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Two-dimensional dodecagonal and decagonal quasiperiodic photonic crystals in the microwave region

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We experimentally and numerically demonstrate dodecagonal and decagonal quasiperiodic photonic crystals in the microwave region. The results show that only a small number of rows are needed to create a frequency gap in the transmission spectrum, and that the gap position is insensitive to the incident angle. In particular, there is no defect state at the first gap of a dodecagonal quasiperiodic photonic crystal when there is a cavity in the center region, even though the cavity volume is much larger than the square of the wavelength. The decagonal quasiperiodic crystal can offer great possibilities for creating defect states. These quasiperiodic photonic crystals may find some important applications in photoelectric devices.

I. INTRODUCTION

Photonic band-gap (PBG) materials have received much attention due to their remarkable property that the periodicity gives rise to gaps in the frequency spectrum of electromagnetic waves.[1,2] From a materials point of view, PBG materials are a typical example of material by design. The existence of the band gap opens up many potential applications for, for example, quantum electronic devices,[3] distributed feedback mirrors, light-emitting diodes,[4] optical waveguides,[5] microwave antennae substrates,[6] and so on. Recently, Chan, Chon, and Liu, have proposed a new class of quasiperiodic photonic crystal (QPC).[7] Quasiperiodic crystals have a nonperiodic lattice with long-range bond-orientational order, so that they have only rotational symmetry, while a periodic photonic crystal can have both translational and rotational symmetries. It was predicted theoretically that an octagonal QPC constructed with dielectric cylinders could have several band gaps for S and P polarized waves, respectively,[8] and that the defect modes would be structure dependent. These properties were then demonstrated experimentally.[9,10] Moreover, the phenomenon that the band gaps were independent of the incident direction was observed in an octagonal QPC. To know whether these characteristics are common to all kinds of QPC, the photonic properties of both dodecagonal and decagonal QPC's have been systematically studied. We find that a significant attenuation (>40 dB) in the gap region can be achieved for these crystals composed of just a small number of cylinders, and the gaps in the spectrum are independent of the incident angle. Various defect states can be created in the first gap of the decagonal (DC) quasiperiodic photonic crystal, while for the dodecagonal (DD) quasiperiodic photonic crystal, no corresponding defect modes are observed in the first gap. Our investigation confirms that QPC's are a valuable new photonic structure that possesses many different characteristics from periodic structures.

II. EXPERIMENT

Dodecagonal and decagonal quasiperiodic photonic crystals used in the experiment are formed by placing 2.044 mm radius alumina cylinders with circular cross sections in the vertices of two-dimensional dodecagonal and decagonal quasiperiodic lattices, illustrated in Figs. 1(a) and 1(b), respectively. The DD quasiperiodic pattern is tiled with squares and rhombuses (with an acute angle of 30°) with the equal side length $l=11$ mm. The structure is composed of 333 cylinders. There are 51 rows of cylinders in the incident direction. Meanwhile, the DC quasiperiodic pattern is tiled with squares and rhombuses (with an acute angle of 36°) of the same side length $l=11$ mm. The crystal is constituted of 339 cylinders. There are 55 rows of cylinders in the incident direction. The dielectric constant of the aluminum cylinder is 8.9. The filling fractions of the cylinders are about 13.55% and 13.27% in a Styrofoam template of $\epsilon=1.04$ for the dodecagonal and decagonal quasiperiodic photonic crystals, re-

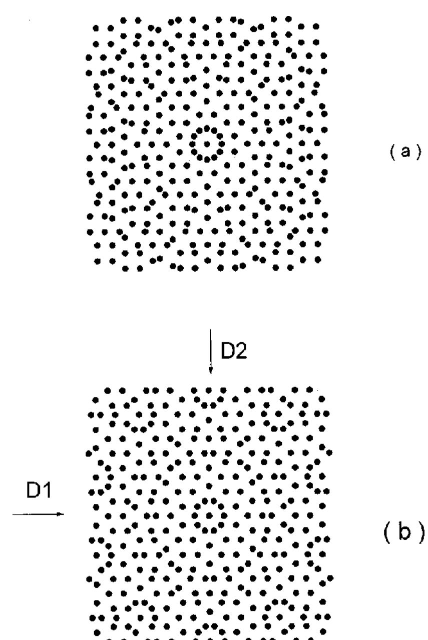


FIG. 1. Schematic figures showing the quasiperiodic arrangement of cylinders: (a) Dodecagonal quasiperiodic pattern; (b) Decagonal quasiperiodic pattern.

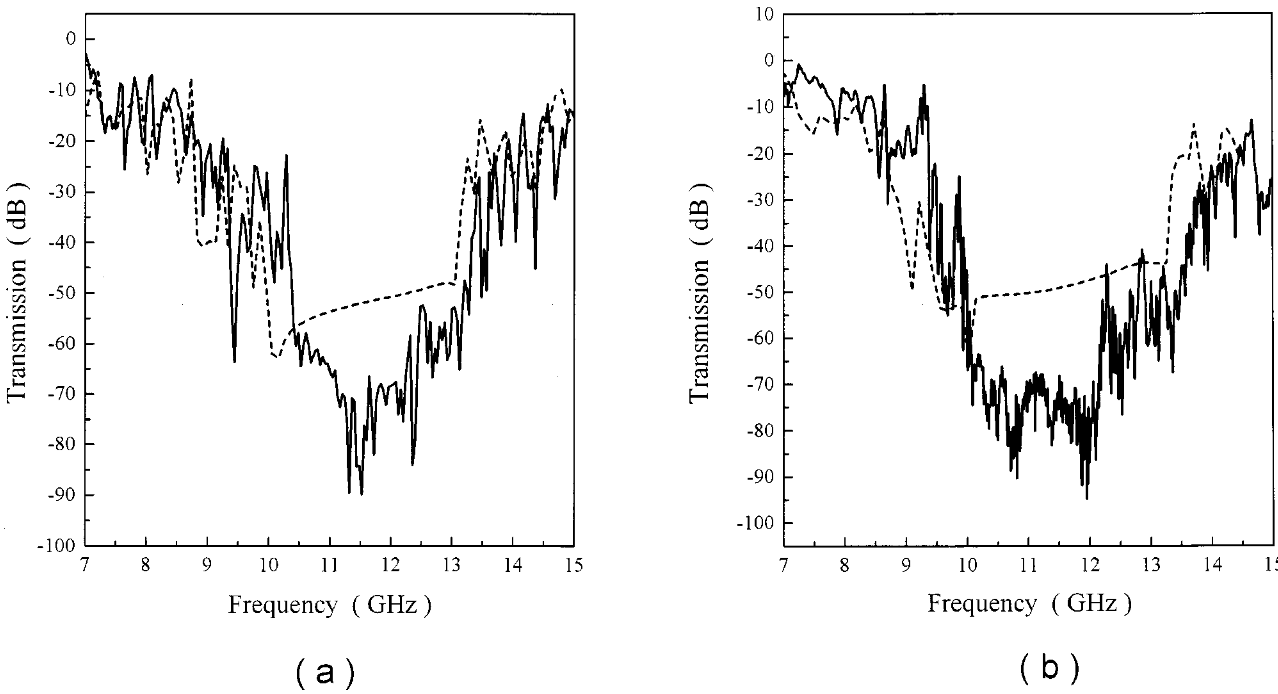


FIG. 2. TM transmission spectra with normal incidence: (a) a dodecagonal quasiperiodic photonic crystal; (b) a decagonal quasiperiodic photonic crystal.

spectively. It should be noted that for the DC QPC we use $D1$ and $D2$ to denote two different normal directions, shown in Fig. 1(b). A detailed description of the measurement system can be found elsewhere.[11] Here we give a brief explanation. We measure the transmission spectrum of the QPC's between 7 and 15 GHz in a wide scattering aluminum chamber using a Hewlett-Packard 8510C vector network analysis system. The chamber is composed of wide waveguides 250 mm wide and 300 mm long, and two H-type 8–12 GHz trumpets that are connected to the back and front ports of the wide waveguides. The photonic crystal is placed in the middle of the wide waveguide scattering chamber. All of the transmission spectra are measured under TM polarized wave incidence.

III. METHOD OF CALCULATION

We have calculated the transmission spectra of quasiperiodic photonic crystals. In our simulations, we assume that the incident beam passes through a slit with a width of $6l$, which is positioned in front of the sample with a distance of $5l$ between the center of the slit and the surface of the sample, and then illuminates the sample. In this case the incident field can be obtained from the Kirchhoff integral formula. In two dimensions, for a plane wave $\exp(ikx)$ incident from $x<0$, the diffracted wave in the region $x>0$ arising from a slit centered at the origin with an opening of width w in the y direction is given by

$$u_{\rm inc}(x,y)=\left(\frac{k_0}{4}\right)\int_{-\omega/2}^{\omega/2}dy'\left[H_0(k_0\rho')+i\frac{x}{\rho'}H_1(k_0\rho')\right],$$

where $\rho'=\sqrt{x^2+(y-y')^2}$, and H_m is a Hankel function of the first kind. A generalized transmission coefficient is defined as the ratio of energy flux to that of the incident wave at $\theta=0$,

$$T=|1+\sqrt{(2\pi/k_0w)}e^{i\pi/4}f_s(0)|^2,$$

where $f_s(0)$ is the total scattering amplitude at $\theta=0$ in the far field, which can be calculated by a multiple-scattering method.[12,13]

IV. RESULTS

A. Transmission spectra of perfect quasiperiodic crystals

Figures 2(a) and 2(b) show the transmission spectra of the dodecagonal and decagonal quasiperiodic photonic crystals, respectively, with normal incidence. The solid and dashed lines denote the measured transmission and simulations, respectively. In the case of the DD QPC, there is a band gap from 9.8 to 13.49 GHz in the experiment, while the simulated band gap is from 9.9 to 13.2 GHz. For the transmission of the DC QPC in the $D1$ direction, the measured band gap is from 9.8 to 13.36 GHz and the simulated one is between 9.5 and 13.36 GHz. The deviation between the experiments and simulations is estimated to be 5%. The difference of the band gap position between experiment and calculation is caused mainly by the small difference of the cylinder radius.

We further measured the transmission spectra for various incident directions, shown in Figs. 3(a) and 3(b) for the DD and DC quasiperiodic photonic crystals, respectively. It should be noted that the dodecagonal quasiperiodic structure has a twelvefold symmetry, and in each 30° sector the distribution of the cylinders has mirror symmetry with respect to the line of 15°. Therefore, for the DD QPC, we need only measure the transmission at incident angles between 0 and 15°. The DC quasiperiodic structure possesses tenfold symmetry, so that transmission spectra measured between 0 and 18° of the incident angle are adequate to reflect the transmission properties of the whole 2π range of incident angles. From Figs. 3(a) and 3(b), it is seen that the location and

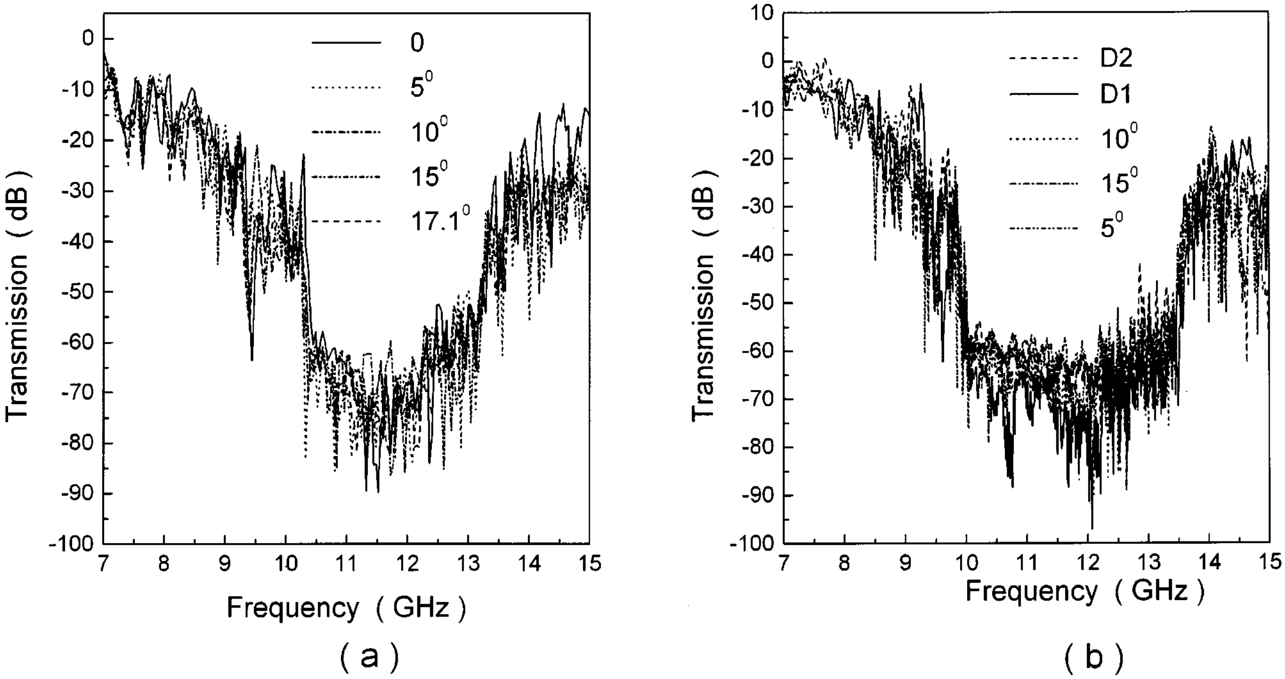


FIG. 3. TM transmission spectra with various incident angles: (a) a dodecagonal quasiperiodic photonic crystal; (b) a decagonal quasiperiodic photonic crystal.

width of the band gaps do not shift either in the dodecagonal photonic crystal or in the decagonal photonic crystal as the incident angle varies. This phenomenon has been found in octagonal quasiperiodic photonic crystals.[9] Based on the previous and present experiments, it may be confirmed that in a two-dimensional (2D) QPC the band gaps of TM waves are independent of the incident direction. This is a quite interesting effect. In a one-dimensional system, the band gaps of the Fibonacci series exhibit a self-similar property as the number of layers increases. Here, in the 2D QPC system, the photonic band gaps possess direction-independent characteristics. This implies that aperiodic structures are an exciting area where various photonic properties may be expected.

It is well known that a periodic two-dimensional photonic crystal with a few rows of cylinders in the incident direction can still have a photonic band gap; the same phenomenon is found in QPC's. First, we examine DD quasiperiodic photonic crystals. We remove 12 rows from the forward and backward sides of the DD QPC. Therefore, the remainder of the photonic crystal has 27 rows of cylinders, instead of the previous 51 rows, and is 92 mm long in the incident direction, which is equal to only about three wavelengths at the lower band gap edge, as shown in the inset of Fig. 4. The corresponding transmission spectrum is plotted in Fig. 4. Both the width and the depth of the gap are the same as in the complete QPC. It is quite surprising that the extinction inside the

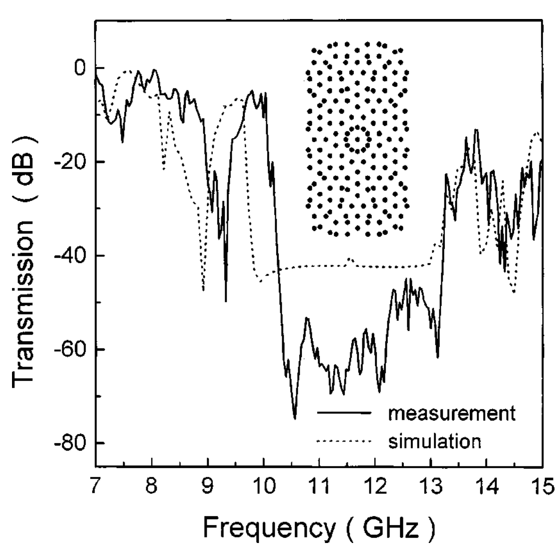


FIG. 4. The experimental and simulated transmission spectra of a photonic crystal. The inset is the schematic pattern of the dodecagonal quasiperiodic photonic crystal with 24 rows removed.

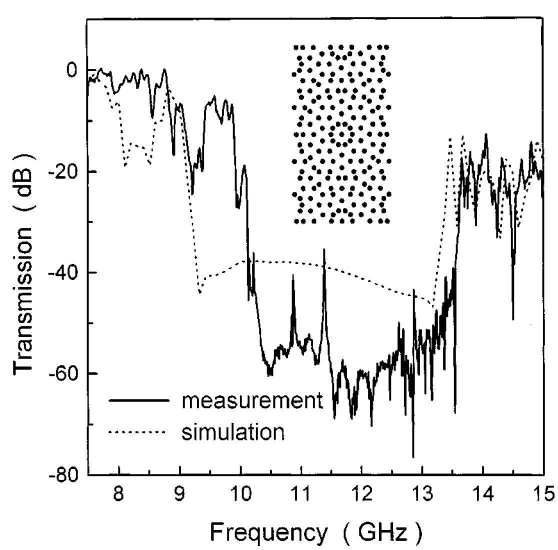


FIG. 5. The experimental and simulated transmission spectra of a photonic crystal. The inset is the schematic pattern of the decagonal quasiperiodic photonic crystal with 24 rows removed.

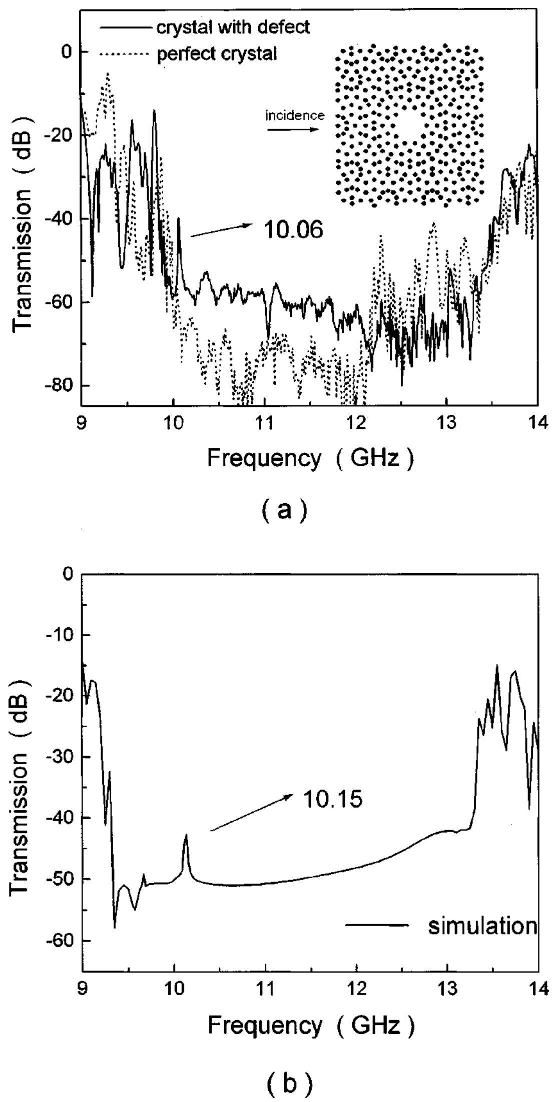


FIG. 6. (a) Experimental and (b) simulated transmission spectra of a photonic crystal. The inset in (a) is a decagonal quasiperiodic photonic crystal with 11 cylinders removed at the center zone.

gap can reach 40 dB when the length of the DD QPC is just three wavelengths. The extinction inside the gap is also near 40 dB in the simulation.

The measurement has been carried out also for the DC QPC. Twelve rows of cylinders are removed simultaneously from the front and back of the DC QPC. The remains of the crystal are only three wavelengths long, which is schematically shown in the inset of Fig. 5. The measured and simulated spectra are given in Fig. 5; a similar result as for the DD QPC is obtained.

From Figs. 4, 5, and previous work on an octagonal QPC,[9] it can be concluded that these quasiperiodic photonic crystals with a few rows in the incident direction can still have photonic band gaps. This characteristic is the same as for a periodic photonic crystal, and is important for applications in photonic devices.

B. Point-defect properties

It has been predicted that an octagonal quasiperiodic photonic crystal can have rich defect modes because each cylinder is different from the others.[8] The defect modes formed by removing different single cylinders are quite complicated. Here we examine a new kind of defect, which can form a large cavity in QPC's. From the structure of QPC's, it is found that a QPC grows ring upon ring from the center point to the outermost ring. Because of the rotational symmetry of QPC's, the cylinders with the same distance to the center may be identical, while those with different distances are distinguishable. Defects are then created by removing the whole ring (the cylinders have the same distance to the center). Here, we observe such defect states of the decagonal quasiperiodic photonic crystal in the first band gap. In our experiment, the electromagnetic wave is incident upon the photonic crystal along the $D2$ direction.

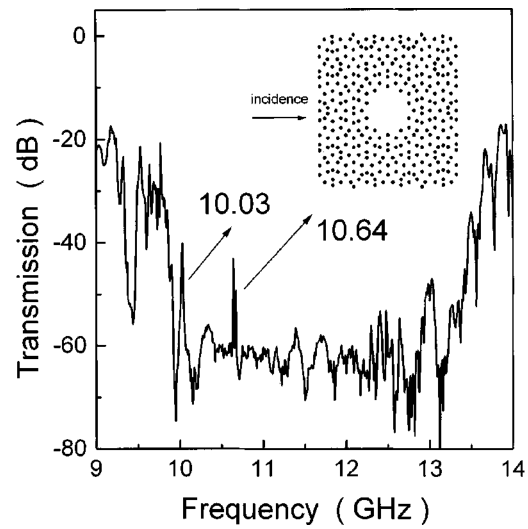


FIG. 7. The experimental transmission spectrum of a photonic crystal. The inset is a decagonal quasiperiodic photonic crystal with 21 cylinders removed at the center zone.

We remove 11 cylinders that include the center one and the nearest 10 cylinders, as shown in the inset of Fig. 6(a). There is a defect state at about $\omega = 10.06$ GHz to be found in

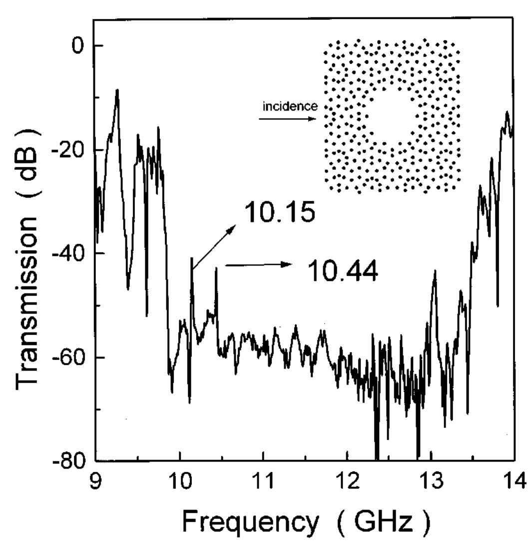
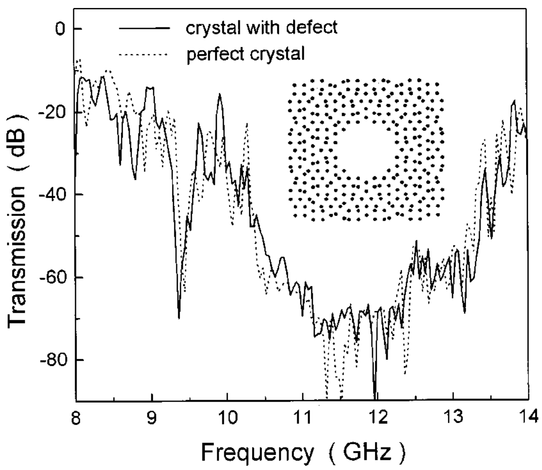
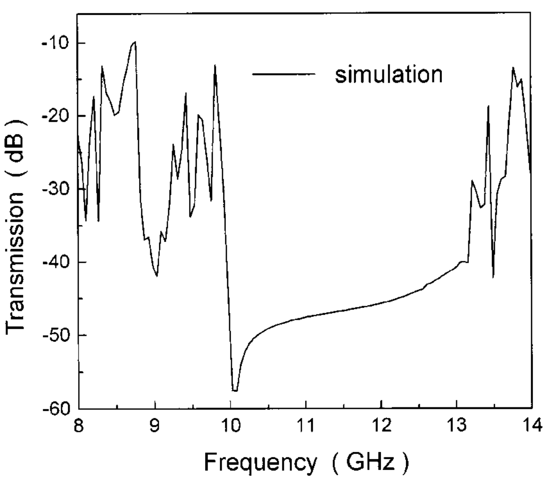


FIG. 8. The experimental transmission spectrum of a photonic crystal. The inset is a decagonal quasiperiodic photonic crystal with 31 cylinders removed at the center zone.



(a)



(b)

FIG. 9. (a) Experimental and (b) simulated transmission spectra of a photonic crystal. The inset is a dodecagonal quasiperiodic photonic crystal with 31 cylinders removed at the center zone.

the transmission spectrum of Fig. 6(a). The simulated defect state appears at about $\omega = 10.15$ GHz, as shown in Fig. 6(b).

When all the cylinders located within the second circle from the center (21 cylinders) are removed, shown in the inset of Fig. 7, two defect states at about $\omega = 10.03$ and 10.64 GHz are observed. When the cylinders in the center four circles (31 cylinders) are all removed, we find two defect states at about $\omega = 10.15$ and 10.44 GHz as shown in Fig. 8. From these transmission spectra, it is clear that the decagonal quasiperiodic structure can offer a great variety of possibilities for creating defect states in different frequency regions in addition to the cavity configuration.

In DD quasiperiodic photonic crystals, however, our measurement shows a different phenomenon. As for the DC QPC, the cylinders of the DD QPC are gradually removed from the center region. We examine three defect structures, i.e., the center cylinder, all cylinders within the second circle, and all cylinders within the third circle are removed, and no defect modes can be found in the first photonic band gap.

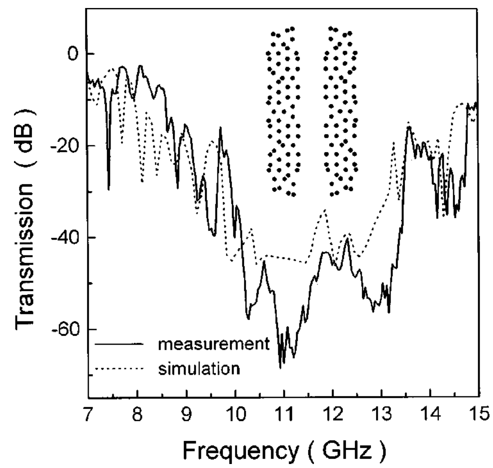


FIG. 10. The experimental and simulated transmission spectra of a photonic crystal. The inset is the schematic pattern of a DD QPC with seven rows removed around the central line.

When all cylinders within the third circle are removed, shown in the inset of Fig. 9(a), no defect mode appears in the experimental and simulated transmission spectra, as shown in Figs. 9(a) and 9(b). We also removed cylinders at different positions in the center region; even when the volume of the microcavity constructed by the defect is more than $3\lambda^2$, where λ is the wavelength at the midgap, we do not find any obvious defect state in the first gap. This marked phenomenon is different from that in a periodic photonic crystal, where when a relatively large volume of material is removed in a unit cell, the acceptor level is deep.[14] This phenomenon has not been found in other QPC's, such as decagonal and octagonal quasiperiodic photonic crystals. It seems that the number of rotational axes has a vital effect on this characteristic. This is a quite useful effect in device application; for example, for cavities and reflectors.

C. Line defect

It is well known that a line defect in a photonic crystal that is parallel to the propagation of the light beam can form a waveguide. These photonic crystal waveguides can guide

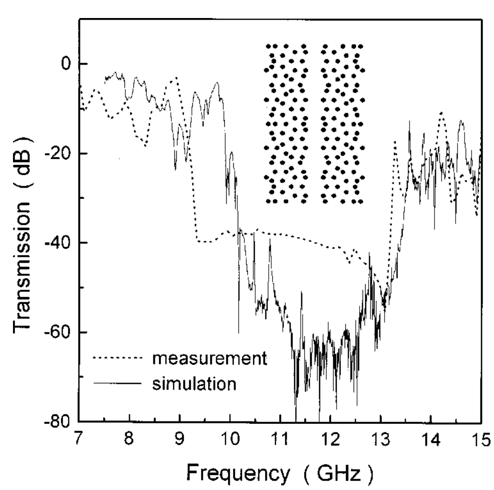


FIG. 11. The experimental and simulated transmission spectra of a photonic crystal. The inset is the schematic pattern of a DC QPC with three rows removed around the central line.

waves around a sharp corner with high efficiency within the region of the photonic band gaps.[5,10] Here we study a line defect that is perpendicular to the propagation of the light beam rather than a waveguide. This line defect can tell us whether a piece of a QPC without the central region can have the properties of a photonic crystal.

For the DD QPC of the inset of Fig. 4 we remove 7 rows around the central line. The remaining 20 rows are shown in the inset of Fig. 10. The measurement spectrum of this QPC is plotted in Fig. 10 by the solid line. It can be seen that a photonic band gap still exists, and as in the original DD QPC the attenuation is over 40 dB within the gap. The simulation has a similar result, as shown in Fig. 10 by the dotted line.

For the DC QPC of the inset of Fig. 5 three rows are removed around the central line, as shown in the inset of Fig. 11. The experimental transmission spectrum of the QPC is shown in Fig. 11 by the solid line, while the simulation is denoted by the dotted line. From Fig. 11, it is obvious that this crystal can realize 40 dB attenuation in the frequency gap zone. The location of the gap does not change when the crystal is shortened and without the central region. Both Figs. 10 and 11 show that a piece of a QPC without the central region can have the same photonic properties as a perfect QPC.

V. SUMMARY

We have experimentally and numerically studied decagonal and dodecagonal photonic crystals in the microwave frequency region. We found that the band gap does not shift when the incident angle changes, and only a few rows are needed to form the frequency gap, even without the central region. This is the same as in a periodic photonic crystal.

For the case of point defects, the decagonal quasiperiodic photonic crystal can offer a great variety of possibilities for creating and controlling the number of defect modes as well as the form of the cavity. On the other hand, the dodecagonal quasiperiodic photonic crystal exhibits an unusual phenomenon. There is no defect state at the first band gap when a cavity is formed by removing cylinders at the center, even though the volume of the cavity is much larger than the square of the wavelength.

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