Influence of divergent electric fields on space-charge distribution measurements by elastic methods

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Elastic methods are presently being used to study the space charge distributions in insulators. They are limited in their interpretation to simple geometries whereas real systems often involve complex structures such as the case for divergent electric field regions. The authors propose a general theory for the interpretation of the data obtained by the pressure-wave-propagation method and the electro-acoustic method. This model makes it possible to study crystals, isotropic solids, or fluid samples of any geometry, containing charge and dipole distributions and also submitted to divergent electric fields such as produced by treeing or defects. A comparison is made with existing models in the case of simple geometries, such as planar or coaxial. It is shown that classical results are justified in the case of a planar sample. However, a correction has to be introduced in more complex geometries, even in coaxial ones. Indeed, some experiments show the associated difference. The model also emphasizes the similarities of the two elastic methods of measurement both for the analytical and experimental point of view.

I. INTRODUCTION

The measurement of space charge distributions in insulators makes it possible to observe the real behavior of electrically stressed insulators. This has many consequences both for the validation of theoretical models and for the design of high-voltage structures. Many studies[1–11] have already been carried out on different systems.

In most cases, breakdown phenomena arise from the buildup of large electric fields such as found in regions where the field is highly divergent. Such regions are, for instance, associated with water trees or defects.[12] Though the measurement methods were initially proposed for planar or coaxial geometries, they can also be applied to divergent electric field geometries[13–17] but the signal analysis is more complex. This is true with thermal, pressure-wave-propagation, and electro-acoustic methods, which are extensively used to give information on the space charge repartitions in insulating samples. In the case of thermal and pressure-wave-propagation methods a local motion of the insulator, produced by a thermal diffusion or an elastic wave, induces a variation of charges on adjacent electrodes connected through a low impedance measuring circuit. This variation is associated with a current that is the measured signal. In the case of the electro-acoustic method a variation of the applied voltage produces a variation of the force acting on the charges. This variation of force initiates an elastic wave that is transformed by an adjacent transducer into the measured signal. In all cases the measured signal depends on the geometry of the system, which may lead to large uncertainties on the shape and on amplitudes of the charge distribution.

The aim of this paper is to propose an analytical model, taking into account the sample geometry, applicable both to the pressure-wave-propagation method or the electro-acoustic method. Hence, knowing the sample geometry it is possible to link the measured signal to the amplitude and shape of the charge distribution in the insulator with an improved accuracy. The equation proposed for the pressure-wave-propagation method can also be applied with minor modifications to thermal methods.

In the next section we define the notations. Section III is devoted to the pressure-wave-propagation method. This method is briefly recalled in the case of a planar geometry. Then the variation of the charge on the electrodes is analyzed for given deformations and charge and dipole distributions. In Sec. IV the electro-acoustic method is briefly exposed. The force density in the material, which is the source of the elastic waves, is studied. In Sec. V, the models built for the analysis of the two former methods are applied to simple geometries, such as planar or coaxial ones, and the signals obtained in samples with diverging field are shown.

II. NOTATIONS

The sample, of any geometry, is made with an insulator of dielectric constant ε and two electrodes, 1 and 2, the first enclosing the other, at least at infinity. This ensures that all electric field lines starting from electrode 1 go to electrode 2 or reciprocally. In static conditions the potentials on the interfacial surfaces S_1 and S_2 are, respectively, V_1 and V_2 as illustrated in Fig. 1.

The potential $V(M)$ and the electric field $\vec{E}(M)$ at a position M in the volume between S_1 and S_2 for an applied voltage V_2-V_1 is supposed to be known when the sample is free of charges and dipoles. Since $\vec{E}(M)$ is proportional to V_2-V_1, we define the electric field, normalized in applied voltage, as

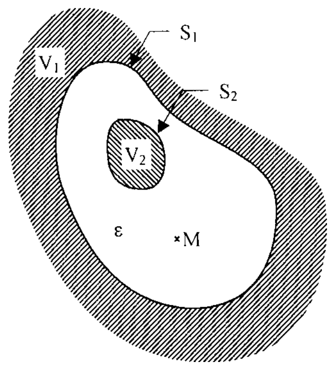

FIG. 1. Sample geometry.

$$\vec{\xi}(M)=\frac{\vec{E}(M)}{V_2-V_1}. \quad (1)$$

This quantity, having the units of inverse meters, will appear as a factor describing the sample geometry. The space charge and the permanent dipole distributions are, respectively, $\rho(M)$ and $\vec{P}(M)$. These charges and dipoles produce, when $V_1=V_2$, an electric field $\vec{E}'(M)$. In the general case the electric field $\vec{E}(M)$ is then expressed as

$$\vec{E}(M)=(V_2-V_1)\vec{\xi}(M)+\vec{E}'(M). \quad (2)$$

Since the insulator may be either a fluid or a solid material, subscripts are used when required to make expressions clearer as many times as necessary. Any term is understood to be summed over all values of any subscript that appears twice. For instance, the displacement field is expressed as $D_i=\varepsilon_{ij}E_j+P_i$ and the Poisson's equation is written as

$$\frac{\partial D_i}{\partial x_i}=\rho. \quad (3)$$

III. PRESSURE-WAVE-PROPAGATION METHOD

A. Brief description of the method

In this subsection, a usual setup for the pressure-wave-propagation method[2,12,18,19] is presented in the case of a planar sample. It is summarized in Fig. 2.

The insulator has two adjacent electrodes. A plane elastic wave transmitted into the insulator travels at the velocity of sound. We assume that the elastic wave is a pulse of very short duration as compared to the transit time. During its propagation, the pulse moves the charges locally which in turn produces a variation of the image charges on the electrodes. Thus a current proportional to the displaced space charges appears in the external circuit. The dependence on time of this current is similar to the space charge distribution inside the insulator since time and position are connected by the speed of sound.

B. Preliminary

The following analysis will be made with the assumption that the electrodes are connected through a low impedance measuring circuit that allows us to assume short-circuit conditions. This is indeed the mostly used measuring condition.

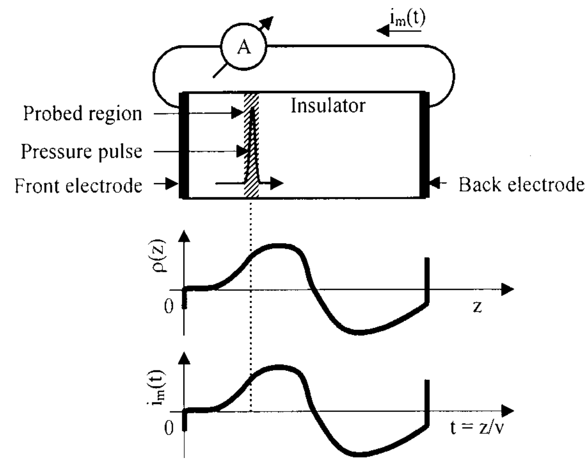

FIG. 2. The pressure-wave-propagation method in planar geometry.

As recalled above, the propagation of an elastic wave induces a variation of the charges on the electrodes. In order to evaluate the signal produced by this variation of charges, we decompose the effects of the material displacement into independent simple situations. We assume that the electrical response of the system to the mechanical perturbation is short as compared to the time variations of the elastic wave. Thus it is possible to describe the situation with electrostatic equations at any time.

The displacement modifies the electric field, the dielectric constant, and the charge and dipole distributions, which become, respectively, at a given time $\vec{E}+\overrightarrow{\delta E}$, $\varepsilon+\delta\varepsilon$, $\rho+\delta\rho$, and $\vec{P}+\overrightarrow{\delta P}$. When the displacement is sufficiently small, so that second-order terms may be neglected, Gauss's equation leads to

$$\mathrm{div}(\varepsilon\,\overrightarrow{\delta E}+\vec{E}\,\delta\varepsilon+\overrightarrow{\delta P})=\delta\rho. \quad (4a)$$

Equation (4a) shows that the material displacement modifies locally the electric field as if the space charge distribution $\delta\rho$ and the permanent dipole distribution $\vec{E}\,\delta\varepsilon+\overrightarrow{\delta P}$ were added into the insulator.

For the application of the boundary conditions it is necessary to take into account the motion of the electrodes. We have already defined S_1 and S_2 as the interfacial surfaces before the application of the deformation. We now introduce S_1' and S_2' as the interfaces under the deformation. Gauss's theorem makes it possible to advantageously replace the electrodes by equipotential surfaces without modifying the value of the potential between the electrodes. On these equipotential surfaces there is no superficial charge and the potential is the same as on the electrodes. Then, the electrode displacements are equivalent to the equipotential motions. The potential changes from V_1 to $V_1+\delta V_1(M)$ on the surface S_1 and from V_2 to $V_2+\delta V_2(M)$ on the surface S_2 whereas it remains unchanged on equipotentials S_1' and S_2'. The variations of potential $\delta V_1(M)$ and $\delta V_2(M)$, which depend on the position, represent the motion of the electrodes. These expressions are valid independently of the magnitude of the deformations. However the expression that will be

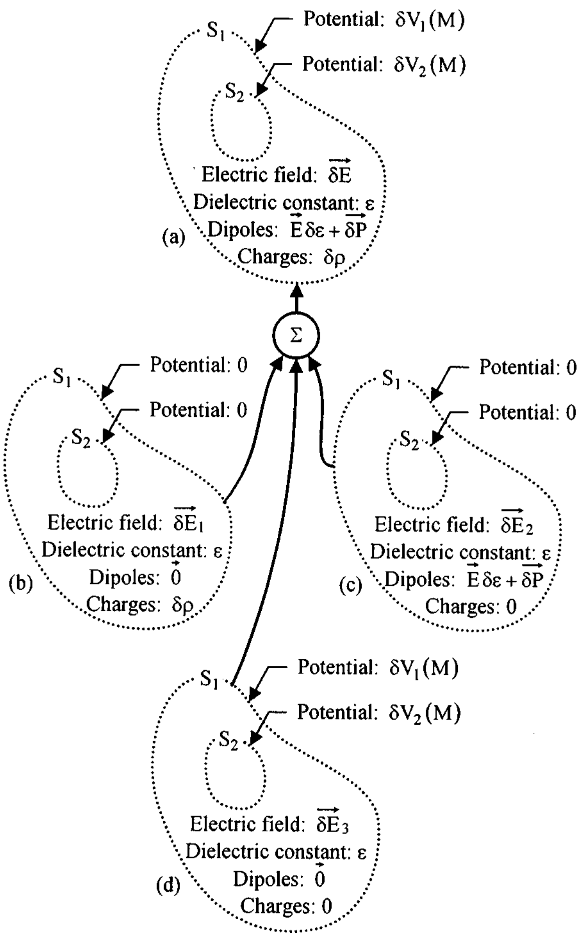

FIG. 3. The different equivalent equilibrium states describing the incidence on the electric field of the material displacement. They are associated with the modification of the dielectric constant, of the charge and the permanent dipole densities, and of the electrode surfaces. (a) describes all modifications due to a material displacement. (b) describes the modification of the space charge distribution. (c) describes the modification of the permanent dipole distribution and the dielectric constant. (d) describes the modification of the electrode surfaces.

developed in Sec. III E assumes a linear dependence justified by the small-considered deformations.

Consequently one can define the variation $\overrightarrow{\delta E}$ of the electric field due to a material displacement by the equilibrium state of Fig. 3(a). By splitting $\overrightarrow{\delta E}$ into three components so that $\overrightarrow{\delta E}=\overrightarrow{\delta E}_1+\overrightarrow{\delta E}_2+\overrightarrow{\delta E}_3$ one may decompose the equilibrium state of Fig. 3(a) into three independent equilibrium states such as

$$\mathrm{div}(\varepsilon\,\overrightarrow{\delta E}_1)=\delta\rho, \tag{4b}$$

$$\mathrm{div}(\varepsilon\,\overrightarrow{\delta E}_2+\vec{E}\,\delta\varepsilon+\overrightarrow{\delta P})=0, \tag{4c}$$

$$\mathrm{div}(\varepsilon\,\overrightarrow{\delta E}_3)=0. \tag{4d}$$

In the first and second states, the potentials on S_1 and S_2 are taken equal to zero; in other words, the electrodes are not deformed. This is illustrated, respectively, in Figs. 3(b) and 3(c). In the third state, as shown in Fig. 3(d), the potential on S_1 and S_2 varies from one point to the other in order to reflect the deformation of the electrodes. In the next three subsections we evaluate the signal contribution of each of these three equilibrium states.

C. Charge displacements

The displacement of charges in a domain limited by the two surfaces S_1 and S_2 is now analyzed. The charges that appear on the electrodes during the displacement of the internal charges are equivalent to the charges on the electrodes of a sample with the internal electric field $\overrightarrow{\delta E}_1$ [Fig. 3(b)]. In that equilibrium state, the sample contains the space charge distribution $\delta\rho$ and the potential zero is applied to each electrode. We suppose that the charge $q=\rho(M)dv$ at the position M is moved to a new position M' close to M under the effect of a material displacement $\vec{u}=\overrightarrow{MM'}$. Then the charge density $\delta\rho$ is reduced to a charge $-q$ at the position M and a charge $+q$ at the position M'. In order to evaluate the charge quantity δQ_1 on the electrode 1 and δQ_2 on the electrode 2 by using Gauss's identity (see the Appendix), we consider another equilibrium state. In that state, the insulator is free of charges and the potentials V_1 and V_2 are, respectively, applied on electrodes 1 and 2. Thus the electric field in the insulator, which derives from the potential $V(M)$, is given by Eq. (1). Gauss's identity connects the two equilibrium states by

$$\delta Q_1 V_1+\delta Q_2 V_2+q[V(M')-V(M)]=0. \tag{5}$$

The conservation of the charges implies that $\delta Q_2 =-\delta Q_1$. Thus Eq. (5) becomes

$$\delta Q_1=q\,\frac{V(M')-V(M)}{V_2-V_1}. \tag{6}$$

Furthermore, because the distance $\vec{u}$ from M to M' is small, $V(M')-V(M)=\overrightarrow{\mathrm{grad}}(V)\cdot\vec{u}$ at first order. This leads to

$$\delta Q_1=-q\vec{\xi}\cdot\vec{u}=-\rho\vec{\xi}\cdot\vec{u}\,dv. \tag{7}$$

Finally, the displacement of charges at the position M for a given space charge distribution ρ produces a signal proportional to the moved charges. Moreover, the signal is maximized when the material displacement is parallel to the direction of the field that would exist in the insulator if no space charge were present.

D. Dipole displacements and dielectric constant modifications

The displacement of permanent dipoles and the modification of the dielectric constant induced by a material deformation are now analyzed. We have seen that the dipole displacements and the dielectric constant modifications result in an equilibrium state in which the internal electric field is $\overrightarrow{\delta E}_2$ [see Fig. 3(c)]. In that state, the sample contains the permanent dipole distribution $\overrightarrow{\delta P}+\vec{E}\,\delta\varepsilon$ and both electrodes are at the potential zero. Since a dipole $q\,\overrightarrow{dl}$ is equivalent to a negative charge $-q$ and a positive charge $+q$ separated by the distance $\overrightarrow{dl}$, its apparition in the insulator produces a signal identical to that produced by a displacement $\overrightarrow{dl}$ of a charge $+q$. Thus δQ_1 can be derived from Eq. (7) as

$$\delta Q_1 = -q\,\overrightarrow{dl}\cdot\vec{\xi}. \tag{8}$$

Furthermore, the apparition of the dipole distribution $\overrightarrow{\delta P}+\vec{E}\,\delta\varepsilon$ in the insulator leads to the variation of charge δQ_1 on electrode 1:

$$\delta Q_1 = -(\overrightarrow{\delta P}+\vec{E}\,\delta\varepsilon)\cdot\vec{\xi}\,dv. \tag{9}$$

In order to take into account the possible anisotropy of the dielectric constant, δQ_1 can also be expressed as

$$\delta Q_1 = -(\delta P_1\xi_i + E_j\xi_i\delta\varepsilon_{ij})dv. \tag{10}$$

The variations δP_i and $\delta\varepsilon_{ij}$ derive from two effects. First the dipole density and the dielectric constant at a position M' become that which existed at position M prior to the material displacement $\vec{u}=\overrightarrow{MM'}$. Thus, a variation of permanent dipole density or of dielectric constant appears only if a gradient of these quantities existed before the material displacement. The first component of the variation is then

$$(\delta P_i)_1 = -u_k\frac{\partial P_i}{\partial x_k} \quad\text{and}\quad (\delta\varepsilon_{ij})_1 = -u_k\frac{\partial\varepsilon_{ij}}{\partial x_k}. \tag{11}$$

Second, the volume that was at position M changes due to the material deformation S_{kl}. The number of permanent dipoles does not change. One has

$$(\delta P_i)_2 dv + P_i\,\delta\,dv = 0, \quad\text{so}\quad (\delta P_i)_2 = -P_i\frac{\delta\,dv}{dv}. \tag{12}$$

Since the relative variation of volume is given by the divergence of the material displacement $\vec{u}$, which is also S_{kk}, the second component of the variation of the permanent dipole density is

$$(\delta P_i)_2 = -P_i\frac{\partial u_k}{\partial x_k}. \tag{13}$$

If the deformation is sufficiently small, the dielectric constant can be assumed to be linearly dependent on the deformation. The product of the electrostrictive tensor a_{ijkl} with the deformation S_{kl} gives the variation $\delta\varepsilon_{ij}$. One has

$$(\delta\varepsilon_{ij})_2 = a_{ijkl}S_{kl} = \tfrac{1}{2}a_{ijkl}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right), \tag{14}$$

Noticing that $a_{ijkl}=a_{ijlk}$, the variation δQ_1 on the electrode 1 is

$$\delta Q_1 = \left(\frac{\partial(P_i u_k)}{\partial x_k}\,\xi_i + E_j\xi_i u_k\frac{\partial\varepsilon_{ij}}{\partial x_k} - a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}\right)dv. \tag{15}$$

E. Electrode deformations

We have seen that the deformation of the electrodes results in the equilibrium state illustrated in Fig. 3(d). In that state, the electric field is $\overrightarrow{\delta E}_3$ and the potentials on the surface S_1 and S_2 are, respectively, $\delta V_1(M)$ and $\delta V_2(M)$. Furthermore in this case the sample contains no space charge nor permanent dipole distributions. In order to evaluate the charge ΔQ_1 on electrode 1 due to the deformation of the electrodes, we proceed in two steps.

(i) We consider two equilibrium states, one with the potential $\delta V_2(M)$ on the surface S_2 and zero on the surface S_1, and the other with the potential V_1 on the surface S_1 and zero on the surface S_2. In this latter state, the electric field at a position M in the sample is $-V_1\vec{\xi}(M)$ as defined in Eq. (1). Moreover the sample contains only superficial charges on surfaces S_1 and S_2. These superficial charges are given by $-\vec{D}\cdot\vec{n}$, which reduces to $-\varepsilon\vec{E}\cdot\vec{n}$ since we are now in the particular equilibrium state where the sample contains no permanent dipole distribution. In the above expressions $\vec{E}$ is the electric field, and $\vec{n}$ is the unit normal vector outwards the surfaces. One finds with Gauss's identity that

$$\Delta Q_1' = \int_{S_2}\varepsilon\vec{\xi}\cdot\vec{n}\,\delta V_2 ds. \tag{16}$$

In order to express $\delta V_2(M)$, we observe that if the displacement is sufficiently small, the potential on surface S_2 has been modified, in a first approximation, as the opposite of the variation of the voltage along the displacement vector. This variation is given by the dot product of the displacement vector with the electric field. One has

$$\delta V_2 = \vec{E}\cdot\vec{u}. \tag{17}$$

By introducing this result into Eq. (16), one obtains the variation of charge:

$$\Delta Q_1' = \int_{S_2}\varepsilon(\vec{E}\cdot\vec{u})(\vec{\xi}\cdot\vec{n})ds. \tag{18}$$

(ii) We also consider two equilibrium states, one with the potential δV_1 on the surface S_1 and zero on the surface S_2, and the other with the potential V_2 on the surface S_2 and zero on the surface S_1. In this latter state, the electric field at a position M in the sample is $V_2\vec{\xi}(M)$ as defined in Eq. (1). The resulting variation of charge $\Delta Q_2''$ can be obtained by the same development as above. One has

$$\Delta Q_2'' = -\int_{S_1}\varepsilon(\vec{E}\cdot\vec{u})(\vec{\xi}\cdot\vec{u})ds. \tag{19}$$

Since the variation of charge on one electrode is the opposite of that on the other electrode, the total variation of charge ΔQ_1 on electrode 1 when both electrodes are moved is given by $\Delta Q_1 = \Delta Q_1' + \Delta Q_1'' = \Delta Q_1' - \Delta Q_2''$.

$$\Delta Q_1 = \int_S\varepsilon(\vec{E}\cdot\vec{u})(\vec{\xi}\cdot\vec{n})ds, \tag{20}$$

where $S=S_1+S_2$.

F. Synthesis

The signal produced by the deformation of the insulator sample can be determined by integrating over the volume V of the sample Eqations (7) and (15), and by adding Eq. (20). One has

$$\Delta Q_1=\int_V\left[-\rho u_k\xi_k+\frac{\partial(P_1u_k)}{\partial x_k}\xi_i+E_j\xi_i u_k\frac{\partial\varepsilon_{ij}}{\partial x_k}-a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}\right]dv+\int_S\varepsilon_{ij}\xi_j n_i B_k u_k ds. \quad (21)$$

In order to avoid the partial derivatives over space in the case of discontinuous variables, for instance the electric field, the dielectric constant and the dipole distributions, we can split, applying integration by parts, the volume integral itself of Eq. (21) into a surface integral and a volume integral each containing only partial derivatives of the displacement. Using Gauss's equation to express the charge density ρ, it becomes

$$\Delta Q_1=\int_V\left[-\frac{\partial(\varepsilon_{ij}E_j+P_i)}{\partial x_i}u_k\xi_k+\frac{\partial(P_iu_k)}{\partial x_k}\xi_i+E_j\xi_iu_k\frac{\partial\varepsilon_{ij}}{\partial x_k}-a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}\right]dv+\int_S\varepsilon_{ij}\xi_jn_iE_ku_k ds. \quad (22)$$

This expression can also be written as

$$\Delta Q_1=\int_V\left[-\frac{\partial(\varepsilon_{ij}E_ju_k\xi_k)}{\partial x_i}+\varepsilon_{ij}E_j\xi_k\frac{\partial u_k}{\partial x_i}+E_ju_k\frac{\partial(\varepsilon_{ij}\xi_i)}{\partial x_k}+\varepsilon_{ij}E_ju_k\left(\frac{\partial\xi_k}{\partial x_i}-\frac{\partial\xi_i}{\partial x_k}\right)\right]dv+\int_V\left[\frac{\partial(P_iu_k\xi_i)}{\partial x_k}-\frac{\partial(P_iu_k\xi_k)}{\partial x_i}+P_i\xi_k\frac{\partial u_k}{\partial x_i}+P_iu_k\left(\frac{\partial\xi_k}{\partial x_i}-\frac{\partial\xi_i}{\partial x_k}\right)\right]dv-\int_V\left[a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}\right]dv+\int_S[\varepsilon_{ij}\xi_jn_iE_ku_k]ds. \quad (23)$$

Since $\overrightarrow{\text{curl}}(\vec{E})=\vec{0}$, one has

$$\Delta Q_1=+\int_V\left[\varepsilon_{ij}E_j\xi_k\frac{\partial u_k}{\partial x_i}\right]dv+\int_V\left[P_i\xi_k\frac{\partial u_k}{\partial x_i}\right]dv-\int_V\left[a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}\right]dv+\int_V\left[\frac{\partial(\varepsilon_{ij}\xi_iE_ju_k)}{\partial x_k}-\frac{\partial(\varepsilon_{ij}\xi_iE_ku_k)}{\partial x_j}-\varepsilon_{ij}\xi_i\left(E_j\frac{\partial u_k}{\partial x_k}-E_k\frac{\partial u_k}{\partial x_j}\right)+E_ku_k\frac{\partial(\varepsilon_{ij}\xi_i)}{\partial x_j}\right]dv+\int_S[\varepsilon_{ij}n_iu_k(\xi_jE_k-\xi_kE_j)]ds+\int_S[P_iu_k(\xi_in_k-\xi_kn_i)]ds. \quad (24)$$

We recall that ξ_i is the voltage normalized electric field in a sample free of charges and permanent dipoles. Thus the divergence of $\varepsilon_{ij}\xi_i$ is equal to zero. Moreover $\vec{E}$ and $\vec{\xi}$ are collinear to the unit normal vector outwards $\vec{n}$ at the insulator electrode interfaces so that $\xi_jE_k=\xi_kE_j$, $E_jn_k=E_kn_j$, and $\xi_in_k=\xi_kn_i$. Consequently the surface integrals vanish. The variation of charge ΔQ_1 is finally

$$\Delta Q_1=\int_V\left[(\varepsilon_{ij}E_j+P_i)\xi_k\frac{\partial u_k}{\partial x_i}-a_{ijkl}E_j\xi_i\frac{\partial u_k}{\partial x_l}-\varepsilon_{ij}\xi_i\times\left(E_j\frac{\partial u_k}{\partial x_k}-E_k\frac{\partial u_k}{\partial x_j}\right)\right]dv. \quad (25)$$

This general formula can be applied to various materials for instance isotropic solids or fluids. One can derive from Eq. (25) the measured current $i_m(t)$ in short-circuit condition by applying $i_m(t)=\partial\Delta Q_1/\partial t$. In the case of open circuit conditions, the measured voltage $v_m(t)$ can be reconstructed in a similar way.

(i) In isotropic solids, the dielectric constant is a scalar ε and the electrostrictive tensor a_{ijkl} is reduced, due to the invariance of the tensor with any space rotation, to two coefficients a_{11} and a_{12}. Introducing the Kronecker's symbol δ_{ij} which is equal to 1 if $i=j$ and 0 otherwise, one has

$$\varepsilon_{ij}=\varepsilon\,\delta_{ij}\quad\text{and}$$

$$a_{ijkl}=\tfrac{1}{2}(a_{11}-a_{12})(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+a_{12}\delta_{ij}\delta_{kl}. \quad (26)$$

In isotropic solids, the variation of charge ΔQ_1 on electrode 1 is

$$\Delta Q_1=\int_V\left[(\varepsilon E_i+P_i)\xi_k\frac{\partial u_k}{\partial x_i}+(\varepsilon+a_{12}-a_{11})\xi_iE_k\frac{\partial u_k}{\partial x_i}-(\varepsilon+a_{12})E_i\xi_i\frac{\partial u_k}{\partial x_k}\right]dv. \quad (27)$$

so that

$$\Delta Q_1=\int_V\{[(\varepsilon\vec{E}+\vec{P})\cdot\overrightarrow{\text{grad}}](\vec{u})\cdot\vec{\xi}+(\varepsilon+a_{12}-a_{11})\times(\vec{\xi}\cdot\overrightarrow{\text{grad}})(\vec{u})\cdot\vec{E}-(\varepsilon+a_{12})\vec{E}\cdot\vec{\xi}\,\text{div}(\vec{u})\}dv. \quad (28)$$

(ii) In fluids, both the dielectric constant and the electrostrictive tensor a_{ijkl} are scalars since $a_{12}=a_{11}$. The variation of the dielectric constant $(\delta\varepsilon)_2$ of Eq. (14) with the variation of the volume δdv is

$$(\delta\varepsilon)_2=\frac{\partial\varepsilon}{\partial v}\,\delta\,dv. \quad (29)$$

Introducing the mass density m_v it becomes

$$(\delta\varepsilon)_2=\frac{\partial\varepsilon}{\partial m_v}\,\frac{\partial m_v}{\partial v}\,\delta\,dv=-m_v\frac{\partial\varepsilon}{\partial m_v}\text{div}(\vec{u}). \quad (30)$$

Hence the coefficient a_{11} has the value

$$a_{11}=-m_v\frac{\partial\varepsilon}{\partial m_v}. \quad (31)$$

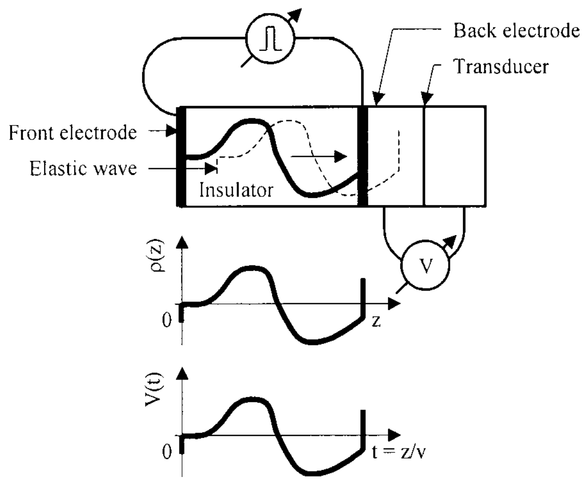

FIG. 4. The electro-acoustic method in planar geometry.

In fluids, the variation of charge ΔQ_1 on electrode 1 is thus

$$\Delta Q_1=\int_V\left\{[(\varepsilon\vec{E}+\vec{P})\cdot\overrightarrow{\mathrm{grad}}](\vec{u})\cdot\vec{\xi}+\varepsilon(\vec{\xi}\cdot\overrightarrow{\mathrm{grad}})(\vec{u})\cdot\vec{E}-\left(\varepsilon-m_v\frac{\partial\varepsilon}{\partial m_v}\right)\vec{E}\cdot\vec{\xi}\,\mathrm{div}(\vec{u})\right\}dv. \tag{32}$$

IV. ELECTRO-ACOUSTIC METHOD

A. Brief description of the method

In this subsection, a usual setup for the electro-acoustic method[4,6,7,9] is presented in the case of a planar sample. It is summarized in Fig. 4.

The insulator has two adjacent electrodes. When a voltage is applied, a Coulombian force acts on its charges. If a short pulsed voltage is superimposed to the applied voltage, the force acting on each charge varies. This creates an elastic wave that travels through the sample at the velocity of sound. A piezoelectric transducer picks up this wave. Since the amplitude of the elastic source is proportional to the charge, the elastic wave is the image of the charge distribution. The time to reach the transducer is proportional to the distance between the charge and the transducer. The dependence on time of the transducer signal is similar to the space charge distribution inside the insulator, since time and position are connected by the speed of sound.

B. Preliminary

The electrostatic forces acting on each charge are in equilibrium with the reaction forces in the material. If the electrostatic force changes with time, via the applied voltage for instance, an elastic wave is created. The associated deformation in the material induces a change of the dielectric constant known as electrostriction. In solids, even when they are isotropic, the dielectric constant becomes anisotropic under the effect of the material deformation.

In order to establish the propagation equation of the elastic waves in the solid, the force densities in the volume must be expressed. In a first stage the static forces are studied and then the impact of the variation of the voltage applied to the sample is analyzed.

C. Electrostatic force densities

In this subsection the local expression of the electrostatic force density is demonstrated in the general case. Electrostatic forces have been extensively studied in many reference books.[20] A simple way to find the force density is to consider the variation δW of the electric energy when the charges are virtually moved by an arbitrary small displacement δu_k. This variation can be expressed either as the work of the electrostatic force density f_k or as the variation of the electric energy W. The first approach leads to

$$\delta W=-\int f_k\delta u_k dv. \tag{33}$$

The second approach requires the knowledge of W, which is the sum of the energy[24] due to the charges and to the permanent dipoles in their own electric field. One has

$$W=\tfrac{1}{2}\int\rho V\,dv-\tfrac{1}{2}\int P_iE_i dv. \tag{34}$$

From the above expression of W, the variation δW can be written as

$$\delta W=\tfrac{1}{2}\int V\delta\rho\,dv+\tfrac{1}{2}\int\rho\,\delta V\,dv-\tfrac{1}{2}\int E_i\delta P_i dv-\tfrac{1}{2}\int P_i\delta E_i dv. \tag{35}$$

Introducing the displacement electric field D_i, whose divergence gives the charge density ρ, δW can also be expressed as

$$\delta W=\int V\delta\rho\,dv-\tfrac{1}{2}\int V\frac{\partial\delta D_i}{\partial x_i}dv+\tfrac{1}{2}\int\frac{\partial D_i}{\partial x_i}\delta V\,dv-\tfrac{1}{2}\int E_i\delta P_i dv-\tfrac{1}{2}\int P_i\delta E_i dv. \tag{36}$$

The second and the third terms of this expression can be split into volume and surface integrals applying integration by parts. Since the potential tends rapidly towards zero as one goes to the infinity, the surface integral is equal to zero. Thus

$$\delta W=\int V\delta\rho\,dv-\tfrac{1}{2}\int E_i\delta D_i dv+\tfrac{1}{2}\int D_i\delta E_i dv-\tfrac{1}{2}\int E_i\delta P_i dv-\tfrac{1}{2}\int P_i\delta E_i dv. \tag{37}$$

Replacing the displacement electric field D_i by its expression and noticing that ε_{ij} is symmetrical so that $\varepsilon_{ij}=\varepsilon_{ji}$, one obtains finally

$$\delta W=\int V\delta\rho\,dv-\tfrac{1}{2}\int E_iE_j\delta\varepsilon_{ij}dv-\int E_i\delta P_i dv. \tag{38}$$

The variation of energy δW contains three terms that depend, respectively, on the variations of the charge density $\delta\rho$, of the dielectric constant $\delta\varepsilon_{ij}$, and of the permanent dipole density δP_i. Each of these variations has two components, the first due to the material displacement and the other to the material deformation. The two components of the variations $\delta\varepsilon_{ij}$ and δP_j have already been expressed in Eqs. (11), (13), and (14). Since the number of charges does not change with the deformation, the variation of the charge density $\delta\rho$ is similar to the variation of the permanent dipole density δP_i. Thus

$$\delta\rho=(\delta\rho)_1+(\delta\rho)_2=-\delta u_k\frac{\partial\rho}{\partial x_k}-\rho\frac{\partial\delta u_k}{\partial x_k}. \tag{39}$$

Noticing as in Eq. (14) that $a_{ijkl}=a_{ijlk}$ and introducing the variations of the dielectric constant, the charge and the permanent dipole densities in Eq. (38), the variation of energy is

$$\begin{aligned}\delta W=&\tfrac{1}{2}\int E_iE_j\frac{\partial\varepsilon_{ij}}{\partial x_k}\delta u_k dv-\tfrac{1}{2}\int a_{ijkl}E_iE_j\frac{\partial\delta u_k}{\partial x_l}dv\\&-\int V\frac{\partial\rho}{\partial x_k}\delta u_k dv-\int\rho V\frac{\partial\delta u_k}{\partial x_k}dv+\int E_i\frac{\partial P_i}{\partial x_k}\delta u_k dv\\&+\int E_iP_i\frac{\partial\delta u_k}{\partial x_k}dv.\end{aligned} \tag{40}$$

Terms having partial derivative of the displacement can be integrated by parts in order to yield volume and surface integrals. Since the potential and the electric field tend rapidly towards zero as one goes to infinity, surface integrals are equal to zero. Thus

$$\begin{aligned}\delta W=&\tfrac{1}{2}\int E_iE_j\frac{\delta\varepsilon_{ij}}{\partial x_k}\delta u_k dv+\tfrac{1}{2}\int\frac{\partial(a_{ijkl}E_iE_j)}{\partial x_l}\delta u_k dv\\&+\int\rho\frac{\partial V}{\partial x_k}\delta u_k dv-\int P_i\frac{\partial E_i}{\partial x_k}\delta u_k dv.\end{aligned} \tag{41}$$

Finally, the two expressions of δW can be compared [Eqs. (33) and (41)] leading to

$$f_k=-\tfrac{1}{2}E_iE_j\frac{\partial\varepsilon_{ij}}{\partial x_k}-\tfrac{1}{2}\frac{\partial(a_{ijkl}E_iE_j)}{\partial x_l}+\rho E_k+P_i\frac{\partial E_k}{\partial x_i}. \tag{42}$$

The result obtained in such a manner leads to an expression of the force density that is valid anywhere in space and in particular inside the finite size dielectrics. The above terms, from left to right, are the components of the force densities due to the variation of the dielectric constant, to the effect of electrostriction, and to the presence of charge and permanent dipole densities in an electric field. The effect of electrostriction is negligible when the electric field is almost uniform whereas it is significant when the electric field is diverging since its variation with space becomes important.

In order to benefit from the general properties of tensors, we use Eq. (42) to determine the Maxwell tensor M_{kl}, recalling that

$$f_k=\frac{\partial M_{kl}}{\partial x_l}. \tag{43}$$

We replace in Eq. (42) the charge density by the divergence of the displacement field and take into account the fact that the electric field derives from a potential. Since $\varepsilon_{ij}=\varepsilon_{ji}$, one has

$$f_k=-\tfrac{1}{2}\frac{\partial(\varepsilon_{ij}E_iE_j)}{\partial x_k}-\tfrac{1}{2}\frac{\partial(a_{ijkl}E_iE_j)}{\partial x_l}+\frac{\partial[(\varepsilon_{ij}E_j+P_i)E_k]}{\partial x_i}. \tag{44}$$

We introduce the Kronecker symbol δ_{ij}, which leads to Maxwell's tensor M_{kl}:

$$M_{kl}=(\varepsilon_{lj}E_j+P_l)E_k-\tfrac{1}{2}(a_{ijkl}+\delta_{kl}\varepsilon_{ij})E_iE_j. \tag{45}$$

D. Voltage variation

During the variation of the voltage applied to the electrodes, the electric field in the insulator changes. Assuming that the displacement of charges and dipoles in the material is negligible, and thus the electric field $\vec{E}$ produced by these distributions is not modified, the electric field $\vec{E}'$ in the insulator submitted to an extra voltage $V(t)$ can be deduced from Eq. (2) by

$$\vec{E}'=\vec{E}+V(t)\vec{\xi}. \tag{46}$$

With this expression of the electric field, Maxwell's tensor M_{kl} can be written as

$$\begin{aligned}M_{kl}=&(\varepsilon_{lj}E_j+P_l)E_k-\tfrac{1}{2}(a_{ijkl}+\delta_{kl}\varepsilon_{ij})E_iE_j\\&+V(t)[\delta_{il}(\varepsilon_{ij}E_j+P_i)\xi_k-a_{ijkl}E_j\xi_i\\&-\varepsilon_{ij}\xi_i(\delta_{kl}E_j-\delta_{jl}E_k)]\\&+V^2(t)[\varepsilon_{lj}\xi_j\xi_k-\tfrac{1}{2}(a_{ijkl}+\delta_{kl}\varepsilon_{ij})\xi_i\varepsilon_j].\end{aligned} \tag{47}$$

As expected the application of an extra voltage modifies Maxwell's tensor. It becomes the sum of the static Maxwell's tensor, of a term proportional to the extra voltage, and of a third term proportional to the square of the extra voltage. If the static electric field $\vec{E}$ is much larger than the electric field produced by the extra voltage, the squared term can be neglected. It can be noticed that the term under the integral of Eq. (25), which is the signal obtained in the case of the pressure-wave-propagation method, is equal to the term proportional to the extra voltage times the partial derivative of u_k over x_l. This shows the great similarity between the pressure-wave-propagation method and the electro-acoustic method.

E. Wave equation

The electrostatic forces are compensated by elastic forces. The elastic force densities are given by the divergence of the stress tensor T_{kl}. Hooke's law specifies that the stress tensor T_{kl} is proportional to the deformation S_{ij}:

$$T_{kl}=c_{klij}S_{ij}, \tag{48}$$

where c_{klij} is the elastic stiffness tensor. Taking into account the relation between the acceleration and the sum of all force densities, it turns out that

$$m_v \frac{\partial^2 u_k}{\partial t^2} = \frac{\partial T_{kl}}{\partial x_l} + \frac{\partial M_{kl}}{\partial x_l} + O_k, \quad (49)$$

where O_k represents all other static force densities, for instance the gravity, t is the time, and u_k is the displacement along the direction x_k. When no extra voltage is applied to the sample, the forces in the insulator are all static. Defining u_i^e as the static material displacement in such an equilibrium situation, one has

$$m_v \frac{\partial^2 u_k^e}{\partial t^2} = \frac{\partial}{\partial x_l}\left[c_{klij}\frac{\partial u_i^e}{\partial x_j}\right] + \frac{\partial}{\partial x_l}[(\varepsilon_{lj}E_j + P_l)E_k - \tfrac{1}{2}(a_{ijkl} + \delta_{kl}\varepsilon_{ij})E_iE_j] + O_k = 0. \quad (50)$$

When an extra voltage is applied, the material displacement is the sum of the static material displacement u_i^e and of the dynamic material displacement u_i. Introducing Eqs. (50) into (49) and using expression (47) for Maxwell's tensor, the elastic wave propagation equation becomes

$$m_v \frac{\partial^2 u_k}{\partial t^2} = \frac{\partial}{\partial x_l}\left[c_{klij}\frac{\partial u_i}{\partial x_j}\right] + V(t)\frac{\partial}{\partial x_l}[\delta_{il}(\varepsilon_{ij}E_j + P_i)\xi_k - a_{ijkl}E_j\xi_i - \varepsilon_{ij}\xi_i(\delta_{kl}E_j - \delta_{jl}E_k)] + V^2(t)\frac{\partial}{\partial x_l}[\varepsilon_{lj}\xi_j\xi_k - \tfrac{1}{2}(a_{ijkl} + \delta_{kl}\xi_i)\xi_i\xi_j]. \quad (51)$$

This equation does not exhibit any coupling between the electric field and the material displacement. We can see that the terms depending on the extra voltage are the force densities that create the elastic sources. Thus introducing the Green's function $\vec{G}_i(M,M',t)$ connecting the displacement u_i at the position M to the force density $\vec{f}$ at the position M', the solution of Eq. (51) is

$$u_i(M,t) = \int_{t'}\int_{V'} \vec{G}_i(M,M',t-t')\cdot\vec{f}(M',t')dv'dt'. \quad (52)$$

In this expression V' represents the entire space from which M' is taken and t' all times. If the Green's function is unknown, this equation must be solved by applying the continuity of the displacement and superficial tensions through interfaces as boundary conditions. Introducing T'_{kl} and M'_{kl} as the stress and the Maxwell's tensor on the other side of the interface having the unit normal vector n_l, the continuity of superficial tension can be written as

$$(T_{kl} - T'_{kl})n_l + (M_{kl} - M'_{kl})n_l = 0. \quad (53)$$

Calculating the terms depending on the extra voltage applied to the sample in Eq. (51), the propagation equation is also expressed as

$$m_v \frac{\partial^2 u_k}{\partial t^2} = \frac{\partial}{\partial x_l}\left[c_{klij}\frac{\partial u_i}{\partial x_j}\right] + V(t)\left[\rho\xi_k + P_i\frac{\partial \xi_k}{\partial x_i} - E_i\xi_j\frac{\partial \varepsilon_{ij}}{\partial x_k} - \frac{\partial(a_{ijkl}E_i\xi_j)}{\partial x_l}\right] - \tfrac{1}{2}V^2(t)\left[\frac{\partial(a_{ijkl}\xi_i\xi_j)}{\partial x_l} + \xi_i\xi_j\frac{\partial \varepsilon_{ij}}{\partial x_k}\right]. \quad (54)$$

In the term proportional to the extra voltage a first source is proportional to the space charge density and is in the direction of the electric field as if the insulator were free of charges and dipoles. A second source is proportional to the dipole density. This source exists only as long as the electric field, in the insulator free of charges and dipoles, is nonuniform at the position of the dipole. A third source depends on the variation of the dielectric constant and a fourth source depends on the variation of the electric field amplitude. This last source may be very important when the electric field is diverging.

This wave equation can be applied to various materials, for instance isotropic solids or fluids.

(i) In isotropic solids, the wave equation is simplified by taking as the dielectric constant and the electrostrictive tensor the values of Eq. (26). The stress tensor is

$$c_{ijkl} = \tfrac{1}{2}(c_{11} - c_{12})(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + c_{12}\delta_{ij}\delta_{kl}. \quad (55)$$

(ii) In fluids, the wave equation is even more simplified since $a_{11} = a_{12}$ and $c_{11} = c_{12}$. In that case the coefficient a_{11} is given by Eq. (31).

V. APPLICATION OF THE GENERAL THEORY TO SIMPLE GEOMETRIES

A. Comparison with existing situations

It is of interest to apply the general theory presented above to simple geometries for which many detailed calculations have been reported and that have been extensively experimentally studied.[4,7,10,18,21] This allows verifying the theory and also to point out discrepancies with presently admitted interpretations. We limit ourselves to situations in which the dielectric constant is uniform and no permanent dipoles are present.

B. Planar sample geometry

In planar geometry, the value of all subscripts present in the above equations is 1. The sample thickness is d and the electrode surface is S.

1. Pressure-wave-propagation method

The signal produced by the pressure-wave-propagation method is calculated from Eq. (25). One has

$$\Delta Q_1 = S\int_0^d \left[(\varepsilon E + P)\xi\frac{\partial u}{\partial x} - a_{11}E\xi\frac{\partial u}{\partial x}\right]dx. \quad (56)$$

The normalized electric field $\vec{\xi}$ is uniform in the insulator. When the current is picked up from the electrode that imposes the high voltage, as illustrated in Fig. 5, the normalized

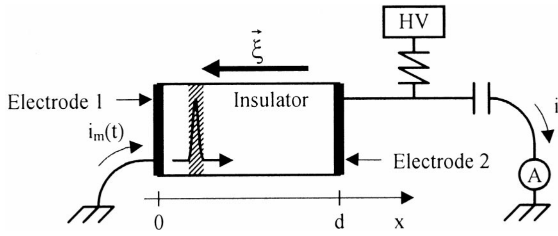

FIG. 5. Pressure-wave-propagation setup for a planar sample.

electric field is $-1/d$. The derivative of the displacement along the sample thickness is equivalent to the opposite of the pressure P in the sample over Young's modulus Y. If the sample contains no permanent dipoles and has a uniform dielectric constant, the variation of charge ΔQ_1 on electrode 1 becomes

$$\Delta Q_1 = \frac{S(\varepsilon - a_{11})}{Yd} \int_0^d EP\, dx. \tag{57}$$

Introducing the capacity C_0 of the sample at rest and the coefficient $G(\varepsilon) = 1 - a_{11}/\varepsilon$, one finds finally the usual expression of the current $i_m(t)$ in the measuring circuit:[2,18]

$$i_m(t) = \frac{C_o G(\varepsilon)}{Y} \int_0^d E \frac{\partial P}{\partial t} dx. \tag{58}$$

2. Electro-acoustic method

In the case of the electro-acoustic technique, the propagation equation in a one-dimensional plate sample is

$$m_v \frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial x} + \frac{\partial M}{\partial x} \quad \text{with} \quad T = c_{11} \frac{\partial u}{\partial x} \quad \text{and}$$

$$M = (\varepsilon - a_{11}) E \xi V(t) + P \xi V(t) + \tfrac{1}{2}(\varepsilon - a_{11}) \xi^2 V^2(t). \tag{59}$$

A transducer, at a distance x_t from the insulator on the side of the electrode 1, produces a signal $S(t)$ proportional to the pressure stress to which it is submitted. When a high voltage is applied to electrode 2, as shown in Fig. 6, the normalized electric field is $-1/d$. We suppose that elastic waves come only from the sample, i.e., the electrodes are of infinite thickness and the transducer is perfectly matched. The transmission coefficient of elastic waves from the insulator to electrode 1 is τ_1. γ_1 and γ_2 are, respectively, the reflection coefficients of elastic waves from the insulator to

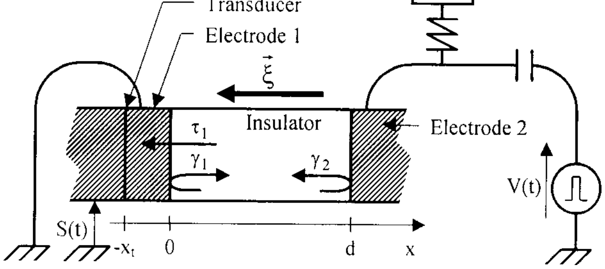

FIG. 6. Electro-acoustic setup for a planar sample.

electrodes 1 and 2. The Green's function $G(x_t, x, t)$ giving the pressure at a point x_t produced by a force density at a position x in the insulator is[22]

$$G(x_t, x, t) = \tfrac{1}{2}\tau_1 \sum_{n=0}^{\infty} (\gamma_1\gamma_2)^n \left[\delta\left(t - \frac{x_t}{v_l} - \frac{2nd + x}{v_s}\right) - \gamma_2 \delta\left(t - \frac{x_t}{v_l} - \frac{2(n+1)d - x}{v_s}\right)\right], \tag{60}$$

where $\delta(x)$ is the Dirac function, v_l and v_s are, respectively, the speed of sound in electrode 1 and in the insulator, and t is the time. The coefficient $\frac{1}{2}$ at the front of Eq. (60) represents the radiating diagram. The energy of the elastic sources is indeed separated in two equal parts, the first giving a wave propagating forward and the other a wave propagating backward. We call H the coefficient that gives the amplitude of the signal produced by the transducer for a given pressure at the position x_t. Then the signal amplitude $S(t)$ measured on the transducer is

$$S(t) = H \int_{t'} \int_{x'} G(x_t, x', t - t') \frac{\partial M(x', t')}{\partial x'} dx'\, dt'. \tag{61}$$

Moreover at each interface of force discontinuity, that is to say in this case at the interface between the insulator and the electrodes, the signal depends on the superficial tensions. The interfacial signal $S(t)$ produced by a tension at the position $x = x_0$ is

$$S(t) = H \int_{t'} G(x_t, x_0, t - t') M(x_0, t') n(x_0) dt', \tag{62}$$

where $n(x_0)$ is the unit normal vector outwards the interface. The useful signal, which is the first measured signal, comes from the direct propagation of elastic waves and not from the reflected waves on the insulator boundaries. If the extra voltage is applied to the sample at time $t = 0$, the first signal is produced at time $t = x_t/v_l$ and originates from the interface between electrode 1 and the insulator at $x = 0$. This interfacial signal is obtained by introducing definitions (59) and (60) into Eqs. (62):

$$S(t) = \tfrac{1}{2} H \tau_1 \left[(\varepsilon - a_{11}) \frac{V(t - x_t/v_l)}{d} E(x = 0) - \tfrac{1}{2}(\varepsilon - a_{11}) \frac{V^2(t - x_t/v_l)}{d^2} \right]. \tag{63}$$

Since the dielectric constant is uniform in the insulator, the electric field derivative over space is also ρ/ε. Hence the signal produced by the direct sources in the insulator, at times $x_t/v_1 < t < x_t/v_1 + d/v_S$, is obtained by introducing definitions (59) and (60) into Eq. (61):

$$S(t) = -H \frac{\tau_1(\varepsilon - a_{11})}{2\varepsilon d} \int_0^d V(t - x_t/v_1 - x/v_S) \rho(x) dx. \tag{64}$$

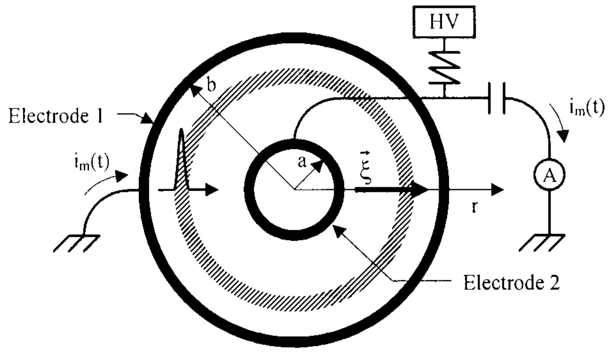

FIG. 7. Pressure-wave-propagation setup for a coaxial sample.

The interfacial signal produced by the other interface localized at the position $x=d$ and taking place at time $t=x_t/v_1+d/v_S$ is obtained by introducing definitions (59) and (60) into Eq. (62):

$$S(t)=\tfrac{1}{2}H\tau_1(1-\gamma_2)\left[(\varepsilon-a_{11})\frac{V(t-x_t/v_1-d/v_S)}{d}E(x=d)-\tfrac{1}{2}(\varepsilon-a_{11})\frac{V^2(t-x_t/v_1-d/v_S)}{d^2}\right]. \quad (65)$$

As expected, the theory applied to this simple case leads to the usually reported results[4,21] in which electrostriction is taken into account.

C. Coaxial sample geometry

In coaxial geometry the electric field E_i, the normalized electric field ξ_i, the dipoles P_i, and the material displacement u_i are all supposed radial. If the sample has a length l, an inner radius a, and an outer radius b, the signal produced by the pressure-wave-propagation method calculated from Eq. (25) is

$$\Delta Q_1=2\pi l\int_a^b\left[(\varepsilon E+P)\xi\frac{\partial u}{\partial r}-a_{11}E\xi\frac{\partial u}{\partial r}-a_{12}E\xi\frac{u}{r}-\varepsilon E\xi\frac{u}{r}\right]r\,dr. \quad (66)$$

The normalized electric field $\vec{\xi}$ is proportional to the inverse of the radius. When the current is picked up from the inner electrode, on which the high voltage is supposed to be applied as shown in Fig. 7, the normalized electric field along the radius is

$$\xi=\frac{1}{r\ln(b/a)}. \quad (67)$$

If the sample contains no permanent dipoles and has a uniform dielectric constant, the signal becomes

$$\Delta Q_1=\frac{2\pi l}{\ln(b/a)}\int_a^b\left[\varepsilon E\left(\frac{\partial u}{\partial r}-\frac{u}{r}\right)-a_{11}E\left(\frac{\partial u}{\partial r}+\frac{a_{12}}{a_{11}}\frac{u}{r}\right)\right]dr. \quad (68)$$

This equation is slightly different from those reported in the literature.[10] This is due to the fact that previous calculations assumed $a_{11}=a_{12}$ as is the case in fluids. Such an approximation may not be justified for some solid dielectrics. In fluids the divergence of the displacement is the opposite of the pressure P times the compressibility χ. In that case the variation of charge ΔQ_1 on the electrode 1 is

$$\Delta Q_1=-\frac{2\pi l}{\ln(b/a)}\left\{(\varepsilon-a_{11})\chi\int_a^b EP\,dr+2\varepsilon\int_a^b\frac{E}{r}u\,dr\right\}. \quad (69)$$

Introducing the capacity C_0 of the sample at rest, the coefficient $G(\varepsilon)=1-a_{11}/\varepsilon$, and exchanging the limits of the integrals, the current $i_m(t)$ in the measuring circuit is

$$i_m(t)=C_0\left\{G(\varepsilon)\chi\int_b^a E\frac{\partial P}{\partial t}dr+2\int_b^a\frac{E}{r}\frac{\partial u}{\partial t}dr\right\}. \quad (70)$$

Finally an additive term has to be added to the usual expression[10] corresponding to the first term of Eq. (70). This additive term makes the signal vanish when no space charge is in the insulator and when $a_{11}=0$, except during the deformation of the electrodes of the sample. The experimental results exposed in the next section show that Eq. (70) is more suitable than the formerly reported expressions.

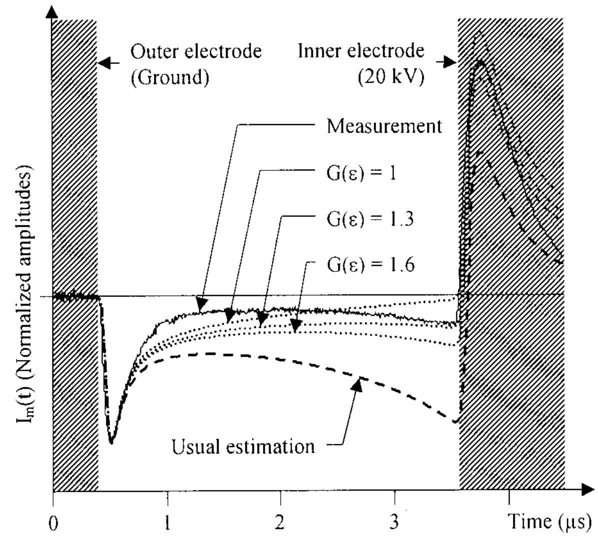

FIG. 8. Comparison between the analytical expressions and the real signal in a coaxial sample in the case of the pressure-wave-propagation method.

D. Signals from divergent field geometry

In this section measurements on divergent electric field samples are exposed. In the first experiment the pressure-wave-propagation method is used on a coaxial sample in order to validate expression (70) of the signal. In the second experiment the pressure-wave-propagation and the electro-acoustic methods are used and compared on the same sample.

Figure 8 shows a measurement using the pressure-wave-propagation method on a coaxial polyethylene sample having no space charge and submitted to 20 kV. This sample has a 5.1-mm inner-radius electrode and an 11.7 mm outer-radius electrode. Close to the inner electrode the deviation of the measured signal, in the solid line of Fig. 8, is due to the

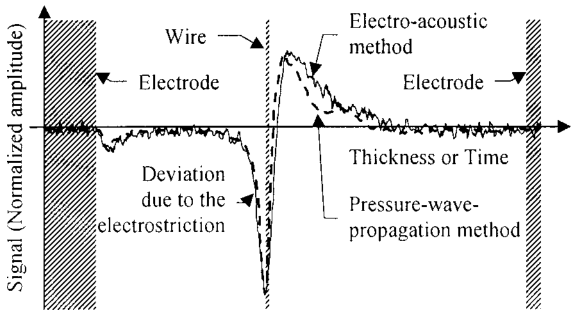

FIG. 9. Effect of a divergent electric field for both the pressure-wave-propagation and the electro-acoustic methods.

variation of the electric field in the insulator. As one can see the usual estimation does not fit properly the signal in this region whereas the analytical expression (70) fits quite well the experimental data taking $a_{11}=-0.3\varepsilon$, which is nearly the electrostrictive coefficient of polyethylene.[23] However close to the outer electrode the models do not follow completely the experimental signal. This is due to the fact that calculations have been carried out with a purely coaxial wave contrary to that used in the experiment. Indeed the wave is initiated from a portion representing 8% of the cylinder perimeter so that diffraction occurs resulting in the modification of the pressure wave profile.

Figure 9 illustrates measurements on a very divergent electric field sample. Both the pressure-wave-propagation and the electro-acoustic methods have been used on the same sample. This sample has two planar electrodes connected to ground enclosing the insulator. A 25-μm-diameter wire embedded in the insulator and submitted to a high voltage produces the divergence of the electric field. It can be seen on each measurement that, before the wave reaches the wire, a large negative signal is detected whereas one would expect the interfacial electric field on the wire to induce a positive signal. This deviation is not produced by the presence of charge but by the effect of electrostriction. As expected the divergence of the electric field produces nonnegligible signals that have comparable shapes for both the pressure-wave-propagation method and the electro-acoustic method.

VI. CONCLUSION

In this paper, the pressure-wave-propagation method and the electro-acoustic method have been analyzed in crystals, isotropic solids or fluids in the case of complex electric field distributions. The solutions proposed show the great similarity of these two methods and are in good agreement with the experiments. The usual expressions already reported in the literature are justified in the case of simple sample geometries such as planar and coaxial geometries. In this later geometry, the solutions proposed predict some changes in the usual formulation of the pressure-wave-propagation method signal. These changes have been physically explained and experimentally verified. In more complex geometries, there is a non-negligible signal in regions containing no charges but where the electric field is diverging. This is the effect of electrostriction, which has been demonstrated for both pressure-wave-propagation method and electro-acoustic method.

The general theory proposed makes it possible to study the behavior of insulators submitted to complex electric field distributions, for instance insulators including water trees, defects, wires, or needles electrodes. It can be applied to the pressure-wave-propagation method and to the electro-acoustic method.

APPENDIX

The Gauss identity connects two equilibrium states A and B by an integral equation involving the charges and the potential V of each state.[24] If ρ denotes a set of any kinds of charges, for instance in a volume, on a surface, on a curve or at a point, one has

$$\int \rho_A V_B dv = \int \rho_B V_A dv. \tag{A1}$$

This relation can easily be demonstrated by replacing the potential by its expression in terms of charge:

$$V(M) = \int \frac{\rho(M')}{4\pi\varepsilon(M')MM'} dv, \tag{A2}$$

where MM' is the distance from the charge at the position M' to the position M where the potential is calculated.

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