  to be comparable to, or less than, the sample size. In this case the loss peak will occur at the same temperature as the diamagnetism appears. In principle there should be a small paramagnetic susceptibility of  $1/(2\kappa)^2$  above the irreversibility field but this is usually too small to see.

## IV. RESULTS

### A. NbTa rod

Figure 1 shows the magnetic moment of a cylindrical (2 mm in diameter and 4.5 mm long) NbTa alloy sample measured in the SQUID magnetometer as a function of increasing and decreasing field. The inset shows the whole hysteresis loop (the drop at around 0.25 T was associated with a flux jump. Note the large difference in scale in the inset). A large difference in the magnitude of  $M$  between the reversible and irreversible magnetic moments of the sample is evident from this figure. The irreversibility field  $B_{irr}$  is identified in the figure as the field at which the magnetic hysteresis collapses.  $M$  shows reversible behavior above  $B_{irr}$  and the gradient has a clear discontinuity at a field that is identified as the upper critical field  $B_{c2}$ .  $M$  changes linearly with increasing field at fields beyond  $B_{c2}$  and is believed to be a background contribution from the sample holder and the normal state of the sample. In the range between  $B_{irr}$  and  $B_{c2}$ , the magnetization appears to show a linear dependence on the applied field, which is in agreement with the theoretical prediction of Abrikosov.<sup>27</sup>

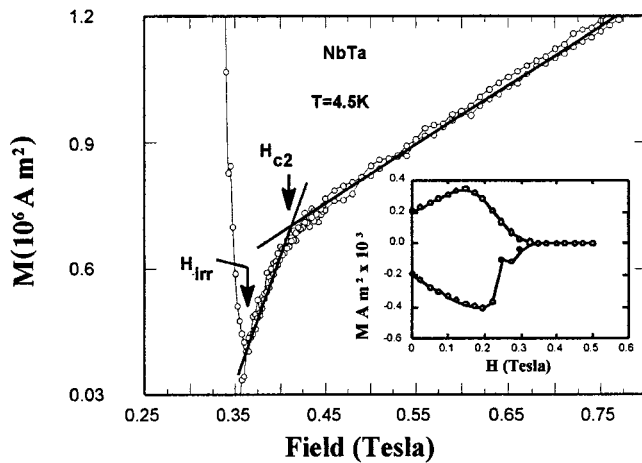


FIG. 1. A detailed view of the variation of magnetic moment with the applied field for a NbTa sample around  $B_{c2}$ . The inset shows the whole  $M$ - $B$  curve. The data were taken in a SQUID magnetometer.

Results of ac susceptibility measurements on the NbTa sample are shown in Fig. 2 along with the dc magnetization data. The dc magnetization data were collected on the VSM and are similar to those measured on the SQUID magnetometer (Fig. 1). The data in Fig. 2 show that  $B_{irr}$  determined from the ac susceptibility is close to that measured in the dc magnetization. For this sample it was found at 4.2 K that  $B_{irr} \sim 0.41$  T and  $B_{c2} \sim 0.45$  T. The difference between these two characteristic fields is small in comparison with the corresponding values observed for the high- $T_c$  superconductors.

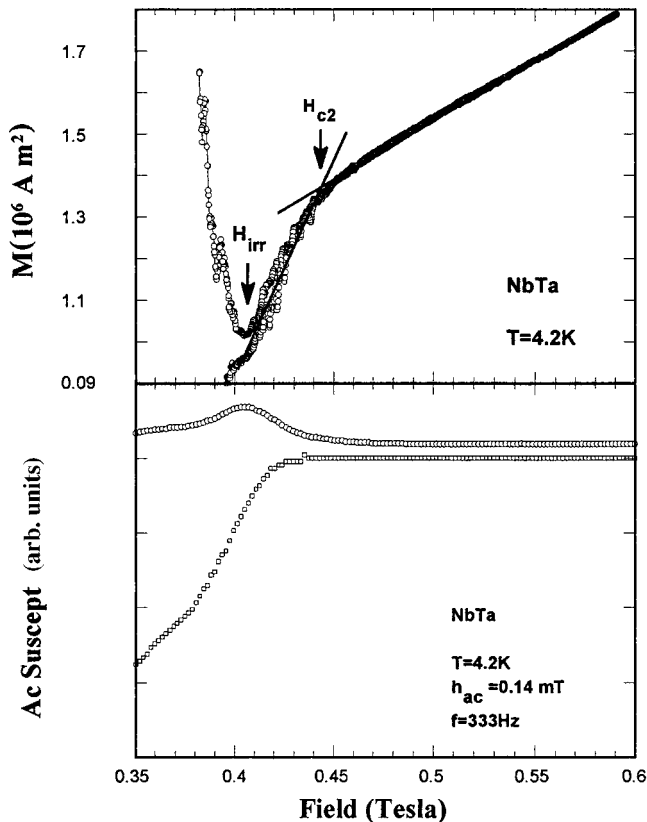


FIG. 2. Experimental dc magnetization (top) and ac susceptibility (bottom) data for a NbTa sample plotted as a function of applied field. The data were taken in a vibrating sample magnetometer.

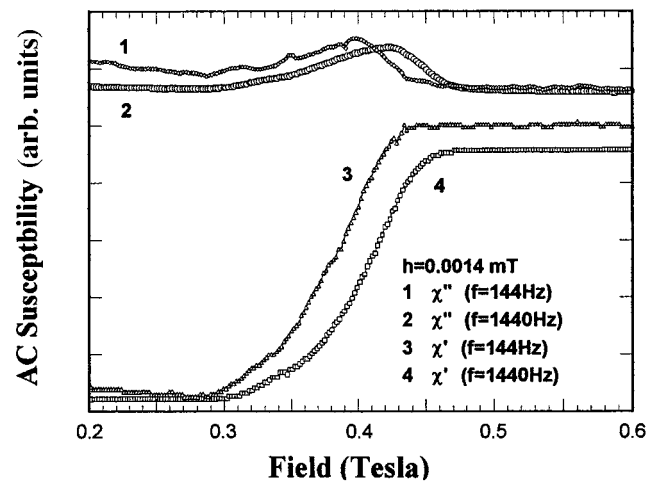
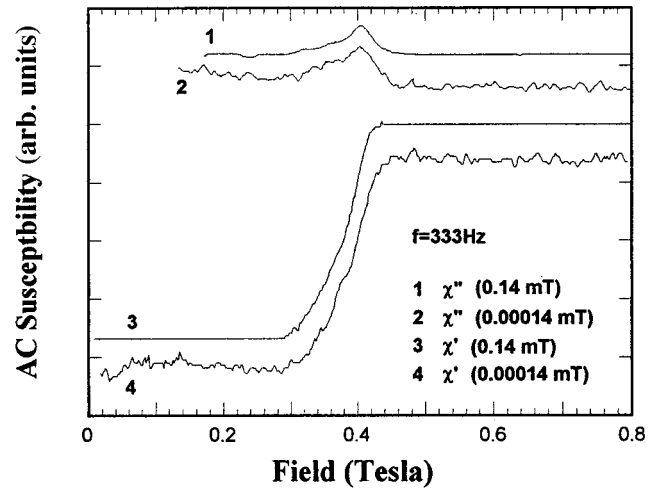


FIG. 3. ac susceptibility data for a NbTa sample measured at two different fields (top) and at two different frequencies (bottom).

Using the skin depth effect to interpret the ac susceptibility data, one would expect the transition field to be independent of the ac field amplitude  $B_{ac}$  (in the Ohmic region) but dependent on the frequency of the ac field. Data for  $\chi'$  and  $\chi''$  measured at different ac fields and frequencies are shown in Fig. 3, the curves have been displaced for clarity. The effect of changing  $B_{ac}$  by a factor of 1000 on the signal is small, while the effect of increasing frequency by a factor of 10 is significant. This shows that the ac susceptibility measurements are in the linear regime, as expected from the low amplitude of the ac field.

Analysis based on the skin depth effect suggests that the height of the peak in  $\chi''$  should be about half the magnitude of the  $\chi'$  transition. ac susceptibility results on high- $T_c$  superconductors show that this indeed is the case.<sup>28,29</sup> The data in Fig. 3, however, show that the peak in  $\chi''$  is much smaller than this value. This difference is due to the fact, discussed above, that at this field  $\lambda'$  is still larger than the sample size, so the diamagnetism it can produce is much less than 100%.

The resistive measurements showed a higher transition and the critical current is shown in Fig. 4. A finite current extends up  $B_{c3}$ , well above  $B_{c2}$ . This current is of the order of 0.06 A/cm, and so would not show up in a magnetization measurement. It is a factor of 60 greater than amplitude of the ac field in the susceptibility measurement, but more rel-



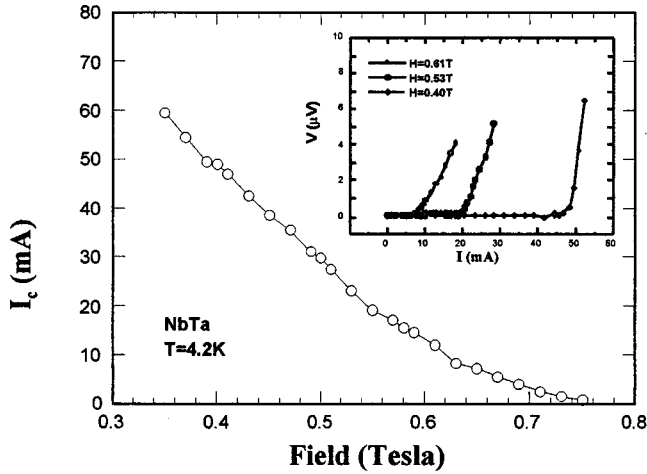


FIG. 4. Variation of the critical transport current of a cylindrical NbTa sample with the applied field. Measurements were performed with the field and current parallel to the longitudinal axis. The inset shows current-voltage characteristics measured at 4.2 K for three different fields.

evant is the value of  $\lambda'$  for a surface current which is an unknown quantity. Since there are about 100 vortex lines contributing to a bulk value of  $\alpha$  we expect  $\lambda'$  for a single layer to be at least ten times larger than the bulk value so it is not too surprising that we do not see the surface current in the susceptibility measurements, but further investigation is needed to see if critical transport surface currents are the same as those in a magnetic experiment.

### B. NbTi multifilamentary wire

Figure 5 shows experimental data from the dc magnetization, ac susceptibility (i.e., the real part,  $\chi'$ ) and resistivity as a function of applied field at 7.6 K for the NbTi multifilamentary wire. A linear background signal, which is present above  $B_{c2}$ , has been subtracted from the dc magnetization data, and hence the curve becomes flat at fields beyond  $B_{c2}$ . The behavior of the dc magnetization is very similar to that observed for the NbTa sample (see Figs. 1 and 2). Hence  $B_{irr}$  and  $B_{c2}$  can be identified in a similar manner as shown in Fig. 5.

$\chi'$  shows a transition with the diamagnetic susceptibility appearing at about  $B_{irr}$ . Also, as the sample becomes reversible, supercurrents can no longer be carried, so that resistance is anticipated to appear at about  $B_{irr}$ . The data displayed in Fig. 5 indeed show a simultaneous appearance of irreversible dc magnetization, ac susceptibility transition and zero resistance.

The peak in  $\chi''$  is not well defined for the NbTi multifilamentary wire sample, due to the small size of the very fine NbTi filaments in the wire. The measurements were performed as the applied dc field was swept slowly and it was found that  $\chi''$  is field-sweep rate  $s (=dB/dt)$  dependent. In a hysteretic regime the dc field should be varied extremely slowly so that  $s$  is much smaller than  $2\pi f B_{ac}$  ( $f$  is the frequency of the ac field). However, this is rather time consuming and impractical in view of the drift in temperature over a long period of time. It was found that, although  $\chi''$  is af-

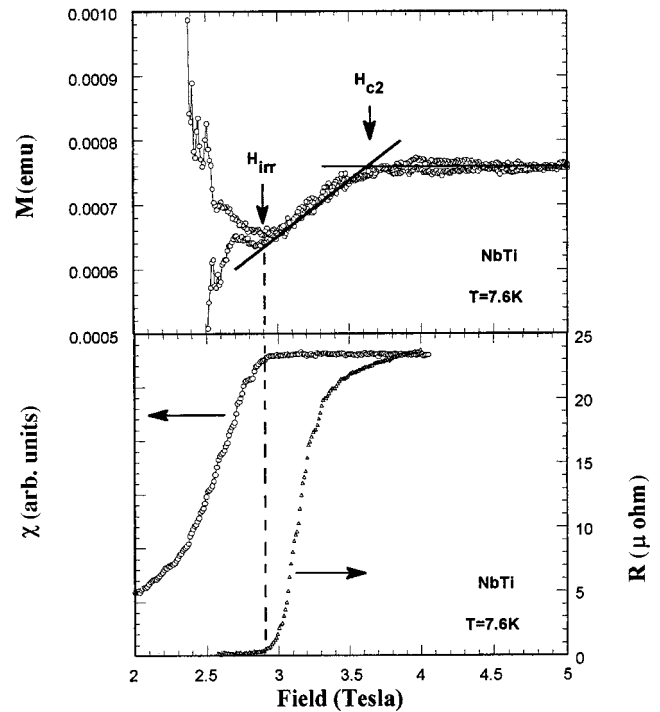


FIG. 5. Experimental dc magnetization, ac susceptibility, and resistivity data for a NbTi multifilamentary wire sample as a function of applied field. The data were taken in a vibrating sample magnetometer (VSM). An ac field of 82 Hz and 0.014 mT was used for ac susceptibility measurements while resistivity was measured with an ac current of 77 Hz and 1 mA.

ected by  $s$ , the onset of the transition in  $\chi'$  is not dependent on this parameter, confirming the linear reversibility of the transition.

Measurements using different values of current were also performed at selected temperatures. The resistive transition remained virtually unchanged as the current was increased by three orders of magnitude, implying a rapid change of critical current as the field approaches  $B_{irr}$ .

In the reversible region, the flux flow resistivity  $\rho f$  was significantly different from the Bardeen-Stephen model<sup>30</sup> which suggests that  $\rho f \sim \rho_n B/B_{c2}(T)$  ( $\rho_n$  is the normal-state resistivity at the given temperature). Thus in a simplified picture, one would expect that at  $B_{irr}$  the sample resistance changes to a value of  $R_n B_{irr}/B_{c2}(T)$  and then increases linearly to the normal-state value  $R_n$  at  $B_{c2}$ . However, the data plotted in Fig. 5 show that this is not the case and that pinning is still affecting the flux flow above the irreversibility line.

The resistive transition showed no substantial change in width over the entire temperature range ( $4.5 < T < 9$  K) in this study. This is in contrast to the observation of Orlando *et al.*<sup>31</sup> who reported a significant broadening in the resistive transition for some of their Nb<sub>3</sub>Sn films in the presence of high magnetic fields. This observation was suggested by Suenaga *et al.*<sup>14</sup> to indicate the existence of the reversible region, but is more likely to be due to material inhomogeneity in a complex material such as Nb<sub>3</sub>Sn.

A cylindrical NbTi rod sample (diameter=1.8 mm) was also measured in addition to the multifilamentary wire. This sample is an intermediate product in the manufacture of mul-

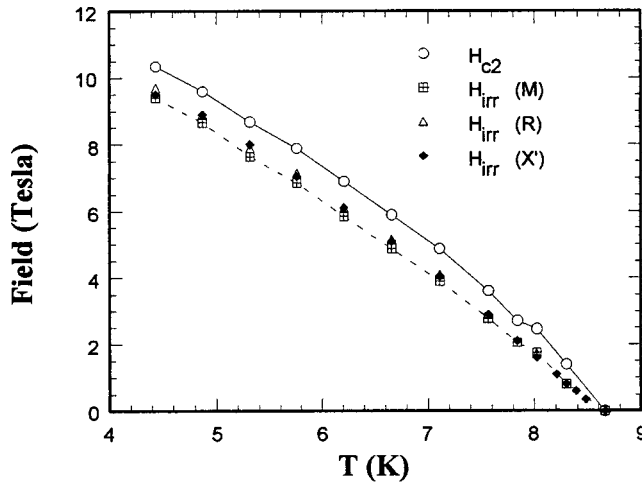


FIG. 6. Temperature dependence of  $B_{c2}$  (determined from the measured dc magnetization) and  $B_{irr}$  (determined from dc magnetization, ac susceptibility and resistivity) for a NbTi multifilamentary wire sample.

filamentary NbTi wires and hence has a similar composition to the multifilamentary specimen. Results show that the width of the reversible region  $B_{c2}-B_{irr}$  for this sample is very close to that of the wire at the same reduced temperature. This result appears to support the notion that the irreversibility line is a flux lattice melting line, since, in the creep model, complete flux penetration of the large bulk sample takes longer than for the fine filamentary samples. The irreversibility line is thus expected to be higher for the larger of the two specimens, which is in contrast to the above observation. However, the exponential creep rate means that the size effect will be difficult to detect, so that a creep model is not inconsistent with the data. As with the NbTa rod surface currents meant that the resistive transition did not coincide with the magnetic irreversibility line in a parallel field. No critical current was observed above  $B_{c2}$  when the field was applied perpendicular to the axis of the sample.

The temperature dependencies of  $B_{irr}$ , determined from dc magnetization, ac susceptibility, and magnetoresistivity measurements, and  $B_{c2}$ , determined from dc magnetization measurements, are plotted in Fig. 6 for the NbTi wire. The figure illustrates an observable region of reversible flux motion in the NbTi wires. The width of the reversible region, as measured by  $B_{c2}(T)-B_{irr}(T)$  or  $T_c(B)-T_{irr}(B)$ , is quantitatively similar to that reported by Suenaga *et al.*<sup>14</sup> Interestingly, the two curves of  $B_{irr}(T)$  and  $B_{c2}(T)$  are ‘‘parallel’’ to each other over almost the entire experimental temperature range, except in the vicinity of  $T_c$ . Suenaga *et al.* and Schmidt, Israeloff, and Goldman<sup>15</sup> found that measured irreversibility line data could be fitted well to the flux-line melting model of Houghton, Pelcovits, and Sudbø<sup>4</sup> for NbTi, Nb<sub>3</sub>Sn multifilamentary wires, and Nb thin films. These authors suggested that the irreversibility line in low- $T_c$  superconductors is essentially the flux lattice melting line on the basis of their observations. The melting theory also suggests that the melting line predicted by the calculation is essentially parallel to the superconducting-normal phase boundary  $B_{c2}(T)$  over a wide range of field, except in the range close to  $T_c$  where  $B_{irr}$  follows a  $(1-T/T_c)^2$  relation. This is in agreement with the data shown in Fig. 6. A fit of the data

given in Fig. 6 to the theory, however, is not very meaningful due to the relatively large experimental error in the data, particularly at temperatures close to  $T_c$ . Additionally, a linear temperature dependence of  $B_{c2}$  is assumed in the theory of Houghton, Pelcovits and Sudbø while the experimental data of  $B_{c2}$  in Fig. 6 show nonlinear behavior which could be better described by the empirical relation  $B_{c2}(T) = B_{c2}(0)[1 - (T/T_c)^2]$ .

Also as shown in Fig. 6, the  $B_{irr}$  curve tends to saturate at low temperatures, i.e., it has a slight negative curvature. Although this appears different from that seen in high- $T_c$  materials it is not entirely unexpected since on almost any model the irreversibility line cannot be far from the  $B_{c2}$  line in low- $T_c$  material.

It has been reported<sup>14,15</sup> that the flux creep effect increases very rapidly near  $B_{irr}$  or  $T_{irr}$  confirming the easy motion of the flux lines in this field or temperature range. In particular Schmidt, Israeloff, and Goldman<sup>15</sup> analyzed their flux creep data within the framework of flux-line depinning theory<sup>1</sup> and vortex glass transition theory<sup>2</sup> and found none of these theories could account for the experimental data consistently. In their study, flux creep measurements were performed at selected temperatures. No detectable decay in magnetization was observed within the resolution of the experiment when the applied field was significantly less than  $B_{irr}$ . Flux creep, however, became apparent as the field was increased to  $B_{irr}$  which indicates that flux lines may be moved relatively easily in this field region. Because of the large noise level compared with the variation of magnetization decay, it is difficult to say if the decay follows a  $\ln t$  relation as seen in previous reports.<sup>14,15</sup>

## V. DISCUSSION

Consider first the NbTa. The magnetization curve would detect a critical current density of 5 A/cm<sup>2</sup> compared with 10<sup>4</sup> A/cm<sup>2</sup> at half  $B_{c2}$ . Within 10% of  $B_{c2}$  we might expect  $J_c$  to be lower by a factor of about 100, so this is evidence of an irreversibility line. However, the most convincing evidence comes from the susceptibility measurements since the loss peak at the irreversibility line shows that there is a linear resistivity between this field and  $B_{c2}$ . The resistive measurements do not show an irreversibility line due to surface superconductivity and something that needs explaining is how a surface current which must be borne by pinned pancake vortices can carry a current so far above the irreversibility line of the bulk. It may be because the current is parallel to the field so there is little force on the pancakes. It is, however, interesting to note that a similar effect has been seen in BSCCO crystals which have been shown to carry surface currents in fields well above the irreversibility field, although still much smaller than  $B_{c2}$ .<sup>32</sup>

The very small size of the NbTi filaments make detecting an irreversibility line magnetically rather difficult. The loss peak was ill defined and  $J_c$  would need to be above 5000 A/cm<sup>2</sup> before the hysteresis would show up. However, the bulk sample showed similar behavior to NbTa and the irreversibility field found in it was similar to the field at which the filaments became diamagnetic. But the most convincing evidence here comes from the resistive measurements. Here the resistive transition coincided with the magnetic irrevers-

ibility line and seems to be the only example of a coincident resistive and magnetic irreversibility line in low- $T_c$  materials. This is different from the behavior of PbBi where plating the surface reduced the transition to  $B_{c2}$ .

We conclude therefore that there is strong evidence of a reversibility line in NbTi and NbTa at about 10% of  $T_c$  or  $B_{c2}$ . The cause is a matter for speculation. Since in NbTi the pinning is caused by a mass of extended defects we would expect a vortex liquid to be pinned as strongly as a solid. There is no significant anisotropy so decoupling cannot occur, although it is possible that flux cutting becomes easy so effectively splitting the vortices into short sections.<sup>33</sup>

The most natural explanation is that the energy wells are becoming comparable in depth with  $kT$ , which is consistent with the rapid flux creep near  $B_{irr}$ . Simple flux creep theory gives a resistivity  $B\omega d \exp(-U/kT)\sinh(J/J_c)$  where  $d$  is the

distance between centers and  $\omega$  is an attempt frequency. For the observed linear resistivity at low current densities  $U/kT \sim 5$  if  $\omega$  is the atomic vibration frequency. If  $U$  varies as  $B_c^2$  and  $B_c$  as  $1 - (T/T_c)^2$  then  $U/kT$  at half  $T_c$  must be about 200 which is a long way below the value of 9000 found by Beasley. However, the theory of flux creep is not straightforward and it is difficult to exclude this mechanism completely. It is clear that a good deal more work is required before a self-consistent picture can emerge.

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