Self-similar magnetoconductance fluctuations in quantum dots

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Self-similarity of magnetoconductance fluctuations in quantum dots is investigated by means of a tight binding Hamiltonian on a square lattice. Regular and chaotic dots are modeled by either a perfect $L\times L$ square or introducing diagonal disorder on a number of sites proportional to L. The conductance is calculated by means of an efficient implementation of the Kubo formula. The degree of opening of the cavity is varied by changing the width W of the connected leads. It is shown that the fractal dimension D is controlled by the ratio W/L. The fractal dimension decreases from 2 to 1 when W/L increases from $1/L$ to 1, and is almost independent of other parameters such as Fermi energy, leads configuration, etc. This result is consistent with recent experimental data for soft-wall cavities, which indicate that D decreases with the degree of wall softening (or, alternatively, cavity opening). The results hold for both regular and chaotic cavities.

I. INTRODUCTION

Conductance fluctuations in quantum dots have been the subject of much current interest.[1–5] Experimental evidence indicates that the graph of conductance versus the applied magnetic field (perpendicular to the cavity) have a self-similar character.[2,3] In particular, self-similar patterns down to the mT scale were reported in Ref. 2. More recently, it has been observed that magnetoconductance fluctuations have a fractal character with a fractal dimension which decreases with the gate voltage applied to the cavity.[3] The effect of the latter is to decrease the potential well experienced by the carriers (soft-wall potential).

These experimental findings seem to be in agreement with theoretical studies which predict a self-similar character of conductance fluctuations in mixed-phase-space systems.[4] Although the theoretical foundation of these findings seems sound, few detailed calculations are available in the literature. The purpose of this paper is to present and discuss the results of an extensive numerical study of the fractal character of magnetoconductance fluctuations in quantum dots. We will not consider the low-magnetic-field region, where weak-localization effects have been observed[1,2] and investigated theoretically,[6–8] but rather we shall look at conductance fluctuations over a fairly wide range of magnetic fields. The opening of the cavity, that was achieved in the experiments by means of a gate voltage,[3] is simulated by increasing the ratio W/L, where W is the width of the leads and L the linear dimension of the dot. The results are consistent with the experimental result of Ref. 3, namely, that the fractal dimension decreases with the degree of opening of the cavity.

The paper is organized as follows. The results of a semiclassical theory of magnetoconductance fluctuations and the main features of the experimental results are discussed in the remaining part of this section. Section II is devoted to a description of the model and the theoretical tools. Particular attention is given to the method we use to calculate the current. The results are described and compared to experiments in Sec. III, while some concluding remarks are included in Sec. IV.

A. Theoretical background

It has been argued that in purely chaotic systems classical particles describe trajectories which sweep the phase space in an ergodic fashion.[4,9] As a consequence, escape from the cavity occurs exponentially fast. Instead, in mixed systems (like most real billiards; see below) the probability of staying longer than a time t within the cavity follows a power law.[4,10] In the presence of a magnetic field B, the particles acquire a phase $\exp(2\pi iAB/\Phi_0)$, which varies with the number AB/Φ_0 of magnetic flux quanta $\Phi_0=h/e$ enclosed by a trajectory of total swept area A. Again, in mixed systems the probability distribution of enclosed areas larger than A follows a power law,

$$P(A)\sim A^{-\gamma}. \tag{1}$$

It is interesting to consider the possible existence of bounds to the exponent γ. The available information corresponds to the survival probability (the probability of staying in the cavity a time longer than t), which is also expected to follow a power law with an exponent similar to that of the distribution of areas.[4] Numerical results indicate that the survival probability follows a power law with an exponent in the range $1<\gamma<2$.[10,11] This result is consistent with the analysis of Meiss *et al.*[12] These authors proved that the survival time distribution for the entry set obeyed a power law with an exponent greater than 1.

It was suggested by Ketzmerick[4] that power laws such as that of Eq. (1) lead to self-similarity and fractality. This author used a semiclassical approach[4] to show that the change in the conductance (at a fixed energy E) due to a small change in the magnetic field,

$$\Delta G=G(E,B+\Delta B)-G(E,B), \tag{2}$$

is a random variable with zero mean and a variance which, assuming a power law such as that of Eq. (1), is proportional to a power of ΔB. The result, reported in Refs. 4 and 13, is

$$\langle (\Delta G)^2 \rangle \sim (\Delta B)^\gamma \tag{3}$$

(see Ref. 14). Random processes having such a variance and zero mean are known as fractional Brownian motion, either persistent or antipersistent having $1<\gamma<2$ or $0<\gamma<1$, respectively.[15] The corresponding graph is fractal with a fractal dimension D given by

$$D = 2 - \gamma/2. \tag{4}$$

Taking account of the bounds for γ discussed above, we conclude that D varies in the range 1–1.5.

It should be remarked that the mixed character of a cavity can have several sources.[16,17] First note that in the presence of a magnetic field only the circle remains integrable no matter how large the field is. Square cavities become nonintegrable as soon as the field is switched on. Moreover, connecting the leads can also break down a regular behavior,[18] even in the case of a circular dot. On the other hand, it is also widely accepted that the magnetic field regularizes fully chaotic cavities.[16] Thus it is likely that most experimental, real, situations can fit into the framework described above.

B. Survey of experimental results

The most detailed study of the fractal character of magnetoconductance fluctuations in quantum dots is that of Sachrajda *et al.*[3] The experiments were carried out on a stadium cavity with an area of 5.3×10^{-12} m^{-2}, a lithographic radius of 1.1 μm, and an electron density $n = 2.5\times 10^{15}$ m^2 (we take the average of densities before and after illumination, see Ref. 3). Leads having a rather large width of 0.7 μm were used. The magnetic field B was varied from 0 to 0.2 T (note that the conductance is symmetric relative to $B=0$). Introducing the numerical value of the flux quantum $\Phi_0 = h/e = 4.14\times 10^{-15}$ Wb, the corresponding flux range is $\Phi \approx (0\text{–}260)\Phi_0$. On the other hand, the fractal analysis was carried out over increments of the magnetic field in the range $\Delta B \approx 10^{-4}\text{–}10^{-1}$ T which corresponds to $\Delta\Phi \approx (0.13\text{–}130)\Phi_0$, although the fittings were actually done over a narrower range $\Delta\Phi \approx (0.9\text{–}65)\Phi_0$. The fractal dimension of the magnetoconductance curves varied with increasing gate voltage approximately in the range 1.35–1.0. The authors of Ref. 3 carried out a similar fractal analysis for the data of Ref. 2 obtaining a fractal dimension $D = 1.3$. In Ref. 2, narrower leads 0.2 μm wide were used on Sinai cavities produced from 1-μm squares.

To figure out to what extent the range of magnetic field explored in the experiments can be considered small as concerns the validity of the semiclassical approximation,[16] we have calculated the cyclotron radius r_c corresponding to the maximum flux reached in the experiments.[3] In terms of the magnetic flux, the classical cyclotron radius r_c is written as

$$r_c = \frac{mvA}{h}\frac{\Phi_0}{\Phi}, \tag{5}$$

where m and v are the mass and velocity of electrons, A is the area of the cavity, and h is Planck's constant. The electron velocity can be obtained from the two-dimensional electron density, $v = (\hbar/m)\sqrt{2\pi n}$. Introducing the values of the parameters given above and $\Phi = 260\Phi_0$, we obtain $r_c = 2.5$ μm, which is approximately twice the typical radius of the stadium.

II. MODEL AND METHODS

A. Hamiltonian

The quantum dot is described by means of a tight-binding Hamiltonian with a single atomic level per lattice site on $L\times L$ clusters of the square lattice,

$$\hat{H} = \sum_{m,n\,\in\, IS} \epsilon_{m,n} |m,n\rangle\langle m,n| - \sum_{\langle m,n;m',n'\rangle} t_{m,n;m',n'} \times |m,n\rangle\langle m',n'|, \tag{6}$$

where $|m,n\rangle$ represents an atomic orbital on site (m,n). Indexes run from 1 to L, and the symbol $\langle\ \rangle$ denotes that the sum is restricted to nearest neighbors. Using Landau's gauge the hopping integral is given by

$$t_{m,n;m',n'} = t\exp\left(2\pi i \frac{m}{(L-1)^2}\frac{\Phi}{\Phi_0}\right), \quad m = m'$$
$$= t \quad \text{otherwise}, \tag{7}$$

where $(L-1)^2$ is the area of the cluster, and $\Phi_0 = h/e$ the quantum of magnetic flux. The hopping integral t will be used as the unit of energy, whereas lengths will be measured in terms of the lattice constant a ($t=1$ and $a=1$). The energy $\epsilon_{m,n}$ of atomic levels at impurity sites is randomly chosen between $-\Delta/2$ and $\Delta/2$, whereas at other sites $\epsilon_{m,n}=0$. Impurities were taken on $2L$ sites forming a cross centered on the square dot. The latter procedure has been checked to give chaotic behavior in isolated cavities.[19] Placing the impurities on the surface sites[20,21] would require avoiding the leads' entrance sites (in order to avoid excessively strong scattering effects). This choice would give an amount of impurities that decreases with the width of the leads, and thus will hinder a study of the change of the fractal dimension with W/L.

In our model the electrons see a potential that jumps abruptly at the surface sites. This potential cannot of course model the parabolic potential produced by the gate voltage used in the experiments of Ref. 3. The *opening* of the cavity induced by this softening of the walls was simulated in the present case by increasing the width of the leads. As shown below, this procedure gives results for the variation of the fractal dimensions with the degree of opening of the cavity, in qualitative agreement with experimental observations.

B. Conductance

The conductance (measured in units of the quantum of conductance $G_0 = e^2/h$) was computed by using an efficient implementation of Kubo formula. A detailed description of the method can be found in Ref. 23, whereas applications to mesoscopic systems are described in Refs. 24 and 18. For a current propagating in the x direction, the static electrical conductivity is given by

$$G = -2\left(\frac{e^2}{h}\right)\mathrm{Tr}[(\hbar\hat{v}_x)\,\mathrm{Im}\,\hat{\mathcal{G}}(E)(\hbar\hat{v}_x)\,\mathrm{Im}\,\hat{\mathcal{G}}(E)], \tag{8}$$

where $\mathrm{Im}\,\hat{\mathcal{G}}(E)$ is obtained from the advanced and retarded Green functions,

$$\mathrm{Im}\,\hat{\mathcal{G}}(E)=\frac{1}{2i}[\hat{\mathcal{G}}^R(E)-\hat{\mathcal{G}}^A(E)], \tag{9}$$

and the velocity (current) operator $\hat{v}_x$ is related to the position operator $\hat{x}$ through the equation of motion $\hbar\hat{v}_x=[\hat{H},\hat{x}]$, $\hat{H}$ being the Hamiltonian.

Numerical calculations were carried out connecting quantum dots to semi-infinite leads of width W in the range $1-L$. The hopping integral inside the leads and between leads and dot at the contact sites is taken equal to that in the quantum dot (ballistic case). Assuming the validity of both the one-electron approximation and the linear response, the exact form of the electric field does not change the value of G. An abrupt potential drop at one of the two junctions provides the simplest numerical implementation of the Kubo formula[23] since, in this case, the velocity operator has finite matrix elements on only two adjacent layers and Green functions are just needed for this restricted subset of sites. Assuming this potential drop to occur at the left contact side, the velocity operator can be explicitly written as

$$i\hbar v_x=-\sum_{j=1}^{W}(|l,j\rangle\langle 1,j|-|1,j\rangle\langle l,j|) \tag{10}$$

where ($|l,j\rangle$ are the atomic orbitals at the left contact sites nearest neighbors to the dot.

Green functions are given by

$$[E\hat{I}-\hat{H}-\hat{\Sigma}_1(E)-\hat{\Sigma}_2(E)]\hat{\mathcal{G}}(E)=\hat{I}, \tag{11}$$

where $\hat{\Sigma}_{1,2}(E)$ are the self-energies introduced by the two semi-infinite leads.[25] The explicit form of the retarded self energy due to the mode of wave vector k_y is

$$\Sigma(E)=\frac{1}{2}\{E-\epsilon(k_y)-i\sqrt{4-[E-\epsilon(k_y)]^2}\}, \tag{12}$$

for energies within its band $|E-\epsilon(k_y)|<2$, where $\epsilon(k_y)=2\cos(k_y)$ is the eigenenergy of the mode k_y which is quantized as $k_y=(n_{k_y}\pi)/(W+1)$, n_{k_y} being an integer from 1 to W. The transformation from the normal modes to the local tight-binding basis is obtained from the amplitudes of the normal modes, $\langle n|k_y\rangle=\sqrt{2/(W+1)}\sin(nk_y)$.

C. Numerical procedures

The linear size of the square clusters was varied in the range $L=47$–197, and the energy was fixed at $E=-\pi/3$. Some calculations at an energy near the bottom of the band, and through the whole energy band, were also done. Input-output leads having a width of W were attached at opposite corners of the square: from site (1,1) to site $(1,W)$, and from (L,L) to $(L,L-W)$. We checked that, as far as fluctuations are concerned, the position of the leads is not a relevant parameter. In particular some calculations were done with the leads connected to contiguous corners as follows: first lead as before, and second from $(L,1)$ to (L,W). In order to allow for a reliable analysis of the graph of conductance vs magnetic flux, we have swept a range of Φ larger than that covered in the experiments. We have checked that fluctuations do not change much with the magnetic field, provided that the range of fluxes at which quantum interference effects are no longer present is not reached (see below). The flux

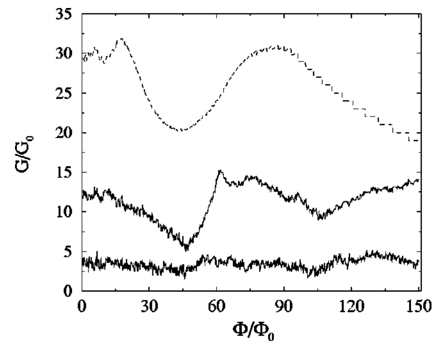

FIG. 1. Magnetoconductance vs magnetic flux (both in units of their respective quanta) in 97×97 regular dots ($\Delta=0$), with leads having widths W of 9 (lower curve), 25 (middle curve), and 57 (upper curve) and connected to opposite corners of the dot as described in the text. Calculations were done at an energy $E=-\pi/3$.

was varied in the range $(0-150)\Phi_0$. The corresponding cyclotron radii can be calculated from the expression for r_c rewritten in the units used here, namely, $\hbar^2/(2ma^2)=t=1$, where t is the hopping integral. Then

$$\frac{r_c}{L/2}=\hbar v(E)\frac{L}{2\pi}\frac{\Phi_0}{\Phi}, \tag{13}$$

where $\hbar v(E)=\langle 2\sqrt{\sin^2 k_x+\sin^2 k_y}\rangle_E$. For $E=-\pi/3$ $\hbar v(E)\approx 2.1$.[24] The minimum $r_c\approx 0.1(L/2)$ is reached for the smallest cavity ($L=47$) and the largest flux ($\Phi=150\Phi_0$), and is more than an order of magnitude smaller than those attained in the experimental work (see above).

It is interesting to estimate how the magnetic flux scales over which fluctuations are expected change with the system size. A small flux scale can be derived by equating the magnetic energy $\hbar\omega_c$ to the average level spacing which we approximate by $8/L^2$. This gives $\Delta\Phi=(2/\pi)\Phi_0$, suggesting that fluctuations are not expected to occur at scales much lower than one magnetic flux quanta, independently of the system size. As concerns the largest magnetic field, Eq. (13) indicates that it increases linearly with the linear size of the system L.

In doing the fractal analysis of the magnetoconductance graphs we note that they are self-affine (two different magnitudes in the two axes as in the Brownian motion; see Ref. 15). Thus we use the method proposed by Hurst,[15,22] also followed in Ref. 3. The application of the Hurst algorithm to the present case consists of (i) dividing the magnetic flux range in intervals of length $\Delta\Phi$; and (ii) determining $N(\Delta\Phi)$ as the difference between the maximum and minimum conductances in each interval summed over all intervals in the explored range, and divided by $\Delta\Phi$. This procedure corresponds, in the standard box-counting algorithm, to taking rectangular boxes with an infinitely small size in the conductance axis.

III. RESULTS

A. Magnetoconductance fluctuations

The effect of the leads width on magnetoconductance fluctuations is illustrated in Fig. 1. As the width decreases the graph shows stronger fluctuations, mainly due to the reduction of the momentum constraint. This applies to cavities

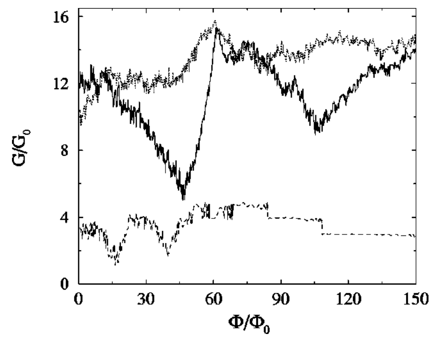

FIG. 2. Magnetoconductance vs magnetic flux (both in units of their respective quanta) in 97×97 dots with $\Delta=0$. The results correspond to energies $E=-3.5$ (dashed line) and $E=-\pi/3$ (continuous and dotted lines), and leads of width 25 attached at opposite (dashed and continuous lines) or contiguous (dotted line) corners of the square as described in the text.

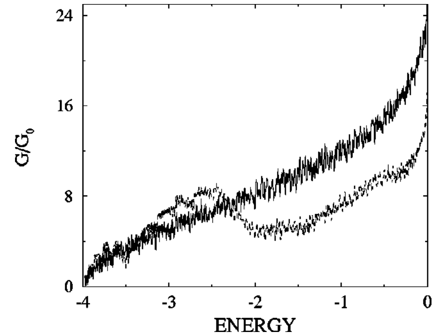

FIG. 3. Conductance fluctuations in the whole energy band of 97×97 dots with no disorder, and leads having a width of 25 and connected to opposite corners of the cavity as described in the text. The results correspond to $\Phi=0$ (continuous line) and $\Phi=40\Phi_0$ (dashed line).

with or without disorder, and independently of other parameters of the model (energy, leads configuration, etc.). For large W and above a given flux, the conductance coincides with that obtained for $W=L$ (a case without interference effects). In the latter case G decreases stepwise, each step (of one conductance quantum) being a consequence of transversal modes successively crossing the energy at which G is calculated. As the energy difference between the bottom of the transversal modes bands increases with the magnetic field[25] (actually in $\hbar\omega_c$, where $\omega_c=eB/m$ is the cyclotron frequency) the width of the steps in Fig. 1 also increases with B. The results for the energy near the bottom of the band show that the transition to the regime in which quantum interference is negligible occurs at a lower magnetic field (see Fig. 2). This is surely a consequence of the smaller velocity and, thus, cyclotron radius, of electrons at that energy.

For moderately wide leads the magnetoconductance shows two minima (probably a long period oscillation which extends up to higher magnetic fields). The period of the oscillation is almost independent of W (see Figs. 1 and 2), although it increases with the linear size of the system. For instance at $E=-\pi/3$ the flux increment between the two minima for $L=47$ and 97, are 31 and 60 flux quanta, respectively. The amplitude of the oscillation, however, decreases as the leads width W decreases: for $L=97$, and at the same Fermi energy, the conductance drop from zero flux to the first minimum is approximately two, seven, and ten conductance quanta for $W=9$, 17, and 25, respectively. The period also varies with the Fermi energy, being 60 and 24 for $E=-\pi/3$ and 3.5, respectively (the results correspond to $L=97$ and $W=25$). Although we do not have a fully satisfactory explanation for this long period oscillation, we suggest that it could be related to changes in the density of states induced by the magnetic field. As the magnetic field increases the energy levels segregate into bundles whose width, average energy, and number of levels vary with the magnetic field.

The dependence of the conductance G on energy E is illustrated in Fig. 3. The figure shows numerical results for $\Phi=0$ and $40\Phi_0$ on a cavity of linear size 97 and leads 25 wide. In the absence of magnetic field G increases, on average, as a function of E. This is a consequence of the increasing number of transversal modes that contribute to the current. Fluctuations produced by quantum interference are also to be noted. At a finite magnetic field the conductance shows a more complex behavior, probably as a consequence of the oscillation discussed in the preceding paragraph.

Cavities with disorder (chaotic) also show fluctuations at a small flux scale, as illustrated in Fig. 4. The magnetoconductance curve is less spiky than in regular cavities, albeit globally they are not much different. Fluctuations remain even for leads as wide as the dot itself $(W=L)$ as a consequence of the lack of the momentum constraint induced by disorder. However, the long-period oscillation observed in regular cavities is no longer present.

B. Fractal dimension

The analysis of the magnetoconductance curves is illustrated in Fig. 5, where results for the number of boxes $N(\Delta\Phi)$ required to cover the curves and the variance of Eq. (3) are plotted versus the flux increment $\Delta\Phi$. Fittings for the number of boxes are far better than for the variance. This has also been reported to occur in the analysis of the fractional Brownian motion.[15] In any case the fitted exponents (shown in the caption of the figure) fulfill Eq. (4) within 10%. The number of decades where a power law is followed increases with the system size rather slowly. Thus we have not gone

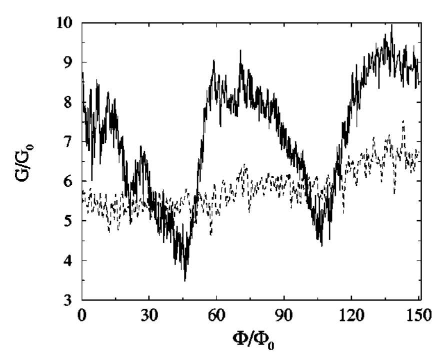

FIG. 4. Magnetoconductance vs magnetic flux in 97×97 dots either with no disorder $\Delta=0$ (continuous line) or with disorder $(\Delta=6)$ on $2L$ impurity sites in cross (broken line). The results correspond to an energy $E=-\pi/3$ and leads of width 17 connected to opposite corners of the dot, as described in the text.

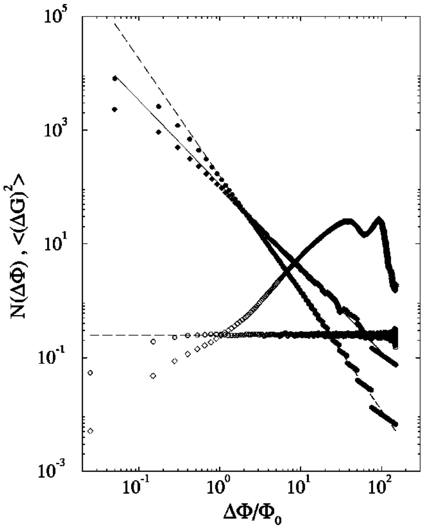

FIG. 5. Fractal analysis of magnetoconductance fluctuations in 97×97 regular dots ($\Delta=0$), with leads of widths 1 (circles) and 33 (diamonds) attached at opposite corners of the dot as discussed in the text, and an energy $E=-\pi/3$. Filled symbols correspond to the number of boxes $N(\Delta\Phi)$, and empty symbols to the variance of conductance increments $\langle(\Delta G)^2\rangle$. The numerical results were fitted by $N(\Delta\Phi)\propto(\Delta\Phi)^{-D}$ and $\langle(\Delta G)^2\rangle\propto(\Delta\Phi)^{\gamma}$, obtaining $W=1$, $D=2.06$, $\gamma=0.004$, $W=33$, $D=1.47$, and $\gamma=1.42$. All fittings were carried out in the whole range, but for the variance of dots with $W=33$ which was done for $\Delta\Phi=1$–$10\ \Phi_0$.

beyond $L=197$. On the other hand, and in order to do a similar analysis for all sizes, we have carried out fittings in the range $\Delta\Phi=(1$–$10)\Phi_0$. We have checked for the larger sizes (as in Fig. 5) that increasing this range does not significantly alter the exponents. It is also interesting to note that the fittings are poor for small values of $\Delta\Phi$, as expected.[4]

A technical point is worthy of comment. It concerns the determination of the fractal dimension in cases where the flux range in which interference effects are absent is reached within the explored range of magnetic fields, such as those of Figs. 1 and 2. In those cases the region where the conductance decreases stepwise, if included in the analysis, may affect D in a rather spurious way. We have analyzed this point on the results for the two cases of those figures which show that behavior. In the case of $E=-3.5$ and leads 25 wide (Fig. 1) the results are $D=1.49$ and 1.55 for fits done over either $\Phi=0$–150 or $\Phi=0$–70, while for $E=-\pi/3$ and leads of $W=57$ (Fig. 2) $D=1.13$ no matter whether the mentioned flux range was or was not included in the fittings. The effect is larger in the low-energy case, as the smaller electron velocity (and, thus, smaller cyclotron radius) induces the entering into that regime at lower degrees of opening (smaller W) when conductance fluctuations are still important and, consequently, D is large. In any case the effect can be considered as small, and fittings can be safely done in the whole flux range (induced errors are always well below 10% in the cases investigated here).

The dependence on energy and lead configuration has been explored on $L=97$ dots with leads of width $W=25$ (Fig. 2). For leads at opposite corners fittings in the flux range $\Delta\Phi=(1$–$10)\Phi_0$ give fractal dimensions of 1.46 and 1.49 for $E=-\pi/3$ and $E=-3.5$ (see Fig. 6). If the fittings are done over the whole range of the figure these results are

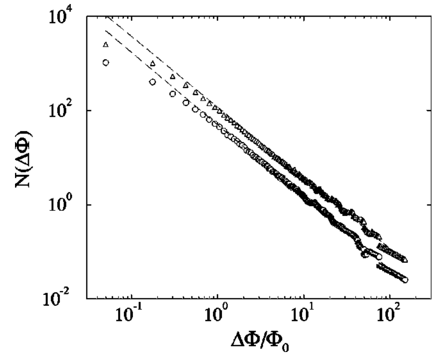

FIG. 6. Fractal analysis of magnetoconductance fluctuations in 97×97 regular dots ($\Delta=0$), with leads having a width $W=25$ and connected to opposite corners of the square dot as discussed in the text. Filled circles correspond to an energy $E=-3.5$ and triangles to $E=-\pi/3$. The numerical results for the number of boxes $N(\Delta\Phi)$ were fitted by $N(\Delta\Phi)\propto(\Delta\Phi)^{-D}$, with $D=1.46$ for $E=-\pi/3$ and $D=1.49$ for $E=-3.5$ for fittings done in the range $\Delta\Phi=1-10\ \Phi_0$, and $D=1.50$ and 1.53 if done over the whole range of the figure.

changed to 1.50 and 1.53, respectively. These results illustrate the almost null dependence of the fractal dimension on the Fermi energy, suggesting that the number of open modes is not an important factor as far as fluctuations are concerned. If leads are placed at contiguous corners the results are not changed significantly: $D=1.57$ for both energies if the fitting is done over the restricted range, and $D=1.52$ and 1.58 for $E=-\pi/3$ and $E=-3.5$ when fits are carried out over the whole range. Differences are in the range 5–10%. We can conclude that D is not much affected either by the energy at which G is calculated (or measured) or by lead configuration.

In Fig. 7 we plot the fractal dimension for three cluster sizes versus the ratio W/L. The collapse of the numerical results indicate that the relevant parameter is in fact the ratio W/L. D monotonically decreases from 2 to 1 as W/L increases from $1/L$ to 1. In the latter case we do not obtain $D=1$ for regular cavities due to numerical inaccuracies. Our results clearly show that results for D greater than 3/2 are possible, in apparent contradiction with the remarks made in Ref. 3. Introducing disorder has no apparent effect on D for

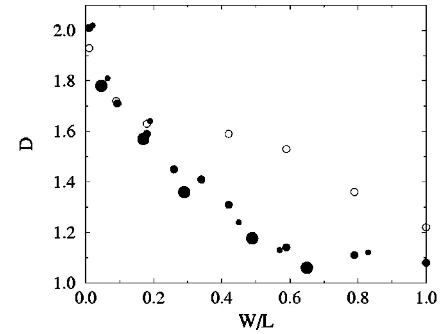

FIG. 7. Fractal dimension of magnetoconductance fluctuations, vs the ratio W/L, where W is the width of the leads and L the linear dimension of the system. The size of the circles is proportional to the system size. The results correspond to regular (filled circles) and chaotic (empty circles) cavities of linear sizes $L=47$, 97, and 197. For chaotic cavities, $\Delta=6$ was used.

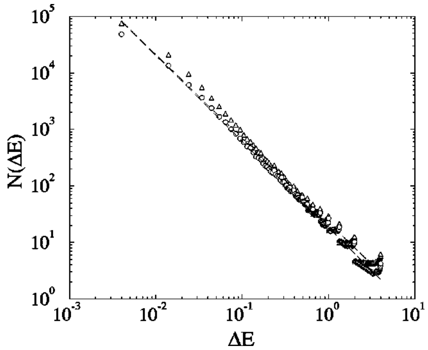

FIG. 8. Fractal analysis of the conductance vs energy curves of Fig. 3. The results are for a 97×97 regular dots with no disorder $(\Delta=0)$, with leads of width $W=25$ attached at opposite corners of the square dot. Triangles correspond to $\Phi=0$ and circles to $\Phi=40\Phi_0$. The numerical results for the number of boxes $N(\Delta E)$ were fitted by $N(\Delta E)\propto(\Delta E)^{-D}$, with $D=1.49$ for $\Phi=0$ and $D=1.52$ for $\Phi=40\Phi_0$. The fittings were done in the whole range.

W/L smaller than 0.2. For sufficiently wide leads, however, disorder increases noise and thus D. The overall behavior of D versus W/L is, in any case, very similar to that found for regular cavities. We have checked that the spatial distribution of the impurities does not qualitatively change these results, although actual values of D may of course be affected. This indicates that the nature of the closed cavity at $B=0$ is not a determining factor as far as the self-similarity of fluctuations is concerned. For very closed cavities (small leads width, low wall softening, etc.) the fractal dimension is near 2 (no fractal character) no matter the type of the cavity, and D decreases down to ≈ 1 with the degree of opening, also for all cavities, although the limit $D=1$ is probably never reached in chaotic cavities (see above). It is also interesting to note that the fact that D does not depend on the cavity size indicates that it is not affected by the strength of the magnetic field. In fact the cyclotron radius varies in a factor of 4 from the smallest to the largest cavity investigated in this work, without a significant change in D.

The above discussion suggests that what matters as far as the fractality of fluctuations is concerned is the degree of opening of the cavity. In particular in our case this is controlled by the parameter W/L. To provide a further illustration of this point, we have calculated the fractal dimension of the conductance versus energy curves of Fig. 3. Using E as a tunable parameter can be attained experimentally by changing a gate voltage. The fractal analysis shown in Fig. 8 gives fractal dimensions $D=1.49$ and 1.52 for $\Phi=0$ and 40, respectively. These numbers are quite close to those obtained for curves of the conductance versus the magnetic flux (see Fig. 6), giving support to our conclusions.

C. Comparison with experiment

The results reported in Fig. 7 are in line with the decrease in D observed experimentally as the degree of softness of the cavity walls was increased.[3] A more detailed comparison with experimental results is worthwhile. In Ref. 3 a fractal dimension of 1.35 was reported for the less opened cavity (this is the result for the most confining voltage reported in Ref. 3). The cavity had a leads width to lithographic radius ratio of ≈ 0.64. For such a ratio our results give $D\approx 1.4$ and 1.1 for a square cavity with or without disorder, respectively. As the cavity investigated in Ref. 3 had a stadium shape, the results should be compared with those corresponding to cavities with disorder.[20] The agreement is quite satisfactory.

Concerning the results of Ref. 2, for which the authors of Ref. 3 reported $D\approx 1.3$, we note that the discrepancy with our results is rather appreciable. In Ref. 2 W/L was 0.2, for which the numerical results of Fig. 7 give $D=1.5$–1.6. The slight difference between the fractal dimensions reported for the two experiments (1.35 and 1.3, respectively) cannot be understood in the light of the present analysis, as their respective W/L ratio differ in approximately a factor of 4.

IV. CONCLUDING REMARKS

We have presented a numerical study of magnetoconductance fluctuations in regular and chaotic cavities addressed to investigate how the fractal dimension of the magnetoconductance curves depends on the system parameters. Our results indicate that the relevant parameter as far as the fractal dimension is concerned is the degree of opening of the cavity, which in this work was changed by varying the width of the leads attached to the dot. D decreases with the degree of opening ($D=1$–2 as the ratio W/L decreased from 1 to $1/L$), in line with an experimental investigation in which dot opening was tuned by means of a gate voltage that softened the dot walls. Note that values of D greater 1.5 are incompatible with the semiclassical result of Eq. (4), once the proper bounds for the exponent γ are taken into account (see above and in Ref. 13). We have not yet investigated the classical dynamics of our model (a billiard with boundaries at which reflection is not specular) and, in particular, whether the survival probability decays as a power law and, if so, how the classical power-law exponent γ varies with W/L. Thus, it is not clear whether the semiclassical theory of Ref. 4 should apply at all to the quantum model we have studied numerically.[26,27]

The results also indicate that, for sufficiently wide leads, fluctuations are more important in chaotic than in regular cavities, likely due to a reduction of the momentum constraint in the former. As a consequence the fractal dimension is larger in chaotic than in regular cavities for the same degree of opening.

We have also investigated the effects of other system parameters such as lead configuration and Fermi energy, and showed that they do not significantly alter the fractal dimension. This conclusion is further supported by numerical results for the fractal dimension of conductance versus energy curves. The results for D are practically identical to those obtained for curves of conductance versus magnetic flux in cavities with the same W/L ratio. We hope that these results will help in the analysis of future experimental studies of this topic.

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