

Progressive suppression of spin relaxation in two-dimensional channels of finite width

A. A. Kiselev and K. W. Kim

Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, North Carolina 27695-7911

(Received 9 November 1999)

We have investigated spatiotemporal kinetics of electron spin polarization in a semiconductor narrow two-dimensional (2D) strip and explored the ability to manipulate spin relaxation. Information about the conduction electron spin and mechanisms of spin rotation is incorporated into a Monte Carlo transport simulation program. A model problem, involving linear-in- k splitting of the conduction band responsible for the D'yakonov-Perel' mechanism of spin relaxation in the zinc-blende semiconductors and heterostructures, is solved numerically to yield the decay of spin polarization of an electron ensemble in the 2D channel of finite width. For very wide channels, a conventional 2D value of spin relaxation is obtained. With decreasing channel width, the relaxation time increases rapidly by orders of magnitude. Surprisingly, the crossover point between 2D and quasi-1D behavior is found to be at tens of electron mean-free paths. Thus, classically wide channels can effectively suppress electron spin relaxation.

I. INTRODUCTION

Spintronics,^{1,2} a nontrivial extension of conventional electronics, augments functionality by utilizing the carrier spin degree of freedom. Spin can potentially be used as a much more capacious quantum information storage cell, be involved in the transfer of information, for elaborate schemes of information processing, both quantum mechanical and classical, and be integrated with electric-charge counterparts in combined designs. Electron or nuclear spin manipulation is believed to be the key component of the potential realizations of a quantum computer (see Ref. 3, for example).

So far several devices, including spin hybrid⁴ and field effect transistors,⁵ tunneling structures with magnetic layers,⁶ spin-based memory,⁷ and even systems based on carbon nanotubes⁸ have been proposed. The most notable success was obtained with giant magnetoresistance effect devices that rely on the variation of electron scattering in a multilayer stack of ferromagnetic films separated by non-magnetic materials.^{9,10} Though the variation of the current through the structure is small, it is sufficient for detection, and the excellent sensitivity of the device to weak external magnetic fields (typically 1% change of resistance per oersted) opened the door for massive data storage applications. Spin coherence is a pivotal prerequisite for the operability of prospective spintronic devices.

Most propositions for experimental realizations of spintronic elements rely on the injection of spin-polarized electrons from the ferromagnetic layer. These suffer from the poor quality of the ferromagnet-semiconductor interfaces that reduce the polarization of the injected electrons to a few percent as a result of strong spin relaxation caused by the large number of surface states. In spin-valve designs the importance of the interface problem is drastically amplified because the injected electron crosses the metal-semiconductor interface twice (i.e., at the source and drain terminals of the device). Furthermore, spin relaxation in the active region of the device adds to the difficulties.

In our study, we consider the possibilities for suppressing spin relaxation of the conduction electrons and evaluate different approaches. A potentially important case is alteration

of spin relaxation in a 2D electron gas as a long strip of finite width is formed, e.g., using an electrostatic squeezing split-gate technique. These strips can be used to connect active elements in integrated chip designs and could even become a part of active devices as the size of circuit elements is reduced. Eventually the one-dimensional (1D) limit, exhibiting no transport-induced spin relaxation related to the absence of the inversion center in the elementary crystal cell, will be reached. This transition to 1D behavior was recently documented by Bournel *et al.*¹¹ in their study of the spin transistor. In this paper, we focus on the definition of the crossover points of the spin relaxation regimes and the description of the relaxation time behavior in the broad regions between the crossovers. A Monte Carlo approach is used to simulate electron transport, including the evolution of spin polarization and relaxation, in 2D channels of finite width.

This paper is organized as follows: In Sec. II, we start with a brief account of spin relaxation mechanisms in semiconductors in order to identify the most relevant one. We discuss its transformation to lower-dimensional systems, as well as different possibilities to suppress destructive spin relaxation. In Sec. III, the model of a narrow patterned 2D electron gas channel is described. Results of a Monte Carlo simulation of spin relaxation in the channel are given in the next section (Sec. IV), along with a comprehensive discussion of the relaxation regimes in terms of the channel width and a value of the spin splitting of electron subbands (Sec. V). A brief summary follows at the end.

II. MECHANISMS OF SPIN RELAXATION

There are several mechanisms that can cause spin relaxation of conduction electrons¹² (see Ref. 13 for an up-to-date informal review).

(i) The mechanism of D'yakonov and Perel'¹⁴ (DP) regards the spin splitting of the conduction band in zinc-blende semiconductors at finite wave vectors as equivalent to the presence of an effective magnetic field that causes electron spin to precess. For an electron experiencing random multiple scattering events, the orientation of this effective field is

random, thus leading to spin relaxation.

(ii) Bir-Aronov-Pikus¹⁵ (BAP) processes involve a simultaneous flip of electron and hole spins due to electron-hole exchange coupling.

(iii) Spin relaxation due to momentum relaxation is possible directly through spin-orbit coupling [Elliot-Yafet¹⁶ (EY) process].

(iv) Spin relaxation can take place as a result of hyperfine interaction of electron spins with magnetic momenta of lattice nuclei, the hyperfine magnetic field being randomly changed due to the migration of electrons in the crystal.

The BAP processes require a substantial hole concentration that is not available in the unipolar doped structures. EY processes are suppressed in the 2D environments studied in this work due to the particular structure of the perturbation matrix element responsible for the EY processes (see Ref. 12, p. 84). Random hyperfine fields are much weaker for extended electron states than for highly localized ones; their amplitude and even their presence in the system are determined by the nature of the constituent atoms. Thus, the most relevant mechanism for our case is of the DP type.

A. DP mechanism and change of the dimensionality

As a result of the relatively low zinc-blende crystal symmetry, the effective 2×2 electron Hamiltonian for the conduction electrons contains spin-dependent terms that are cubic in the electron wave vector \mathbf{k}

$$H = \eta' [\sigma_x k_x (k_y^2 - k_z^2) + \text{cycl. perm.}], \quad (1)$$

where $\sigma_i (i=x, y, z)$ are the Pauli matrices. The constant η' reflects the strength of the spin splitting in the conduction band and its value is defined by the details of the semiconductor band structure. Other terms in Eq. (1) are obtained by cyclic permutations of the indices.

Modifications to the character of the spin relaxation also occur as the system dimensionality is changed from the 3D to 2D, as can be achieved, for example, in semiconductor heterostructures.¹⁷ First, the average wave vector in the direction of the quantum confinement (z axis) is large, so the terms in the spin splitting involving k_z^2 will dominate. This reorients the effective magnetic field into the plane of the 2D electron gas (xy plane). Nevertheless, the elementary rotations around random axes, all lying in one plane, do not commute with each other; thus the electrons reaching the same final destination by different trajectories will have different spin orientations. As the transport time increases, more and more distinguishable trajectories will become possible leading to a progressive reduction of the average spin polarization of the electron ensemble. For a structure with $|k_z| > k_F$ (relatively easy to design), the splitting for a typical electron will be larger and the rate of spin relaxation will be enhanced. One might naively expect similar behavior with the further reduction of the dimensionality to the 1D case. Choose the x axis to be along the wire. Again the main terms in Eq. (1) will contain spatial-confinement multipliers k_y^2 , k_z^2 . The principal difference with the 2D case is that all rotations in one dimension are limited to a single axis direction and they commute with each other. Apart from the systematic rotation, spin polarization does not disappear with time. All particles, independent of the number and the sequence of

the scattering events, that reach the same final point B , will have the same spin orientation. This statement is relaxed if one allows intersubband scattering in the 1D system. This type of scattering becomes progressively more important for wider and wider quantum wires with more and more subbands involved. Thus, we will recover the 2D or 3D case in the limit of very thick quantum wires.

In fact, there are two types of linear-in- k terms, with similar functional forms, that appear in the effective-mass spin Hamiltonian for the 2D electron gas:¹⁸ the bulk-asymmetry¹⁹ and structure-asymmetry-induced (Bychkov-Rashba) terms.^{20,21} The former can be derived from Eq. (1) by keeping only the terms with k_z^2 , and the latter is

$$H = \eta \boldsymbol{\sigma} \cdot [k \times \hat{z}] \equiv \eta (\sigma_x k_y - \sigma_y k_x), \quad (2)$$

where \hat{z} is a unit vector in the z direction. We ignore the origin of the spin splitting term in our model problem and simply assume its presence with a wave-vector dependence given by Eq. (2). (See Refs. 22 and 23 for the band-structure calculation of the linear-in- k spin splitting in heterostructures simultaneously treating bulk- and structure-induced asymmetry.)

This form of the spin Hamiltonian is equivalent to the interaction of the spin with the effective magnetic field

$$H = \frac{\hbar}{2} \boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_{\text{eff}}, \quad \text{where } \boldsymbol{\Omega}_{\text{eff}} \equiv \eta_{\text{DP}} \mathbf{v} \times \hat{z}. \quad (3)$$

Here the particle velocity $\mathbf{v} = \hbar \mathbf{k} / m^*$ and an obvious substitution $\eta \rightarrow \eta_{\text{DP}}$ is done for convenience. η_{DP} is expressed in inverse length units. The spin of a particle moving ballistically through distance $1/\eta_{\text{DP}}$ will rotate by the angle $\phi = 1$. The quantum-mechanical description of the evolution of the spin $1/2$ is equivalent to the evolution of the classic momentum \mathbf{S} with the equation of motion

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\Omega}_{\text{eff}} \times \mathbf{S}.$$

The reciprocal effect of electron spin on the orbital motion through spin-orbit coupling can often be ignored due to the large electron kinetic energy in comparison to the typical spin splittings and strong change of the momentum in scattering events.

B. Influencing spin relaxation time

In addition to understanding the physics of carrier spin depolarization, we consider and assess the ability to actively influence these destructive processes in order to improve parameters and gain new functionality of future spintronic devices. Potential approaches for manipulating spin relaxation times are the following:

(i) A simple observation follows directly from the intrinsic nature of the motional narrowing regime (small elementary spin rotations during ballistic electron flights).¹² Reduction of momentum relaxation time τ_p leads to the suppression of spin relaxation, since $\tau_S^{-1} \sim \tau_p \langle \Omega_{\text{eff}}^2 \rangle$. On the other hand, this will lead to broadenings as well as decoherence and can worsen device parameters.

(ii) Spin splitting is absent (small) for structures with principal axis along (011). Recent experiments delivered τ_S in (011) quantum wells more than an order of magnitude longer than that of the (001) counterpart.²⁴

(iii) The bulk-asymmetry- and structure-asymmetry-induced spin splittings are additive with the similar linear-in- k dependence; thus it is possible to tune combined spin splittings in the conduction band to a desired value through manipulation of the external electric field (and fix the axis of the effective magnetic field, as in the 1D case, although it is not possible to cancel the splittings completely).

(iv) Additional spin splitting, which is independent of the electron wave vector, will fix the precession axis. An obvious possibility here is the Zeeman effect. The time of spin relaxation scales in the presence of the external magnetic field B as²⁵

$$\frac{1}{\tau_S(B)} = \frac{1}{\tau_S(0)} \frac{1}{1 + (\Omega_L \tau_p)^2},$$

where $\hbar\Omega_L = g\mu_B B$ is a Zeeman splitting of electron spin sublevels. This suggests that spin relaxation time will double for $\Omega_L \tau_p = 1$.

(v) Spin relaxation of the conduction electrons can be controlled by doping. The first realization of this was reported in δ -doped heterostructures²⁶ and enhancement in spin memory by several orders of magnitude has recently been observed in n -doped structures.²⁷

(vi) When the channel width L is comparable to the magnitude of the electron mean free path L_p the sequential alteration of one of the wave vector components should effectively reduce spin relaxation (reflective boundaries).¹¹ Scattering on the boundaries (diffusive boundaries) will also decrease τ_p and can potentially influence spin relaxation. Quantum mechanically, channel narrowing leads to the quantization of electron transverse motion in the strip and the absence of spin relaxation without intersubband scattering.

The first five possibilities have been considered to some extent in the scientific literature or are just obvious consequences of the relaxation mechanism. The sixth deserves a more thorough analysis and is the subject of our study. To examine the effect of the patterning of the 2D electron gas into strips of large width L , we developed a simple Monte Carlo simulator encompassing random scattering of the electrons in the channel, reflection from the boundaries, and spin rotation during free flight due to spin splitting of the conduction band.

III. THE MODEL

As a model system we consider a strip of the 2D electron gas. The third dimension (z axis) is quantized and irrelevant to the issue of electron movement in real space since the intersubband gap is larger than all other energy scales involved. We make the following assumptions.

(i) All particles have the same velocity $|\mathbf{v}|$.

(ii) Scattering is considered to be elastic and isotropic in order to retain model simplicity; the former assumption preserves velocity modulus, and the later one eliminates any correlations between directions of the particle velocity before and after the scattering event.

(iii) We neglect electron-electron interactions and consider electrons to be independent.

(iv) An assumption that all scatterers are completely uncorrelated leads to an exponential distribution of times between any two consecutive scattering events. Their average is called the momentum relaxation time τ_p , which corresponds to the mean free path $L_p = |\mathbf{v}| \tau_p$.

(v) The spin Hamiltonian is given in the form of Eq. (3) and influences only the spin coordinate. We ignore the reciprocal effect of the spin on the motion in real space.

(vi) The width of the channel is large, several or even tens of mean free paths, so that it is permissible to consider classically the electron real-space movement.

(vii) Initially, we consider reflecting boundary conditions at the borders of the 2D strip. Reflecting channel boundaries preserve the longitudinal component of the particle velocity and change sign of the normal component in collisions. Later, we compare our results obtained with reflective and diffusive boundaries.

Now we consider the types of possible spin polarization measurements. For simplicity, in all of our experiments, particles will be injected into the system at some particular point A (input terminal) at time $t=0$ with spin S . As the particle experiences multiple scattering events, a diffusive pattern of motion is formed with a Gaussian distribution (for an isotropic system)

$$\Gamma(r) \sim \frac{1}{\langle r \rangle} \exp\left(-\frac{r^2}{\langle r \rangle^2}\right),$$

that broadens as time increases: $\langle r \rangle \sim L_p \sqrt{t/\tau_p}$.

Clearly, there are multiple possibilities for experimental setups. The definition of spin relaxation time τ_S obtained in these experiments will vary correspondingly. Several important possibilities are as follows.

(i) The most informative type of experiment would be to measure the average spin $\langle S \rangle$ as a function of time t at each point B (output terminal). Measurement results for all other experimental configurations can be derived by partial integration of this correlation function. In our computer simulation, we give preference to this type of measurement. Real experimental realization can be technically complicated, since it requires measurement locality in both space and time domains.

(ii) At time t , $\langle S \rangle$ is taken as an average over the whole ensemble independent of the real-space position of electrons. Optical experiments are considered likely to deliver information of this type, because of the limited possibilities to focus optical systems and fundamental restrictions.

(iii) Electrons are removed from the system immediately upon arrival at the output terminal. $\langle S \rangle$ is measured, for example, as a function of the interterminal distance. Individual particles can spend substantial time in the system, depending on their trajectories. This type of experiment is the most probable variant in electric experiments where points A and B can be identified as real device gates. Made from ferromagnetic materials, gates can inject polarized electrons and sense the polarization of the drain flux, delivering information about the average spin of carriers. As it was already

pointed out, the possibility of experimental realization of these measurements is severely restricted at present by limitations of the technology.

We now show that our result for spin relaxation does indeed depend on the type of measurement. As a simple example we consider a pure 1D case. From the point of view of the first and third experiments, there is no spin relaxation. In these cases there exists a systematic rotation of the spin proportional to the distance from the injection point A . All particles reaching point B will have exactly the same spin orientation, independent of the number of scattering events and individual trajectories, but different for different choices of point B . Thus, the transverse component of the spin is given by $S_y(x) = S_y^0 \cos(\eta_{\text{DP}} x)$. For the second realization, we readily conclude that the average spin is expressed as

$$\begin{aligned} \langle S_y \rangle &= \int dx S_y(x) \Gamma(x) \sim S_y^0 \exp\left(-\frac{1}{4} \eta_{\text{DP}}^2 \langle x \rangle^2\right) \\ &\sim S_y^0 \exp\left(-\frac{1}{4} \eta_{\text{DP}}^2 L_p^2 \frac{t}{\tau_p}\right) \end{aligned}$$

and shows an exponential decay of spin polarization in the spatially broadening electron distribution.

Note that in the 2D case, the same phenomenon of the systematic rotation of the electron spin takes place in addition to the (2D-3D specific) DP spin relaxation. This systematic rotation by the angle (vector form) $\phi = \eta_{\text{DP}} \mathbf{r} \times \hat{\mathbf{z}}$ for the particle real-space transfer \mathbf{r} is again independent on the details of individual trajectories.

IV. SIMULATION RESULTS

Figure 1(a) shows the time dependence of average spin polarization $2\langle S_y \rangle$ in the channel. It is found that, apart from the systematic rotation, this dependence is essentially the same for all points inside the channel that have a substantial electron occupation for a reliable calculation of spin polarization. The calculation is performed for the DP parameter $\eta_{\text{DP}} L_p = 0.05$ and different channel widths. For this particular plot the trajectories of $N = 5 \times 10^3$ electrons are traced giving a standard deviation on the order of $\sqrt{N} \sim 10^{-2}$ (throughout our investigation $N = 10^3 - 10^5$).

As can be seen from the figure, a strong dependence of polarization decay on the channel width is obtained. The decay is found to be approximately exponential, apart from the small initial interval. This is the region where the diffusive regime of electron motion and spin rotation is not yet established ($\sqrt{t}/\tau_p \sim 1$). It overlaps or is followed by the region where the typical trajectory of a randomly scattered particle does not reach channel boundaries yet. Here the relaxation should be essentially the same as in the unpatterned 2D gas. The generally exponential temporal behavior of spin polarization justifies an introduction of the spin relaxation time τ_S . The dependence of this decay time on the channel width is presented in Fig. 1(b). In the case of sufficiently wide channels, the relaxation time defined in this way approaches the 2D limit of $\tau_S^{2\text{D}}$, while τ_S scales as L^{-2} for narrow channels.

Figure 1(c) summarizes our simulation results for channels with a fixed width and different values of DP term η_{DP} .

As the constant η_{DP} becomes larger and larger, the characteristic behavior gradually deviates from the regime of motional narrowing since elementary rotations on the elementary free flights are no longer small. This leads to quick spin relaxation, as reflected by the reduction and saturation of τ_S . On the other hand, there is a steep increase in the relaxation time as η_{DP} decreases. We have found an intermediate region where τ_S^{-1} scales as a second power of the DP constant, and that is followed by a fourth-power dependence for very small values of η_{DP} . Thus, in the well-established regime of motional narrowing and in the limit of sufficiently narrow channels, we propose an asymptotic formula $\tau_S \sim \eta_{\text{DP}}^{-4} L^{-2}$.

To identify an effect of the particular choice of the boundary conditions we have simulated a channel with diffusive boundaries as well. The particle that reaches the channel border is scattered back into the channel with equal probabilities for all directions of the new particle velocity. Calculation results repeat closely the case of the reflective boundaries up to the very narrow channel with widths of only several mean free paths. No systematic deviation of the spin relaxation time is observed for wider channels.

V. REGIMES OF THE DP RELAXATION IN A CHANNEL

We distinguish the following regimes of spin relaxation in channels of finite width as we vary both independent parameters η_{DP} and L of the problem in consideration. First, for very large spin splitting ($\eta_{\text{DP}} L_p \geq 1$) we violate a general condition for the motional narrowing regime for the DP spin relaxation. Each elementary rotation is not small and the information about the spin polarization is lost after the first random scattering event (see Fig. 2 as a guide). For this regime, τ_S is on the order of τ_p and is the shortest of all regimes.

When η_{DP} is small ($\eta_{\text{DP}} L_p < 1$), we return to the regime of motional narrowing and a well-known equation $\tau_S^{2\text{D}} \sim \tau_p (\eta_{\text{DP}} L_p)^{-2}$ defines the time of spin relaxation in the 2D system. As we narrow the strip of the 2D electron gas, we find that the behavior is unchanged until $\eta_{\text{DP}} L \sim 1$. For smaller channel widths ($\eta_{\text{DP}} L < 1$), DP spin relaxation is suppressed very effectively with $\tau_S \sim \tau_S^{2\text{D}} (\eta_{\text{DP}} L)^{-2}$. For $L < L_p$ the channel width L acts as a new mean free path in the system, substituting L_p in equations. In fact, this region does not satisfy our assumption about classical motion of particles in real space; the transverse motion is quantized and this system should be considered as a quantum wire with multiple subbands. The crossover points define broad regions of mixed behavior, that becomes more definitive as we leave them.

While our conclusions are based on the results of the Monte Carlo simulation, other approaches are possible. In a recent publication,²⁸ Mal'shukov and Chao, considering an equation of spin diffusion, were able to identify special waveguide diffusion modes that determine the propagation of spin density in long channels. Their finding, that spin relaxation in this mode slows as L^{-2} in very narrow channels, directly confirms one of our statements.

Thus far, the analysis has been presented using symbolic notation for a generalized discussion. Hence, it is necessary to examine whether the chosen range of parameters is well within the reasonable limits of contemporary heterostruc-

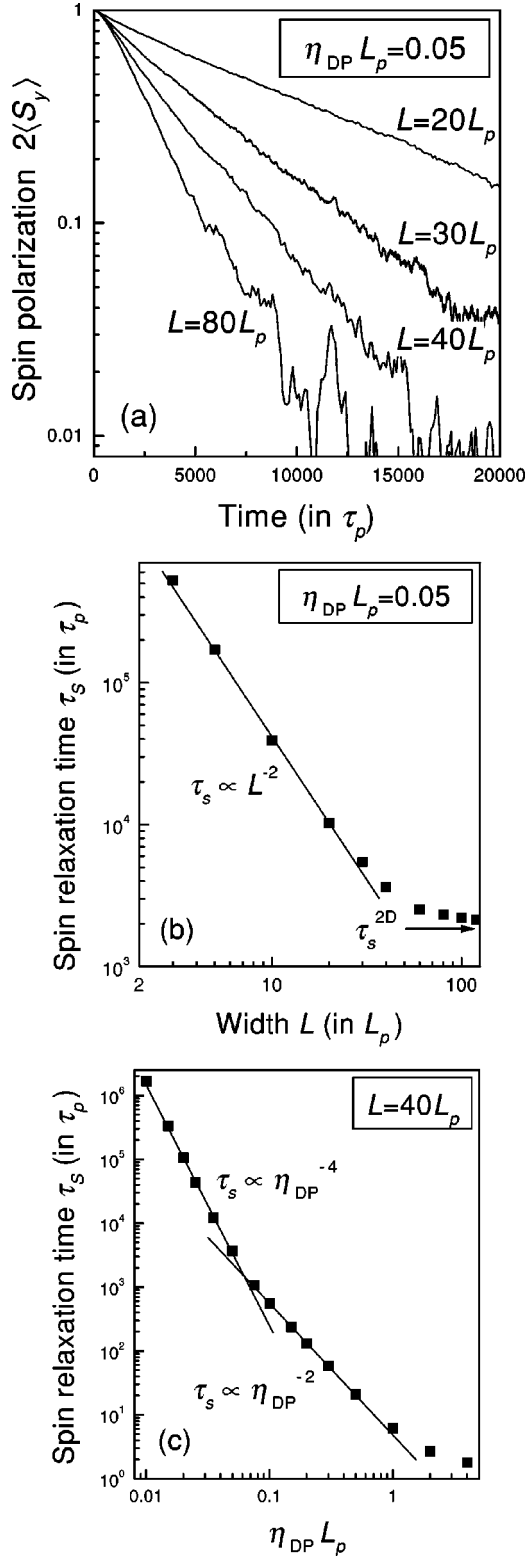


FIG. 1. Spin relaxation in a channel. (a) Time dependence of the spin polarization, calculated for different channel widths. The DP constant η_{DP} is fixed to be 0.05. The trajectories of 5×10^3 particles are traced yielding a standard deviation $\sim 10^{-2}$ for the calculated averages. Close-to-exponential decays of the polarization permit the definition of the spin relaxation time τ_s ; (b) τ_s as a function of the channel width L ; (c) spin relaxation time dependence on the DP spin splitting constant η_{DP} at fixed channel width $L=40L_p$.

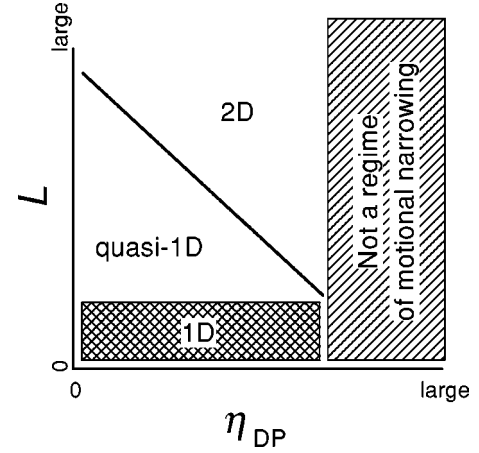


FIG. 2. Different regimes of the spin relaxation on the plane (η_{DP}, L) of model parameters: $\eta_{DP}L_p \geq 1$ (elementary rotations during free flights are not small), $\tau_s \sim \tau_p$; $\eta_{DP}L_p < 1$, $\eta_{DP}L \geq 1$ (2D spin relaxation), $\tau_s^{2D} \sim \tau_p (\eta_{DP}L_p)^{-2}$; $\eta_{DP}L < 1$ (suppression of spin relaxation, quasi-1D regime), $\tau_s \sim \tau_s^{2D} (\eta_{DP}L)^{-2} \sim \tau_p \eta_{DP}^{-4} L^{-2}$; $L \leq L_p$ (L substitutes L_p , quantum-mechanical quantization in the channel).

tures. For the asymmetric GaAs/Al_xGa_{1-x}As quantum well with channel electron concentration of 10^{12} cm^{-2} (corresponding to $k_F \approx 3 \times 10^5 \text{ cm}^{-1}$), the spin splitting values in the conduction band are measured to be on the order of 0.2–0.3 meV at the Fermi energy (see Ref. 29 and the references therein), corresponding to $\eta_{DP} = 6-9 \times 10^4 \text{ cm}^{-1}$. Samples with $L_p \gg 1/q = 3 \times 10^{-6} \text{ cm}$ are readily available in the laboratories. This set of parameters verges on the regimes of motional narrowing and large elementary rotations. Reducing the spin splitting (i.e., η_{DP}) will push parameters into the well-established regime of motional narrowing, suitable for the experimental observation of the described phenomena. This can be accomplished by lowering the electron concentration in the 2D electron gas resulting in a reduced structure asymmetry due to the internal electric field. Datta and Das⁵ give an estimate of $\pi/\eta_{DP} = 7 \times 10^{-5} \text{ cm}$ for some particular In_xGa_{1-x}As/In_xAl_{1-x}As heterostructures.

This analysis used an assumption that other mechanisms of spin relaxation are not important. As we change channel parameters and the DP spin relaxation time increases by orders of magnitude, other relaxation mechanisms can come into play, determining the value of τ_s .

VI. SUMMARY

We have investigated spin-dependent transport in semiconductor narrow 2D channels and explored the possibility of suppressing spin relaxation. Our approach is based on a Monte Carlo transport model and incorporates information on conduction-band electron spins and spin rotation mechanisms. Specifically, an ensemble of electrons experiencing multiple scattering events is simulated numerically to study the decay of electron spin polarization in channels of finite width due to the DP mechanism. We have identified different regimes of the spin relaxation in the 2D channels of finite

width and established the dependencies of spin relaxation time on the width L and DP parameter η_{DP} . The most attractive regime for future spintronic applications is the regime of the suppressed spin relaxation with the relaxation time τ_S scaling as L^{-2} .

ACKNOWLEDGMENTS

This work was supported in part by the Office of Naval Research. The authors thank M. A. Stroschio for the critical reading of the manuscript.

-
- ¹G. A. Prinz, Phys. Today **48**(4), 58 (1995); Science **282**, 1660 (1998).
- ²J. Gregg, W. Allen, N. Viart, R. Kirschman, C. Sirisathitkul, J.-P. Schille, M. Gester, S. Thompson, P. Sparks, V. DaCosta, K. Ounadjela, and M. Skvarla, J. Magn. Magn. Mater. **175**, 1 (1997).
- ³B. E. Kane, Nature (London) **393**, 133 (1998).
- ⁴D. J. Monsma, J. C. Lodder, J. A. Popma, and B. Dieny, Phys. Rev. Lett. **74**, 5260 (1995).
- ⁵S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
- ⁶J. Moodera, L. Kinder, T. Wong, and R. Meservey, Phys. Rev. Lett. **74**, 3273 (1995).
- ⁷M. Dax, Semicond. Int. **20**, 84 (1997).
- ⁸K. Tsukagoshi, B. W. Alphenaar, and H. Ago, Nature (London) **401**, 572 (1999).
- ⁹J.-Ph. Ansermet, J. Phys.: Condens. Matter **10**, 6027 (1998).
- ¹⁰P. B. Allen, Solid State Commun. **102**, 127 (1997).
- ¹¹A. Bournel, P. Dollfus, P. Bruno, and P. Hesto, Eur. Phys. J.: Appl. Phys. **4**, 1 (1998).
- ¹²G. E. Pikus and A. N. Titkov, in *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (Elsevier, New York, 1994), p. 73.
- ¹³J. Fabian and S. Das Sarma, J. Vac. Sci. Technol. B **17**, 1708 (1999).
- ¹⁴M. I. D'yakonov and V. I. Perel', Sov. Phys. Solid State **13**, 3023 (1972).
- ¹⁵G. L. Bir, A. G. Aronov, and G. E. Pikus, Zh. Éksp. Teor. Fiz. **69**, 1382 (1975) [Sov. Phys. JETP **42**, 705 (1976)].
- ¹⁶R. J. Elliott, Phys. Rev. **96**, 266 (1954); Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1963), Vol. 14.
- ¹⁷E. L. Ivchenko and G. E. Pikus, *Superlattices and Other Heterostructures. Symmetry and Optical Phenomena*, 2nd ed., Springer Series in Solid State Sciences, Vol. 110 (Springer-Verlag, Heidelberg, 1997).
- ¹⁸E. L. Ivchenko, A. A. Kiselev, and M. Willander, Solid State Commun. **102**, 375 (1997).
- ¹⁹G. Lommer, F. Malcher, and U. Rössler, Phys. Rev. B **32**, 6965 (1985).
- ²⁰Yu. A. Bychkov and E. I. Rashba, Pis'ma Zh. Éksp. Teor. Fiz. **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)].
- ²¹G. Lommer, F. Malcher, and U. Rössler, Phys. Rev. Lett. **60**, 728 (1988).
- ²²P. Pfeffer and W. Zavadski, Phys. Rev. B **52**, 14 332 (1995).
- ²³P. Pfeffer, Phys. Rev. B **55**, 7359 (1997).
- ²⁴Y. Ohno, R. Terauchi, T. Adachi, F. Matsukura, and H. Ohno, Phys. Rev. Lett. **83**, 4196 (1999).
- ²⁵E. L. Ivchenko, Fiz. Tverd. Tela (Leningrad) **15**, 1566 (1973) [Sov. Phys. Solid State **15**, 1048 (1973)]; E. L. Ivchenko, P. S. Kop'ev, V. P. Kochereshko, I. N. Uraltsev, and D. R. Yakovlev, Pis'ma Zh. Éksp. Teor. Fiz. **47**, 407 (1988) [JETP Lett. **47**, 486 (1988)].
- ²⁶J. Wagner, H. Schneider, D. Richards, A. Fischer, and K. Ploog, Phys. Rev. B **47**, 4786 (1993).
- ²⁷J. M. Kikkawa, I. P. Smorchkova, N. Samarth, and D. D. Awschalom, Science **277**, 1284 (1997); J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).
- ²⁸A. G. Mal'shukov and K. A. Chao, Phys. Rev. B **61**, 2413 (2000).
- ²⁹L. Wissinger, U. Rössler, R. Winkler, B. Jusserand, and D. Richards, Phys. Rev. B **58**, 15 375 (1998).