Vortex-boson analogy and the nonlinear response function in high-T_c superconductors near the phase transition

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The physics of flux lines in a high-T_c superconductor pinned by strongly correlated defects can be mapped onto charged bosons localized in two dimensions (2D). Considering the viscous dissipation of moving vortices, we derive a nonlinear response function. This function is compatible with so far suggested different model barriers $U(J)$ and able to make a consistent description of the vortex system near transition. A comparison with the scaling behavior of the measured isothermal current-voltage curves with $YBa_2Cu_3O_{7-\delta}$ (YBCO) samples shows fair agreement. This nonlinear response function also shows an empirical fit with I-V behavior of some 2D charge systems.

I. INTRODUCTION

The static and dynamic response of the flux in the high-temperature superconductors has been the subject of numerous recent experimental and theoretical investigations.[1] One of the most striking phenomena is the solid-liquid transition in the vortex system. The analysis of current-voltage (I-V) characteristics measured in twinned $YBa_2Cu_3O_{7-\delta}$ (YBCO) (Refs. 2 and 3) with the magnetic field applied in the c direction, in terms of the vortex-glass scaling theory[4] provide impressive evidence of a second-order phase transition.

However, Nelson and Vinokur raised the question, whether these transitions are really caused by uncorrelated ''point'' disorder as assumed in the original vortex-glass phenomenology,[4] because twin boundary may offer much stronger pinning.[5] The vortex dynamics with strongly correlated pinning can be studied efficiently by exploiting the mapping between vortices and two-dimensional (2D) bosons.[6] Similar to the physics of flux lines in a pure system,[7] the statistical mechanics of vortices interacting with columnar pinning centers that are aligned parallel to the magnetic field may be mapped into the quantum mechanics of charged bosons in two dimensions. Table I summarizes the analogy between the vortices system, with the tilt modules $\tilde{\varepsilon}_1$ and thickness L (length of vortex), and the corresponding 2D charged bosons system.[8] This new low-temperature glassy phase stabilized by correlated defects is called Bose glass. For fields parallel to c axis, the Bose-glass theory[5] predicts for the I-V characteristics with similar critical exponent relation as those given by the vortex-glass theory. But when the magnetic field is rotated off the c axis the Bose-glass theory predicts a critical state with a different universality class. This has been experimentally observed recently by the measurements of the electrical transport properties of twinned YBCO crystals.[9]

Another attempt to explain the experimental data of Koch *et al.* in Ref. 2 has been made by Coppersmith *et al.*[10] in terms of the flux-creep-flow model.[11] Though this model reproduces the qualitative features of the data, it fails to give a quantitative fit. The exponent values needed to collapse the I-V curves in the model of Coppersmith *et al.* are $z=13.5$ and $\nu=0.6$ (Ref. 10) in contrast to $z=4.8$ and $\nu=1.7$ from the experimental data of Ref. 2. Thus, this reinterpretation has been reasonably questioned.[12] However, Wen *et al.* reported later the exponent values $z=14$ and $\nu=0.7$ from the scaling of I-V isotherms of $YBa_2Cu_3O_{7-\delta}$ thin films.[13]

This paper is organized as follows. In the subsequent section we introduce a common nonlinear response function starting from the Bose glass $U(J)$. This function is also shown in connection with the Anderson-Kim model and the stationary solution of Brownian motion in a periodic potential. In Sec. III, our equation is compared with the widely quoted experimental results observed by Koch *et al.*[12] In view of the analogy of Table I, we discuss the nonlinear transport of some 2D charge systems in Sec. IV. Finally, a short summary concludes this work.

II. NONLINEAR RESPONSE FUNCTION

For the macroscopic description of the mixed state in type-II superconductors we have to deal with the Maxwell equations combined with the materials equation $J(E,B)$ describing the electromagnetic response of superconductors.

Bardeen and Stephen studied the motion of vortices, when they are not subjected to pinning force.[14] In the case of ideal type-II superconductors the materials can be characterized by the relation

$$E=\rho_f(B,T)J, \tag{1}$$

TABLE I. Boson analogy applied to vortex transport.

Charged bosons	Mass	$\hbar$	$\hbar/T$	Pair potential	Charge	Electric field	Current
Vortices	$\tilde{\varepsilon}_1$	T	L	$2\varepsilon_0 K_0(r/\lambda)$	ϕ_0	$\vec{z}\times\vec{J}/c$	$E(J)$

where the flux-flow resistivity $\rho_f \approx \rho_n(B/B_{c2})$. On the other hand, in a nonideal superconductor with considerable pinning the material is described by set of equations,

$$E = B \times v,$$

$$v = v_0 e^{-U(J)/kT},$$

or

$$E(J) = J\rho_f e^{-U(J)/kT}, \tag{2}$$

where the activation barrier U additionally depends on the temperature T and magnetic field B. Different types of $U(J)$ have been suggested to approximate the real barrier, for instance, the Anderson-Kim model[11] with $U(J) = U_c(1 - J/J_c)$, the logarithmic barrier $U(J) = U_c \ln(J_c/J)$ (Ref. 15) and the inverse power law with $U(J) = U_c[(J_c/J)^{\mu} - 1]$.[4,5]

In view of Eq. (1), for the steady state of flux motion in nonideal type-II superconductor the mean transport current density J can be phenomenologically expressed as

$$J = J_p + J_f \tag{3}$$

with

$$J_f \equiv E(J)/\rho_f \tag{4}$$

the component due to the moving vortices of uniform density. J_p is the contribution from the pinned vortices with nonuniform distribution.

We find, if one makes a common modification to the different model barriers $U(J)$ as

$$U(J) \rightarrow U(J_p \equiv J - E/\rho_f), \tag{5}$$

the corresponding modified materials equation

$$E(J) = J\rho_f e^{-U(J_p)/kT} \tag{6}$$

leads to a common normalized form as

$$y = x \exp[-\gamma(1+y-x)^p] \tag{7}$$

with x and y the normalized current density and electric field, respectively. γ is a parameter characterizing the symmetry breaking of the pinned vortices system and p is an exponent.

For an example, in following we show the derivation of the unified materials equation in connection with the inverse power-law model $U(J)$.[4,5,16] We start from the expression widely used for the highly nonlinear $E(J)$ characteristics of Bose-glass phases,[5,8]

$$E(J) = \rho_f J \exp\left[-\left(\frac{E_k}{kT}\right)\left(\frac{J_0}{J}\right)^p\right], \tag{8}$$

where E_k is a typical vortex kink energy, J_0 sets the current scale, and p is a glass exponent. Equation (8) can also be expressed as

$$E(J) = \rho_f J \exp\left[-\left(\frac{E_k}{kT}\right)\left(\frac{2R^*(J)}{d}\right)\right], \tag{9}$$

with d the average distance between strongly correlated defects, e.g., columnar damage tracks, twin boundaries, etc. $R^*(J)$ is the typical hopping range of vortex at the current density J. Since the hopping range can not exceed the sample size L, Eqs. (8) and (9) can be used in the range $J > J_L$ with

$$R^*(J_L) \equiv L. \tag{10}$$

The vortex kink energy E_k relates to the tilt modules $\tilde{\varepsilon}_1$ and the average pinning potential U_0 as

$$E_k = d\sqrt{\tilde{\varepsilon}_1 U_0}. \tag{11}$$

Substituting $J_p \equiv J - E(J)/\rho_f$ for the current density J in the brackets on the right-hand sides of Eqs. (8) and (9) and taking its logarithm we get

$$J - J_f = \left(\frac{E_k}{kT}\right)^{1/p} J_0\left[\ln\left(\frac{J}{J_f}\right)\right]^{-(1/p)}$$
$$= \left(\frac{E_k}{kT}\right)^{1/p} J_0(1+\eta)^{-1}\left[\ln\left(\frac{J_L}{J_{Lf}}\right)\right]^{-(1/p)}, \tag{12}$$

where $J_{Lf} \equiv E(J_L)/\rho_f$, which is much smaller than J_L, and

$$\eta \equiv \frac{-[\ln(J_L/J_{Lf})^{1/p} - \ln(J/J_f)^{1/p}]}{\ln(J_L/J_{Lf})^{1/p}}, \quad |\eta| < 1. \tag{13}$$

Using the approximation $(1+h)^{-1} \approx 1 - h$ for $|h| < 1$, finally we find Eq. (12) in the form

$$x - y = 1 - \ln(x/y)^{1/p}\gamma^{-(1/p)}, \tag{14}$$

which is exactly the general normalized form of the materials equation Eq. (7), here we have with

$$\gamma \equiv 2^p \ln\frac{J_L}{J_{Lf}} = 2^p\left(\frac{E_k}{kT}\right)\left(\frac{J_0}{J_L - J_{Lf}}\right)^p \approx 2^p\left(\frac{E_k}{kT}\right)\left(\frac{J_0}{J_L}\right)^p,$$

$$x \equiv \frac{1}{2}\left(\frac{E_k}{kT}\right)^{-(1/p)}\left(\ln\frac{J_L}{J_{Lf}}\right)^{1/p}\left(\frac{J}{J_0}\right) = \frac{1}{2}\left(\frac{J}{J_L - J_{Lf}}\right) \approx \frac{J}{2J_L},$$

$$y \equiv \frac{1}{2}\left(\frac{E_k}{kT}\right)^{-(1/p)}\left(\ln\frac{J_L}{J_{Lf}}\right)^{1/p}\left(\frac{E(J)}{J_0\rho_f}\right) = \frac{1}{2}\left(\frac{E(J)}{(J_L - J_{Lf})\rho_f}\right)$$
$$\approx \frac{E(J)}{2J_L\rho_f}. \tag{15}$$

In an earlier work this materials equation for type-II superconductors has been shown in connection with the Anderson-Kim model as[17]

$$E(J) = 2v_0 B \exp[(-U_c - W_\nu)/kT]\sinh(W_L/kT)$$

or

$$E(J) = J\rho_f \exp[(-U_c - W_\nu + W_L)/kT], \tag{16}$$

where v_0 is a prefactor with dimension of velocity and U_c is the pinning potential, $W_\nu = \eta v A = E(J) \times BA/\rho_f$ is the viscous dissipation term of flux motion with viscosity coefficient $\eta = B \times B_{c2}/\rho_n = B^2/\rho_f$,[14] W_L the energy due to Lorentz driving force, $W_L = J \times B \times A$, the parameter A is a product of the volume of the moving flux bundle and the range of the force action.

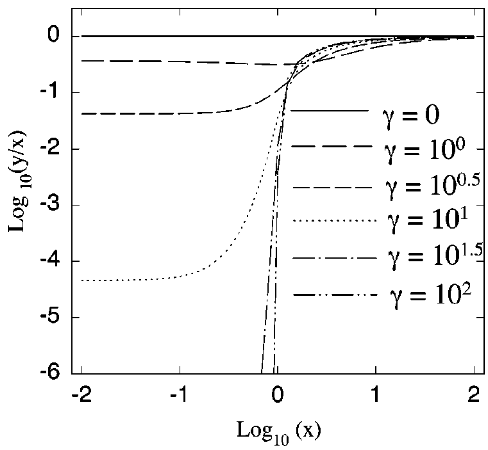

FIG. 1. Numerical solution of Eq. (7) for $p=1$.

Equation (16) can be expressed with a general reduced form

$$y=xe^{-\gamma(1+y-x)}, \tag{17}$$

where $\gamma \equiv U_c/kT$, $x \equiv W_L/U_c$, and $y \equiv W_\nu/U_c$.

A numerical solution of Eq. (7) with $p=1$ is shown in Fig. 1, which unifies consistently three regimes of flux motion, i.e., the Anderson-Kim regime, the critical state, and the flux flow.

It is also interesting to note that the materials equation (16) has the same form as the stationary solution of Brownian motion in a periodic potential, which is a typical example of nonlinear stochastic equations. The one-dimensional equation of motion for this problem has the stationary solution in the form of integrals[18] that can be reformulated by using the mean-value theorem in the form

$$\langle v\rangle = 2v_0 \exp\left[\frac{-2\varepsilon_0-\eta^*\langle v\rangle l^*}{\Theta}\right]\sinh[Fl^*/\Theta] \tag{18}$$

with the velocity prefactor v_0 and the effective mean-value barrier crossing free path l^* and friction η^* (see Ref. 19).

Equation (7) gives us a relation between the parameter γ and the slope

$$S\equiv\frac{d\ln y}{d\ln x}=\frac{1+p\gamma x(1+y-x)^{p-1}}{1+p\gamma y(1+y-x)^{p-1}}. \tag{19}$$

The maximal slope S_{max} occurs at the inflection point (x_i, y_i) of isotherm $\ln y \sim \ln x$, where

$$p^2\gamma^2 x_i y_i (1+y_i-x_i)^{2p-1}=1-p(x_i-y_i), \tag{20}$$

and we have the power law $V \propto I^{S_{max}}$.

From Eq. (19) and Eq. (20) we get

$$S_{max}=\frac{1+\gamma x_i\zeta}{1+\gamma y_i\zeta}\approx\left(\frac{x_i}{y_i}\right)^{1/2} \tag{21}$$

and

$$\gamma^2 x_i y_i = 1/\zeta^2, \tag{22}$$

where $\zeta^2 \equiv p^2(1+y_i-x_i)^{2p-1}/[1-p(x_i-y_i)]\approx 1$. In view of the numerical solution of Eq. (7) for $p=1$ as shown in Fig. 1, one finds $x_i \approx 1$. Thus from Eq. (21) and Eq. (22), we have approximately the relation

$$\gamma \approx 2S_{max}^2 \ln S_{max}. \tag{23}$$

III. SCALING BEHAVIOR OF ISOTHERMAL $E(J)$ CURVES

Now we compare our Eq. (7) with the scaling behavior of the experimental measured isothermal $E(J)$ curves obtained by Koch *et al.* with YBCO samples.[2,12] At different temperatures and magnetic fields, they found that for each field at a single well defined temperature T, the I-V curves shows a power-law behavior $V \propto I^S$. This temperature is defined as T_g. All the iostherms can be collapsed onto two scaling functions, for $T>T_g$ and $T<T_g$ correspondingly, by plotting V/I scaled by $|T-T_g|^{\nu(z-1)}$ vs I scaled by $|T-T_g|^{2\nu}$, where ν is the exponent of the coherence length ξ, $\xi \sim |T-T_g|^{-\nu}$, and z is the dynamical exponent of the coherent time ξ^z. Based on their experimental data they found $\nu=1.7$ and $z=4.8$ for $B=2$, 3, and 4 T. The slop $S\equiv(d\ln V)/(d\ln I)$ at $T=T_g$ is reported of the value 2.9 ± 0.3 in all their measurements. Since $E_k=d\sqrt{\tilde{\varepsilon}_1 U_0}$, one may reasonably assume $E_k(T)\propto(T^*-T)^\delta$, $J_L(T)\propto(T^*-T)^\alpha$, and $\rho_f \propto T$ with T^* being the irreversibility temperature where tilt modules $\tilde{\varepsilon}_1$ vanishes, so according to Eq. (15) we expect

$$\gamma(T)=\gamma_0(T^*-T)^{\delta-\alpha p}/kT, \tag{24}$$

$$I\propto xJ_L(T)\propto x(T^*-T)^\alpha, \tag{25}$$

$$V\propto yJ_L(T)\rho_f \propto y(T^*-T)^\alpha T. \tag{26}$$

In accordance with the observed $S_{max}\approx 2.5$ for the case of $B=4T$ (Ref. 2), one may expect $\gamma(T_g)\approx 11.5$. Assuming $\delta=2.5$, $\alpha=3$, and $p=0.6$, we get from Eq. (7) more than 100 $E(J)$ isotherms near the $T_g \approx 78$ K (as observed in Refs. 2 and 12). All the isotherms collapsed nicely onto two curves ($T>T_g$ and $T<T_g$), consistent with the scaling of $\nu=1.7$, $z=4.8$, as shown in Fig. 2. The similar scaling result of Refs. 2 and 12 is shown in Fig. 2(b) with open circles, which has the same scaling exponents of $\nu=1.7$ and $z=4.8$.

This rather quantitative agreement between our nonlinear equation (7) and the widely quoted pertinent experimental results of Koch *et al.* in wide temperature range shows that this equation may provide an advantageous basis for describing nonlinear electromagnetic phenomena in type-II superconductors.

IV. DISCUSSION

In view of the flux-charge analogy shown in Table I, a natural conjecture is whether there exists some similar equations in 2D charge systems. Recently, Grayson *et al.* reported a systematic study of the current versus voltage (I-V) relation when tunneling into the fractional quantum Hall effect (FQHE) sample at different values of B over a continuum of filling factors ν from 1/4 to 1.[20] The series of log-log I-V

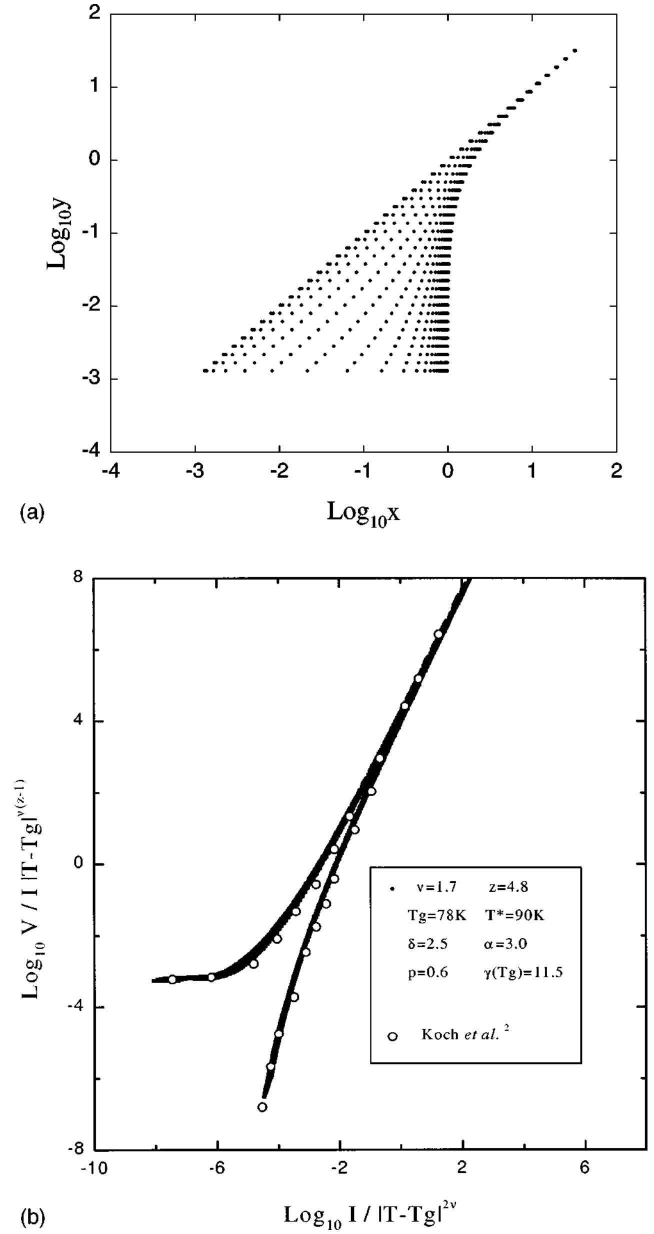

FIG. 2. (a) The *I-V* curves derived from Eq. (7). (b) ●: The collapsed data derived from Eq. (7), with $\nu=1.7$, $z=4.8$, $\delta=2.5$, $\alpha=3$, $p=0.6$, $T_g\approx 78$ K (as observed in Ref. 2) and $\gamma(T_g)\approx 11.5$. More than 100 curves are plotted for $T>T_g$ and $T<T_g$. ○: The original experimental result of Ref. 2.

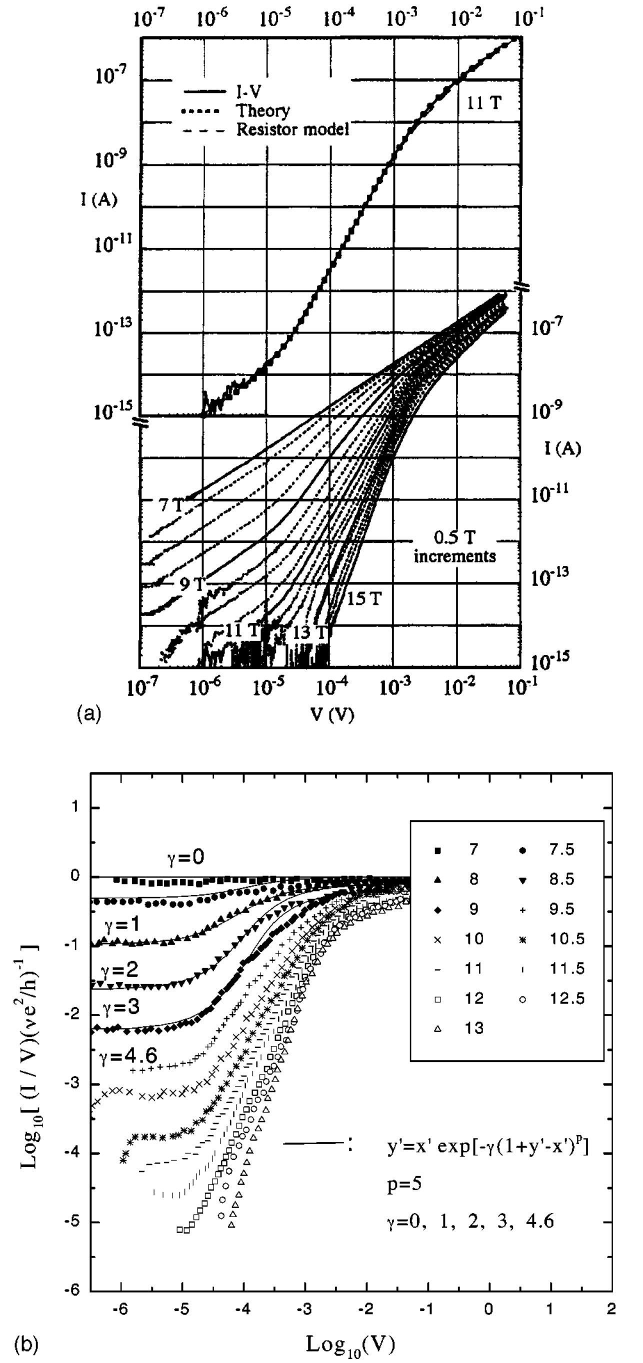

FIG. 3. (a) *I-V* response of FQHE (Ref. 20). (b) $\log[(I/V)(\nu e^2/h)^{-1}]$-$\log V$ plot of data in Fig. 3(a). $y'=(eIR_{xy})/(kT_s)$, $x'=(eV)/(kT_s)$.

curves over the whole range of B field manifest a power-law region in the middle of the curve whereas they soften to linear behavior at lower and higher bias voltages. In Fig. 3 we show these data in $\log[(I/V)(\nu e^2/h)^{-1}]$-$\log V$ plot analogous with Fig. 1 and try to compare with the phenomenological nonlinear response function $y'=x'\exp[-\gamma(1+y'-x')^p]$. In the B field range from 7.0 to 9.0 T, where $R_{xy}\propto B$ we see a rather fair agreement between the empirical nonlinear response function

$$I=\frac{V}{R_{Xy}}\exp\left[-\frac{T_s}{T}\left(1+\frac{eIR_{xy}}{kT_s}-\frac{eV}{kT_s}\right)^5\right] \qquad (27)$$

and the measured *I-V* data of Ref. 20. At B values due to the fractional Hall plateaus in R_{xy}-B plot (for example, $B=11.0$ T, $\nu=1/3$), the theoretical *I-V* relationship of Chamon and Fradkin for the problem of tunneling between a chiral Fermi liquid and a chiral Luttinger liquid[21] fits the experimental data with remarkable precision.

It is important to note, though the FQHE sample is a 2D charge system, there are still essential differences between the physical condition of the experimental of Grayson *et al.* and Table I, for instance, the high magnetic field in the former, so Fig. 3 is only an empirical fit at present.

V. SUMMARY

We suggest common nonlinear response function that can well describe the electromagnetic response of high-temperature superconductors with inhomogenities or defects as pinning centers. This function is compatible with so far suggested different model barriers $U(J)$ and is able to make a consistent description of the vortex system near transition. A comparison with the scaling behavior of the experimental measured isothermal current-voltage curves with YBCO samples shows fair agreement. In view of the flux-charge analogy, this nonlinear response function may also be applied to the transport of 2D charge system.

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