Five-loop renormalization-group expansions for the three-dimensional n-vector cubic model and critical exponents for impure Ising systems

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The renormalization-group (RG) functions for the three-dimensional n-vector cubic model are calculated in the five-loop approximation. High-precision numerical estimates for the asymptotic critical exponents of the three-dimensional impure Ising systems are extracted from the five-loop RG series by means of the Padé-Borel-Leroy resummation under $n=0$. These exponents are found to be $\gamma=1.325\pm0.003$, $\eta=0.025\pm0.01$, $\nu=0.671\pm0.005$, $\alpha=-0.0125\pm0.008$, and $\beta=0.344\pm0.006$. For the correction-to-scaling exponent, the less accurate estimate $\omega=0.32\pm0.06$ is obtained.

I. INTRODUCTION

The critical thermodynamics of cubic crystals and weakly disordered systems has remained an area of extensive theoretical work during past decades. Considerable progress in studying the random critical behavior was achieved 25 years ago when Harris and Lubensky[1,2] and Khmelnitskii[3] attacked the problem using the field-theoretical renormalization-group (RG) approach based on the Euclidean scalar φ^4 theory in $(4-\epsilon)$ dimensions. As a result, the regular method for calculating critical exponents and other universal quantities of the impure Ising model—the famous $\sqrt{\epsilon}$ expansion—was invented. The numerical power of this technique, however, stayed for a long time unclear since only lower-order contributions to critical exponents and the equation of state have been found.[1–5] Recently, starting from the five-loop RG series obtained for the $(4-\epsilon)$-dimensional cubic model by Kleinert and Schulte-Frohlinde,[6] the calculation of the $\sqrt{\epsilon}$ expansions for critical exponents was performed up to the $\sqrt{\epsilon}^4$ and $\sqrt{\epsilon}^5$ terms.[7] As was found, these series possess a rather irregular structure making them unsuitable for subsequent resummation and, hence, practically useless for getting numerical estimates.[8]

On the other hand, there exists an alternative field-theoretical approach that proved to be very efficient when used for evaluation of universal critical quantities. We mean the perturbative renormalization group in three dimensions yielding most accurate numerical estimates for critical exponents, critical amplitude ratios, and universal higher-order couplings of the $O(n)$-symmetric systems.[9–20] The impure Ising model at criticality is known to be described by the n-vector field theory with the quartic self-interaction having a hypercubic symmetry, provided $n\to 0$ (the replica limit) and the coupling constants have proper signs. In the 1980s, the RG expansions for three-dimensional (3D) cubic and impure Ising models have been calculated in the two-loop,[21] three-loop,[22,23] and four-loop[24,25] approximations paving the

way for estimating the critical exponents and other universal quantities.[21–34] The numerical results thus obtained were found to agree, in general, with the most accurate experimental and simulation data.

In the course of this study, it was revealed, however, that even the highest-order available, four-loop 3D RG expansions, when resummed by means of the generalized Padé-Borel-Leroy method do not allow us, in fact, to optimize the resummation procedure, i.e., to choose the best Padé approximant and the optimal value of the tune parameter since there is the only approximant—[3/1]—that does not suffer from positive axis poles. Moreover, an account for four-loop terms in the 3D RG expansions shifts the fixed point coordinates and the value of the correction-to-scaling exponent ω appreciably with respect to their three-loop analogs, indicating that at this step the RG-based iterations do not still achieve their asymptote. This prevents the four-loop RG approximation from being thought of as sufficient, i.e., providing, within the perturbation theory, the accurate theoretical predictions.

In such a situation, a calculation of the higher-order contributions to the RG functions looks very desirable. In this paper, the five-loop RG expansions for the three-dimensional cubic model are obtained and the resulting numerical estimates for the critical exponents of the weakly disordered Ising systems are found.

II. RG EXPANSIONS FOR β FUNCTIONS AND CRITICAL EXPONENTS

The Landau-Wilson Hamiltonian of the three-dimensional n-vector cubic model reads

$$H = \frac{1}{2}\int d^3x\left[m_0^2\varphi_\alpha^2+(\nabla\varphi_\alpha)^2+\frac{u_0}{12}\varphi_\alpha^2\varphi_\beta^2+\frac{v_0}{12}\varphi_\alpha^4\right], \quad (1)$$

where φ is an n-component real order parameter, m_0^2 being the reduced deviation from the mean-field transition tem-

perature. In the replica limit, this Hamiltonian describes the critical behavior of the impure Ising model provided $u_0 < 0$ and $v_0 > 0$.

We calculate the β functions for the Hamiltonian Eq. (1) within a massive theory. The renormalized Green function $G_R(p,m)$, the φ^2 insertion, and four-point vertices $U_R(p_i,m,u,v,)$, $V_R(p_i,m,u,v,)$ are normalized at zero external momenta in a conventional way:

$$G_R^{-1}(0,m)=m^2, \quad \left.\frac{\partial G_R^{-1}(p,m)}{\partial p^2}\right|_{p^2=0}=1,$$

$$\Gamma_R^{(1,2)}(p,q,m,u,v)\big|_{p=q=0}=1,$$

$$U_R(0,m,u,v)=mu, \quad V_R(0,m,u,v)=mv. \tag{2}$$

The value of the one-loop vertex graph including the factor $(n+8)$ is absorbed in u and v in order to make the coefficient for the u^2 term in β_u equal to unity.

The four-loop RG expansions for the functions of interest have been found earlier.[24] To extend these series to the five-loop order, we calculate corresponding tensor (field) factors generated by the $O(n)$-symmetric and cubic interactions. Taking, then, numerical values of the 3D integrals from Ref. 35, we arrive at the following five-loop expansions:

$$\frac{\beta_u}{u}=1-u-\frac{6v}{(n+8)}+\frac{4}{27(n+8)^2}[(41n+190)u^2+300uv+69v^2]-\frac{1}{(n+8)^3}[(1.348\,942\,76n^2+54.940\,377\,0n$$

$$+199.640\,417)u^3+(19.940\,635\,0n+493.841\,548)u^2v+(1.865\,667\,61n+302.867\,786)uv^2+65.937\,285\,1v^3]$$

$$+\frac{1}{(n+8)^4}[(-0.155\,645\,891n^3+35.820\,203\,8n^2+602.521\,231n+1832.206\,73)u^4+(-4.058\,645\,97n^2$$

$$+546.221\,669n+6192.512\,10)u^3v+(81.751\,008\,6n+6331.226\,42)u^2v^2$$

$$+(11.619\,156\,5n+2777.394\,24)uv^3+495.005\,747v^4]$$

$$-\frac{1}{(n+8)^5}[(0.051\,236\,175\,5n^4+3.237\,876\,19n^3+668.554\,337n^2+7819.564\,76n+20\,770.176\,97)u^5$$

$$+(1.876\,564\,22n^3+65.418\,109\,9n^2+11\,485.347\,19n+89\,807.669\,84)u^4v+(21.050\,525\,8n^2+3858.044\,76n$$

$$+130\,340.905\,33)u^3v^2+(630.460\,362n+90\,437.636\,44)u^2v^3$$

$$+(79.535\,942\,1n+33\,088.222\,88)uv^4+5166.392\,01v^5], \tag{3}$$

$$\frac{\beta_v}{v}=1-\frac{(12u+9v)}{(n+8)}+\frac{4}{27(n+8)^2}[(23n+370)u^2+624uv+231v^2]-\frac{1}{(n+8)^3}[(-1.251\,107\,31n^2$$

$$+41.853\,902\,1n+469.333\,970)u^3+(2.239\,058\,86n+1228.605\,91)u^2v+957.781\,662uv^2$$

$$+255.929\,737v^3]+\frac{1}{(n+8)^4}[(0.574\,652\,520n^3-0.267\,107\,207n^2+584.287\,672n+5032.692\,26)u^4$$

$$+(0.172\,125\,857n^2+322.925\,039n+17\,967.850\,60)u^3v+(-49.482\,007\,8n$$

$$+21\,964.393\,81)u^2v^2+11\,856.956\,86uv^3+2470.392\,52v^4]-\frac{1}{(n+8)^5}[(-0.318\,104\,330n^4$$

$$-3.629\,821\,62n^3+139.264\,889n^2+9324.600\,54n+64\,749.281\,95)u^5+(-1.144\,541\,68n^3$$

$$-122.339\,901n^2+10\,376.558\,04n+294\,450.703\,68)u^4v+(12.614\,708\,9n^2$$

$$+233.955\,446n+493\,917.036\,78)u^3v^2+(-1363.287\,87n+407\,119.306\,75)u^2v^3+$$

$$170\,403.119\,05uv^4+29\,261.585\,18v^5], \tag{4}$$

$$\gamma^{-1}=1-\frac{(n+2)u+3v}{2(n+8)}+\frac{1}{(n+8)^2}[(n+2)u^2+6uv+3v^2]-\frac{1}{(n+8)^3}[(0.879\,558\,892n^2+6.485\,476\,86n$$

$$+9.452\,718\,16)u^3+(7.916\,030\,03n+42.537\,231\,7)u^2v+(1.155\,056\,03n$$

$$+49.298\,205\,7)uv^2+16.817\,753\,9v^3]+\frac{1}{(n+8)^4}[(-0.128\,332\,104n^3+7.966\,740\,70n^2+51.844\,213\,0n$$

$$+70.794\,806\,3)u^4+(-1.539\,985\,25n^2+98.680\,858\,9n+424.768\,838)u^3v+(30.815\,175\,5n$$

$$+752.049\,392)u^2v^2+(5.642\,961\,22n+516.266\,750)uv^3+130.477\,428v^4]$$

$$-\frac{1}{(n+8)^5}[(0.049\,096\,605\,5n^4+4.288\,152\,49n^3+108.361\,822n^2+537.813\,610n+675.699\,608)u^5$$

$$+(0.736\,449\,089n^3+62.849\,389\,3n^2+1499.728\,55n+5067.747\,06)u^4v+(9.542\,864\,09n^2+1059.534\,69n$$

$$+12\,193.045\,34)u^3v^2+(295.911\,053n+12\,966.211\,84)u^2v^3+(43.227\,584\,5n$$

$$+6587.833\,86)uv^4+1326.212\,29v^5],\tag{5}$$

$$\eta=\frac{8}{27(n+8)^2}[(n+2)u^2+6uv+3v^2]+\frac{1}{(n+8)^3}\left[(0.024\,684\,001\,4(n^2+10n+16)u^3\right.$$

$$+0.222\,156\,013(n+8)u^2v+1.999\,404\,12uv^2+0.666\,468\,039v^3]+\frac{1}{(n+8)^4}[(-0.004\,298\,562\,6n^3$$

$$+0.667\,985\,921n^2+4.609\,221\,06n+6.512\,109\,94)u^4+(-0.051\,582\,750\,7n^2+8.118\,996\,555n$$

$$+39.072\,659\,6)u^3v+(2.044\,846\,77n+68.665\,263\,4)u^2v^2+47.140\,073\,4uv^3$$

$$+11.785\,018\,4v^4]-\frac{1}{(n+8)^5}[(0.006\,550\,922\,14n^4-0.132\,451\,063n^3+1.891\,139\,27n^2+15.188\,093\,4n$$

$$+21.647\,206\,4)u^5+(0.098\,263\,832\,1n^3-2.183\,293\,61n^2+32.733\,676\,2n+162.354\,048)u^4v$$

$$+(-0.164\,297\,296n^2+4.145\,870\,79n+382.023\,815)u^3v^2+(-3.991\,318\,84n$$

$$+389.996\,707)u^2v^3+193.002\,694uv^4+38.600\,538\,9v^5].\tag{6}$$

These expansions will be used to evaluate the critical exponents of the impure Ising model.

III. RESUMMATION AND NUMERICAL ESTIMATES

Numerical values of critical exponents are known to be determined by the coordinates of a relevant fixed point. In our case, i.e., for $n=0$, a point of interest is the random fixed point. To find its location and, more generally, to extract the physical information from divergent RG series, a proper resummation procedure should be applied. Here, we use the Padé-Borel-Leroy resummation technique, which demonstrates high numerical effectiveness both for the $O(n)$-symmetric models[9,11,15] and for anisotropic systems preserving their internal symmetries (see, e.g., Ref. 36 for details). Since the expansions of quantities depending on the two variables u and v are dealt with, the Borel-Leroy transformation is taken in a generalized form:

$$f(u,v)=\sum_{ij} c_{ij}u^iv^j=\int_0^\infty e^{-t}t^bF(ut,vt)dt,$$

$$F(x,y)=\sum_{ij}\frac{c_{ij}x^iy^j}{(i+j+b)!}.\tag{7}$$

To perform an analytical continuation, the resolvent series

$$\tilde{F}(x,y,\lambda)=\sum_{n=0}^\infty \lambda^n\sum_{l=0}^n\frac{c_{l,n-l}x^ly^{n-l}}{(n+b)!}\tag{8}$$

is constructed, which is a series in powers of λ with coefficients being uniform polynomials in x, y and then Padé approximants $[L/M]$ in λ at $\lambda=1$ are used.

For the resummation of the five-loop RG expansions, we employ three different Padé approximants: $[4/1]$, $[3/2]$, and $[2/3]$. The first of them, being pole-free, is known to give good numerical results for 3D $O(n)$-symmetric models while the others are near-diagonal and should reveal, *a priori*, the best approximating properties. The coordinates of the random fixed point resulting from the series Eqs. (3) and (4) resummed using these approximants under $b=0$ and $b=1$ are presented in Table I, which also contains analogous estimates given by the four-loop RG expansions. The four-loop series were processed on the base of the Padé approximant $[3/1]$, since use of the diagonal approximant $[2/2]$ leads to the integrand in Eq. (7) that has a dangerous pole in the vicinity of the random fixed point both for β_u and β_v.[37] The fixed-point location given by the approximant $[2/3]$ is presented for $b=0$ only, because for $b=1$ this approximation predicts no random fixed point.

As seen from Table I, Padé approximants $[4/1]$ and $[3/2]$ yield numerical values of the random fixed-point coordinates, which are remarkably close to each other. Moreover, for $b=0$, they are also close to those given by the approximant $[3/1]$: the largest difference between the five-loop and four-loop estimates does not exceed 0.026. With increasing b, corresponding numbers diverge, indicating that $b=0$ is an optimal value of the tune parameter. On the contrary, Padé approximant $[2/3]$ gives the random fixed-point location,

TABLE I. Numerical estimates for the random fixed-point location and correction-to-scaling exponent ω obtained from the five-loop RG expansions (3,4) resummed by the Padé-Borel-Leroy technique using approximants [4/1], [3/2], and [2/3]. The last column contains results given by the four-loop RG series processed on the base of the approximant [3/1]. The superscript "c" denotes that the exponent ω is complex and its real part is presented. The superscript "p" stands to mark that the Padé approximant has a "nondangerous" positive axis pole, i.e., a pole well remoted from the origin ($t > 40$) that affects neither the procedure of numerical evaluation nor the value itself of the Borel integral.

	b	[4/1]	[3/2]	[2/3]	[3/1]
u_c	0	-0.7200	-0.7148	-0.6871	-0.6991
	1	-0.7445	-0.7385^p		-0.6839
v_c	0	2.0182	2.0125	2.0571	1.9922
	1	2.0296	2.0236^p		1.9877
ω	0	0.266	0.303	0.462^c	0.376
	1	0.263	0.325^p		0.361

which deviates appreciably from those predicted by approximants [4/1], [3/2], and [3/1]. This approximant, however, is found to lead to poor numerical results even for simpler systems, e.g., for the Ising model. Indeed, when used to evaluate the coordinate of the Ising fixed point, it results in $v_c = 1.475$ (under $b = 0$) while the best estimate today is known to be $v_c = 1.411$.[14] This forces us to reject the data obtained on the base of the approximant [2/3].

So, to determine the random fixed-point coordinates, we have to average the numerical data given by three working Padé approximants at $b = 0$. This procedure yields the values

$$u_c = -0.711, \quad v_c = 2.008, \tag{9}$$

which are claimed to be the final results of our search of the random fixed-point location. Let us estimate their accuracy. It seems unlikely that the deviations of these numbers from the exact ones would exceed the differences between them and the four-loop estimates since, among all proper estimates, the four-loop ones most strongly differ from the averaged values (9). Hence, the error bounds for u_c and v_c are believed to be not greater than ± 0.012 and ± 0.016, respec-

tively. Another way to estimate an apparent accuracy is to trace how the averaged values of the random fixed-point coordinates vary with the variation of b. We calculate u_c and v_c using the pole-free approximants [4/1] and [3/1] for $0 \leqslant b \leqslant 15$; $b = 15$ is chosen as a largest reasonable value of the tune parameter, since for greater b the saturation of dependences of various quantities on b becomes visible. Running through this interval, the averaged coordinates change their values by about 0.02 indicating that an accuracy of the estimates (9) is of the order of 0.01–0.02, in accord with that found above.

With the numbers (9) in hand, we can evaluate the critical exponents for the 3D impure Ising model. The exponent γ is estimated by the Padé-Borel-Leroy resummation of the RG series (5) for γ^{-1} and of the inverse series, i.e., the RG expansion for γ. The Fisher exponent is also evaluated in two different ways: via the estimation of the critical exponent $\eta_2 = (2 - \eta)(\gamma^{-1} - 1)$ having the RG expansion, which exhibits a good summability, and by direct substitution of the fixed point coordinates into the series (6) with rapidly diminishing coefficients. The estimates for η originating from η_2 were obtained under the central value of γ: $\gamma = 1.325$. The numerical results thus found are collected in Table II.

As is seen from this table, two methods of evaluating the susceptibility exponent γ lead to remarkably close numerical results, which very weakly depend on the tune parameter. Indeed, with increasing b from 0 to 15, the estimates for γ obtained by the resummation of the RG series for γ and γ^{-1} on the base of the pole-free approximant [4/1] vary by less than 0.0036, while the difference between them never exceeds 0.0013. Under the same variation of b, the value of γ averaged over these two most reliable approximations remains within the segment [1.3240, 1.3266]. On the other hand, the accuracy of determination of the critical exponents depends not only on a quality of the resummation procedure but also on the accuracy achieved in the course of locating of the relevant fixed point. That is why we investigated to what extent the numerical estimates for γ vary when coordinates of the random fixed point run through their error bars. It was found that the susceptibility exponent calculated at the optimal value of tune parameter $b = 2$ (see Table II) does not leave the segment [1.3228, 1.3263]. This enables us to con-

TABLE II. Numerical estimates for the critical exponents γ and η obtained from the five-loop RG expansions (5,6) resummed by the Padé-Borel-Leroy technique using approximants [4/1] and [3/2]. DS stands for "direct summation," the symbol $(\gamma^{-1})^{-1}$ means that the RG series for γ^{-1} was resummed. The superscript "p" denotes, as in Table I, that the Padé approximant has a "nondangerous" pole, while empty cells are due to the dangerous ones spoiling corresponding approximations. The estimates for η standing in the fifth and sixth lines were obtained under $\gamma = 1.325$ by the resummation of the RG series for η_2.

b		0	1	2	3	4	5	10	15
$(\gamma^{-1})^{-1}$	[4/1]	1.323 6	1.324 4	1.325 0	1.325 4	1.325 7	1.326 0	1.326 8	1.327 2
	[3/2]				$1.325\,3^p$	$1.325\,7^p$	1.326 0	1.326 5	1.326 7
γ	[4/1]	1.324 5	1.324 8	1.325 0	1.325 2	1.325 3	1.325 4	1.325 7	1.325 9
	[3/2]	$1.324\,6^p$	$1.325\,1^p$	$1.325\,4^p$	$1.325\,7^p$	$1.325\,9^p$	$1.326\,1^p$	$1.326\,7^p$	$1.327\,0^p$
η (via η_2)	[4/1]	0.031 22	0.027 65	0.025 06	0.023 11	0.021 58	0.020 36	0.016 65	0.014 78
	[3/2]				$0.028\,70^p$	$0.024\,19^p$	$0.021\,68^p$	$0.016\,66^p$	0.014 87
η (DS)					0.027 2				

clude that the error bounds for the theoretical value of γ obtained in this work would be about ± 0.003 or smaller.

Less stable numerical results, with respect to a variation of b, are found for the Fisher exponent η. As one can see from Table II, the values of η given by the resummation of the RG series for η_2 with use of the pole-free Padé approximant [4/1] spread from 0.0148 to 0.0312. The average over this interval is equal to 0.023. The direct summation of the RG expansion for η at the random fixed point gives 0.027. It is natural to suppose that 0.025 should play a role of the most likely value of exponent η. Since the estimates for η found via the evaluation of η_2 are sensitive to the accepted value of γ, the apparent accuracy achieved in this case is not believed to be better than ± 0.01.

Having estimated γ and η, we can evaluate other critical exponents using well-known scaling relations. So, the final results of our five-loop RG analysis are as follows:

$$\gamma = 1.325 \pm 0.003, \quad \eta = 0.025 \pm 0.01,$$

$$\nu = 0.671 \pm 0.005,$$

$$\alpha = -0.0125 \pm 0.008, \quad \beta = 0.344 \pm 0.006. \tag{10}$$

These numbers are thought to be the most accurate theoretical estimates for the critical exponents of the 3D impure Ising model known today. It is interesting to compare them with those obtained earlier within the lower-order RG approximations. For the exponent γ, previous RG calculations in three dimensions gave the values 1.337 (two-loop),[21,27] 1.328 (three-loop),[23] 1.326 (four-loop),[24] and 1.321 (four-loop).[25] Being found by means of the different resummation procedures, they are, nevertheless, centered around our estimate, which is thus argued to be very close to the exact value of γ or, more precisely, to the true asymptote of the RG iterations.

At the end of this work, we employ our technique to evaluate the correction-to-scaling exponent ω. The exponent ω is known to be equal to the stability matrix eigenvalue that has a minimal modulus. The derivatives $\partial \beta_u / \partial u$, $\partial \beta_u / \partial v$, $\partial \beta_v / \partial u$, and $\partial \beta_v / \partial v$ entering this matrix are evaluated numerically at the random fixed point on the base of the resummed RG expansions for β_u and β_v, and then the matrix eigenvalues are found. Such a procedure leads to the estimates for ω presented in Table I (lower lines). They are seen to be considerably scattered and sensitive to the tune parameter. The average over three working Padé approximants, however, being equal to 0.315 at $b=0$ and to 0.316 at $b=1$ turns out to be stable under the variation of b unless b becomes large. It is natural therefore to accept that

$$\omega = 0.32 \pm 0.06. \tag{11}$$

This number is smaller by 0.05–0.07 than its counterparts given by recent Monte Carlo simulations[38] and the alterna-

tive RG analysis,[33] but their central values lie within the declared error bounds (11). Hence, an agreement between the results discussed exists. On the other hand, because of its low accuracy, the estimate (11) would not be thought of as satisfactory. It certainly needs to be improved, along with the estimates for the small exponents η and α also exhibiting appreciable uncertainties. The only way to do this is the studying of the critical behavior of the 3D impure Ising model in the next perturbative order that includes calculations of the six-loop terms in the relevant RG expansions. Such calculations are now in progress.

IV. CONCLUSION

To summarize, we have calculated the five-loop RG expansions for the β functions and critical exponents of the 3D n-vector cubic model. The resummation of the RG series by the Padé-Borel-Leroy technique in the replica limit ($n=0$) has enabled us to obtain high-precision numerical estimates for the random fixed-point coordinates and the critical exponents γ, η, α, ν, and β of the 3D impure Ising model. For the correction-to-scaling exponent ω, the resummed five-loop RG expansions turned out to give much less accurate numerical results. The values of the critical exponents obtained earlier from the lower-order RG expansions in three dimensions are shown to be centered by our five-loop estimates. This indicates that the five-loop RG approximation provides numerical data very close to the asymptotic ones, i.e., to those representing the point of convergence of the RG-based iterations. At the same time, an accuracy achieved when evaluating the exponent ω would not be referred to as sufficient making the next-order, six-loop RG calculations, very desirable.

Note added. After this paper had been submitted for publication, Ref. 39 appeared where the six-loop RG expansions for the 3D cubic model are calculated. The five-loop terms obtained in this remarkable work are found to agree with ours. Among other quantities, in Ref. 39 the marginal value of n, n_c, separating different regimes of critical behavior of the cubic model, is estimated. To clear up how sensitive to the method of resummation the five-loop RG results are, we evaluate n_c using the Padé-Borel-Leroy technique. Exploiting the near-diagonal Padé approximant [3/2] under b varying from 0 to 20, the values of n_c are obtained which lie between 2.89 and 2.92. They agree quite well with the estimate $n_c = 2.91(3)$ extracted from the five-loop RG series by means of the conformal-mapping-based resummation machinery.[39]

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