Ginzburg-Landau calculations for a superconducting cylinder in a magnetic field

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Self-consistent solutions of the Ginzburg-Landau system of nonlinear equations, which describe the behavior of the order parameter ψ and the magnetic-field distribution B in a long superconducting cylinder of finite radius R in external magnetic field H, provided that, there are no vortices inside the superconductor, are studied by using numerical method. The lower and upper critical fields of the cylinder, $H_{c1}^{(0)}$ and $H_{c2}^{(0)}$, are found as functions of the radius R, temperature T, and parameter κ of the Ginzburg-Landau theory. For type-I superconductors one has $H_{c1}^{(0)}=H_{c2}^{(0)}$; for type-II superconductors one has $H_{c1}^{(0)}<H_{c2}^{(0)}$. In small fields $H<H_{c1}^{(0)}$ the superconductor is in stable Meissner phase (with $\psi\sim 1$ and $B\sim 0$). It is found, that for type-II superconductors the state with $\psi\sim 1$ is unstable in the fields $H>H_{c1}^{(0)}$, and the superconductor passes to a new stable state. In this state the external field begins to penetrate freely into a superconductor in a form of a finite width ring, which is situated near the surface of the cylinder, where the order parameter is strongly suppressed (the rim-suppressed state). The field $H_{c1}^{(0)}$ differs from the lower critical field H_{c1}, at which the field begins to penetrate into the bulk superconductor in a form of vortices. When the field H is increased further, this ring layer (or, the rim) widens, while the order parameter remaines finite ($\psi\neq 0$) only near the center of the cylinder. In the field $H=H_{c2}^{(0)}$ the order parameter finally vanishes everywhere and the metal passes into the normal state. For $R\gg\lambda$ the field $H_{c2}^{(0)}$ coincides with the upper critical field H_{c2}, at which the mixed vortex state terminates. The intervals of R, T, and κ, where the rim-suppressed state can exist, are found.

I. INTRODUCTION

The behavior of small size superconductors in a magnetic field, in the scope of the nonlinear system of the Ginzburg-Landau equations,[1] was studied in numerous publications (see, for instance, Refs. 2 and 3). Recently, much of the attention was devoted also to the finite-size superconductors of different geometries (see, for instance, the theoretical papers[4]). The main attention in these papers was paid to the case, when the magnetic vortex, carrying $m\phi_0$ flux quanta, is rested in the center of superconducting cylinder (here $m=0$, 1, 2, . . . , $\phi_0=hc/2e$ is the flux quantum). Some of the results, obtained in Ref. 4, were used to explain some anomalies in the behavior of small size superconductors, placed in the magnetic field, which were detected recently.[5] In the present paper the vortex-free Meissner state ($m=0$) is studied in more detail, as compared to Ref. 4.

The main result of the present investigation is, that for a superconducting cylinder of finite radius R placed in an external magnetic field H, two characteristic fields exist (if $\kappa>1/\sqrt{2}$): the lower, $H=H_{c1}^{(0)}$, and the upper, $H=H_{c2}^{(0)}$ [the upper index (0) denotes the vortex-free state, with $m=0$]. If the external field is small, $H<H=H_{c1}^{(0)}$, then $\psi(r)\approx 1$ everywhere, the internal magnetic field B is also small (this corresponds to the Meissner state, with almost total expulsion of the external field from the superconductor interior). When H increases and reaches the value $H=H_{c1}^{(0)}$, a sharp reconstruction of the order parameter occurs: its value on the axis, ψ_0 continues to be very close to unity, but the function $\psi(r)$ suffers strong suppression in a ring-shaped layer, situated near the outer surface of the cylinder (this may be referred to as a rim-suppressed state). The magnetic field inside this layer is equal to the external field H, while at the center $B_0\equiv B(0)<H$. When H is increased further, the rim layer widens, the value ψ_0 gradually diminishes, and the field $B(r)\to H$ everywhere. At $H=H_{c2}^{(0)}$ the final transition to the normal state takes place, with $\psi(r)\equiv 0$ and $B(r)\equiv H$.

The upper critical field $H=H_{c2}^{(0)}$ (for $R\gg\lambda$, where $\lambda=\kappa\xi$, ξ is the coherence length) coincides with the field $H_{c2}^{(v)}\equiv H_{c2}=\phi_0/(2\pi\xi^2)$, which marks the endpoint of the mixed state of a bulk superconductor.[2,3] This is natural, because both these fields correspond to the normal state, with $\psi\to 0$. The lower critical field $H=H_{c1}^{(0)}$ (even for $R\gg\lambda$) does not coincide with the field $H_{c1}^{(v)}=H_{c1}\equiv H_c\cdot(\sqrt{2})^{-1}(\ln\kappa+0.08)$ (the field H_{c1} marks the beginning of vortex penetration into massive type-II superconductor[2,3]). This is also natural, because we consider vortex-free state ($m=0$), when vortices are forbidden. The fields $H_{c1}^{(0)}$ and $H_{c2}^{(0)}$ are found below as functions of κ, R and T. If $\kappa>1/\sqrt{2}$, one has $H_{c2}^{(0)}>H_{c1}^{(0)}$; if $\kappa=1/\sqrt{2}$, both fields coincide: $H_{c1}^{(0)}=H_{c2}^{(0)}$; if $\kappa<1/\sqrt{2}$, there exists a single critical field, $H_c^{(0)}$. For $R\gg\lambda$ the field $H_{c1}^{(0)}\to H_c\equiv\phi_0/(2\pi\sqrt{2}\lambda\xi$, where H_c is the superconductor thermodynamic critical field.[2,3] Some other characteristics of the system are found as well: the magnetic moment (or, magnetization) of the cylinder, its free energy, the order parameter and magnetic field values on the cylinder axis, the mean field value in the speciment, the width of the magnetization curve tail, etc. (Note that the numeric algorithm, used in the present paper to obtain self-consistent solutions of the nonlinear system of the Ginzburg-Landau equations, does not allow us to study the unstable ''supercooled'' and ''superheated'' states,[6–8] which are characteristic for type-I superconductors with $\kappa<1/\sqrt{2}$ and require different methods of investigation.)

The paper is organized as follows. In Sec. II, the basic equations of the problem are written, and the method of finding the self-consistent solution of the nonlinear system of equations is briefly described. Section III contains the results of numerical calculations. In Sec. IV, the discussion and comparison with the previous publications on this problem are given.

II. THE SETTING OF THE PROBLEM

Consider the long superconducting cylinder of radius R in the external magnetic field H, which is parallel to the cylinder element. The basic system of the Ginzburg-Landau equations[1] is of the form

$$\mathrm{rot\,rot}\,\mathbf{A}=\frac{4\pi}{c}\mathbf{j}_s, \quad \frac{4\pi}{c}\mathbf{j}_s=\frac{\psi^2}{\lambda^2}\left(\frac{\phi_0}{2\pi}\nabla\Theta-\mathbf{A}\right),$$

$$\nabla^2\psi-\left(\nabla\Theta-\frac{2\pi}{\phi_0}\mathbf{A}\right)^2\psi+\frac{1}{\xi^2}(\psi-\psi^3)=0, \qquad (1)$$

where $\mathbf{A}$ is the vector potential of the magnetic field ($\mathbf{B}=\mathrm{rot}\,\mathbf{A}$), $\mathbf{j}_s$ is the current density inside the superconductor, λ is the field penetration depth, ξ is the correlation length, $\lambda=\kappa\xi$. The order parameter, in a general case, is written as $\Psi=\psi e^{i\Theta}$, where ψ is the modulus and Θ is the phase of the order parameter. From the single valuedness of Ψ the condition follows

$$\oint_C \nabla\Theta\, d\boldsymbol{l}=2\pi m,$$

where the contour C embraces the vortex axis, m is an integer (the topological invariant), which shows how many vortices are present inside the contour R. We consider the case $m=0$ and set $\Theta=0$, so the function $\Psi=\psi$ is real.

In the cylidrical system of coordinates r,φ,z, with the z axis directed along the cylinder element [when the vector-potential has only one component, $\mathbf{A}=\mathbf{e}_\varphi A(r)$], these equations may be written in the dimensionless form

$$\frac{d^2U}{d\rho^2}-\frac{1}{\rho}\frac{dU}{d\rho}-\psi^2U=0, \qquad (2)$$

$$\frac{d^2\psi}{d\rho^2}+\frac{1}{\rho}\frac{d\psi}{d\rho}+\kappa^2(\psi-\psi^3)-\frac{U^2}{\rho^2}\psi=0. \qquad (3)$$

Here, instead of the dimensioned potential A, field B and current j_s, the dimensionless quantities $U(\rho)$, $b(\rho)$, and $j(\rho)$ are introduced:

$$A=\frac{\phi_0}{2\pi\lambda}\frac{U}{\rho}, \quad B=\frac{\phi_0}{2\pi\lambda^2}b, \quad b=\frac{1}{\rho}\frac{dU}{d\rho}, \qquad (4)$$

$$j(\rho)=j_s\bigg/\frac{c\phi_0}{8\pi^2\lambda^3}=-\psi^2\frac{U}{\rho}, \quad \rho=\frac{r}{\lambda}.$$

[The field B in Eq. (4) is normalized by H_λ, $b=B/H_\lambda$; frequently the normalization by $H_{\mathrm{cm}}=\kappa H_\lambda/\sqrt{2}$ is used, where H_{cm} is the thermodynamical critical field.[4,5]]

The magnetic flux, confined inside the contour of the radius r, is

$$\Phi=\int \mathbf{B}d\mathbf{s}=\oint_r \mathbf{A}d\boldsymbol{l}=\phi_0(U+m), \quad U=U(\rho), \quad \rho=\frac{r}{\lambda}.$$

Thus, the potential $U(\rho)$, in the normalization adopted above, is related to the flux $\Phi(\rho)$ by a simple formula $\phi\equiv\Phi/\phi_0=U$.

Because the magnetic flux through the vanishing contour is zero, and the field $B|_{r=R}=H$, the following boundary conditions correspond to Eq. (2):

$$U|_{\rho=0}=0, \quad \left.\frac{dU}{d\rho}\right|_{\rho=\rho_1}=h, \qquad (5)$$

where $\rho_1=R/\lambda$, $h=H/H_\lambda$, $H_\lambda=\phi_0/(2\pi\lambda^2)$.

As to the Eq. (3), we shall take the usual boundary condition on the external surface:[1] $d\psi/d\rho|_{\rho=\rho_1}=0$. The order parameter at the center is maximal (if $m=0$), thus, the following boundary conditions correspond to Eq. (3):

$$\psi|_{\rho=0}=0, \quad \left.\frac{d\psi}{d\rho}\right|_{\rho=\rho_1}=0. \qquad (6)$$

The magnetic moment (or, magnetization) of the cylinder, related to the unity volume, is

$$\frac{M}{V}=\frac{1}{V}\int\frac{B-H}{4\pi}dv=\frac{B_{av}-H}{4\pi}, \qquad (7)$$

$$B_{av}=\frac{1}{V}\int B(\mathbf{r})dv=\frac{1}{S}\Phi_1,$$

where B_{av} is the field mean value inside the superconductor, $\Phi_1=\Phi(R)$, $S=\pi R^2$. In normalization (4), denoting $\overline{b}=B_{av}/H_\lambda$, $h=H/H_\lambda$, $M_\lambda=M/H_\lambda$, one finds from Eq. (7):

$$4\pi M_\lambda=\overline{b}-h, \quad \overline{b}=\frac{2}{\rho_1^2}U_1, \qquad (8)$$

$$U_1=U(\rho_1), \quad \rho_1=\frac{R}{\lambda}.$$

The difference of the Gibbs free energies of the system in superconducting and normal states, $\delta G=G_s-G_n$, can be expressed through the magnetic moment:[6]

$$\Delta G=\mathcal{F}_{s0}-\frac{1}{2}MH, \qquad (9)$$

$$\mathcal{F}_{s0}=\frac{H_{\mathrm{cm}}^2}{8\pi}\int\left[\psi^4-2\psi^2+\xi^2\left(\frac{d\psi}{dr}\right)^2\right]dv,$$

where $\mathcal{F}_{s0}$ corresponds to the superconductor condensation energy (see, also, Ref. 9). Using Eq. (4), one finds from Eq. (9) the normalized expression

$$\Delta g=\Delta G\bigg/\left(\frac{H_{\mathrm{cm}}^2}{8\pi}V\right)=g_0-\frac{8\pi M_\lambda}{\kappa^2}h, \qquad (10)$$

$$g_0=\frac{2}{\rho_1^2}\int_0^{\rho_1}\rho d\rho\left[\psi^4-2\psi^2+\frac{1}{\kappa^2}\left(\frac{d\psi}{d\rho}\right)^2\right].$$

Expressions (8)–(10) will be used later in Sec. III.

We remind the reader, that the field penetration depth λ and the coherence length $\xi=\lambda/\kappa$ depend on temperature. Thus, the expressions above depend implicitly on temperature, and, formally, are valid for arbitrary values of T. (Though, the Ginzburg–Landau equations themselves are applicable only in the limit $T\rightarrow T_c$.[2,3,10])

It is appropriate here to make a comment on the iteration procedure we used to obtain the self-consistent solution of the system of equations (2)–(6). First, some trial function $\psi(\rho)$ was chosen, and the solution for $U(\rho)$ was found from Eq. (2), with account of conditions (5). Then, the function $U(\rho)$ was introduced into Eq. (3), which was solved taking into account the boundary conditions (6). Further, Eq. (2) was solved again, and all the procedure was repeated, until the functions $\psi(\rho)$ and $U(\rho)$ ceased to change. In every step of our iteration procedure, instead of the two-sided boundary value conditions (5) and (6), we have used the one-sided Cauchy data to specify the solutions (see Ref. 11 for more details). In this manner (in accordance with the Cauchy theorem), for every value of H we found the unique final functions $\psi(\rho)$ and $U(\rho)$, which constitute a real self-consistent solution. Evidently, the solution, found in this way, is stable, because it is not sensitive to small perturbations, introduced during iterations. However, some other solutions may exist (for instance, representing states with magnetic quantum numbers $m>0$, or states without central symmetry), which may have smaller Gibbs free energy, than in the state $m=0$. That would mean that the state with $m=0$ may pass to a different state (with $m>0$), and in this sense it would be metastable. The accompanying physical changes in the system, in principle, may be observed experimentally, but this topic requires special investigation (see, also, the discussion in Sec. IV below). The results of our calculations are presented below.

III. THE NUMERICAL RESULTS

The functions $U(\rho)$ and $\psi(\rho)$, found by the procedure described above, were used to calculate the quantities (8)–(10), and other characteristics of the system.

In Fig. 1 we present, as functions of h, the values of (a) the magnetic moment, $-4\pi M_\lambda$; (b) the difference of free energies Δg; (c) the magnetic field on the cylinder axis, b_0; (d) the order parameter on the axis, ψ_0; (e) the mean value of the magnetic field inside the cylinder, $\overline{b}$; (f) the order parameter at the surface, $\psi(\rho_1)$ (all the curves correspond to the case $\rho_1=R/\lambda=5$ with $\kappa=0.5$; 1; 1.5; 2).

First of all, consider Fig. 1(a), where two branches of magnetization $-4\pi M_\lambda(h)$ are seen (these branches are shown by solid lines). The initial linear part of the first branch (at small h) corresponds to the Meissner effect (the field is expelled from the superconductor interior). When h increases, the magnetization begins to deviate from the linear law (simultaneously, the order parameter at the cylinder surface, $\psi_1=\psi(\rho_1)$, gradually diminishes, and the external field partly penetrates into the specimen). This Meissner branch ends up with an abrupt fall of the magnetization curve at $h=h_{c1}^{(0)}$ (shown by the vertical broken line), with the system passing to the second branch (solid line at $h>h_{c1}^{(0)}$). The order parameter value at the surface ψ_1 diminishes here by a jump [Fig. 1(f)]. When h is increased further, $h>h_{c1}^{(0)}$, the magnetization, $-4\pi M_\lambda$, diminishes gradually, and finally it vanishes at $h=h_{c2}^{(0)}$. As can be seen from Figs. 1(c) and 1(d), for $h=h_{c2}^{(0)}$ one has simultaneously $\psi=0$ and $\overline{b}=h$, which means the transition to normal state.

The presence of two superconducting states (at $h<h_{c1}^{(0)}$ and at $h_{c1}^{(0)}<h<h_{c2}^{(0)}$) can be seen on all the curves, depicted in Fig. 1. In the field interval $h_{c1}^{(0)}<h<h_{c2}^{(0)}$ the superconducting state gradually degenerates with h increasing, and it vanishes finally at $h_{c2}^{(0)}$ via second-order phase transition (if $\kappa>1/\sqrt{2}$). If κ diminishes, the interval between the fields, $\Delta h_c=h_{c2}^{(0)}-h_{c1}^{(0)}$, diminishes also, and for $\kappa=1/\sqrt{2}$ this interval vanishes, $\Delta h_c=0$. For $\kappa<1/\sqrt{2}$ there exists a single critical field $h_c^{(0)}$ at which the superconductivity is destroyed via first-order phase transition.

Figure 2 illustrates the behavior of the solutions ψ and b of Eqs. (2), (3), as functions of the coordinate, $\rho=R/\lambda$, for the case $\rho_1=R/\lambda=5$, $\kappa=2$. The curves *1–5* correspond to different values of the external field $h=H/H_\lambda$: (*1*) $h=1$; (*2*) $h=1.6837$; (*3*) $h=1.6838$; (*4*) $h=2$; (*5*) $h=3.9$. As can be seen from Figs. 2(a) and 2(b), with the field h increasing from $h=1$ (the curve *1*) to $h=1.6837$ (the curve *2*), the order parameter at the surface, $\psi_1=\psi(\rho_1)$, gradually diminishes [see also Fig. 1(f))], and the field $b(\rho_1)$ gradually increases, however, the value $b(0)\approx 0$ changes insignificantly. This regime corresponds to the Meissner screening, with external field expulsion from the specimen. In passing from the field $h=1.6837$ (the curve *2*) to $h=1.6838$ (the curve *3*) the sharp change of the regime happens: the field at the surface $b(\rho_1)$ remains practically the same, however, the field at the center of a superconductor, $b(0)$, suffers a jump, i.e., the Meissner regime is violated. In addition, near the superconductor surface, a ring layer forms, into which the external field penetrates freely, without screening [Fig. 2(b)]. The order parameter in this layer is suppressed [Fig. 2(a)], but it remains finite near the cylinder axis. The superconducting screening current in this layer is absent [Fig. 2(f), curve *4*].

When the field h is increased further, the outer ring layer (or, the rim) gradually widens, with the order parameter remaining finite only near the cylinder axis [Fig. 2(a), the curve *5*]. [Note, however, that the order parameter in the ring layer, $\psi(\rho)$, is not exactly zero, but only exponentially small. Thus, strictly speaking, this layer is not a normal metal.] At $h\rightarrow h_{c2}^{(0)}=4.0063$ the superconductivity finally vanishes via second-order phase transition to normal state ($\psi\rightarrow 0$, $b\rightarrow h$ everywhere).

It is expedient to clarify the mathematical reason why the solutions suffer transformation and pass from one branch to another. One can verify that at $h\rightarrow h_{c1}^{(0)}$ the second derivative $\psi''(\rho)$ strongly increases in vicinity of $\rho=\rho_1$ [see, in particular, Fig. 2(e), the curve *2*]. This is accompanied by a quick rise of the first derivative, $\psi'(\rho)$ [see Fig. 2(c), the curve *2*]. At $h>h_{c1}^{(0)}$ the derivative ψ' becomes positive at the point $\rho=\rho_1$ ($\psi'|_{\rho=\rho_1}>0$), so now it is impossible to satisfy the second of the boundary conditions (6). In result, the solution $\psi(\rho)$ transforms and passes from a branch, represented by the curves *1*, *2*, to the branch, represented by the curves *3*, *4*. Simultaneously, the field $b(\rho)$ displays the flat

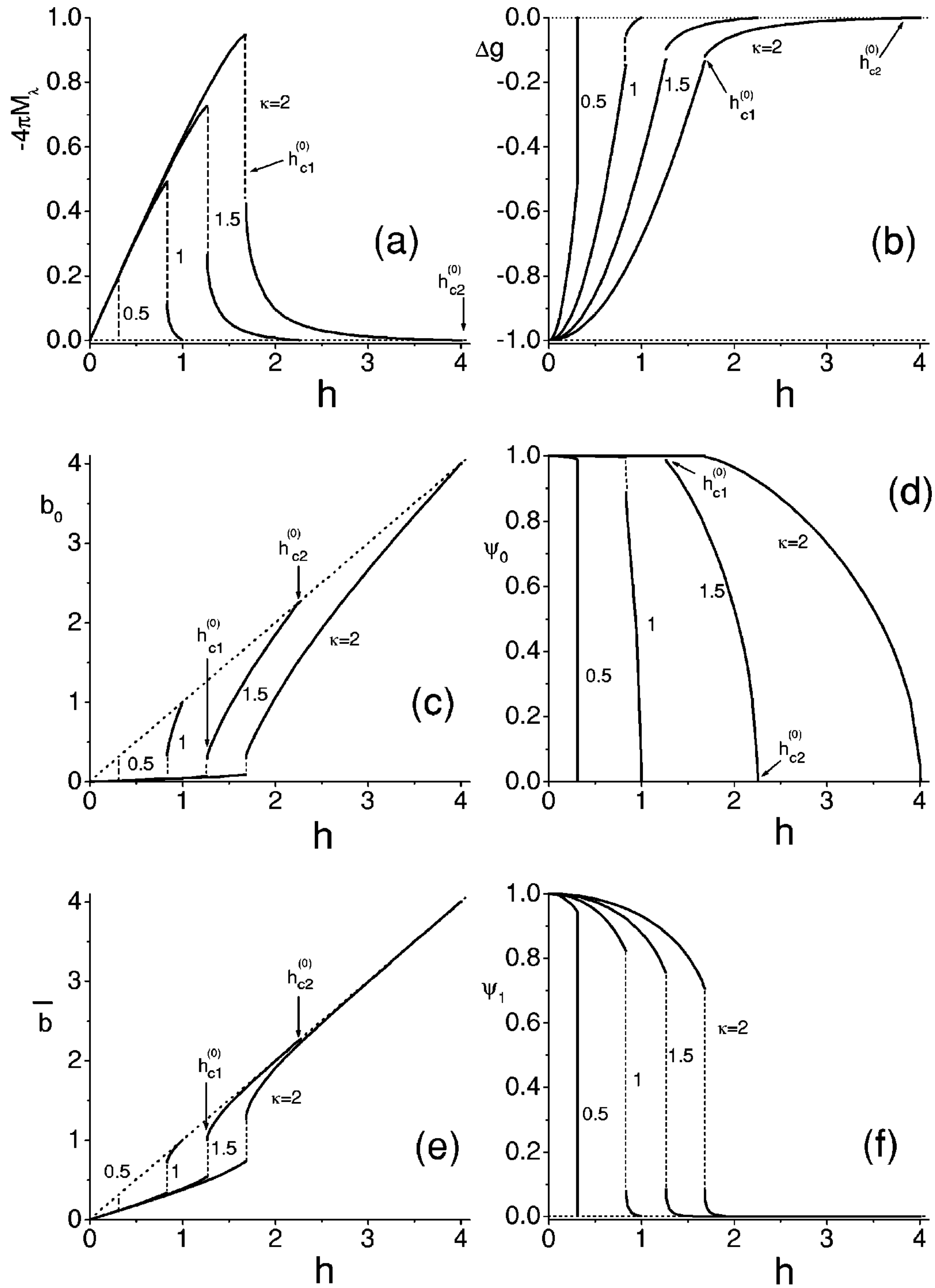

FIG. 1. (a) The magnetic moment (or, magnetization) of the specimen, $-4\pi M_\lambda$, as a function of $h=H/H_\lambda$. The lower critical field h_{c1} corresponds to the jump in magnetization curve. The upper critical field h_{c2} was found from the condition $\psi_0<1\times10^{-5}$. (b) The free energy difference, Δg Eq. (10), as a function of h. (c) The magnetic field on the cylinder axis b_0 as a function of h. (d) The value of the order parameter on the cylinder axis ψ_0 as a function of h. (e) The mean value of the field in the specimen, $\overline{b}=B_{av}/H_\lambda$ Eq. (7), as a function of h. (f) The value of the order parameter at the cylinder surface, ψ_1, as a function of h. All the curves are calculated for $\rho_1=R/\lambda=5$; the numerals at the curves are the values of the parameter κ.

ring layer [see Fig. 2(b)], with no screening current flowing inside, $j(\rho)=0$ [see Fig. 2(b), the curves *3, 4*]. In fact, the existence of two branches of the magnetization, $-4\pi M_\lambda$, is a consequence of the nonlinearity of the system (2)–(6).

The solutions behave analogously, if the parameter ρ_1 is increased, $\rho_1=R/\lambda>5$. The solutions behavior at smaller ρ_1 is illustrated in Fig. 3, which is analogous to Fig. 1, but corresponds to the case $\rho_1=3,\ \kappa=2$.

The dependences $\psi(\rho)$ and $b(\rho)$ for $R/\lambda=3,\ \kappa=2$, are shown in Figs. 4(a) and 4(b) for various h (the numerals at the curves). One can see that the process of the solutions transformation and the system transfer from one branch to another proceeds now more smoothly, as compared with Figs. 2(a) and 2(b).

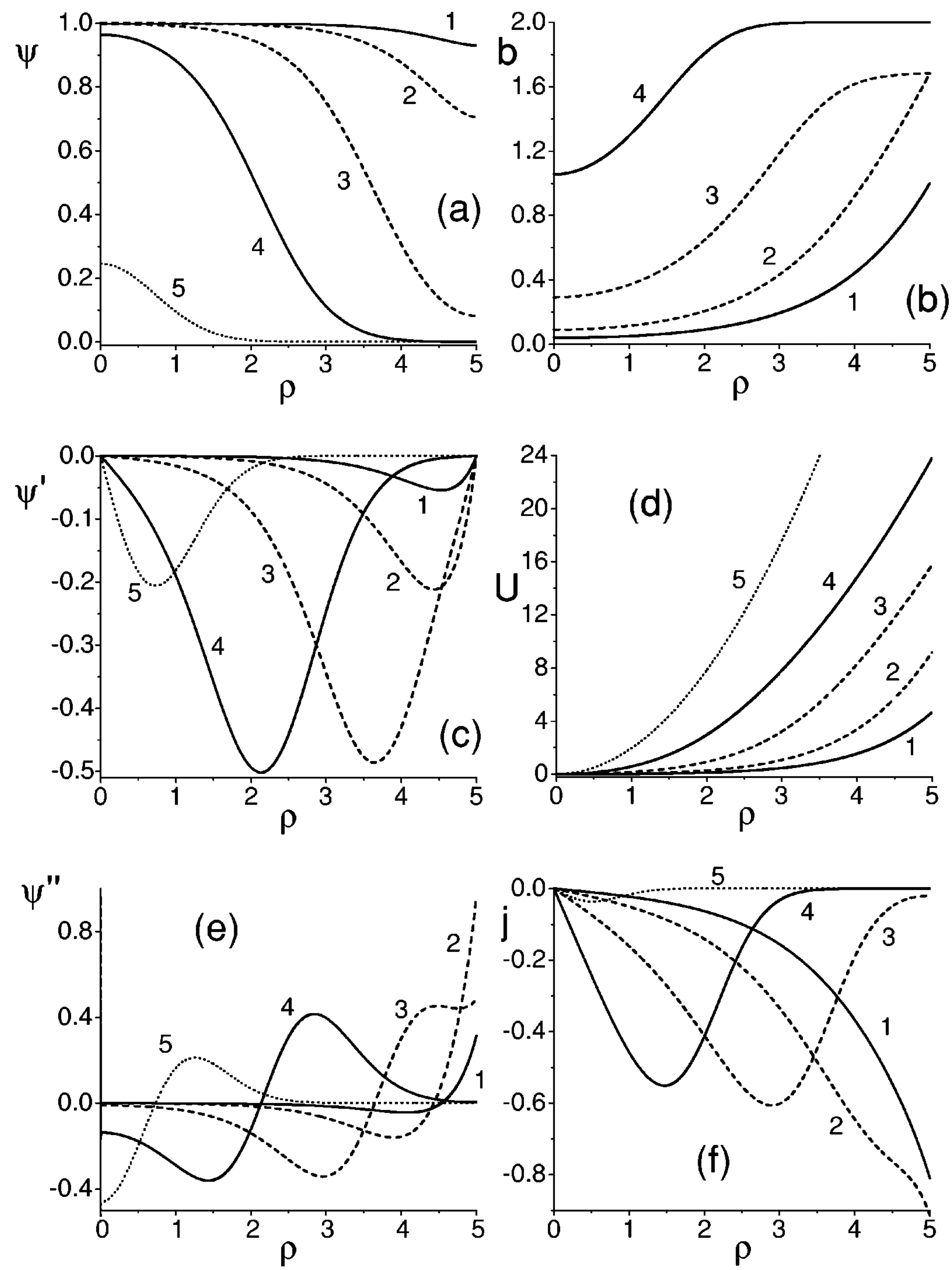

FIG. 2. Depicted are, as a functions of the distance from the cylinder axis, $\rho = r/\lambda$: (a) the order parameter, $\psi(\rho)$; (b) the magnetic field, $b(\rho)$; (c) the first derivative, ψ'; (d) the potential U (or, the magnetic flux, $\phi \equiv \Phi/\phi_0$); (e) the second derivative, ψ''; (f) the superconducting current, j Eq. (4). All the curves are drawn for $\rho_1 = R/\lambda = 5$, $\kappa = 2$. The numerals at the curves correspond to different values of the external field: (*1*) $h=1$; (*2*) $h=1.6837$; (*3*) $h=1.6838$; (*4*) $h=2$; (*5*) $h=3.9$.

As can be seen from Figs. 1(a) and 2(a), the type-I superconductors pass to normal state via first-order phase transition (if the radius ρ_1 is not too small). Figure 5 illustrates the behavior of the type-I superconductor (with $\kappa = 0.1$) for $\rho_1 = 5, 7, 10, 20, 30$ (see numerals at the curves) at the critical field values $h = h_{c1}^{(0)}$, which preceed the jump to normal state [$h_{c1}^{(0)} = 0.0567$ for $\rho_1 = 5$; $h_{c1}^{(0)} = 0.0406$ for $\rho_1 = 7$; $h_{c1}^{(0)} = 0.0286$ for $\rho_1 = 10$; $h_{c1}^{(0)} = 0.0149$ for $\rho_1 = 20$; $h_{c1}^{(0)} = 0.0111$ for $\rho_1 = 30$]. If the field h is increased by 1×10^{-4}, the function $\psi(\rho)$ vanishes by a jump (first-order phase transition to normal state). (However, in the cylinders of very small radiuses, $R/\lambda < 2$, the order parameter vanishes gradually, showing second-order phase transition.)

By comparing Figs. 1(a) and Fig. 3(a), one can see that the magnetization jumps [represented in Fig. 1(a) by the broken vertical lines at $h = h_{c1}^{(0)}$], are replaced (with ρ_1 diminishing) by smooth dependences. In the case of small ρ_1 (when the abrupt jumps are not noticable), one can define the value h_{c1} as a point of maximally quick fall of the magnetization curve [such a point is marked by an arrow in Fig. 3(a);

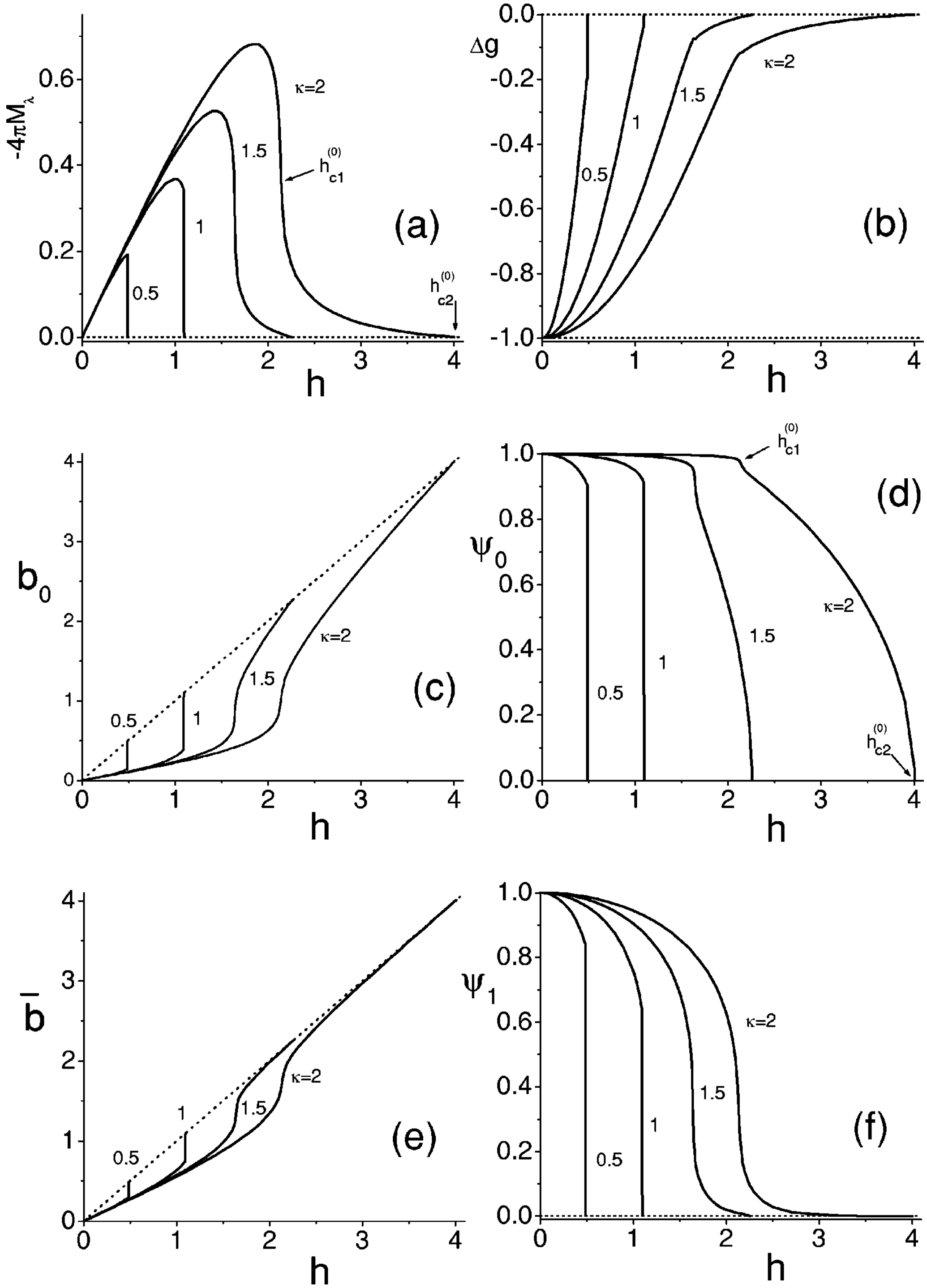

FIG. 3. The same, as in Fig. 1, but for $\rho_1 = R/\lambda = 3$. The lower critical field, h_{c1}, was found as a point of maximal derivative of the magnetization, dM_λ/dh [is shown by an arrow in (a)]. The upper critical field, h_{c2}, was found from the condition $\psi_0 < 1\times 10^{-5}$.

the value h_{c2}, as before, can be found from the conditions $\psi\equiv 0$, $b\equiv h$]. The dependences $h_{c1}(\rho_1,\kappa)$, and $h_{c2}(\rho_1,\kappa)$, determined in this manner, are presented in Figs. 6(a) and 6(c).

As can be seen from Figs. 1(a) and 3(a), the ''tail'' of the magnetization, $-4\pi M_\lambda$, lays within the field interval $\Delta h = h_{c2} - h_{c1}$. In Fig. 6(c) the width of the magnetization tail Δh is depicted as a function of ρ_1 for different κ. For $\kappa \leqslant 1/\sqrt{2}$, the magnetization curve has no tail, so the superconductor passes to the normal state via first-order phase transition (by a jump). (However, as was mentioned above, in the case of very small radiuses, $R < 2\lambda$ and $\kappa \ll 1$, the transition to normal state is of second order.)

Note the universal character of the curves, depicted in Figs. 5 and 6. These curves differ only by the value of κ; they describe superconductors with arbitrary values of parameters R, ξ_0, λ_0, T, T_c [the dependence on these parameters are implicitly contained in H_λ, λ, and ρ_1 in the normalization (4)].

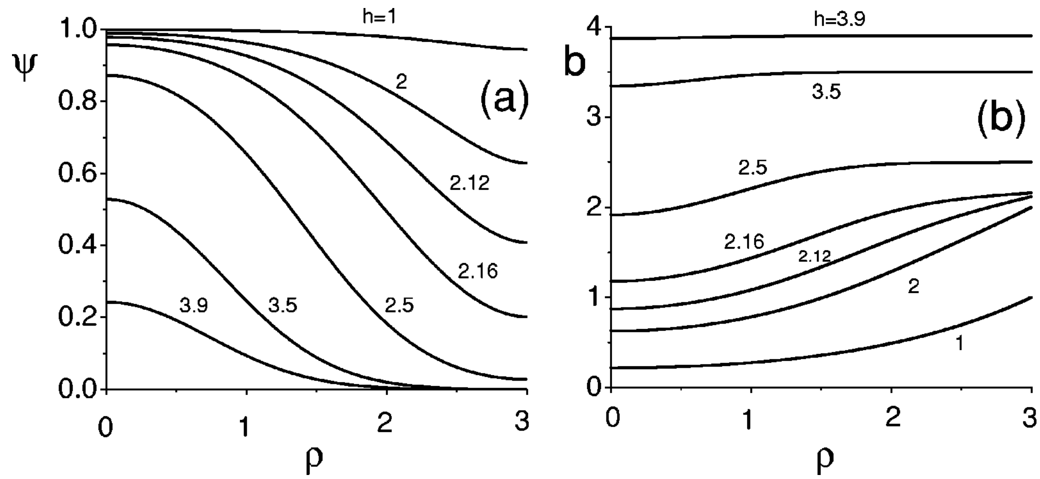

FIG. 4. Shown are, as functions of the distance from the cylinder axis: (a) the order parameter, $\psi(\rho)$; (b) the magnetic field, $b(\rho)$. All the curves are depicted for $\rho_1 = R/\lambda = 3$, $\kappa = 2$. The numerals at the curves are different values of the external field, h.

IV. COMMENTS AND CONCLUSIONS

We would like to make some comments. The usual mechanism of the superconducting state destruction in increasing magnetic field H is, that, after reaching the value $H = H_{c1}$, the localized quantum vortices are formed near the superconductor surface. Gradually, they are pushed into the bulk of superconductor, forming there the more and more dense vortex lattice.[2,3] When the field reaches the value $H = H_{c2}$, the vortex normal cores overlap, and the transition to normal state occurs.[2,3]

As was shown in the present paper, with H increasing, the process of the magnetic field penetration into superconductor can begin by forming a ring-shaped layer of finite width, situated near the superconductor surface (a rim). The order parameter in this layer is strongly suppressed, with the field in the layer $B(r) = H$. This layer begins forming at $H = H_{c1}^{(0)}$ (assuming no vortices inside the superconductor). With the field increasing, $H > H_{c1}^{(0)}$, the ring widens, and in the field $H = H_{c2}^{(0)} = H_{c2}$ the superconductivity is totally suppressed and the transition to normal state occurs.

Thus, in the finite dimension superconductors, there exist two competitive mechanisms for the destruction of superconductivity in the increasing magnetic field: by forming quantized vortices, or by forming a ring layer with the suppressed order parameter (the rim-suppressed state). The evidence of the ring layer existence, possibly, may be found experimentally, by measuring the magnetization curve with the help of a superconducting quantum interferometer, having a very low threshold for registration of the onset of the magnetic flux penetration into the superconductor. However, it is possible that the origination of the vortices in the bulk may start before the superconductivity is destroyed by the formation of the ring layer. In that case, it would be more difficult to find the experimental evidence of its existence. In the present paper the main attention was devoted to a formal description of the possible self-consistent solutions of the nonlinear system of Ginzburg-Landau equations. The physical stability of the ring layer was not studied in detail, and no comparison of the Gibbs energies of the competitive states (with and without vortices) is given. The corresponding tedious calculations are in progress; the results, as well as a more detailed discussion of a possible connection with the experiment, will be presented elsewhere.

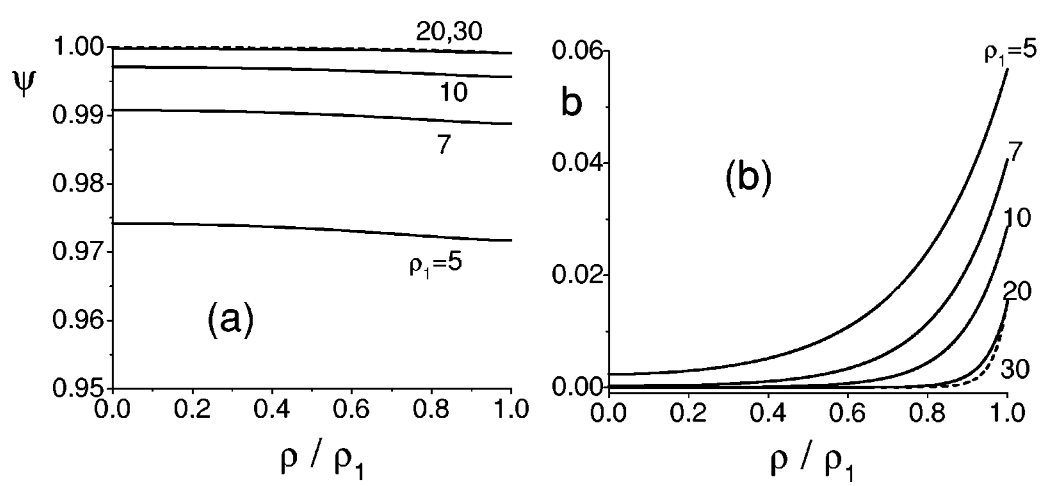

FIG. 5. The dependences on the reduced coordinate ρ/ρ_1 of (a) the order parameter, ψ; (b) the magnetic field b. All the curves correspond to $\kappa = 0.1$. The numerals at the curves are the radii ρ_1. The magnetic field is equal to the critical field $h = h_{c1}^{(0)}$, at the point, preceding the first-order jump to the normal state. (The values of $h = h_{c1}^{(0)}$ for each ρ_1 are given in the text.)

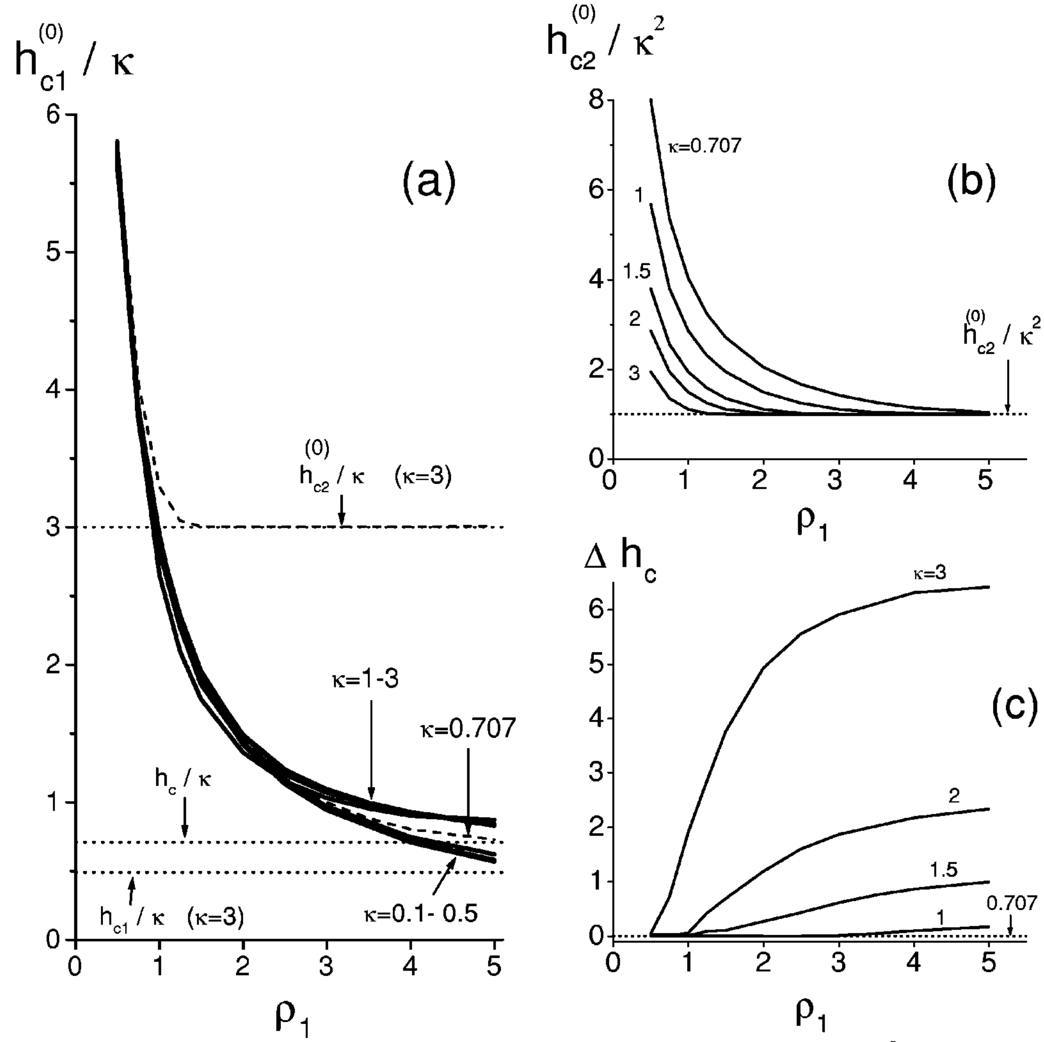

FIG. 6. (a) The lower critical field, h_{c1}/κ, as a function of $\rho_1 = R/\lambda$. (b) The upper critical field, h_{c2}/κ^2, as a function of $\rho_1 = R/\lambda$. The numerals at the curves are the values of the parameter κ. (c) The width of the magnetization tail, $\Delta h = h_{c2} - h_{c1}$, as a function of $\rho_1 = R/\lambda$. The numerals at the curves are the values of the parameter κ.

In conclusion, we would like to elucidate, why in the preceding theoretical papers on this problem there is no mention of the possible existence of the rim-suppressed state. In principle, it was possible to arrive at such a conclusion immediately, after the Ginzburg-Landau theory was formulated.[1] However, as was mentioned above, the ring layer is a consequence of the nonlinearity of Eqs. (2)–(6). To find the self-consistent solution of these equations, it is necessary to carry out rather cumbersome calculations. In one of the papers on this subject, Fink and Presson [Ref. 4(a)] have found the numerical solution of the system (2)–(6) (written in different notations), using the analog computer, which lead to rather pure computation accuracy (about 2%). They have reported one particular solution [for the case $m=0$, $R/\lambda = 3$, $\kappa = 1$; see Fig. 3 in Ref. 4(a)], from which one may guess that the authors were unable to trace the solution in a case of small order parameter values, ψ_1, and thus, were unable to find the corresponding critical field $H_{c1}^{(0)}$.

In recent paper, Moshchalkov, Qiu, and Bruyndoncx[4] studied the behavior of the finite dimension superconductors on the base of Eqs. (2)–(6). They have found the temerature dependence of the cylinder magnetic moment (for $m=0$) in one particular case [$R=\sqrt{3}\lambda(0)$, $\kappa=5$]. Their algorithm of numerical calculations was not described, but they have not analyzed the dependence $M(H,T,\kappa,R)$ in detail and have not searched for the critical field $H_{c1}^{(0)}$. One particular example of the magnetization dependence $M(H)$ (for $m=0$) may also be found in the paper of Deo *et al.*[4] We do not know of other papers where the behavior of the vortex-free finite radius cylinder was studied on the base of self-consistent solution of the nonlinear system of Ginzburg-Landau equations (2)–(6). Note also, in this connection, Ref. 12, where the case of a vortex-free semi-infinite slab, placed in an external magnetic field of small magnitude, $H \ll H_c$, was considered within the perturbation approach, and some evidence of a nonlinear behavior of the magnetization was found.

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