Quantized conductance, circuit topology, and flux quantization

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It is shown that elementary quantum mechanical considerations determine whether the resistive features of a closed electric circuit are governed by the well-known Landauer-Büttiker conductance formula. In particular, it is argued that the latter results from the interplay between the topology of the transport electric field and the quantization of the magnetic flux trapped by the circuit.

I. INTRODUCTION

Exploiting the possibility of down scaling the sizes of modern semiconductor devices, their insulating layers and the connecting wires, one has gradually entered the range where quantum physics dominates the descriptive base for understanding experimental results. In particular, coherent transport through so-called quantum devices such as quantum wires, quantum dots, and quantum point contacts has been regarded as a substantial support for the Landauer picture of transport in disordered solids.[1–5] Moreover, for more than two decades numerous authors have investigated charge transport through large normal or superconducting metallic contacts that are separated by narrow semiconducting or insulating layers the size of which is below the mean free paths related to typical scattering mechanisms.[6] In spite of the agreement between the electrical characterization of mesoscopic conductors and the Landauer-Büttiker theory predicting the conductance in those devices, conceptual doubt has persisted regarding the derivation of the famous Landauer-Büttiker formula from first principles of quantum mechanics and statistical physics.[7–9]

In this paper, we demonstrate that the Landauer-Büttiker formula for a multiply connected electric circuit can be derived straightforwardly if the following two conditions are met: (a) quantization of the magnetic flux generated by the electric current flowing through the circuit, (b) spatial localization of the driving electric field in a simply connected subregion of the circuit. Because the present paper is strictly focussing on the current-voltage relationship in the absence of decoherence effects, we have restricted our investigations to an ensemble of noninteracting electrons, which are allowed to move freely through the circuit. Consequently, the main question to be answered is whether the application of an external voltage will give rise to either infinite currents steadily extracting energy from the driving field or to finite currents being limited by the Landauer-Büttiker conductance. In order to prevent any ambiguity in distinguishing between those two conductance regimes, we have systematically omitted both elastic and inelastic scattering mechanisms in the present treatment.

The paper is organized as follows. In Sec. II the electric circuit is introduced as a multiply connected manifold with a single hole. In Sec. III the transient current response of the electron gas moving freely through the circuit under the action of an irrotational, but non-conservative electric field is investigated and the Landauer-Büttiker conductance formula is recovered. Finally, the obtained results including the proposed conduction mechanism are discussed and compared with more conventional theoretical approaches in Sec. IV.

II. CIRCUIT TOPOLOGY, NONCONSERVATIVE ELECTRIC FIELDS, AND DISSIPATION-LESS TRANSPORT

We consider a closed electric circuit that comprises a three-dimensional multiply connected region Ω encircling exactly one hole, i.e., a torus-shaped region confining an ensemble of electrons. Consequently, all one-electron wavefunctions and the electron field operator $\psi(\vec{r},t)$ are assumed to vanish at the boundary surface $\partial\Omega$:

$$\psi(\vec{r},t)=0 \quad \forall \vec{r} \in \Omega. \tag{1}$$

As shown in Fig. 1, the circuit consists of four regions: A so-called active region Ω_A that is restricted by the cross-sectioned surfaces Σ_{1A} and Σ_{2A}, representing any arbitrary mesoscopic area such as a quantum point contact, a quantum dot or a narrow energy barrier, a ‘‘battery’’ region Ω_B representing the seat of an externally applied dc electromotive force (emf) V_ϵ, two ideally conducting leads Ω_{1L} and Ω_{2L} connecting the battery to the active region.

The motion of the electron ensemble is driven by an electric field $\vec{E}(\vec{r})$, which is invoked by the external emf V_ϵ

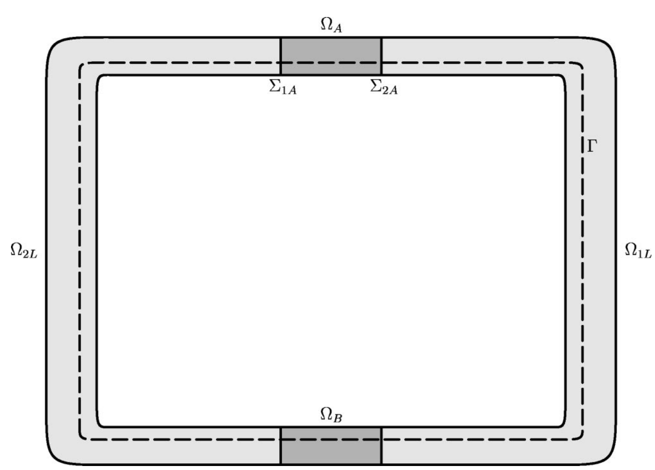

FIG. 1. Torus-shaped electric circuit.

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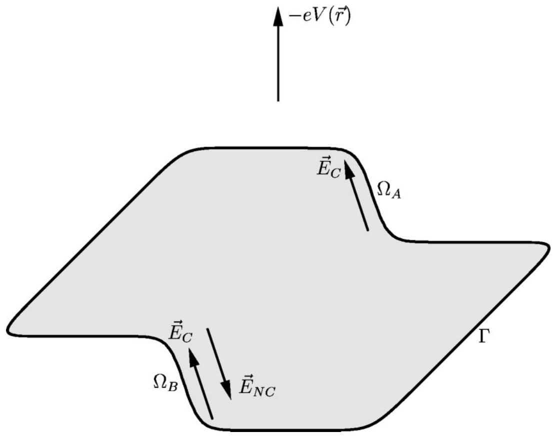

FIG. 2. Potential energy profile along Γ.

$$V_\epsilon = \oint_\Gamma \vec{E}(\vec{r}) \cdot d\vec{r}, \tag{2}$$

where Γ is an arbitrary closed curve in the interior of Ω, encircling the ''hole'' of the circuit once and only once. Under the assumptions that no magnetic field lines are penetrating in the circuit region Ω, we may conclude from the third Maxwell equation that in the interior of Ω the total electric field must be irrotational yet nonconservative, since it exists in a multiply connected region. Moreover, due to Stokes' theorem, the emf, which is nothing but the circulation of the electric field along the closed curve Γ must be independent on any particular choice of Γ. As even the leads are assumed to have no resistance, the electric field is identically vanishing in both lead regions. In general, the electric field in the circuit region may be decomposed into a conservative and nonconservative part:

$$\vec{E}(\vec{r}) = \vec{E}_C(\vec{r}) + \vec{E}_{NC}(\vec{r}) \tag{3}$$

with

$$\vec{E}_C(\vec{r}) = -\vec{\nabla} V(\vec{r})$$
$$\oint_\Gamma \vec{E}_{NC}(\vec{r}) \cdot d\vec{r} = V_\epsilon. \tag{4}$$

Here, the conservative component $\vec{E}_C$ is derived from an electrostatic potential V taking fixed values V_1 and V_2 within the equipotential volumes Ω_{1L} and Ω_{2L} and exhibiting relatively large drops in the active region and the battery region (see Fig. 2).

However, as the entire circuit is assumed to be scattering free, we may as well neglect the internal resistance of the battery and choose the nonconservative component $\vec{E}_{NC}$ to counteract the conservative field in the battery region

$$\vec{E}_{NC}(\vec{r}) = -\vec{E}_C(\vec{r}), \quad \text{for} \quad \vec{r} \in \Omega_A$$
$$= 0, \quad \text{elsewhere}. \tag{5}$$

In other words, the total transport field $\vec{E}$ vanishes everywhere in the circuit except for the active region where the electric field strength may take rather huge values. Furthermore, the potential difference $V_1 - V_2$ is maintained by the emf as can be seen from

$$V_\epsilon = \oint_\Gamma \vec{E}(\vec{r}) \cdot d\vec{r} = \int_{\Sigma_{1A}}^{\Sigma_{2A}} \vec{E}_C(\vec{r}) \cdot d\vec{r} = V_1 - V_2. \tag{6}$$

Clearly, the present transport model requires a totally different approach compared with conventional studies of coherent transport that treat the electric circuit as an open system of charge carriers. The latter ones are injected from a reservoir and, after some ballistic propagation through the mesoscopic area they are seen to disappear in another reservoir. On the contrary, our model describes the electric circuit as a closed, torus-shaped region confining the charge carriers to its interior—as is essentially realized in a real circuit—and allowing them to extract energy from an external electric field which is forcing the carriers back to the active region.

III. HAMILTONIAN AND CURRENT RESPONSE

Under the effective mass approximation the most general second-quantized Hamiltonian describing an ensemble of free electrons acted upon by an electromagnetic field reads

$$H_E = \int_\Omega d\tau \psi^\dagger(\vec{r}) \left\{ \frac{1}{2m} [\vec{p} + e\vec{A}(\vec{r},t)]^2 + U(\vec{r}) - eV(\vec{r}) \right\} \psi(\vec{r}), \tag{7}$$

where m is the electron mass and $U(\vec{r})$ represents any internal potential profile (such as a built-in potential or an energy barrier). The vector potential $\vec{A}$ consists of an irrotational part $\vec{A}_{ex}$, related to the nonconservative, external dc field $\vec{E}_{NC}$ and an induced component $\vec{A}_{in}$ corresponding to the magnetic field which is generated by the moving electrons and the induced electromotive forces.

Under the assumption that the emf is switched on at some initial time $t=0$, we may write according to elementary electrodynamics

$$\vec{A}(\vec{r},t) = \vec{A}_{ex}(\vec{r},t) + \vec{A}_{in}(\vec{r},t) \tag{8}$$

$$\vec{A}_{ex}(\vec{r},t) = -\vec{E}_{NC}(\vec{r})t \tag{9}$$

$$\vec{B}(\vec{r},t) = \vec{\nabla} \times \vec{A}_{in}(\vec{r},t). \tag{10}$$

The current response can be obtained from a self-consistent solution of Maxwell's equations and the quantum dynamical equation yielding the time-dependent ensemble average of the gauge invariant current density operator

$$\vec{J}_t(\vec{r}) = \frac{ie\hbar}{2m_e} \{ \psi^\dagger(\vec{r}) \vec{\nabla} \psi(\vec{r}) - [\vec{\nabla} \psi^\dagger(\vec{r})] \psi(\vec{r}) \}$$
$$- \frac{e^2}{m} \psi^\dagger(\vec{r}) \psi(\vec{r}) \vec{A}(\vec{r},t). \tag{11}$$

Defining the total current $I(t)$ in the ususal way as the electron charge passing the cross section Σ_{1A} per unit time, we may express $I(t)$ in the Heisenberg picture as follows:

$$I(t)=\int_{\Sigma_{1A}}\langle\vec{J}(\vec{r},t)\rangle_0\cdot d\vec{S} \tag{12}$$

where $\langle\vec{J}(\vec{r},t)\rangle_0$ is the time-dependent ensemble average of the Heisenberg operator $\vec{J}(\vec{r},t)$ and $\langle\ \dots\rangle_0$ represents the Gibbs ensemble average describing the equilibrium state of the circuit for a given temperature $T=1/k_B\beta$ and a given chemical potential μ at all times $t\leqslant 0$:

$$\langle\vec{J}(\vec{r},t)\rangle_0=\frac{\mathrm{Tr}\{\vec{J}(\vec{r},t)\exp[-\beta(H_E-\mu\hat{N})]\}}{\mathrm{Tr}\,\exp[-\beta(H_E-\mu\hat{N})]}$$

$$\hat{N}=\int_\Omega d\tau\psi^\dagger(\vec{r})\psi(\vec{r}) \tag{13}$$

$$N=\langle\hat{N}\rangle_0.$$

For the present investigation however, it proves convenient to focus on the equation of motion for $\langle H_E(t)\rangle_0$ describing the rate at which the electron ensemble is extracting energy from the power supply. A lengthy and cumbersome but straightforward calculation based on the the Heisenberg equation governing the time evolution of the electron field operator[10]

$$\mathrm{i}\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left\{\frac{1}{2m_e}[\vec{p}+e\vec{A}(\vec{r},t)]^2+U(\vec{r})-eV(\vec{r},t)\right\}\psi(\vec{r},t) \tag{14}$$

and its Hermitian conjugate yields the quantum-mechanical representation of the classical energy rate equation:

$$\frac{d\langle H_E(t)\rangle_0}{dt}=\int_\Omega d\tau\langle\vec{J}(\vec{r},t)\rangle_0\cdot\vec{E}(\vec{r},t)-\frac{\mathrm{i}}{\hbar}\langle[H_E(t),H'(t)]\rangle_0;\quad t>0, \tag{15}$$

where $\vec{E}(\vec{r},t)$ generally denotes the total electric field, including the externally applied transport field, the self-consistent field in the active region and the induced electric field due to possible changes of the magnetic flux trapped by a curve in the circuit region. As was announced in the introduction, the interaction term H' generally representing all elastic and inelastic scattering mechanisms has been switched off in the present work in order to investigate the existence of another current limiting mechanism.

At a first glimpse of Eq. (15) in which we have put $H'=0$, it might appear that the absence of any dissipative scattering mechanism would inevitably lead to both an unlimited increase of the electron energy and an unbounded acceleration of the electron ensemble due to steady absorption of energy supplied by the nonconservative electric field. In a classical circuit, this is the expected scenario. Indeed, if H' could be switched off in some Gedankenexperiment, for instance, if one considered a simple LR circuit governed by the well-known formula

$$I(t)=\frac{V_\epsilon}{R}(1-\mathrm{e}^{-Rt/L}), \tag{16}$$

the current would trivially diverge as $V_\epsilon t/L$ if the series resistance R were put equal to zero. For a quantum circuit, however, we propose the existence of a closed curve Γ in the interior of Ω such that (1) $\vec{A}$ and the electron field operator are not identically vanishing along Γ and (2) the magnetic flux Φ trapped by Γ is quantized. In particular, the magnetic flux trapped by Γ is supposed to be an integer multiple of the elementary flux quantum $\Phi_0=h/e$. Clearly, the above proposed flux quantization is very well known to occur in superconducting rings and plays an important role also when it comes to trace quantum interference phenomena such as the Aharonov-Bohm effect.[11] Flux quantization for a superconducting ring is a direct consequence of the requirement that paired electrons be described by single-valued wave functions together with the observation that the presence of an irrotational vector potential (Meissner effect) can be fully absorbed in a phase shift $\Delta\phi=(2e/\hbar)\oint_\Gamma\vec{A}(\vec{r})\cdot d\vec{r}=2e\Phi/\hbar$ acquired by an electron after a virtual revolution along Γ.[12]

In this paper, we have omitted all interactions that may destroy the coherence of the electron transport in the circuit and in this respect one might be tempted to consider Ω as an artificial superconducting circuit for which flux quantization need not be imposed as an external constraint as it is already enforced through the superconducting features—upon a simple replacement of the Cooper pair charge $2e$ by a single electron charge e.

On the other hand, we wish also to exploit the results of this work to understand the conductance mechanisms of mesoscopic structures that are embedded in more realistic circuits, containing nonperfectly conducting leads and areas which are exposed to the magnetic field caused by the current flow. Since the total vector potential will no longer be irrotational $(B\neq 0)$ in the interior of those circuits, flux quantization cannot simply emerge from the phase shift argument in the very same way as for the superconducting circuit, and therefore it needs to be postulated explicitly.

As a major consequence of the universality of the flux quantization, the total magnetic flux trapped by the closed loop Γ can only increase or decrease with steps of magnitude Φ_0. Hence the electric current carried by the free electron gas of our model can change only after a minimal time τ_0 required to realize exactly one creation or absorption of an elementary flux quantum, since if such a current change took place at an earlier instant, it would produce a proportional magnetic flux change which would be smaller than Φ_0. (A similar argument is proposed by 't Hooft in Ref. 13).

In other words, the transient built-up of the electric current, after the power is switched on, will be characterized by a discrete time series $\{t_1,t_2,\dots,t_n,\dots|t_n=n\tau_0\}$ such that all dynamical quantities remain constant between two subsequent time instants and energy extraction from the power supply takes place at these discrete time instants only.

The characteristic time τ_0 can be easily calculated by comparing the energy $\Delta H_{E,n}$ extracted from the external field during a time interval $[t_n-\frac{1}{2}\tau_0,t_n+\frac{1}{2}\tau_0]$ with the corresponding magnetic energy increase ΔU_M of the circuit. Integrating the energy rate Eq. (15) from $t_n-\frac{1}{2}\tau_0$ to $t_n+\frac{1}{2}\tau_0$, we may express $\Delta H_{E,n}$ as follows :

$$\Delta H_{E,n}=\int_{t_n-1/2\tau_0}^{t_n+1/2\tau_0}dt\langle\vec{J}(\vec{r},t)\rangle_0\cdot\vec{E}(\vec{r},t). \tag{17}$$

During $[t_n-\frac{1}{2}\tau_0,t_n+\frac{1}{2}\tau_0]$, the charge density remains unchanged before and after the jump at $t=t_n$ and consequently, the current density is solenoidal, while the external electric field is irrotational. Hence, according to a recent integral theorem for multiply connected regions,[14] we may disentangle the right-hand side of Eq. (17):

$$\int_{t_n-1/2\tau_0}^{t_n+1/2\tau_0} dt\langle\vec{J}(\vec{r},t)\rangle_0\cdot\vec{E}(\vec{r},t)=\frac{1}{2}[I_{n-1}+I_n]V_\epsilon\tau_0, \tag{18}$$

where $I_n=\int_{\Sigma_{1A}}\langle\vec{J}(\vec{r},t_n)\rangle_0\cdot d\vec{S}$ is the net current entering the cross section Σ_{1A} at a time t_n. On the other hand, the flux change $\Delta\Phi_n$ associated with the jump $\Delta I_n\equiv I_n-I_{n-1}$, reads

$$\Delta\Phi_n=L\Delta I_n, \tag{19}$$

where L is the inductance of the circuit. Since $\Delta\Phi_n$ has to equal Φ_0, we obtain the increased magnetic energy of the circuit

$$\Delta U_M=\frac{1}{2}LI_n^2-\frac{1}{2}LI_{n-1}^2$$

$$=\frac{1}{2}(I_{n-1}+I_n)\Phi_0. \tag{20}$$

Combining Eqs. (17), (18), and (20) and putting $\Delta U_M=\Delta H_{E,n}$, we derive the following result:

$$\tau_0=\frac{\Phi_0}{V_\epsilon}. \tag{21}$$

We are now in a position to show that the interplay between flux quantization and the topology of the transport electric field explains the limitation of the electric current even in the absence of scattering. Since the total transport field is nonzero only in the active region Ω_A, only the charge Q_n consisting of electrons residing in Ω_A at $t=t_n$ will feel the action of the electric field during the interval $[t_n-\frac{1}{2}\tau_0,t_n+\frac{1}{2}\tau_0]$. For the sake of simplicity, we have considered a circuit in which the electric current is carried by M occupied transverse modes at low temperatures:[15]

$$I_n=\sum_{k=1}^{M} I_{nk}\quad n=1,2,\ldots \tag{22}$$

For instance, in a mesoscopic one dimensional ring, these modes would be simply the energy eigenstates of the ring, whereas they would correspond to the discrete resonances emerging in the continuous spectrum of a circuit with large leads connected by a quantum point contact. Apart from the current sequence $I_{1k},I_{2k},I_{3k},\ldots$ that is increasing due to the steady energy supply, we may also define for each transmission mode k a sequence of charge packets $Q_{1k},Q_{2k},Q_{3k},\ldots$ that are brought into the active region by the current carried by the kth mode as well as a sequence of ''dwell times'' $\{\Delta t_{1k},\Delta t_{2k},\Delta t_{3k},\ldots\}$ representing the time spent by the charge packets in the active region after having entered the latter at $t=t_n$. The superposition of all charge packets is nothing but the total charge residing in the active region between $t=t_n$ and $t=t_{n+1}$:

$$Q_n\equiv-e\int_{\Omega_A}d\tau\langle\psi^\dagger(\vec{r},t_n)\psi(\vec{r},t_n)\rangle_0=\sum_{k=1}^{M} Q_{nk}. \tag{23}$$

Obviously, the three sequences are linked through

$$\Delta t_{nk}=\frac{Q_{nk}}{I_{nk}};\quad n=1,2,\ldots;\quad k=1,2,\ldots,M. \tag{24}$$

Considering for simplicity a ballistic conductor for which each transmission mode provides full transmission, we have $Q_{nk}=-2e$ where the factor 2 accounts for spin degeneracy. Hence, the increasing current sequence translates to a decreasing sequence of dwell times:

$$\Delta t_{1k}\geqslant\Delta t_{2k}\geqslant\Delta t_{3k}\geqslant\ldots. \tag{25}$$

Since τ_0 is non-negative, for each k there must exist a positive integer n_k such that

$$\Delta t_{n_k k}\geqslant\tau_0\geqslant\Delta t_{n_k+1k}. \tag{26}$$

The intuitive interpretation is clear: at $t=t_{n_k}$, the electrons of the kth mode are still residing sufficiently long in the active region to generate a final elementary flux change; as of $t=t_{n_k+1}$, electrons are travelling too fast through the finite active region and are no longer able to collect energy from the external field because that would require them to spend at least a time τ_0 in Ω_A. As a consequence, the current is no longer increasing and $I_{n_k k}=I_{n_k+1k}=I_{n_k+2k}=\ldots$. Together with the inequalities (26) or, equivalently, with

$$\frac{2e}{|I_{n_k k}|}\geqslant\tau_0\geqslant\frac{2e}{|I_{n_k+1k}|} \tag{27}$$

we are left with

$$|I_{n_k k}|=|I_{n_k+1k}|=\frac{2e}{\tau_0}=2e\frac{eV_\epsilon}{h} \tag{28}$$

from which we may finally infer a quantized conductance G

$$G\equiv\frac{1}{V_\epsilon}\sum_k |I_{n_k k}|=\frac{2e^2}{h}M, \tag{29}$$

which is the well-known Landauer-Büttiker formula.

IV. DISCUSSION

In this section, we discuss both the approach itself as it was adopted to derive Eq. (29) and its relation to other, more conventional theories leading to Landauer-Büttiker type conductance formulas.

A. Open versus closed circuits

To the best of our knowledge, most—if not all—theoretical investigations of quantum transport in mesoscopic structures are based on the reservoir concept[16–19] in which a mesoscopic structure, representing the active area, is squeezed between two huge, half-open particle reservoirs. This remark does not apply to superconducting devices and mesoscopic rings carrying persistent currents,[20] for which the torus-like topology is a natural feature. Playing the role of leads, the reservoirs contain two distinct, thermalized electron gases characterized by two different chemical potentials, the difference of which is assumed to equal the applied voltage.[8] We believe that there are at least three good reasons to abandon the reservoir concept when quantum transport is addressed, even if the latter seems to take place only in a small region of the electric circuit. First, the artificial subdivision of the circuit in three distinct parts amounts to the assignment of three sets of quantum states to the three regions: two continuous spectra which are supposed to mimic the semi-infinite reservoirs and a discrete spectrum for the mesocopic structure. The latter is generally confining a relatively small number of electrons to reside in a nanometer scale area. If the three regions were perfectly separated by infinite walls, they would constitute three distinguished quantum systems requiring also separate quantum descriptions and the definition of distinct Hilbert spaces. However, if a communication channel is established, no matter how narrow it is, the three regions should be regarded as one single quantum mechanical entity the dynamics of which is to be described in a unique Hilbert space of states where discrete transmission modes are appearing as sharp, enumerable resonances of a continuous spectrum as is explained in any decent textbook on quantum mechanics.[21–23] From the many-particle point of view, the study of the system dynamics is a formidable task since one has to deal with an open-ended system which is losing and gaining particles in a rather uncontrollable way. Nevertheless, ever since the introduction of the transfer Hamiltonian formalism by Bardeen,[24] numerous transport calculations have been relying on the possibility of treating carrier transport as a set of transitions between ''reservoir states'' and ''mesoscopic area states.'' Here, we do not want to contribute to the on-going discussion as to whether the transfer Hamiltonian formalism is appropriate for studying quantum transport or not, and we definitely do not criticize results which are corroborated by experiment, but we strongly believe that the formalism is descriptive rather than explanatory.

A second, even more striking observation reveals that the open-circuit topology is not accounting for the non-conservative nature of the driving electric field. Being transmitted from reservoir 1 to reservoir 2 by the local electrostatic field of the active region, the charged particles are never returning to reservoir 1. Consequently, the pumping action of the battery mimicing both the energy supply and the maintenance of the electrostatic potential difference is not incorporated at all. On the contrary, the explicit requirement that the reservoirs be thermalized and have fixed chemical potentials is supposed to maintain the *chemical* potential difference and hence the applied voltage. To our feeling, such an approach is hardly appropriate to probe energy limiting mechanisms as any conservative field is already limiting the energy increase itself. This point can be illustrated by the example of a billiard-ball moving without friction in a gravitational field. If the ball is leaving a horizontal platform (reservoir 1) with some velocity v_1 to roll down from a frictionless hill of finite height and width, thereafter arriving at another horizontal platform (reservoir 2) with velocity v_2, the velocity increase is trivially finite as it is acquired at the expense of a finite potential energy decrease. This observation however does not teach us anything about the time evolution of the velocity of the ball in a non-conservative field, if the ball is bound to continue forever its rectilinear motion on platform 2 of the open system.

Finally, the approach adopted in this work completely avoids the necessity of introducing the concept of ''contact resistance.'' The latter refers to the interface between the mesoscopic area and the huge contact reservoirs which should be responsible for the dissipation of energy that cannot be relaxed to the environment when the particles are still residing in the scattering free mesoscopic area. Although it is at least unclear how to identify such interfaces, it is nevertheless generally proposed[16] that the dissipation results from a mismatch between the continuous spectra of the reservoirs and the discrete spectrum of the mesoscopic area, comparable to traffic jam due to a local reduction of the number of available lanes. In other words, the whole explanation of conductance quantization would have to rely on the questionable division of the (open) circuit into spatial subregions. Furthermore, even if the dissipation is related to inelastic scattering events taking place in the reservoirs in the close vicinity of such an interface, it remains an open question how the interaction between the charge carriers and the scattering agents (phonons, impurities, alloys, etc.) and the corresponding coupling strengths which are typical material parameters, can give rise to a resistance that can be expressed solely in terms of fundamental constants $(e,\hbar)$ and a set of transmission matrix elements. As a matter of fact, we have found the last problem a strong incentive to look for alternatives mechanisms to explain the phenomenon of quantized conductance.

B. Energy dissipation versus current limitation

Quite remarkably, the establishment of a stationary state in which a finite current is flowing through an electric circuit in response to a given electromotive force V_ϵ is relying on current limitation, or equivalently on the phenomenon that the electrons can extract energy from the external V_ϵ only for a limited number of cycles. As was explained in the previous section, this limitation in turn relies on the existence of a characteristic time τ_0 an electron should spend in the field region to induce a flux jump and to extract from the external field the corresponding energy packet. This is probably the most striking difference with other treatments which still allow for energy dissipation in the conductance process whereas, in our model, unlimited gain of energy is prohibited by the selection rule for energy extraction.

C. Flux quantization

The flux quantization postulate which is clearly the price we had to pay in this work is inspired by recent work in the field of the fractional quantum Hall effect in two-dimensional gases acted upon by a perpendicular magnetic field where each electron is viewed as composite of a charged boson and a flux tube containing an odd number of flux quanta.[25] Also the argument of Laughlin's Gedankenexperiment[26] invokes quantized flux changes to recover the von Klitzing resistance in a metallic ribbon bent into a circular loop. While this author is considering flux changes associated with the external magnetic field and being related to a flow of electrons from one edge to the other (in the direction of the Hall voltage), this work is addressing the magnetic field produced by the current flowing through the loop. Also in Laughlin's work the flux increment, appearing in the adiabatic derivative of the total energy of the system with respect to the magnetic flux trapped by the ribbon, is taken to be quantized although the vector potential is not irrotational as the magnetic field is piercing the ribbon everywhere.

Moreover, recent successful attempts[27] to discover striking connections between the quantum Hall effect and superconductivity have suggested that magnetic fields impinging on (2D) electron systems may be characterized by a number of flux quanta and therefore seem to corroborate the flux quantization picture.

On the other hand, we do realize that the assumption of flux quantization along a characteristic circuit trajectory may lead to far-reaching consequences on both the theoretical and experimental level. On the theoretical side, we may justify the basic assumption on topological grounds as follows. Since we are explicitly addressing closed circuits rather than open systems, the charge carriers are experiencing strict spatial confinement in the transverse directions, i.e. perpendicular to the transport direction. Suppose concretely that we may introduce local coordinates, say x_1, x_2, and x_3, such that x_3 is a cyclic coordinate defining the transport direction and all electron wave functions are either even or odd in x_1 and x_2. Then the nodes of the transverse wave functions $\chi_k(x_1,x_2)$—the full wave functions being factorized into an expression of the form $\chi_k(x_1,x_2)f_k(x_3)$—are symmetrically located with respect to the closed curve $\Gamma:(x_1=0,x_2=0)$. Hence, the electric current density carried by each transverse eigenstate (below the Fermi level) is symmetric w.r.t. Γ and, accordingly, the generated magnetic field tends to zero on the curve Γ. The latter therefore defines a region where $\vec{A}$ is irrotational whereas neither $\vec{A}$ nor the field operators are identically vanishing, so that the phase argument leading to a quantized flux threaded by Γ can be repeated.

From the experimental point of view, thorough investigations should be conducted to trace directly or indirectly the presence of flux quantization in closed circuits subjected to localized driving electric fields. Obviously, macroscopic circuits in which mesoscopic active areas are embedded cannot be experimentally accessed as a whole without the disturbing presence of scattering events in the conducting leads. On the other hand, mesoscopic metallic rings interrupted by one or two tunnel barriers—such as the Aharanov-Bohm interferometers discussed in Refs. 28 and 29— may provide an appropriate experimental setup for studying changes of the total magnetic flux. In such devices one may generate along a closed trajectory a constant V_ϵ by a linearly growing magnetic field piercing the ring.

D. Localization of the electric field

Besides flux quantization, the topology of the electric field plays also a crucial role when it comes to realizing current limitation. In particular, it is required that the driving electric field governing the electron motion in the circuit be localized in a finite, simply connected region of the circuit. This observation has been made already a few years ago by Fenton[8] who pointed out that for an arbitrary open circuit, the Landauer-Büttiker conductance regime can be realized only for strictly localized transport fields, whereas a uniform field would inevitably yield the Drude-Lorenz conductivity, which, in the absence of scattering, would lead to zero resistance. The same conclusions can be drawn from our model. The connection between the Landauer-Büttiker conductance regime and the requirement of having localized fields is already demonstrated in the previous section, where the finiteness of the active region Ω guarantees that the dwell times become lower than τ_0 after a finite number of cycles. Clearly, this situation cannot occur if the electric field is uniform along the circuit or at least nonvanishing in the whole circuit region, since then the dwell times would increase without limit and the quantized flux changes would be unable to prevent the electrons from unlimited energy extraction.

E. Suggestions for future work: Quantum circuit theory

The postulation of flux quantization proposed in this work, should be properly embedded in a suitable quantum field theory, the dynamical solution of which should encompass the Laundauer-Büttiker conductance regime in a natural way. Nevertheless, the quantization of the corresponding fields will follow another path than that of familiar quantum electrodynamics, the main reason being that, apart from the local field operators associated with the electrons, we have now also to quantize global canonical variables associated with the electromagnetic field, whereas other components of the latter may or may not remain classical. In the light of this work, the magnetic flux is an obvious example of such a global quantity to be quantized. We believe that an appropriate choice of global canonical variables will eventually lead to a useful quantum circuit theory unifying all well-known features of classical circuits as well as the characteristics of quantum devices, which are to be included in real circuits. Finally, it should be noticed that the above mentioned quantization procedure explicitly affects the Maxwell equations expressed in integral form. In particular, Faraday's induction law relates the total emf in a circuit (external and induced) to the change of a quantized flux, the discrete time evolution of which is given by

$$\Phi(t)=\Phi_0\sum_{n=0}^{\infty}\alpha_n\theta(t-n\tau_0), \tag{30}$$

where the coefficients α_n can take only the values 0, ± 1 and have to be determined by a full time-dependent solution of the dynamical equations. As an illustration, we may obtain a quantum mechanical version of Lenz' law by taking the time derivative of Eq. (30)

$$L\frac{dI(t)}{dt}=\Phi_0\sum_{n=0}^{\infty}\alpha_n\delta(t-n\tau_0). \tag{31}$$

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