Nature of spin-charge separation in the *t-J* model

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Quasiparticle properties are explored in an effective theory of the *t-J* model which includes two important components: spin-charge separation and unrenormalizable phase shift. We show that the phase shift effect indeed causes the system to be a non-Fermi liquid as conjectured by Anderson on general grounds. But this phase shift also drastically changes a conventional perception of quasiparticles in a spin-charge separation state: an injected hole will remain *stable* due to the confinement of spinon and holon by the phase shift field despite the background is a spinon-holon sea. True *deconfinement* only happens in the *zero-doping* limit where a bare hole will lose its integrity and decay into holon and spinon elementary excitations. The Fermi-surface structure is completely different in these two cases, from a large band-structure-like one to four Fermi points in one-hole case, and we argue that the so-called underdoped regime actually corresponds to a situation in between, where the ''gaplike'' effect is amplified further by a microscopic phase separation at low temperature. Unique properties of the single-electron propagator in both normal and superconducting states are studied by using the equation of motion method. We also comment on some influential ideas proposed in the literature related to the Mott-Hubbard insulator and offer a unified view based on the present consistent theory.

I. INTRODUCTION

High-T_c cuprates are regarded by many as essentially a doped Mott-Hubbard insulator.[1] At half-filling such an insulator is a pure antiferromagnet with only the spin degrees of freedom not being frozen at low energy. And a metallic phase with gapless charge degrees of freedom emerge after holes are added to the filled lower Hubbard band. To characterize the doped Mott-Hubbard insulator in the metallic regime, two important ideas were originally introduced by Anderson: spin-charge separation[2] and the unrenormalizable phase shift effect.[3,4] The first one is about elementary excitations of such a system and the second one is responsible for its non-Fermi-liquid behavior.

The spin-charge separation idea may be generally stated as the existence of two independent elementary excitations, charge-neutral spinon and spinless holon, which carry spin 1/2 and charge $+e$, respectively. It can be easily visualized in a short-range resonating-valence-bond (RVB) state[5] and has become a widely used terminology in literature, often with an additional meaning attached to it. For example, a spin-charge separation may be mathematically realized in the so-called slave-particle representation[6] of the *t-J* model,

$$c_{i\sigma}=h_i^{\dagger}f_{i\sigma}, \tag{1.1}$$

where the no-double-occupancy constraint, reflecting the Hubbard gap in its extreme limit, is handled by an equality $h_i^{\dagger}h_i+\Sigma_{\sigma}f_{i\sigma}^{\dagger}f_{i\sigma}=1$ which commutes with the Hamiltonian. Here one sees the close relation of the spin-charge separation and the constraint condition through the counting of the quantum numbers. But the spin-charge separation also acquires a *new* meaning here: If those holon ($h_i^{\dagger}$) and spinon ($f_{i\sigma}$) fields indeed describe elementary excitations, the hole (electron) is no longer a stable object and must decay into a holon-spinon pair once being injected into the system as shown by Fig. 1(a). This instability of a quasiparticle will be later referred to as the *deconfinement*, in order to distinguish it from the narrow meaning of the *spin-charge separation* about elementary excitations. We will see later that these two are generally *not* the same thing.

The second idea, the so-called ''unrenormalizable phase shift,''[3,4] may be described as follows. In the presence of an upper Hubbard band, adding a hole to the lower Hubbard band could change the whole Hilbert space due to the on-site Coulomb interaction: The *entire* spectrum of momentum k's may be shifted through the phase shift effect. It leads to the orthogonality of a bare doped hole state with its true ground state such that the quasiparticle weight $Z\equiv 0$, the key criterion for a non-Fermi liquid. In general, it implies

$$c_{i\sigma}=e_{i\sigma}e^{i\Theta_i}, \tag{1.2}$$

where $e_{i\sigma}$ is related to elementary excitation fields, e.g., $h_i^{\dagger}f_{i\sigma}$ in a spin-charge separation framework. Such an expression means that in order for a bare hole created by $c_{i\sigma}$ to become low-lying elementary excitations, a *many-body* phase shift Θ_i must take place in the background. In momentum space, it is easy to see how such a phase shift changes the Hilbert space by shifting k values. Note that $e_{\mathbf{k}\sigma}=\Sigma_{\mathbf{k}'}h_{\mathbf{k}'}^{\dagger}f_{\mathbf{k}+\mathbf{k}'\sigma}$ where $\mathbf{k}$ and $\mathbf{k}'$ belong to the *same* set of

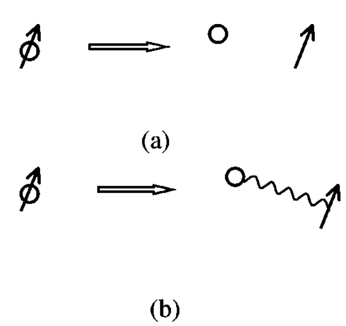

FIG. 1. Schematical illustration of the quasiparticle deconfinement (a) and confinement (b) due to the phase shift field.

quantized values [for example, in a two-dimensional (2D) square sample with size $L\times L$, the momentum is quantized at $k_\alpha=(2\pi/L)n$ under the periodic boundary condition with $\alpha=x$, y and $n=$ integer]. But because of a nontrivial Θ_i, $c_{\mathbf{k}\sigma}$ and $e_{\mathbf{k}\sigma}$ generally may no longer be described by the same set of $\mathbf{k}$'s or in the same Hilbert space, which thus constitutes an essential basis for a possible non-Fermi liquid.

The 1D Hubbard model serves as a marvelous example in favor of the decomposition (1.2) over Eq. (1.1). The quantitative value of the phase shift was actually determined by Anderson and Ren[2,7] using the Bethe-ansatz solution[8] in the large U limit, and Z was shown to decay at large sample size with a finite exponent. An independent path-integral approach[9] without using the Bethe ansatz also reaches the same conclusion which supports Eq. (1.2) as the *correct* decomposition of *true* holon and spinon.

Another important property of the Mott-Hubbard insulator, which is well known but has not been fully appreciated, is the *bosonization* of the electrons at half-filling. Namely, the *fermionic* nature of the electrons completely disappears and is replaced by a *bosonic* one. This is one of the most peculiar features of the Mott-Hubbard insulator due to the strong on-site Coulomb interaction. In fact, under the no-double-occupancy constraint, the t-J model reduces to the Heisenberg model in this limit. Its ground state for any *finite* bipartite lattice is singlet according to Marshall[10] and the wave function is real and satisfies a *trivial* Marshall sign rule as opposed to a much complicated ''sign problem'' associated with the fermionic statistics in a conventional fermionic system. This bosonization is the reason behind a very successful bosonic RVB description of the antiferromagnet: The variational bosonic RVB wave functions can produce strikingly accurate ground-state energy[11,12] as well as an elementary excitation spectrum over the whole Brillouin zone.[12] A mean-field bosonic RVB approach,[13] known as Schwinger-boson mean-field theory (SBMFT), provides a fairly accurate and mathematically useful framework for both zero- and finite-temperature spin-spin correlations.

Starting from either the slave-boson[14] or slave-fermion[15] representation, a 2D version of the decomposition (1.2) has been previously constructed such that the electron bosonization can be naturally realized at half-filling to restore the correct antiferromagnetic (AF) correlations. In the 1D case, this decomposition also recovers the aforementioned spinon-holon decoupling and reproduces the correct Luttinger-liquid behavior.[15] Even in the two-leg ladder system where holons and spinons are recombined together to form quasiparticles in the strong rung case,[16] a many-body phase shift field in this decomposition still exists at finite doping, playing a nontrivial role. Such a decomposition form can be generally written as

$$c_{i\sigma}=h_i^\dagger b_{i\sigma}e^{i\hat{\Theta}_{i\sigma}}. \tag{1.3}$$

It may be properly called a *bosonization* formulation as the holon operator $h_{i\sigma}^\dagger$ and spinon operator $b_{i\sigma}$ are both *bosonic* fields here. They still satisfy the no-double-occupancy constraint $h_i^\dagger h_i+\Sigma_\sigma b_{i\sigma}^\dagger b_{i\sigma}=1$. The *fermionic* nature of $c_{i\sigma}$ is to be represented through the phase shift field $\hat{\Theta}_{i\sigma}$ which replaces the description of Fermi-surface patches *and* Fermi-surface fluctuations in the usual bosonization language.[17,4]

The phase shift field in 2D is related to a nonlocal vortexlike operator by $e^{i\hat{\Theta}_{i\sigma}}\equiv(-\sigma)^i e^{i\Theta_{i\sigma}^{string}}$ [the sign $(-\sigma)^i$ keeps track of the Marshall sign just for convenience] with the vorticity given by[15]

$$\oint_\Gamma d\mathbf{r}\cdot\nabla\Theta_\sigma^{string}=\pm\pi\sum_{l\in\Gamma}\left[\sum_\alpha \alpha n_{l\alpha}^b-1-\sigma n_l^h\right], \tag{1.4}$$

where Γ is an arbitrary closed loop without passing any lattice site except the site i and the summation on the right-hand side (RHS) of Eq. (1.4) runs over lattice sites l within the loop Γ. Here $n_{l\alpha}^b$ and n_l^h are spinon and holon number operators, respectively.

Such a vortexlike phase shift originates from the fact that a doped hole moving on an AF spin background will always pick up sequential $+$ and $-$ signs, $(+1)\times(-1)\times(-1)\times\cdots$, first identified in Ref. 18. These signs come from the Marshall signs hidden in the AF background which are scrambled by the hopping of the doped hole on its path, determined by simply counting the index σ of each spin exchanged with the hole during its hopping.[18] The significance of such a phase string is that it represents the *sole* source to generate phase frustrations in the t-J model (at finite doping, the only additional signs coming from the fermionic statistics of doped holes in the original slave-fermion representation are also counted in). Namely, the ground-state wave function would become real and there should be no ''sign problem'' only if such a phase string effect is absent (like in the zero-doping case). The phase shift field in Eq. (1.3) precisely keeps track of such a phase string effect[15] and therefore can be considered to be a general consequence of the t-J model.

The decomposition (1.3) *defines* a unique spin-charge separation theory where the relation between the physical electron operator and the internal elementary excitations, holon and spinon, is explicitly given. The thermodynamic properties will be obtained in terms of the energy spectra of holon and spinon fields, while the physically observable quantities will be determined based on Eq. (1.3) where the singular phase shift field with vorticities is to play a very essential role in contrast to the conventional spin-charge separation theories in the slave-particle decompositions of Eq. (1.1).

Note that the total vorticity of Eq. (1.4) is always equal to $2\pi\times$integer due to the no-double-occupancy constraint such that the phase shift factor $e^{i\hat{\Theta}_{i\sigma}}$ be single valued. Such a phase shift field will play different roles in different channels. For example, in the spin channel one has $S_i^+=b_{i\uparrow}^\dagger b_{i\downarrow}(-1)^i e^{-i[\Theta_{i\uparrow}^{string}-\Theta_{i\downarrow}^{string}]}$ where the total vorticity

$$-\oint_\Gamma d\mathbf{r}\cdot\nabla(\Theta_\uparrow^{string}-\Theta_\downarrow^{string})=\pm 2\pi\sum_{l\in\Gamma} n_l^h. \tag{1.5}$$

It obviously vanishes at $\delta\rightarrow 0$ (δ is the doping concentration) so that the aforementioned bosonization is naturally realized. And at finite doping, the vorticity shown in Eq. (1.5) reflects the recovered fermionic effect and is responsible for a doping-dependent incommensurate momentum structure[19] in the dynamic spin susceptibility function which provides a unique reconciliation of neutron scattering and NMR measurements in the cuprates. On the other hand, in the *singlet* pairing channel, the phase shift field appearing in the local pairing operator will contribute to a vorticity

$$\oint_{\Gamma} d\mathbf{r}\cdot\nabla(\Theta_{\uparrow}^{string}+\Theta_{\downarrow}^{string})=\pm 2\pi\sum_{l\in\Gamma}\left[\sum_{\alpha}\alpha n_{l\alpha}^{b}-1\right], \tag{1.6}$$

which decides the *phase coherence* of Cooper pairs. According to Eq. (1.6), besides a trivial 2π flux quantum per site, each spinon also carries a fictitious $(\pm)2\pi$ flux tube. To achieve the phase coherence or superconducting condensation, $\uparrow$ and $\downarrow$ spinons have to be *paired* off to remove the vorticities associated with individual spinons in Eq. (1.6), which then connects[20,21] T_c to a characteristic spinon energy scale, in consistency with the experimental result of cuprate superconductors and resolving the issue why T_c is too high in usual RVB theories.

The purpose of the present work is to explore the consequences of the bosonization decomposition (1.3) in 2D *quasiparticle* channel. First of all, we show that the phase shift field indeed causes the quasiparticle weight Z to vanish. Namely, this spin-charge separation state *is* a 2D non-Fermi liquid, a fact almost trivial in such a particular formulation. A surprising ''by-product'' of this phase shift field is that it also plays a role of *confinement force* to ''glue'' spinon and holon constituents together inside a quasiparticle, as illustrated in Fig. 1(b). In other words, a hole injected into this system generally does *not* break up into spinon-holon elementary particles, even though the background is a spinon-holon sea. Such a quasiparticle may be regarded as a spinon-holon bound state or more properly a *collective* mode but will generally remain *incoherent* due to the same phase shift field.

Due to the confinement, an equation-of-motion description of the quasiparticle excitation is constructed, in which the dominant ''scattering'' process is described as the ''virtual'' decay of the quasiparticle into holon-spinon composite. In the superconducting phase, the composite nature of the quasiparticle predicts a unique non-BCS structure for the single-electron Green's function which is consistent with the experimental measurements. In particular, we find the restoration of the quasiparticle coherence with regard to the incoherence in the normal state.

A true deconfinement or instability of the quasiparticle only happens in the zero-doping limit where an injected hole indeed can decay into a holon and spinon pair, which provides[22] a consistent explanation of angle-resolved photoemission spectroscopy (ARPES) measurements.[23] The contrast of a large band-structure-like Fermi surface in the confinement phase to the four Fermi points in the deconfinement phase at the zero-doping limit may provide a unique explanation for the ARPES experimental measurements in cuprate superconductors. In the weakly doped regime, a ''partial'' deconfinement of the quasiparticle between full-blown deconfinement and confinement will be reflected in the single-electron Green's function which may explain the ''spin-gap'' phenomenon.[24]

The remainder of the paper is organized as follows. In Sec. II, we first briefly review the effective spin-charge separation theory based on the decomposition (1.3). Then in Sec. II B, we show that the phase shift field leads to $Z=0$, i.e., a non-Fermi-liquid state. In Sec. II C, we demonstrate how the phase shift field causes the confinement of the holon and spinon within a quasiparticle except in the zero-doping limit. In Sec. II D, we study the single-electron propagator in both normal and superconducting states based on an equation-of-motion approach. We then discuss an underdoped case as a crossover regime from a Fermi-point structure in the one-hole case with the holon-spinon deconfinement to a large Fermi surface in the confinement case. Finally, Sec. III is devoted to discussing some of the most influential ideas proposed in the literature related to the Mott-Hubbard insulator and high-T_c cuprates and offers a unified view based on the present consistent theory.

II. PROPERTIES OF A QUASIPARTICLE IN THE SPIN-CHARGE SEPARATION STATE

A. Effective spin-charge separation theory

The decomposition (1.3) determines an effective spin-charge separation theory of the t-J model in which spinon and holon fields constitute the elementary particles. Before proceeding to the discussion of the quasiparticle properties in next subsections, we first briefly review some basic features of this theory based on Refs. 15 and 20.

In the operator formalism, the phase shift field $\Theta_{i\sigma}^{string}$ satisfying Eq. (1.4) can be explicitly written down in a specific gauge as follows:[15]

$$\Theta_{i\sigma}^{string}\equiv\frac{i}{2}[\Phi_i^b-\sigma\Phi_i^h], \tag{2.1}$$

where

$$\Phi_i^b=\sum_{l\neq i}\theta_i(l)\left(\sum_{\alpha}\alpha n_{l\alpha}^b-1\right) \tag{2.2}$$

and

$$\Phi_i^h=\sum_{l\neq i}\theta_i(l)n_l^h. \tag{2.3}$$

Here $\theta_i(l)$ is defined as an angle

$$\theta_i(l)=\mathrm{Im}\ln(z_i-z_l), \tag{2.4}$$

with $z_i=x_i+iy_i$ representing the complex coordinate of a lattice site i.

In 2D, an effective Hamiltonian based on the decomposition (1.3) after a generalized mean-field decoupling[20] in the t-J model can be written down:

$$H_{eff}=H_h+H_s, \tag{2.5}$$

where the holon Hamiltonian

$$H_h=-t_h\sum_{\langle ij\rangle}(e^{iA_{ij}^f})h_i^{\dagger}h_j+\mathrm{H.c.} \tag{2.6}$$

and the spinon Hamiltonian

$$H_s=-J_s\sum_{\langle ij\rangle\sigma}[(e^{i\sigma A_{ij}^h})b_{i\sigma}^\dagger b_{j-\sigma}^\dagger+\text{H.c.}]$$
$$-\sum_{ij\sigma}J_{ij}^s(e^{i\sigma A_{ij}^h})b_{i\sigma}^\dagger b_{j\sigma}, \qquad (2.7)$$

with $t_h\sim t$, $J_s\sim J$. In the second term of H_s, $J_{ij}^s\sim\delta t\neq 0$ only for i and j on the same sublattice sites, which originates from H_t where a phase shift occurs[20] in the spinon mean-field wave function and results in the same-sublattice hopping of spinons. The lattice gauge fields A_{ij}^f and A_{ij}^h in the specific gauge choice of Eqs. (2.2) and (2.3) are given by

$$A_{ij}^f=\frac{1}{2}\sum_{l\neq i,j}[\theta_i(l)-\theta_j(l)]\left(\sum_\sigma \sigma n_{l\sigma}^b-1\right)\equiv A_{ij}^s-\phi_{ij}^0 \qquad (2.8)$$

and

$$A_{ij}^h=\frac{1}{2}\sum_{l\neq i,j}[\theta_i(l)-\theta_j(l)]n_l^h. \qquad (2.9)$$

In general, A_{ij}^s and A_{ij}^h can be regarded as ''mutual'' Chern-Simons lattice gauge fields as, for example, A_{ij}^h is determined by the density distribution of holons but only seen by spinons.

The above effective theory is based on a RVB pairing order parameter[20]

$$\Delta^s=\sum_\sigma\langle e^{-i\sigma A_{ij}^h}b_{i\sigma}b_{j-\sigma}\rangle, \qquad (2.10)$$

which in the zero-doping limit $\delta\to 0$ reduces to the well-known bosonic RVB order parameter[13] as $A_{ij}^h=0$. And H_{eff} recovers the Schwinger-boson mean-field Hamiltonian[13] of the Heisenberg model. So this theory can well describe AF correlations at half-filling. At finite doping, the ''mutual'' Chern-Simons gauge fields A_{ij}^f and A_{ij}^h will play important roles in shaping superconductivity, magnetic, and transport properties, and some very interesting similarities with cuprate superconductors have been discussed based on this model.[20] In contrast to the slave-fermion approach,[25] Δ^s remains the only order parameter at finite doping, controlling the short-range spin-spin correlations as $\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle=-1/2|\Delta^s|^2$ for nearest-neighboring i and j. It is noted that due to the presence of the RVB pairing (2.10), the conventional gauge fluctuations[26,27] are suppressed as the gauge invariance $h_i^\dagger b_{i\sigma}=[h_i^\dagger e^{i\theta_i}][b_{i\sigma}e^{-i\theta_i}]$ is apparently broken by Δ^s. Here spinons no longer contribute to transport and are really charge-neutral particles.

In the ground state of the uniform-phase solution (Ref. 20), A_{ij}^f and A_{ij}^h both become simplified: A_{ij}^f simply describes a π flux per plaquette, $\Sigma_\Box A_{ij}^f\approx-\Sigma_\Box\phi_{ij}^0=-\pi$ since A_{ij}^s is suppressed due to the spinon pairing in the ground state; A_{ij}^h describes a uniform flux $\Sigma_\Box\overline{A_{ij}^f}=\pi\delta$ due to the Bose condensation of holons. Self-consistently, H_h [Eq. (2.6)] determines a Bose-condensed ground state of holons where the π flux produced by A_{ij}^f merely enlarges the effective mass near the band edge by $\sqrt{2}$. On the other hand, $\overline{A_{ij}^h}$ in H_s [Eq. (2.7)] leads to a ''resonancelike'' energy structure in the spinon spectrum and the corresponding dynamic spin susceptibility function at the AF vector (π,π) is illustrated in Fig. 2. Note that E_g in Fig. 2 is twice as large as the corresponding spinon energy E_s. The doping dependence of E_g is shown in the inset. Finally, the superconducting order parameter $\Delta_{ij}^{SC}=\langle\Sigma_\sigma\sigma c_{i\sigma}c_{j-\sigma}\rangle$ has a finite value (for nearest-neighboring i and j) in the ground state,[20]

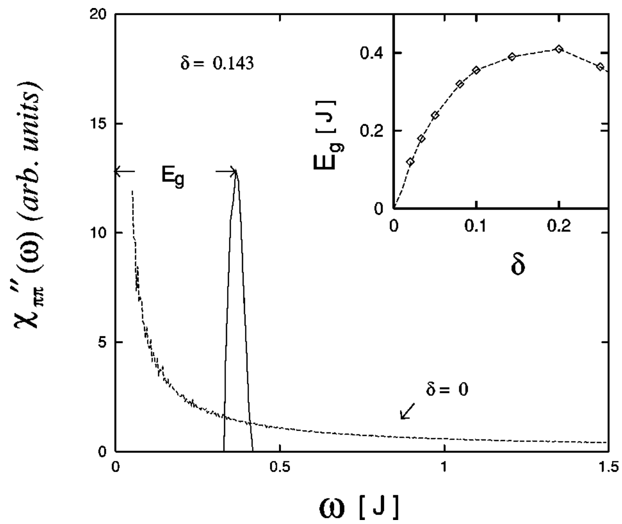

FIG. 2. Dynamical spin susceptibility at the AF vector (π,π) in the uniform phase for $\delta=0$ and 0.143, respectively. The inset: the doping dependence of the characteristic spin ''resonancelike'' energy $E_g=2E_s$ (E_s denotes the corresponding spinon energy).

$$\Delta_{ij}^{SC}=\Delta^s(-1)^i\langle h_i^\dagger e^{i(\Phi_i^b+\Phi_j^b)/2}h_j^\dagger\rangle\neq 0, \qquad (2.11)$$

due to the Bose condensation of holons as well as the pairing of spinons which leads to $\Delta^s\neq 0$ and the vortex-antivortex confinement in $e^{i(\Phi_i^b+\Phi_j^b)/2}$. Thus the ground state is always superconducting condensed with a pairing symmetry of d-wave-like.[20]

Besides the above uniform ground state, possible nonuniform solutions characterized by the coexistence of the Bose condensations of holons and spinons have been also discussed in Ref. 20 where the spinon spectrum has only a pseudogap. In any case, the ground state can be regarded as a spinon-holon sea, and low-lying elementary excitations are described in terms of spinons and holons. What we are mainly interested in this paper is to answer the question how a hole (electron) as a composite of spinon and holon behaves in this spin-charge separation state. This is one of the most fundamental questions not only because it can be directly tested in an ARPES measurement, but also because it will make a crucial distinction between a conventional Fermi liquid and a non-Fermi liquid. Let us begin with the question: if this spin-charge separation state is a non-Fermi liquid.

B. Non-Fermi liquid with $Z=0$

The definition of the quasiparticle weight $Z_{\mathbf{k}}$ at momentum $\mathbf{k}$ is given by $Z_{\mathbf{k}}=|\langle\Psi_G(N_e-1)|c_{\mathbf{k}\sigma}|\Psi_G(N_e)\rangle|^2$, and it measures the overlap of a bare hole state at momentum $\mathbf{k}$, created by $c_{\mathbf{k}\sigma}$ in the ground state of N_e electrons, with the ground state of N_e-1 electrons. For a Fermi-liquid state, one always has $Z_{k_f}\neq 0$ at the Fermi momentum k_f. If $Z_{\mathbf{k}}=0$ for any $\mathbf{k}$, then the system is a non-Fermi liquid by definition. In the following we will show that the bare hole state $c_{\mathbf{k}\sigma}|\Psi_G(N_e)\rangle$ acquires an ''angular'' momentum due to the vorticities in $e^{\hat{\Theta}_{i\sigma}}$ in the decomposition (1.3). Due to such a distinct symmetry, it is always orthogonal to the ground state $|\Psi_G(N_e-1)\rangle$, leading to $Z\equiv 0$.

One can construct a ''rotational'' operation by making a transformation

$$\theta_i(l)\to\theta_i(l)+\phi. \tag{2.12}$$

It corresponds to a simple change of reference axis for the angle function $\theta_i(l)$ defined in Eq. (2.4). It is easy to see that the Hamiltonians (2.6) and (2.7) are invariant since the gauge fields, in which $\theta_i(l)$ appears, are obviously not changed: $A_{ij}^{h,f}\to A_{ij}^{h,f}$. Both the ground state $|\Psi_G\rangle$ as well as single-valued $h_i^\dagger$ and $b_{i\sigma}$ fields are apparently independent of ϕ. But a bare hole state will change under the transformation (2.12) as follows:

$$c_{i\sigma}|\Psi_G\rangle\to e^{i\phi P_i^\sigma}\times c_{i\sigma}|\Psi_G\rangle \tag{2.13}$$

due to the phase shift factor $e^{i\hat{\Theta}_{i\sigma}}$ with $P_i^\sigma=S^z-\sigma N^h/2-(N-1-\sigma)/2$ which remains an integer for a bipartite lattice (S^z and N^h denote total spin and hole numbers, respectively, and N is the lattice size). This implies that $c_{i\sigma}|\Psi_G\rangle$ indeed has a nontrivial ''angular'' momentum in contrast to $|\Psi_G\rangle$ which carries none.

It is probably more transparent to see the origin of the angular momentum if one rewrites, for example,

$$e^{i\hat{\Theta}_{i\downarrow}}=\prod_{l\neq i}(z_i-z_{l\uparrow})^{1/2}\prod_{l\neq i}(z_i^*-z_{l\downarrow}^*)^{1/2}$$
$$\times\prod_{l\neq i}(z_i-z_{lh})^{1/2}\prod_{l\neq i}(z_i^*-z_l^*)^{1/2}\times F_i, \tag{2.14}$$

where $z_{l\uparrow}$, $z_{l\downarrow}$, and z_{lh} denote the complex coordinates of ↑, ↓ spinons, and holons, respectively. And $F_i=\Pi_{l\neq i}|z_i-z_l|$ is a constant (which is obtained by using the no-double-occupancy constraint). It is important to note that despite the fractional (''semion'') exponents of 1/2 in Eq. (2.14), it can be directly verified that the phase shift field $e^{i\hat{\Theta}_{i\downarrow}}$ remains *single valued* under the no-double-occupancy constraint. Generally the vortex field (2.14) introduces an extra angular momentum which can be easily identified as

$$l=S^z+\frac{N^h}{2}. \tag{2.15}$$

Here l is always an integer. Then one has

$$\langle\Psi_G(N_e-1)|c_{i\sigma}|\Psi_G(N_e)\rangle=0 \tag{2.16}$$

due to the *orthogonal* condition[28] as $l\neq 0$ [$S^z=O(1)$, $N^h=O(N)$ at finite doping] for $c_{i\sigma}|\Psi_G\rangle$. By extending the same argument, one can quickly see that the bare hole state $c_{i\sigma}|\Psi_G(N_e)\rangle$ has no overlap not only with $|\Psi_G(N_e-1)\rangle$ but also with all the elementary excitations composed of simple holons and spinons with $l=0$. So $c_{i\sigma}$ is more like a creation operator of a ''collective'' mode whose quantum number l is different from a simple spinon-holon pair.

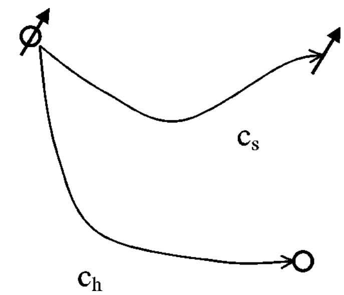

FIG. 3. Schematical illustration of the case when a quasiparticle decays into spinon and holon constituents.

C. Quasiparticle: Spinon-holon confinement

The difference in symmetry between a quasiparticle and a holon-spinon pair implies that the former cannot simply decay into the latter even though they share the same quantum numbers of charge and spin. In this section, we demonstrate that generally the holon and spinon constituents will be *confined* by the phase shift field within a quasiparticle although the background is a spinon-holon sea.

Intuitively such a confinement is easy to understand: If the holon and spinon constituents inside a quasiparticle could move away independently by themselves, as schematically shown in Fig. 3, the vortex phase shift field $e^{i\hat{\Theta}_{i\sigma}}$ left behind would cost a logarithmically *divergent* energy as to be shown below. But a quasiparticle state $c_{i\sigma}|\Psi_G\rangle$ as a local excitation should only cost a finite energy relative to the ground-state energy. Such a discrepancy can be reconciled only if the holon and spinon constituents no longer behave as free elementary excitations: They have to absorb the effect of the vortexlike phase shift and by doing so make themselves bound together.

Let us consider $|\Psi'\rangle\equiv e^{i\hat{\Theta}_{i\sigma}}|\Psi_G\rangle$ and compute the energy cost for the vortexlike phase shift:

$$\langle\Psi'|H_{eff}|\Psi'\rangle-\langle\Psi_G|H_{eff}|\Psi_G\rangle. \tag{2.17}$$

We first focus on the contribution from the holon part H_h [Eq. (2.6)]. Define $E_G^h=\langle\Psi_G|H_h|\Psi_G\rangle$. One has $-t_h\langle\Psi_G|h_l^\dagger h_m e^{iA_{lm}^f}|\Psi_G\rangle=E^h/4N$ for any nearest-neighbor link (lm) due to the translational symmetry. Then a straightforward manipulation leads to

$$\langle\Psi'|H_h|\Psi'\rangle-E_G^h=-\frac{E_G^h}{2N}\sum_{\langle lm\rangle}\{1-\cos[\theta_i(l)-\theta_i(m)]/2\}. \tag{2.18}$$

Notice that if the (lm) link (say, along the $\hat{x}$ direction) is far away from the site i, then one has $|\theta_i(l)-\theta_i(m)|\to a|\sin\theta|/r$ where r denotes the distance between the center of the link and the site i and θ is the azimuth angle. Then it is easy to see that the summation over those links on the RHS of Eq. (2.18) will contribute as $\int r\,dr\,d\theta\sin^2\theta/r^2\propto\ln R$ (R denotes the sample size). Namely, the vortexlike phase shift will cost a logarithmically diverged energy if it is left alone. It should be noted that the same conclusion still holds if one replaces H_{eff} by the exact t-J model in the representation of Eq. (1.3) (Ref. 15).

Hence the vortexlike phase shift field has to be absorbed by the holon and spinon fields in order to keep the quasiparticle energy finite. In the following, let us illustrate how this will happen. We first use the vortex phase $\frac{1}{2}\Phi_i^b$ in Eq. (2.1) as an example. Let us write down the following identity:

$$\exp\left(i\frac{1}{2}\Phi_i^b\right)=\left[\exp\left(i\sum_{c_h}A^f\right)\exp[iK^b(c_h)]\right]\exp\left(i\frac{1}{2}\Phi_j^b\right), \tag{2.19}$$

in which

$$\sum_{c_h}A^f\equiv\sum_s A^f_{m_sm_{s+1}}, \tag{2.20}$$

where $m_0=i,\ m_1,\ \ldots,m_{k_{c_h}}\equiv j$ are sequential lattice sites on an arbitrary path c_h connecting i and j. And $K^b(c_h)\equiv\frac{1}{2}\Sigma_s[\theta_{m_{s-1}}(m_s)-\theta_{m_{s+1}}(m_s)](\Sigma_\alpha\alpha n^b_{m_s\alpha}-1)$ which is a stringlike field only involving spinons on the path c_h. By contrast, the line summation $\Sigma_{c_h}A^f$ is contributed by spinons from the whole system nonlocally. Note that, according to H_h [Eq. (2.6)], holons see the gauge field A^f in the Hamiltonian, and if a holon moves from site i to j via the same path c_h, it should acquire a phase factor $\exp(-i\Sigma_{c_h}A^f)$ which can exactly compensate the similar phase in Eq. (2.19). In other words, if the vortex phase factor $e^{i\Phi_i^b/2}$ is bound to a holon to form a new composite object $\tilde{h}_i^\dagger=h_i^\dagger e^{i\Phi_i^b/2}$, then there will be no more vortex effect as it moves on the path c_h shown in Fig. 3, except for a phase string field $K^h(c_h)$ left on its path, and the new object should cost only a finite energy.

Similarly, for the vortex field $(-\sigma/2)\Phi_i^h$ in Eq. (2.1) one can rewrite

$$\exp\left(-i\frac{\sigma}{2}\Phi_i^h\right)=\left[\exp\left(-i\sigma\sum_{c_s}A^h\right)\exp[-i\sigma K^h(c_s)]\right]\times\exp\left(-i\frac{\sigma}{2}\Phi_{j'}^h\right), \tag{2.21}$$

in which

$$\sum_{c_s}A^h\equiv\sum_s A^h_{m_sm_{s+1}}, \tag{2.22}$$

where the line summation runs over a sequential lattice sites on an arbitrary path c_s connecting i and j' shown in Fig. 3. And then we can similarly see that the spinon constituent of the quasiparticle also has to be bound to $e^{-i\sigma\Phi^h/2}$ in order to compensate the logarithmically divergent energy cost by the vortex structure, and the new composite will only leave a phase string behind given by $K^h(c_s)\equiv\frac{1}{2}\Sigma_s[\theta_{m_{s-1}}(m_s)-\theta_{m_{s+1}}(m_s)]n^h_{m_s}$.

However, there is one problem in the above argument about the absorption of the vortex phase factors $e^{i\Phi_i^b/2}$ and $e^{i\sigma\Phi_i^h/2}$ by the holon and spinon constituents. Namely, these two phase factors are not single valued except in the zero-doping limit. In fact, only the total phase factor $e^{i\hat{\Theta}_{i\sigma}}$ is always well defined and single valued as mentioned before. It thus means that both holon and spinon constituents have to be bound to the total phase shift fields together to eliminate the divergent energy while maintaining single-valuedness. There is another way to see this. Note that $\theta_{m_{s-1}}(m_s)-\theta_{m_{s+1}}(m_s)$ describes the angle between the nearest-neighboring links (m_{s-1},m_s) and (m_{s+1},m_s); it can have an uncertainty by $\pm2\pi\times$integer, and it is easy to see that the phase string factors $e^{iK^b(c_h)}$ and $e^{-i\sigma K^h(c_s)}$ in Eqs. (2.19) and (2.21) are not well defined by themselves as they are multivalued except at $\delta=0$. On the other hand, if one chooses $c_h=c_s=c$ in Fig. 3, the mathematical ambiguity is eliminated in the total phase string field,

$$\begin{aligned}K_\sigma(c)&\equiv K^b(c)-\sigma K^h(c)\\&=\sum_{s=1}^{k_c}[\theta_{m_s-1}(m_s)-\theta_{m_s+1}(m_s)]\\&\quad\times\frac{1}{2}\left(\sum_\alpha\alpha n^b_{m_s\alpha}-1-\sigma n^h_{m_s}\right),\end{aligned} \tag{2.23}$$

since by using the no-double-occupancy constraint, one can show that $\frac{1}{2}(\Sigma_\alpha\alpha n^b_{m_s\alpha}-1-\sigma n^h_{m_s})=-(1+\sigma)/2+\sigma n^b_{m_s\sigma}$ which is an integer such that $e^{iK_\sigma(c)}$ remains single valued.

Physically, it is because a *fermionic* quasiparticle may not decay into two *bosonic* holon and spinon elementary excitations in 2D. The only exception is in the *zero-doping* limit. We have pointed out in the Introduction that at half-filling the ''fermionic'' nature of the electrons essentially disappears and is replaced by a ''bosonic'' one due to the no-double-occupancy constraint. Then it is not surprising that in the one-hole doped case which is adjacent to the half-filling, the deconfinement of holon-spinon can happen as a result of the electron ''bosonization.'' Indeed, in the zero-doping limit Φ_i^h defined in the gauge (2.3) *vanishes*. Without Φ_i^h, the original reason for inseparable $e^{-i\sigma\Phi_i^h/2}$ and $e^{i\Phi_{i\sigma}^b/2}$ in $e^{i\hat{\Theta}_{i\sigma}}$ is no longer present: In this case, the phase shift field $e^{i\hat{\Theta}_{i\sigma}}$ reduces to $e^{i\Phi_i^b/2}$ which itself becomes well defined, and can solely accompany the holon during the propagation. As for the spinon part, H_s in Eq. (2.7) reduces to the well-known SBMFT Hamiltonian with $A^h=0$ and the corresponding line summation Σ_cA^h is also absent in the propagator. Without leading to the multivalue ambiguity, the quasiparticle will break into a spinon and a composite of holon-vortex phase which can propagate independently. More discussions of the one-hole problem can be found in Sec. II D 3.

How a quasiparticle behaves in a spinon-holon sea as a single entity at finite doping will be the subject of discussion in the next subsection. In the following we will make several remarks on some implications of the confinement before concluding the present subsection. First of all, we note that a quasiparticle generally remains an incoherent excitation in contrast to the coherent spinons and holons and we assume that it will not contribute significantly to either thermodynamic and dynamic properties. In the equal-time limit $t=0^-$, the single-electron propagator can be expressed as

$$G_e(i,j;0^-)=i(-\sigma)^{i-j}\left\langle\left[b^\dagger_{j\sigma}\exp\left(i\frac{\sigma}{2}\sum_c A^h\right)b_{i\sigma}\right]\times\left[h_j\exp\left(-i\sum_c A^f\right)h_i^\dagger\right]e^{iK_\sigma(c)}\right\rangle. \tag{2.24}$$

At finite t, temporal components have to be added to the line summations, $\Sigma_c A^h$ and $\Sigma_c A^f$, as well as in the phase string field $K_\sigma(c)$ above. Even though mathematically the path c can be chosen arbitrarily in Eq. (2.24), a natural choice is for c to coincide with the real path of the quasiparticle such that the line summations can be precisely compensated by the phases picked up by the holon and spinon constituents as mentioned above. In this case, all the singular phase effect will be tracked by $e^{iK_\sigma(c)}$ which is nothing but the previously identified phase string effect,[18] where it has been shown that the phase string effect is nonrepairable and represents the dominant phase interference at low energy. Physically, it reflects the *fermionic* exchange relation between the quasiparticle under consideration and those electrons in the background. Such a phase string field accompanying the propagation of the quasiparticle is a many-body operator in terms of elementary holon and spinon fields. Even in the one hole case, such a phase string effect results in the incoherency of the quasiparticle as has been discussed in Ref. 22.

One may also see how a Fermi-surface structure is generated from the phase shift $\hat{\Theta}_{i\sigma}$ in some limits. For example, in the 1D case (where $A_{ij}^{h,f}=0$[15]), since one may always define $\theta_{m_s-1}(m_s)-\theta_{m_s+1}(m_s)=\pm\pi$, the phase string factor $e^{iK_\sigma(c)}$ in Eq. (2.24) can be written as $(-\sigma)^{i-j}e^{i\sigma k_f(x_i-x_j)}e^{i\delta k_f^\sigma(x_i-x_j)}$ which produces the 1D Fermi surface at $k_f=\pm\pi(1-\delta)/2$ (here δk_f^σ denotes Fermi-surface fluctuations with $\langle \delta k_f^\sigma\rangle=0$ which is crucial to the Luttinger-liquid behavior[15]). In the 2D one-hole case, $\hat{\Theta}_{i\sigma}$ also leads to a ''remnant'' Fermi-surface structure in the equal-time limit while it gives rise to four Fermi points $\mathbf{k}_0$ *at low energy* as discussed in Ref. 22. At finite doping, the doping-dependent incommensurate peaks in the dynamic spin susceptibility function has been also related to such a phase shift field.[19] In general, the Luttinger-volume theorem may even be understood based on $e^{iK_\sigma(c)}$ as it involves the counting of the background electron numbers. Nevertheless, the *precise* Fermi-surface topology will not be solely determined by the phase shift field in 2D and one must take into account of the dynamic effect.

Finally, a stable but incoherent quasiparticle excitation in which a pair of holon and spinon are confined means that a photoemission experiment, in which such a quasiparticle excitation can be created through ''knocking out'' an electron by a photon, does not directly probe the *intrinsic* information of coherent elementary excitations anymore, and the energy-momentum structure of the single-electron Green's function is no longer a basis as fundamental and useful as in the case of conventional Fermi-liquid metals to understand superconductivity, spin dynamics, and transport properties in other channels.

D. Description of the quasiparticle: Equation-of-motion method

Now imagine a bare hole is injected into the ground state of N_e electrons. By symmetry, such a state should be orthogonal to the ground state of N_e-1 electrons. Its dissolution into a holon and a spinon is also prohibited by the symmetry introduced by the phase shift field [Eq. (1.3)] and the latter would otherwise cost a logarithmically divergent energy if being left alone unscreened. Therefore, one has to treat a quasiparticle as an *independent* collective excitation in this spin-charge separation system.

Involving *infinite-body* holons and spinons, a quasiparticle cannot be simply described by the mean-field theory of individual holon and spinon. The previously discussed confinement is one example of the *nonperturbative* consequences caused by the infinite-body phase shift field. But such a confinement of the holon and spinon inside a quasiparticle will enable us to approach this problem from a different angle.

Here it may be instructive to recall how a low-lying collective mode is determined in the BCS theory. In BCS mean-field theory, quasiparticle excitations are well defined with an energy gap. But quasiparticle excitations do not exhaust all the low-lying excitations, and there exists a collective mode in the absence of long-range Coulomb interaction, which may be also regarded as a ''bound'' state of a quasiparticle pair due to the *residual* attractive interaction. A correct way[29] to handle this ''bound'' state is to use the *full* BCS Hamiltonian to first write down the equation of motion for a quasiparticle pair and *then* apply the BCS mean-field treatment to linearize the equation to produce the gapless spectrum, which is equivalent to the random phase approximation (RPA) scheme.[30] Including the long-range Coulomb interaction[29] will turn this collective mode into the well-known plasma mode.

Similarly we can establish an equation-of-motion description of the quasiparticle as a ''collective mode,'' which moves on the background of the mean-field spin-charge separation state. For this purpose, let us first write down the *full* equation of motion of the hole operator in the Heisenberg representation: $-i\partial_t c_{i\sigma}(t)=[H_{t-J},c_{i\sigma}(t)]$, based on the *exact t-J* model, either in the decomposition (1.3) or simply in the original c-operator representation, as follows:

$$[H_t,c_{i\sigma}]=\frac{t}{2}(1+n_i^h)\sum_{l=NN(i)}c_{l\sigma}+t\sum_{l=NN(i)}(c_{l\sigma}\sigma S_i^z+c_{l-\sigma}S_i^{-\sigma}) \qquad (2.25)$$

and

$$[H_J,c_{i\sigma}]=\frac{J}{4}c_{i\sigma}\sum_{l=NN(i)}(1-n_l^h)-\frac{J}{2}\sum_{l=NN(i)}(c_{i\sigma}\sigma S_l^z+c_{i-\sigma}S_l^{-\sigma}). \qquad (2.26)$$

Note that the above equations hold in the restricted Hilbert space under the no-double-occupancy constraint: $\Sigma_\sigma c_{i\sigma}^\dagger c_{i\sigma}\leq 1$.

There are many papers in literature dealing with the t-J model in the c-operator representation, in which the no double occupancy $\Sigma_{i\sigma}c_{i\sigma}^\dagger c_{i\sigma}\leq 1$ is disregarded. As a consequence, there is only a conventional *scattering* between the quasiparticle and spin fluctuations as suggested by Eqs. (2.25) and (2.26). This leads to a typical spin-fluctuation theory, which usually remains a Fermi-liquid theory with well-defined coherent quasiparticle excitations near the

Fermi surface in contrast to the $Z=0$ conclusion obtained here. The problem with the spin-fluctuation theory is that the crucial role of the no-double-occupancy constraint hidden in Eqs. (2.25) and (2.26) has been completely ignored which, in combination with the RVB spin pairing, is actually the key reason resulting in a spin-charge separation state in the present effective theory of the t-J model. In such a backdrop of the holon-spinon sea, the scattering terms in Eqs. (2.25) and (2.26) will actually produce a virtual ''decaying'' process which is fundamentally different from the usual spin-fluctuation scattering in shaping the single-electron propagator.

By using the decomposition (1.3) and the mean-field order parameter Δ^s defined in Eq. (2.10), the high-order spin-fluctuation-scattering terms on the RHS of Eqs. (2.25) and (2.26) can be ''reduced'' to the same order of linear $c_{i\sigma}$, and we find

$$-i\partial_t c_{i\sigma}(t)\approx\frac{t}{2}(1+\delta)\sum_{l=NN(i)}c_{l\sigma}+J(1-\delta)c_{i\sigma}$$
$$-\frac{1}{4}tB_0\sum_{l=NN(i)}e^{i\hat{\Theta}_{l\sigma}}h_l^\dagger b_{i\sigma}e^{-i\sigma A_{il}^h}$$
$$+\frac{3}{8}J\Delta^s\sum_{l=NN(i)}e^{i\hat{\Theta}_{i\sigma}}h_i^\dagger b_{l-\sigma}^\dagger e^{i\sigma A_{il}^h}+\cdots, \tag{2.27}$$

where B_0 is the modified (but not an independent) order parameter for the hopping term introduced in Ref. 20. In the following we will discuss some unique quasiparticle properties based on this equation.

So in the spin-charge separation (mean-field) background, the leading order effect of the ''scattering'' terms correspond to the decay of the quasiparticle: The terms in the second and third lines of Eq. (2.27) clearly indicate the tendency for the quasiparticle to break up into holon and spinon constituents. This is in contrast to the conventional *scatterings* between the quasiparticle and spin fluctuations, as Eqs. (2.25) and (2.26) would have suggested. Generally, the quasiparticle is expected to have an intrinsic broad spectral function extended over the whole energy range

$$E_{\text{quasiparticle}}>E_{\text{holon}}+E_{\text{spinon}} \tag{2.28}$$

because of the decomposition process. But the presence of the phase factor $e^{i\hat{\Theta}}$ in these ''decaying'' terms of Eq. (2.27) prevents a real decay of the quasiparticle since such a vortex field would cost a logarithmically divergent energy as has been discussed before. Thus, even in the case of Eq. (2.28), the decaying of a quasiparticle remains only a virtual process which is an another way to understand the confinement discussed in Sec. II C.

Without the ''decaying'' terms, the equation of motion (2.27) would become closed with an eigenspectrum in momentum-energy space (besides a constant which can be absorbed into the chemical potential):

$$\epsilon_{\mathbf{k}}=-2t_{eff}(\cos k_x+\cos k_y), \tag{2.29}$$

with

$$t_{eff}=\frac{t}{2}(1+\delta). \tag{2.30}$$

Generally the ''decaying'' terms do not contribute to a coherent **k**-dependent correction due to the nature of the holon and spinon excitations as well as the ''smearing'' caused by $e^{i\hat{\Theta}}$ in Eq. (2.27). But in the ground state, which is also superconducting, the ''decaying'' terms in Eq. (2.27) do produce a coherent contribution due to the composite nature of the quasiparticle which will modify the solution of the equation-of-motion.

1. Ground state: A superconducting state

In the mean-field ground state, the bosonic holons are Bose condensed with $\langle h_i^\dagger\rangle=h_0\sim\sqrt{\delta}$ and the superconducting order parameter $\Delta^{SC}\neq 0$ (see Sec. II A). The decomposition (1.3) then implies that the electron c operator may be rewritten in two parts:

$$c_{i\sigma}=h_0 a_{i\sigma}+c'_{i\sigma}, \tag{2.31}$$

where $a_{i\sigma}\equiv b_{i\sigma}e^{i\hat{\Theta}_{i\sigma}}$ and $c'_{i\sigma}=(:h_i^\dagger:)b_{i\sigma}e^{i\hat{\Theta}_{i\sigma}}$ with $:h_i^\dagger:\equiv h_i^\dagger-h_0$. Correspondingly, a coherent term will emerge from the ''decaying'' terms in Eq. (2.27) which is linear in $a^\dagger$:

$$J\text{-scattering term in Eq. (2.27)}$$
$$\to\frac{3}{8}J\sum_{l=NN(i)}\left(\frac{\Delta_{il}^{SC}}{h_0^2}\right)\sigma h_0 a_{l-\sigma}^\dagger+\text{high order}. \tag{2.32}$$

In obtaining the RHS of the above expression, the superconducting order parameter defined in Eq. (2.11) is used.

Note that the t-scattering term in Eq. (2.27) gives rise to a term $\propto h_0 a_{i\sigma}$ which can be absorbed by the chemical potential μ added to the equation. Then one finds

$$-i\partial_t a_{i\sigma}\simeq t_{eff}\sum_{l=NN(i)}a_{l\sigma}+\mu a_{i\sigma}$$
$$+\frac{3}{8}J\sum_{l=NN(i)}\left(\frac{\Delta_{il}^{SC}}{h_0^2}\right)\sigma a_{l-\sigma}^\dagger+\text{high order}, \tag{2.33}$$

where the connection between a and c' has been assumed to be in high order and thus is neglected in the leading order approximation to get a closed form in linear a and $a^\dagger$. Finally, introducing the Bogoliubov transformation in the momentum space

$$a_{\mathbf{k}\sigma}=u_{\mathbf{k}}\gamma_{\mathbf{k}\sigma}-\sigma v_{\mathbf{k}}\gamma_{-\mathbf{k}-\sigma}^\dagger, \tag{2.34}$$

we find that Eq. (2.33) can be reduced to

$$-i\partial_t\gamma_{\mathbf{k}\sigma}^\dagger=E_{\mathbf{k}}\gamma_{\mathbf{k}\sigma}^\dagger, \tag{2.35}$$

where $\gamma_{\mathbf{k}\sigma}^\dagger$ represents the creation operator of an eigenstate of quasiparticle excitations with the energy spectrum

$$E_{\mathbf{k}}=\sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+|\Delta_{\mathbf{k}}|^2}. \tag{2.36}$$

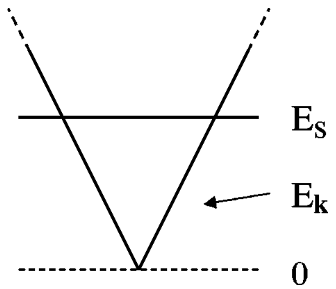

FIG. 4. Low-lying excitations in the superconducting phase: The ''V'' shape quasiparticle spectrum and the discrete spinon energy at E_s.

Here $\Delta_{\mathbf{k}}$ is defined by

$$\Delta_{\mathbf{k}}=\frac{3}{4}J\sum_{\mathbf{q}}\Gamma_{\mathbf{q}}\left(\frac{\Delta^{SC}_{\mathbf{k+q}}}{h_0^2}\right), \tag{2.37}$$

with $\Gamma_{\mathbf{q}}=\cos q_x+\cos q_y$. Like in BCS theory, $u^2_{\mathbf{k}}=[1+(\epsilon_{\mathbf{k}}-\mu)/E_{\mathbf{k}}]/2$ and $v^2_{\mathbf{k}}=[1-(\epsilon_{\mathbf{k}}-\mu)/E_{\mathbf{k}}]/2$.

The large ''Fermi surface'' is defined by $\epsilon_{\mathbf{k}}=\mu$ and $\Delta_{\mathbf{k}}$ then represents the energy gap opened at the Fermi surface. Note that $\Delta_{\mathbf{k}}$ changes sign as

$$\Delta_{\mathbf{k+Q}}=-\Delta_{\mathbf{k}}, \tag{2.38}$$

with $\mathbf{Q}=(\pm\pi,\pm\pi)$, by noting $\Gamma_{\mathbf{q+Q}}=-\Gamma_{\mathbf{q}}$ in Eq. (2.37). It means that $\Delta_{\mathbf{k}}$ has opposite signs at $\mathbf{k}=(\pm\pi,0)$ and $(0,\pm\pi)$, indicating a d-wave symmetry near the Fermi surface. In fact, since the pairing order parameter $\Delta^{SC}_{\mathbf{k}}$ is d-wave-like,[20] $\Delta_{\mathbf{k}}$ should be always d-wave-like with node lines $k_x=\pm k_y$ according to Eq. (2.37).

Comparing to the conventional BCS theory with the d-wave order parameter, there are several distinct features in the present case. First of all, besides the d-wave quasiparticle spectrum illustrated in Fig. 4 by the ''V'' shape lines along the Fermi surface, there exists a discrete spinon excitation level at $E_s\sim\delta J$ (horizontal line in Fig. 4) which leads to $E_g=2E_s\sim 41$ meV (if $J\sim 100$ meV) magnetic peak at $\delta\sim 0.14$ as reviewed in Sec. II A. This latter spin collective mode is *independent* of the quasiparticle excitations at the mean-field level.

Second, even though the superconducting *order parameter* $\Delta^{SC}_{\mathbf{k}}$ and the *energy gap* $\Delta_{\mathbf{k}}$ in the quasiparticle spectrum have the same symmetry, both are d-wave-like, and they cannot be simply identified as the same quantity as in BCS theory. For example, while $\Delta^{SC}_{\mathbf{k}}$ apparently scales with the doping concentration δ ($h_0\propto\sqrt{\delta}$) and vanishes at $\delta\rightarrow 0$, the gap $\Delta_{\mathbf{k}}$ defined in Eq. (2.37) is not, and can be *extrapolated* to a finite value in the zero-doping limit where $T_c=0$. It means

$$\frac{2\Delta_{\mathbf{k}}(T=0)}{T_c}\rightarrow\infty \tag{2.39}$$

at $\delta\rightarrow 0$, whereas the BCS theory predicts a constant ~ 4.28 (d-wave case[31]). The result (2.39) is consistent with the ARPES measurements.[32]

Third, the quasiparticle gains a ''coherent'' part $h_0 a$ which should behave similarly to the conventional quasiparticle in BCS theory as it does not further decay at $E_{\mathbf{k}}<E_s$ (see Fig. 4). In this sense, the quasiparticle partially restores its coherence in the superconducting state. Such a coherent part will disappear as a result of vanishing superfluid density. According to Eq. (2.31) one may rewrite the single-particle propagator as

$$G_e\simeq h_0^2 G_a+G'_e, \tag{2.40}$$

where G_a denotes the propagator of a particles with omitting the crossing term between a and c' which is assumed negligible. Then $h_0^2 G_a$ emerges as the ''coherent'' part of the Green's function in superconducting state against the ''normal'' part G'_e:

$$h_0^2 G_a(\mathbf{k},\omega)\sim h_0^2\left(\frac{u^2_{\mathbf{k}}}{\omega-E_{\mathbf{k}}}+\frac{v^2_{\mathbf{k}}}{\omega+E_{\mathbf{k}}}\right). \tag{2.41}$$

Correspondingly, the total spectral function as the imaginary part of G_e in our theory can be written as

$$A_e(\mathbf{k},\omega)=h_0^2 A_a(\mathbf{k},\omega)+A'_e(\mathbf{k},\omega). \tag{2.42}$$

So at $h_0\rightarrow 0$, even though $\Delta_{\mathbf{k}}$ does not scale with h_0, the superconducting coherent part $h_0^2 A_a$ vanishes altogether, with A_e reduced to the normal part A'_e at $T>T_c$.

Finally, in the present case, $A'_e(\mathbf{k},\omega)$ as a normal part has nothing to do with the procedure that leads to the spectrum (2.36) with a d-wave gap, which is different from the slave-boson approach where the fermionic spinons are all paired up such that even the part of the spectral function corresponding to A'_e should also look like in a d-wave pairing state. Due to the sum rule $\int(d\omega/2\pi)A^e(\mathbf{k},\omega)=1$, the ''normal'' part $A'_e(\mathbf{k})$ is expected to be sort of suppressed by the emergence of the ''coherent'' $h_0^2 A_a$ part, but since the latter is in order of δ, A'_e should be still dominant away from the Fermi surface at small doping. It implies that even in superconducting state, a normal-state dispersion represented by the peak of A'_e may still be present as a ''hump'' in the total spectral function A_e. Recent ARPES experiments have indeed indicated[33] the existence of a ''hump'' in the spectral function which clearly exhibits normal-state dispersion in the Fermi-surface portions near the areas of $(\pm\pi,0)$ and $(0,\pm\pi)$ where the d-wave gap is maximum.

2. Normal state

In the normal state, without any coherent contribution, the scattering terms in Eq. (2.27) only give rise to the virtual process for a quasiparticle to decay into the holon-spinon pairs. Based on Eq. (2.27), the propagator can be determined according to the following standard equation of motion for the single-particle Green's function $G_e(i,j;t)$:

$$\partial_t G_e(i,j;t)=\theta(t)\langle[H_{t-J},c_{i\sigma}(t)]c^{\dagger}_{j\sigma}(0)\rangle - \theta(-t)\langle c^{\dagger}_{j\sigma}(0)[H_{t-J},c_{i\sigma}(t)]\rangle - i\delta(t)\delta_{i,j}. \tag{2.43}$$

If we simply neglect the scattering terms in zero-order approximation, a closed form for G_e can be obtained in momentum-energy space:

$$G_e(\mathbf{k},\omega)\sim\frac{1}{\omega-(\epsilon_{\mathbf{k}}-\mu)}. \tag{2.44}$$

Here the quasiparticle spectrum $\epsilon_{\mathbf{k}}$ [defined in Eq. (2.29)] is essentially the same as the original *band-structure* spectrum except for a factor of $2/(1+\delta)\approx 2$ enhancement in effective mass. It is noted that in the t-J model if the hopping term described by the tight-binding model is replaced by a more realistic band-structure model, like introducing the next-nearest-neighbor hopping terms, the above conclusion about the factor-of-2 enhancement of the effective mass still holds, in good agreement with ARPES experiments.[34] Here the reason for the mass enhancement is quite simple: At each step of hopping, the probability is roughly one-half for a hole not to change the surrounding singlet spin configuration, which in turn reduces the ''bandwidth'' of the quasiparticle by a factor of 2.

The expression (2.44) shows a ''quasiparticle'' peak at $\epsilon_{\mathbf{k}}-\mu$ and defines a large ''Fermi surface'' as an equal-energy contour at $\epsilon_{\mathbf{k}}=\mu$. Here μ is determined such that $-i2\Sigma_{\mathbf{k}}G_e(\mathbf{k},t=-0)=N_e$.[35] So the ''Fermi-surface'' structure should look similar to that of a noninteracting band-structure fermion system as long as the virtual decaying process in Eq. (2.27) does not fundamentally alter it. [As mentioned before, we do not expect such ''decaying'' terms to significantly modify the $\mathbf{k}$ dependence of $\epsilon_{\mathbf{k}}$ since, for example, the spinon no longer has a well-defined spectrum in momentum space (see Sec. II A) and, in particular, the vortex phase $e^{i\hat{\Theta}}$ in Eq. (2.27) will further ''smear out'' the $\mathbf{k}$-dependent correction, if any, from Eq. (2.27).]

So far we have not discussed the finite lifetime effect of a quasiparticle due to the ''decaying'' terms in Eq. (2.27). Even though the true breakup of a quasiparticle is prevented by the phase shift field as discussed before, the virtual decaying process should remain a very strong effect since the phase shift field only costs a logarithmically divergent energy at a large length scale. The corresponding confinement force is rather weak and the virtual decaying process should become predominant locally to cause an intrinsic broad feature in the spectral function at high energy. Such a broad structure reflecting the decomposition in the one-hole case has been previously discussed in Ref. 22. At finite doping, how the ''decaying'' effect shapes the broadening of the quasiparticle peak will be a subject to be investigated elsewhere.

3. Destruction of Fermi surface: Deconfinement of spinon and holon

The existence of a large Fermi surface, coinciding with the *noninteracting* band-structure one, can be attributed to the integrity of the quasiparticle due to the confinement of spinon and holon. But as pointed out in Sec. II C, such a confinement will disappear in the zero-doping limit. The Fermi surface structure will then be drastically changed.

In this limit, the single-electron propagator may be expressed in the following *decomposition* form

$$G_e\approx iG_f\cdot G_b, \tag{2.45}$$

where

$$G_f(i,j;t)=-i\langle T_t h_i^\dagger(t)(e^{i\Phi_i^b(t)/2}e^{-i\Phi_j^b(0)/2})h_j(0)\rangle \tag{2.46}$$

and

$$G_b(i,j;t)=-i(-\sigma)^{i-j}\langle T_t b_{i\sigma}(t)\times(e^{-i\sigma\Phi_i^h(t)/2}e^{i\sigma\Phi_j^h(0)/2})b_{j\sigma}^\dagger(0)\rangle, \tag{2.47}$$

without the multivalue problem because Φ_i^h in Eq. (2.3) vanishes and $e^{i\Phi_i^b/2}$ becomes well defined as discussed in Sec. II C. At $\delta\to 0$, H_s in Eq. (2.7) reduces to the SBMFT Hamiltonian with $A^h=0$ and G_b becomes the conventional Schwinger-boson propagator. Such a deconfinement can be also seen from the equation of motion (2.27) by noting that $e^{i\hat{\Theta}_{i\sigma}}\to e^{i\Phi_i^b/2}$ can be absorbed by $h_i^\dagger$, while $A_{il}^h=0$, so that the scattering term becomes a pure decaying process for the quasiparticle without any confining force. Due to such a true decaying, Eq. (2.27) actually describes in real time the first step towards dissolution for the quasiparticle. In particular, the large Fermi-surface structure originating from the *bare* hopping term in Eq. (2.27) will no longer appear in the decomposition form of the electron propagator (2.45), where the residual Fermi surface (points) will solely come from the oscillating part of the phase shift field $e^{i\Phi_i^b/2}$ in G_f.

The single-electron propagator for the one-hole case has been discussed in detail in Ref. 22. Here the large Fermi surface is gone except for four *Fermi points* at $\mathbf{k}_0=(\pm\pi/2,\pm\pi/2)$ with the remaining in part looking like they are all ''gapped.'' In fact, in the convolution form of Eq. (2.45) the ''quasiparticle'' peak (edge) is essentially determined by the spinon spectrum $E_{\mathbf{k}}^s=2.32J\sqrt{1-s_{\mathbf{k}}^2}$ with $s_{\mathbf{k}}=(\sin k_x+\sin k_y)/2$ in SBMFT through the spinon propagator G_b, since the holon propagator G_f is incoherent.[22] The intrinsic broad feature of the spectral function found in Ref. 22 is due to the convolution law of Eq. (2.45) and is a direct indication of the composite nature of the quasiparticle, which is also consistent with the ARPES results[23] as well as the earlier theoretical discussion in Ref. 36.

Note that the Fermi points $\mathbf{k}_0$ coming from G_f at low energy is due to the phase shift field $e^{i\Phi_i^b/2}$ appearing in it. In Ref. 22, this is shown in the slave-fermion formulation which is related to the present formulation through a unitary transformation[15] with $h_i^\dagger e^{i\Phi_i^b/2}$ being replaced by a new holon operator $f_i^\dagger$. And the f holon will then pick up a phase string factor $(-1)^{N_c^\downarrow}$ ($N_c^\downarrow$ denotes the total number of ↓ spins exchanged with the holon during its propagation along the path c connecting sites i and j) at low energy which can be written as

$$(-1)^{N_c^\downarrow}\equiv e^{\pm i\pi N_c^\downarrow}=e^{i\mathbf{k}_0\cdot(\mathbf{r}_i-\mathbf{r}_j)}e^{\pm i\delta N_c^\downarrow}, \tag{2.48}$$

where $\delta N_c^\downarrow=N_c^\downarrow-\langle N_c^\downarrow\rangle$, and $\langle n_{l\downarrow}^b\rangle=1/2$ is used. On the other hand, in the equal-time ($t\sim 0^-$) limit, the singular oscillating part of $e^{iK_\sigma(c)}$ in Eq. (2.24) will also contribute to a large ''remnant Fermi surface'' in the momentum distribution function $n(\mathbf{k})$ which can be regarded as a precursor of the large Fermi surface in the confining phase at finite doping, and is also consistent with the ARPES experiment as discussed in Ref. 22.

The above one-hole picture may have an important implication for the so-called pseudo-gap phenomenon[24] in the underdoped region of the high-T_c cuprates. Even though the confinement will set in once the density of holes becomes finite, the ''confining force'' should remain *weak* at small doping, and one expects the virtual ''decaying'' process in Eq. (2.27) to contribute significantly at weak doping to bridge the continuum evolution between the Fermi-point structure in the zero-doping limit to a full large Fermi surface at larger doping. Recall that in the one-hole case decaying into spinon-holon composite happens around $\mathbf{k}_0$ at zero energy transfer, while it costs *higher* energy near $(\pi,0)$ and $(0,\pi)$, which should not be changed much at weak doping. In the confinement regime, the quasiparticle peak in the electron spectral function defines a quasiparticle spectrum and a large Fermi surface as discussed before. Then due to the virtual ''decaying'' process in the equation of motion (2.27) [as shown in Fig. 1(b)], the spectral function will become much broadened with its weight shifted toward higher energy like a gap opening near those portions of the Fermi surface far away from $\mathbf{k}_0$, particularly around four corners $(\pm\pi,0)$ and $(0,\pm\pi)$. With the increase of doping concentration and reduction of the decaying effect, the suppressed quasiparticle peak can be gradually recovered starting from the inner parts of the Fermi surface towards four corners $(\pm\pi,0)$ and $(0,\pm\pi)$. Eventually, a coherent Landau quasiparticle may be even restored in the so-called overdoped regime, when the bosonic RVB ordering collapses such that the spin-charge separation disappears.

Furthermore, at small doping (underdoping), something more dramatic can happen in the model described by Eqs. (2.6) and (2.7). In Ref. 20, a microscopic type of *phase separation* has been found in this regime which is characterized by the Bose condensation of bosonic spinon field. Since spinons are presumably condensed in *hole-dilute* regions,[20] the propagator will then exhibit features looking like in an even *weaker* doping concentration or more ''gap'' like than in a uniform case, below a characteristic temperature T^* which determines this microscopic phase separation. Therefore, the ''spin-gap'' phenomenon related to the ARPES experiments[24] in the underdoped cuprates may be understood as a ''partial'' deconfinement of holon and spinon whose effect is ''amplified'' through a microscopic phase separation in this weakly doped regime. As discussed in Ref. 20, T^* also characterizes other ''spin-gap'' properties in magnetic and transport channels in this underdoping regime.

III. CONCLUSION AND DISCUSSION: A UNIFIED VIEW

In this paper, we have studied the quasiparticle properties of doped holes based on an effective spin-charge separation theory of the *t-J* model. The most unique result is that a quasiparticle remains stable as an independent excitation despite the existence of holon and spinon elementary excitations. The underlying physics is that in order for a doped hole to evolve into elementary excitations described by a holon and spinon, the whole system has to adjust itself globally which would take infinite time under a local perturbation. Such an adjustment is characterized by a vortexlike phase shift as shown in Eq. (1.3). As a consequence of the phase shift effect, the holon and spinon constituents are found to be effectively confined which maintains the integrity of a quasiparticle except for the case in the zero-doping limit. In particular, the quasiparticle weight is zero since there is no overlap between a doped hole state and the true ground state due to the symmetry difference introduced by the vortex phase shift. Such a quasiparticle is no longer a conventional Landau quasiparticle and is generally incoherent due to the virtual decaying process. Only in the superconducting state can the coherence be partially regained by the quasiparticle excitation.

The physical origin of the ''unrenormalizable phase shift'' is based on the fact that a hole moving on an antiferromagnetic spin background will always pick up the phase string composed of a product of + and − signs which depend on the spins exchanged with the hole during its propagation.[18] Such a phase string is nonrepairable at low energy and is the only source to generate phase frustrations in the *t-J* model. The phase shift field in Eq. (1.3) precisely keeps track of such a phase string effect[15] and therefore accurately describes the phase problem in the *t-J* model even at the mean-field level discussed in Sec. II A.

Probably the best way to summarize the present work is to compare the present self-consistent spin-charge separation theory with some fundamental concepts and ideas proposed over years in the literature related to the doped Mott-Hubbard insulator.

RVB pairing. The present theory can be regarded as *one* of the RVB theories,[1,37,4] where the spin RVB pairing is the driving force behind everything from spin-charge separation to superconductivity. The key justification for *this* RVB theory is that it naturally recovers the *bosonic* RVB description at half-filling, which represents[11,12] the most accurate description of the antiferromagnet for both short-range and long-range AF correlations. In the metallic state at finite doping, the RVB order Δ^s defined in Eq. (2.10) reflects a partial ''fermionization'' from the original pure bosonic RVB pairing due to the gauge field A_{ij}^h determined by doped holes. But it is still physically different from a *full* fermionic RVB description.[1,6,37] In contrast to the fermionic RVB order parameter, Δ^s here serves as a ''super'' order parameter characterizes a *unified* phase covering the antiferromagnetic insulating and metallic phases, and normal and superconducting states altogether.[20]

Spin-charge separation. In our theory, elementary excitations are described by charge-neutral spinon and spinless holon fields, and the ground state may be viewed as a spinon-holon sea. Different from slave-particle decompositions, spinons and holons here are all *bosonic* in nature and the conventional gauge symmetry is broken by the RVB ordering. But these spinons and holons in 2D still couple to each other through the mutual Chern-Simons-like gauge interactions which are crucial to T_c, anomalous transport and magnetic properties. The Bose condensation of holons corresponds to the superconducting state, while the Bose condensation of spinons in the *insulating phase* gives rise to an AF long-range order. The spinon Bose condensation can persist into the metallic regime, leading to a pseudogap phase with microscopic phase separation which can coexist with superconductivity.[20]

Bosonization. The electron c operator expressed in terms of bosonic spinons and holons in Eq. (1.3) naturally realizes a special form of bosonization. A 2D bosonization description has been regarded by many[4,17,38–40] as the long-sought technique to replace the perturbative many-body theory in dealing with a non-Fermi liquid. The 2D bosonization scheme has been usually studied, as an analog to the successful 1D version,[41] in momentum space where Fermi surface patches have to be assumed first.[4,17,38–40] In the present scheme, which is also applicable to 1D, the Fermi surface satisfying the Luttinger volume and the so-called Fermi-surface fluctuations are presumably all *generated* by the phase shift field in Eq. (1.3), which guarantees the fermionic nature of the electron. Note that the vortex structure involved in the phase shift field in the 2D case is the main distinction from the Θ function in conventional bosonization proposals.[40]

Non-Fermi liquid. As a consequence of the phase shift field, representing the Fermi surface ''fluctuations,'' the ground state is a non-Fermi liquid with vanishing spectral weight Z, consistent with the argument made by Anderson[3,4] based on a ''scattering'' phase shift description. In 1D both methods are equivalent as the phase shift value in the latter can be determined quantitatively based on the exact Bethe-ansatz solution. But in 2D, the phase shift field $\Theta_{i\sigma}^{string}$, which is obtained by keeping track of the nonrepairable phase string effect induced by the traveling holon, provides a unique many-body version with vorticities, and our model shows how a *concrete* 2D non-Fermi liquid system can be realized.

Quasiparticle: Spinon-holon confinement. In conventional spin-charge separation theories based on slave-particle schemes, a quasiparticle does not exist at all: It always breaks up and decays into spinon-holon elementary excitations. This deconfinement has been widely perceived as a logical consequence of the spin-charge separation in the literature. But in the present paper, we have shown that the phase shift field actually *confines* the spinon-holon constituents (at least at finite doping), which means that the integrity of a quasiparticle is still preserved even in a spin-charge separation state. It may be considered as a U(1) version of quark confinement, but with a twist: the stable quasiparticle as a collective mode is generally not a *coherent* elementary excitation since during its propagation the phase shift field also induces a nonlocal phase string on its path. In other words, the quasiparticle here is not a Landau quasiparticle anymore. The confinement and nonrepairable phase string effect both reflect the fermionic nature of the quasiparticle; namely, the *fermionic* quasiparticle cannot simply decay into a *bosonic* spinon and holon, and the phase string effect comes from sequential signs due to the *exchange* between the propagating quasiparticle and the spinon-holon background—the latter after all is composed of fermionic electrons in the original representation.

The issue of the possible confinement of spinon and holon was already raised by Laughlin[42] along a different line of reasoning. He has also discussed numerical and experimental evidence that the spinons and holons may be seen, more sensibly, in high-energy spectroscopy like the way quarks are seen in particle physics. In recent SU(2) gauge theory,[43] an attraction between spinons and holons to form a bound state due to gauge fluctuations is also assumed in order to explain the ARPES data. But in the present work the essential point is that the holons and spinons are not confined in the ground state but are only bound in quasiparticles as a kind of incoherent (many-body) excitations which have no overlap with elementary holon and spinon excitations as guaranteed by symmetry. These incoherent quasiparticle excitations should not have any significant contribution to the thermodynamic properties (at least above T_c). At short distance and high energy, the composite nature of a quasiparticle will become dominant which may well explain the broad *intrinsic* structure in the spectral function observed in the ARPES experiments. The composite structure of the quasiparticle can even show up at low energy when one navigates through different circumstances like the superconducting condensation, underdoping regime, etc., with some unique features different from the behavior expected from Landau quasiparticles.

Fractional statistics. Laughlin[44] made a compelling argument right after the proposal of spin-charge separation that the holon and spinon should carry fractional statistics, by making an analogy of the spin liquid state with a fractional quantum Hall state. Even though the absence of the time-reversal symmetry-breaking evidence in experiments does not support the original version of fractional statistics (anyon) theories,[44,45] the essential characterization of fractional statistics for the singlet spin liquid state is, surprisingly, present in our theory in the form of the phase string effect as discussed in Ref. 15. But no explicit time-reversal symmetry is broken in this description.[20]

A fractional statistics may sound strange as we have been talking about ''bosonization'' throughout the paper. But as pointed out in Ref. 15, if the phase shift field $\Theta_{i\sigma}^{string}$ is to be ''absorbed'' by the bosonic spinon and holon fields, then the expression (1.3) can be regarded as a slave-semion decomposition with the new ''spinon'' and ''holon'' fields being ''bosonic'' among themselves but satisfying a *mutual* fractional statistics between the spin and charge degrees of freedom. The origin of mutual statistics can be traced back to the nonrepairable phase string effect induced by a hole in the antiferromagnetic spin background. At finite doping, the order parameter Δ^s in Eq. (2.10) actually describes the RVB pairing of spinons with *mutual statistics* which reduces to the bosonic RVB only in the half-filling limit. Furthermore, in the bosonic representation of Eqs. (2.6) and (2.7) the lattice Chern-Simons fields A_{ij}^f and A_{ij}^h precisely keep track of mutual statistics,[15] which are crucial to various peculiar properties exhibited in the model.

Gauge theory. Based on the slave-boson decomposition, it has been shown[26,27] that the gauge coupling is the most important low-energy interaction associated with spin-charge deconfinement there. The anomalous linear-temperature resistivity[27] has become the hallmark for anomalous transport phenomenon based on the scattering between charge carriers and gauge fluctuations. In the present theory, the holons are also subject to strong random flux fluctuations, in terms of the effective holon Hamiltonian (2.7), in a uniform normal state where it leads[14] to the linear-T resistivity in consistency with the Monte Carlo numerical calculation.[46] Other anomalous transport properties related to the cuprates may be also systematically explained in such a simple gauge model based on some effective analytic treatment.[14,47] In contrast to the slave-boson gauge theory, however, the spinon part does not participate in the transport phenomenon due to the RVB condensation ($\Delta^s \neq 0$) persisting over the normal state, and besides the Chern-Simons gauge fields $A_{ij}^{f,h}$, the conventional gauge interaction between spinons and holons is suppressed because of it.

Furthermore, the π-flux phase[48] and commensurate flux phase[49] in the mean-field slave-boson theory, and the recent SU(2) gauge theory[43] at small doping have a very close connection with the present approach: A *fermionic* spinon in the presence of π flux per plaquette is actually a precursor to become a *bosonic* one at half-filling under the lattice and no-double-occupancy constraint. In Ref. 14, how such a statistics transmutation occurs has been discussed, and in fact the bosonization decomposition (1.3) was first obtained there based on the fermionic flux phase. In spite of the physical proximity, however, the detailed mathematical structure of the gauge description[50,51] for the fermionic flux phase and the present bosonic spinon description are obviously rather different.

Superconducting mechanism. Anderson[1] originally conjectured that the superconducting condensation may occur once the RVB spin pairs in the insulating phase start to move like Cooper pairs in the doped case. The superconducting condensation in the present theory indeed follows suit. But there is an important subtlety here. Since the RVB paring order parameter Δ^s covers the normal state as well, there must be an another factor controlling the superconducting transition: the *phase coherence*. Indeed, the vortex phase Φ^b appearing in the superconducting order parameter (2.11) is the key to ensure the *phase coherence* at a relatively low temperature compatible to a characteristic *spin* energy.[20,21] In other words, the phase coherence discussed by Emery and Kivelson[52] is realized by the *phase shift field* in the present theory which effectively resolves the issue why T_c is too high in previous RVB theories.

Furthermore, the interlayer pair tunneling mechanism[53] for superconductivity is also relevant to the present theory from a different angle. Recall that the quasiparticle does exist in the present theory but is always *incoherent* just like the blocking of a coherent single-particle interlayer hopping conjectured in Ref. 53. On the other hand, a pair of quasiparticles in the singlet channel can recover the *coherency* due to the cancellation of the frustration caused by the phase string effect. Thus, if one is to construct a phenomenological theory based on the *electron* representation, the superconductivity can be naturally viewed as due to an *in-plane* kinetic mechanism.[54]

Phase string and Z_2 gauge theory. As pointed out at the beginning of this section, our whole theory is built on the phase string effect identified in the *t-J* model. Namely, the main phase frustration induced by doping is characterized by a sequence of signs $\Pi_c \sigma_{ij}$ on a closed path c where $\sigma_{ij} = \pm 1$ denotes the index of spins exchanged with a hole at a link (ij) during its hopping. Thus, instead of working in the vortex representation of Eq. (1.3) where the singular phase string effect has been built into the wave functions with the nonsingular part described by the Chern-Simons-like lattice gauge fields,[15] one may also directly construct a $2+1$ Z_2 gauge theroy[55] to deal with the singular phase string $\Pi_c \sigma_{ij}$. A discrete Z_2 gauge theory here seems the most natural description of the phase string effect as the sole phase frustration in the *t-J* model, in contrast to the conventional continuum gauge field description.[26,27]

Finally, we emphasize the close connection between the antiferromagnetism and superconductivity as both occur in a unified RVB background controlled by Δ^s. The relation between the AF insulating phase and the superconducting phase here is much more intrinsic than in conventional approaches to the *t-J* model. Especially, the coexistence of holon and spinon Bose condensations in the underdoped regime[20] makes a group theory description of such a phase, in the fashion of SO(5) theory,[56] become possible but with an important modification: Inhomogeneity must play a crucial role here in the Bose-condensed holon and spinon fields in order to describe this underdoped regime. A detailed investigation along this line will be pursued elsewhere.

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