Chiral electromagnetic waves at the boundary of optical isomers: Quantum Cotton-Mouton effect

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We demonstrate that the boundary of two optical isomers with opposite directions of the gyration vectors (both parallel to boundary) can support propagation of electromagnetic waves in the direction perpendicular to the gyration axes (Cotton-Mouton geometry). The components of electromagnetic field in this wave decay exponentially into both media. The characteristic decay length is of the order of the Faraday rotation length for the propagation along the gyration axis. The remarkable property of the boundary wave is its *chirality*. Namely, the wave can propagate only in *one* direction determined by the relative sign of nondiagonal components of the dielectric tensor in contacting media. We find the dispersion law of the boundary wave for the cases of abrupt and smooth boundaries. We also study the effect of asymmetry between the contacting media on the boundary wave and generalize the results to the case of two parallel boundaries. Finally we consider the arrangement when the boundaries form a random network. We argue that at a point, when this network percolates, the corresponding boundary waves undergo quantum delocalization transition, similar to the quantum Hall transition.

Optical properties of gyrotropic and optically active media are well known and described in many textbooks (see, e.g., Ref. 1). A homogeneous gyrotropic medium is characterized by the following relation between the displacement vector and electric field $\mathbf{D}=\hat{\varepsilon}\mathbf{E}$, where the tensor $\hat{\varepsilon}$ has the following form:

$$\hat{\varepsilon}=\begin{pmatrix} \varepsilon_0 & ig & 0 \\ -ig & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_1 \end{pmatrix}. \qquad (1)$$

For a wave propagating along the z direction, the wave equation

$$\nabla(\nabla\mathbf{E})-\nabla^2\mathbf{E}=-\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2} \qquad (2)$$

has two circular polarized solutions characterized by the dispersion laws

$$k_{\pm}=\frac{\omega}{c}\sqrt{\varepsilon_0\pm g}. \qquad (3)$$

A linear polarized wave, incident on this medium along the axis of gyration z splits into two circular polarized waves propagating with velocities $c/(\varepsilon_0\pm g)^{1/2}$. Since g is always much smaller than ε_0, this can be viewed as rotation of the plane of polarization (Faraday effect). The distance at which the plane of polarization is rotated by 90° is equal to $l_\omega=\pi\sqrt{\varepsilon_0}c/g\omega$, where ω is the frequency of the wave. The direction of rotation is determined by the sign of the nondiagonal component g of the tensor (1). Correspondingly, in the case of optical activity, there exist left-rotating (*levorotatory*) and right-rotating (*dextrorotatory*) media. Two modifications of a crystal differing by the sign of g are called optical isomers. For example, the parameters of tensor $\hat{\varepsilon}$ for the quartz crystal are known to be[1] $\varepsilon_0=2.3839, \varepsilon_1=2.4118$ and $g=1.1\times10^{-4}$. Consequently, $l_\omega\approx4$ mm for yellow light ($\lambda=589$ nm). Isomers with tensor $\hat{\varepsilon}$ in the form (1) can only belong to the certain crystalline symmetry classes listed, e.g., in Ref. 2.

When the direction of propagation is perpendicular to the gyration axis, there also exist two types of solutions of the wave equation (2). First, there is a trivial solution for which only the E_z component is nonzero and the spectrum is $\omega=kc/\sqrt{\varepsilon_1}$. The nontrivial solution (Cotton-Mouton effect) corresponds to polarization perpendicular to the axis of gyration

$$\mathbf{E}=E_0\left(1,\frac{c^2k_y^2-\omega^2\varepsilon_0}{c^2k_xk_y+ig\omega^2},0\right)e^{ik_xx+ik_yy}, \qquad (4)$$

with the dispersion law

$$k_x^2+k_y^2=\frac{\omega^2}{c^2}\left(\varepsilon_0-\frac{g^2}{\varepsilon_0}\right). \qquad (5)$$

The phenomenon of surface electromagnetic waves is also a well-studied subject (see, e.g., Ref. 3). Surface waves can propagate along the boundary of two isotropic media with dielectric constants $\varepsilon_a(\omega)$ and $\varepsilon_b(\omega)$ when two conditions are met:[3] (i) $\varepsilon_a(\omega)\varepsilon_b(\omega)<0$ and (ii) $\varepsilon_a(\omega)+\varepsilon_b(\omega)<0$. The polarization of these waves is (transverse-magnetic) TM. For such waves the localization of electromagnetic field at the boundary is provided by the negative sign of ε in one of the contacting media. Interesting examples of surface electromagnetic wave at the boundary of two *transparent media* were given in Refs. 4 and 5. In Ref. 4 the boundary between isotropic and uniaxial media is considered. In Ref. 5 both contacting media represent uniaxial crystals with certain mismatch in the directions of optical axes.

In the present paper we show that the boundary of two optical isomers with opposite directions of axes of optical activity (both parallel to the boundary) can also support propagation of localized electromagnetic waves. In contrast to the conventional surface plasmons,[3] these states exist for *positive* values of components ε_0 and ε_1. The distinguishing feature of these boundary waves is a *unidirectional* character of propagation, i.e., they propagate along the boundary of two media with opposite gyration vectors *only in one direction*, which is determined by the relative sign of g. We also argue that random arrangement of boundaries provides an optical realization of the quantum delocalization transition for the boundary waves, which is similar to the quantum Hall transition for two-dimensional electrons.[6]

Let the plane $x=0$ be a boundary between two isomers. For generality we consider g to be some function of $x, g(x)$, changing its value from $g(x)=g_0$ to $g(x)=-g_0$ within some region around $x=0$. In the Cotton-Mouton geometry, when only x and y components of the electric field are nonzero, the system of equations for E_x, E_y which follows from Eq. (2) takes the form

$$\frac{\partial^2 E_y}{\partial x \partial y} - \frac{\partial^2 E_x}{\partial y^2} = \frac{\omega^2}{c^2}[\varepsilon_0 E_x + i g(x) E_y], \tag{6}$$

$$\frac{\partial^2 E_x}{\partial x \partial y} - \frac{\partial^2 E_y}{\partial x^2} = \frac{\omega^2}{c^2}[\varepsilon_0 E_y - i g(x) E_x]. \tag{7}$$

Let us search for a solution propagating along the y axis: $E_x \propto e^{iky}$, $E_y \propto e^{iky}$. Then from Eq. (6) we can express E_x through E_y,

$$E_x = \frac{ikc^2 \dfrac{\partial E_y}{\partial x} - i\omega^2 E_y g(x)}{\varepsilon_0 \omega^2 - k^2 c^2}. \tag{8}$$

Substituting Eq. (8) into Eq. (7) we get

$$-\frac{\partial^2 E_y}{\partial x^2} + \left[\frac{k}{\varepsilon_0}\frac{\partial g(x)}{\partial x} + \frac{\omega^2}{c^2 \varepsilon_0}[g^2(x) - g_0^2]\right] E_y = \left[\frac{\omega^2}{c^2}\left(\varepsilon_0 - \frac{g_0^2}{\varepsilon_0}\right) - k^2\right] E_y. \tag{9}$$

Note now that Eq. (9) has the form of the Schrödinger equation with an effective potential

$$U(x) = \frac{k}{\varepsilon_0}\frac{\partial g(x)}{\partial x} + \frac{\omega^2}{c^2 \varepsilon_0}[g^2(x) - g_0^2], \tag{10}$$

and energy

$$\mathcal{E} = \frac{\omega^2}{c^2}\left(\varepsilon_0 - \frac{g_0^2}{\varepsilon_0}\right) - k^2. \tag{11}$$

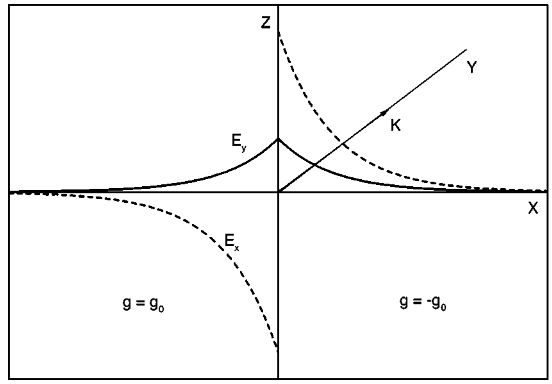

FIG. 1. The boundary of two optical isomers ($x=0$). Boundary wave propagates along the y axis. The decay of E_x (dashed curve) and E_y (solid curve) components of electric field away from the boundary $x=0$ is shown schematically.

Consider first the simplest case when the transitional region between two media is infinitely narrow: $g(x) = -g_0[2\theta(x) - 1]$, where $g_0 > 0$ and $\theta(x)$ is a step function (see Fig. 1). Then $U(x)$ takes the form $U(x) = -2g_0 k \delta(x)/\varepsilon_0$. It is seen that the potential is attractive for the positive values of k and repulsive for the negative k. This illustrates the above statement about the *chirality* of the boundary states. It is well known that there exists only a single bound state in attractive δ potential with an arbitrary magnitude.[7] The corresponding ''binding energy'' is equal to $\mathcal{E} = -g_0^2 k^2/\varepsilon_0^2$. This leads to the following dispersion law for electromagnetic wave propagating along the abrupt boundary between two media with $g = \pm g_0$:

$$\omega = \frac{ck}{\sqrt{\varepsilon_0}}. \tag{12}$$

The origin of the localized boundary state Eq. (12) is the following. It is seen from Eq. (5) that the maximal possible wave vector inside each of the contacting media is equal to $(\omega/c)(\varepsilon_0 - g_0^2/\varepsilon_0)^{1/2}$. The wave vector determined by Eq. (12) exceeds this maximal wave vector. As a result, both components of electric field decay to the left and to the right from the boundary $x=0$:

$$E_y(x) = E_y(0) e^{-q|x|}, \tag{13}$$

$$E_x(x) = sign(x) \frac{i(\varepsilon_0^2 + g_0^2)}{2 g_0 \varepsilon_0} E_y(0) e^{-q|x|},$$

where $q = |k g_0|/\varepsilon_0$ (Fig. 1). The characteristic decay length $1/q$ can be expressed through the length of rotation of plane of polarization of light in the Faraday geometry: $1/q = l_\omega/\pi$.

It is easy to establish certain properties of chiral boundary states.

(a) Asymmetry between the contacting media. Suppose that the mirror symmetry between the contacting media is lifted, say, due to external magnetic field applied in the z direction. This amounts to the following modification of $g(x)$: $g(x) = -g_0[2\theta(x) - 1] + g_1$, where g_1 is proportional to the magnetic field. Then it is straightforward to check that the dispersion law Eq. (12) remains unchanged, while the

magnitude of g_1 remains smaller than g_0. When g_1 exceeds g_0 the bound state *disappears abruptly*. Despite the dispersion law being insensitive to the asymmetry for $g_1<g_0$, the distribution $E_y(x)$ changes strongly. For $g_1\neq 0$ instead of Eq. (13) we have

$$E_y(x)=E_y(0)\exp\left[-\frac{\omega}{c\sqrt{\varepsilon_0}}(g_0-g_1)x\right],\quad x>0, \quad (14)$$

$$E_y(x)=E_y(0)\exp\left[\frac{\omega}{c\sqrt{\varepsilon_0}}(g_0+g_1)x\right],\quad x<0. \quad (15)$$

(b) Two boundaries. Consider now the case of two boundaries

$$g(x)=-g_0,\quad |x|>l/2, \quad (16)$$

$$g(x)=g_0,\quad |x|<l/2. \quad (17)$$

It is straightforward to analyze Eq. (9) in this case. Indeed, the effective potential takes the form

$$U(x)=\frac{2g_0k}{\varepsilon_0}[\delta(x-l/2)-\delta(x+l/2)]. \quad (18)$$

The corresponding dispersion law for the boundary state becomes

$$k^2=\frac{\omega^2}{c^2}\left(\varepsilon_0-\frac{g_0^2}{\varepsilon_0}\right)+q_l^2, \quad (19)$$

where q_l is determined by the equation

$$q_l^2\left[\frac{1}{1-e^{-2q_l l}}-\frac{g_0^2}{\varepsilon_0^2}\right]=\frac{\omega^2g_0^2}{c^2\varepsilon_0}\left[1-\frac{g_0^2}{\varepsilon_0^2}\right]. \quad (20)$$

In the limit $l\to\infty$ we return to the dispersion law Eq. (12), which for positive k corresponds to the wave propagating along the boundary $x=l/2$; negative k corresponds to the wave propagating along the boundary $x=-l/2$. Inspection of Eq. (20) shows that it has solution for *an arbitrary small l*. In the case when the distance between boundaries is much smaller than the localization length l_ω, we get from Eq. (20),

$$q_l=\frac{l}{\pi l_\omega^2}<<l_\omega^{-1}. \quad (21)$$

This means that both states (with positive and negative k) are ''weakly bound.'' This is illustrated in Fig. 2 for different l/l_ω and positive k. For negative k the distribution of electric field corresponds to the change $x\to -x$.

(c) Two boundaries with external magnetic field. We have also studied the suppression of the states associated with a pair of boundaries by an external magnetic field. In this case the critical value of g_1 depends on the distance between boundaries. The electric field in the region $|x|<l$ represents the superposition of exponents $\exp(\pi\kappa x/l_\omega)$ and $\exp(-\pi\kappa x/l_\omega)$, where the dimensionless parameter κ satisfies the equation

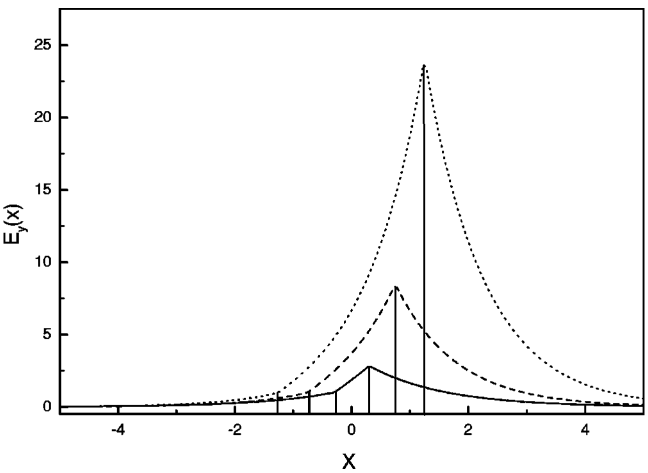

FIG. 2. $E_y(x)$ component of electric field for the case of two boundaries. The location of the boundaries are $x/l_\omega=\pm 1.25$ (dotted line), $x/l_\omega=\pm 0.75$ (dashed line), and $x/l_\omega=\pm 0.3$ (solid line).

$$\kappa^2\left[1+\frac{2}{\kappa^2}\frac{g_1}{g_0}-\frac{2g_0^2}{\varepsilon_0^2}+\frac{1+\exp(-2L^*\kappa)}{1-\exp(-2L^*\kappa)}\left(1+\frac{4g_1}{g_0\kappa^2}\right)^{1/2}\right]$$
$$=2\left[1-\frac{(g_0-g_1)^2}{\varepsilon_0^2}\right], \quad (22)$$

with $L^*=\pi l/l_\omega$. In Fig. 3 we present the critical line of g_1/g_0 as a function of L^*, which determines the domain where the solution of Eq. (22) exists. Note, that the boundaries are asymmetric with respect to the change of the direction of magnetic field (the sign of g_1).

(d) Smooth boundary. Next we consider the case when the boundary is smooth and has a characteristic width of b. This, for example, can be a result of mutual diffusion of isomers. Then $g(x)$ can be modeled by $g(x)=-g_0\tanh(x/b)$. Thus, for the effective potential $U(x)$ we get

$$U(x)=-\frac{g_0(kc^2+\omega^2g_0b)}{\varepsilon_0bc^2\cosh^2(x/b)}. \quad (23)$$

The solutions of the Schrödinger equation with potential Eq. (23) are well known:[7]

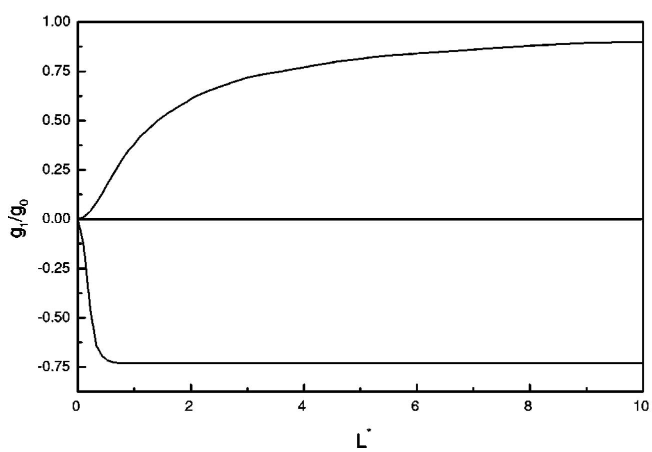

FIG. 3. Critical line g_1/g_0 as a function of L^* for two opposite directions of external magnetic field.

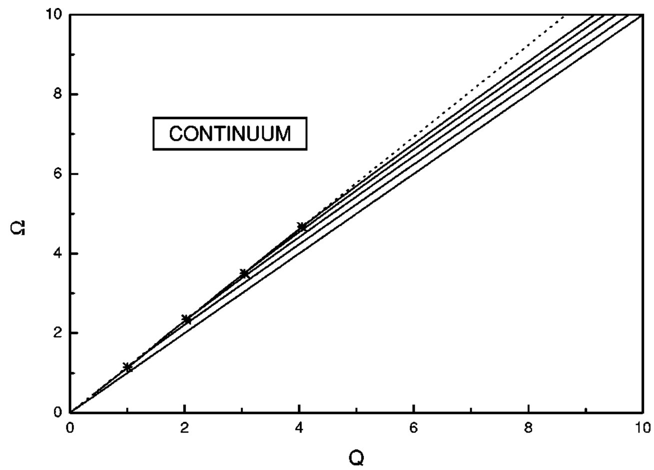

FIG. 4. Dispersion relations for the guided modes with $n=0,1,2,3,4$ are plotted using Eq. (28). For illustration purposes we chose an unrealistically large value of parameter $\tau=0.5$.

$$\mathcal{E}_n=-\frac{1}{4b^2}\left[\sqrt{1+4\left(\frac{kg_0b}{\varepsilon_0}+\frac{g_0^2b^2\omega^2}{\varepsilon_0c^2}\right)}-(2n+1)\right]^2. \tag{24}$$

The mode with $n=0$ has no threshold frequency. The threshold frequencies ω_n for the modes with $n>0$ are determined from the condition $\mathcal{E}_n=0$:

$$\omega_n=\frac{c\sqrt{\varepsilon_0}}{2g_0b}\left[\sqrt{(2n+1)^2-\frac{g_0^2}{\varepsilon_0^2}}-\sqrt{1-\frac{g_0^2}{\varepsilon_0^2}}\right]. \tag{25}$$

The corresponding dispersion laws $\omega_n(k)$ can be conveniently presented after introducing a dimensionless frequency Ω and the wave vector Q:

$$\Omega=\frac{\tau\sqrt{\varepsilon_0}b}{c}\omega, \quad Q=\tau bk, \tag{26}$$

where the dimensionless parameter τ is defined as

$$\tau=\frac{g_0}{\varepsilon_0}. \tag{27}$$

Then from Eqs. (11) and (24) we have

$$\begin{aligned}\Omega(Q)=\frac{1}{2}\{&[(2n+1)\tau^2\\&+\sqrt{(2Q+1-\tau^2)^2-4n(n+1)\tau^2(1-\tau^2)}]^2\\&-(1+4Q)\}^{1/2}.\end{aligned} \tag{28}$$

The dispersion law for the first five modes is shown in Fig. 4.

Let us consider qualitatively the situation when the nondiagonal component of the tensor $\hat{\varepsilon}$ is a random function of both coordinates x and y. Suppose for simplicity that the correlation length of $g(x,y)$ is much bigger than l_ω. Then it is obvious from the above consideration that the boundary waves would circulate along the contours $g(x,y)=0$. If the average value $\overline{g(x,y)}$ is negative [Fig. 5(a)] or positive [Fig. 5(b)], and comaparable to $[\overline{g^2(x,y)}]^{1/2}$, these contours are disconnected, and, correspondingly, the boundary waves are localized. As $\overline{g(x,y)}$ approaches zero, the contours, defined by the condition $g(x,y)=0$, grow in size and form a network. The correlation size of the network diverges as $[\overline{g(x,y)}]^{-\nu_1}$, where $\nu_1\approx 4/3$ is the critical exponent of the percolation problem in two dimensions.[8] Along with decreasing of $\overline{g(x,y)}$, the neighboring contours $g(x,y)=0$ come closer than l_ω, and the scattering between the left- and right-moving boundary waves, encircling these contours, becomes increasingly important. As a result of this scattering, the interference of different unidirectional paths comes into play. The crucial role of interference effects, allowed by the coupling, was first pointed out in a seminal paper (Ref. 9) in relation to the integer quantum Hall effect. In Ref. 9 unidirectional waves modeled the motion of a two-dimensional electron in a strong perpendicular magnetic field and a smooth random potential (edge states). It was demonstrated in Ref. 9 that, with interference taken into account, the delocalization transition in the system of edge states occurs at *discrete* energies, i.e., it remains infinitely sharp (as is the case for the classical percolation). However, due to the coupling and interference, the size of the eigenstates in the critical region (localization radius) is much bigger than the correlation radius of the network and diverges with the exponent[9] $\nu_2\approx 2.3$. The correspondence between the edge states of electrons and the boundary electromagnetic waves

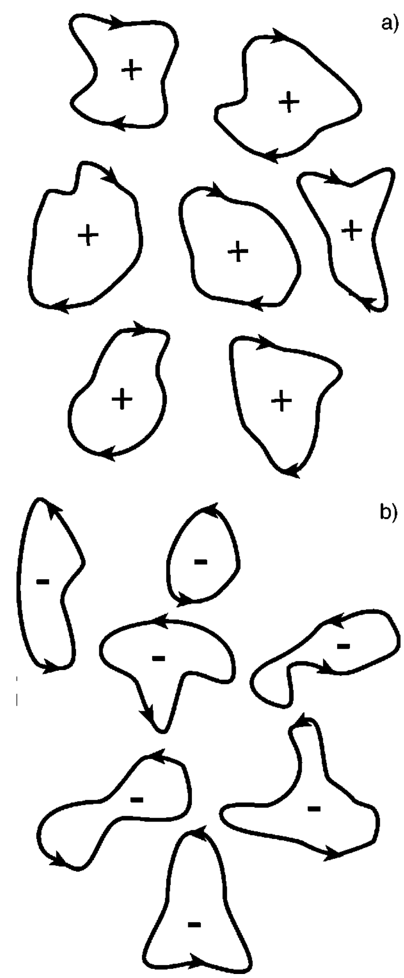

FIG. 5. The contours $g(x,y)=0$ are shown schematically for the cases (a) $\overline{g(x,y)}<0$ and (b) $\overline{g(x,y)}>0$. The arrows show the direction of propagation of boundary waves.

allows us to conclude that with $\overline{g(x,y)} \rightarrow 0$ the radius of localized boundary waves behaves as $[\overline{g(x,y)}]^{-\nu_2}$.

By analogy to the integer quantum Hall transition, which originates from the competition of the unidirectional motion of an electron in a magnetic field and quantum interference,[10,11] the delocalization transition of the boundary waves at $\overline{g(x,y)}=0$ (when optical activity of the system is zero *on average*) can be called the quantum Cotton-Mouton effect.

Note that throughout the paper we assumed that the system is uniform in the z direction. The situation relevant for experiment is a thin-film geometry, in which two contacting optically active media are confined within the region $|z| < d/2$ with the thickness d of the film being much smaller than l_ω. If ε_0 exceeds the dielectric constants of the media, between which the film is sandwiched, the solutions of the Maxwell equations are the waveguide modes, propagating along the plane of the film. The components of electric and magnetic fields in these modes are confined within the region of the order of d. Suppose first, that g is constant within the film: $g(z)=g_0\theta(d/2-|z|)$. To modify the Cotton-Mouton dispersion law [Eq. (5)] to the case of the waveguide mode propagating in the y direction, it is convenient to rewrite the system of Maxwell's equations in the following form:

$$-\frac{\partial^2 E_x}{\partial z^2}-\left(\varepsilon_0(z)\frac{\omega^2}{c^2}-k^2\right)E_x = -\frac{\omega^2}{c^2}\frac{g^2(z)}{\varepsilon_0(z)}E_x-\frac{\omega}{c}\frac{g(z)}{\varepsilon_0(z)}\frac{\partial B_x}{\partial z}, \qquad (29)$$

$$-\varepsilon_0(z)\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon_0(z)}\frac{\partial B_x}{\partial z}\right)-\left(\varepsilon_0(z)\frac{\omega^2}{c^2}-k^2\frac{\varepsilon_0(z)}{\varepsilon_1(z)}\right)B_x = \frac{\omega}{c}\frac{\partial}{\partial z}(g(z)E_x), \qquad (30)$$

where $\varepsilon_0(z)$ and $\varepsilon_1(z)$ describe the profile of the diagonal components of $\hat{\varepsilon}$ in the z direction. For $g_0=0$, the right-hand sides (rhs's) in Eqs. (29) and (30) are zeros, so that the above equations yield the sets of TE and TM waveguide modes, respectively. With the right-hand sides included, the correction to the wave vector k is quadratic in g. For the TE mode with a number n, after some algebra one can get the following dispersion law:

$$k^2=[k_{TE}^{(n)}(\omega)]^2-\frac{\omega^2}{c^2}[(g_{eff}^{(1)})^2-(g_{eff}^{(2)})^2], \qquad (31)$$

where

$$(g_{eff}^{(1)})^2=g_0^2\left[\frac{\int_{-d/2}^{d/2}dz\,\frac{(E_x^{(n)})^2}{\varepsilon_0(z)}}{\int_{-\infty}^{\infty}dz(E_x^{(n)})^2}\right], \qquad (32)$$

and

$$(g_{eff}^{(2)})^2=g_0^2\left[\sum_m \frac{\int_{-\infty}^{\infty}dz\,\frac{B_x^{(m)}E_x^{(n)}}{\varepsilon_0(z)}}{\int_{-\infty}^{\infty}dzB_x^{(m)}E_x^{(n)}\frac{\partial^2}{\partial z^2}[\varepsilon_0(z)^{-1/2}]} \times\frac{\int_{-d/2}^{d/2}dz\,\frac{E_x^{(n)}}{\varepsilon_0(z)}\frac{\partial B_x^{(m)}}{\partial z}\int_{-d/2}^{d/2}dzE_x^{(n)}\frac{\partial}{\partial z}\left(\frac{B_x^{(m)}}{\varepsilon_0(z)}\right)}{\int_{-\infty}^{\infty}dz(E_x^{(n)})^2\int_{-\infty}^{\infty}dz\frac{(B_x^{(m)})^2}{\varepsilon_0(z)}}\right]. \qquad (33)$$

The correction $(g_{eff}^{(1)})^2$ originates from the g^2 term in the rhs of Eq. (29), whereas $(g_{eff}^{(2)})^2$ results from the mixing of the TE mode n with all TM modes. Generally speaking, $g_{eff}^{(1)}$ and $g_{eff}^{(2)}$ are of the same order. This means that, when g depends on x and changes sign, the corresponding boundary wave would represent a mixture of TE and TM modes. The situation simplifies if, for numerical reasons, $g_{eff}^{(1)}$ appears to be much bigger than $g_{eff}^{(2)}$. This is the case when the thickness d of the film is much smaller than the transverse size of the waveguide mode. Then the dispersion law for the boundary wave, analogous to Eq. (12), takes a simple form $k=k_{TE}^{(n)}(\omega)$, and the electromagnetic field decays away from the boundary $x=0$ as $\exp[-(\omega/c)g_{eff}^{(1)}|x|]$.

In conclusion, in this paper we have demonstrated that the Maxwell equations for the wave, propagating along the boundary of two optical isomers, possess a nontrivial solution *localized at the boundary*. This solution is chiral, in the sense that it can propagate only in one direction. We have also traced the analogy between the boundary electromagnetic waves and the edge states in the integer quantum Hall effect. By virtue of this analogy, we argue that in a medium with a random nondiagonal component $g(x,y)$ of the dielectric tensor the boundary waves undergo the delocalization transition when $g(x,y)$ is zero on average.

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