Self-trapping of electromagnetic pulses in narrow-gap semiconductors

V. Skarka
Laboratoire POMA, EP 130 CNRS Université d'Angers, 2 Boulevard Lavoisier, 49045 Angers, Cedex 1, France

V. I. Berezhiani
Institute of Physics, The Georgian Academy of Science, Tbilisi 380077, Georgia

V. Boucher
Laboratoire POMA, EP 130 CNRS Université d'Angers, 2 Boulevard Lavoisier, 49045 Angers, Cedex 1, France

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Dynamics of the self-trapping of short laser pulses in narrow-band-gap semiconductors is studied. The nonparabolicity of the conduction band leads to a nonlinear dielectric response with saturating nonlinearity. Due to the nonlinearity saturation the beam can be trapped in a self-generated guide, and the formation of stable two-dimensional spatial solitons can take place.

Self-focusing and self-guiding of light beams have received much attention in recent years in connection with their important applications, such as soliton propagation, all-optical switching, and logic.[1] Stable spatial solitons with two transverse dimensions can exist in materials characterized by a saturable nonlinearity that exactly compensates for the diffraction.[2]

Narrow-gap semiconductors, in particular III-V alloys with a small effective mass of conduction electrons, exhibit a large degree of nonparabolicity. In fact, in some narrow-gap semiconductors like InSb, the dispersion relation mimics that of a relativistic electron with a small effective mass, and with an effective ''speed of light'' several order smaller than the speed of light in a vacuum. In Kane's model, the dispersion relation can be written in the form[3]

$$\mathcal{E}=\frac{E_g}{2}\left(1+\frac{p^2}{m_*^2c_*^2}\right)^{1/2}-\frac{E_g}{2}, \tag{1}$$

where $\mathcal{E}$ is the energy of a conduction-band electrons, and p is a quasimomentum. Here $c_*=(E_g/2m_*)^{1/2}$ plays the part of the speed of light ($c_*\approx 3\times 10^{-3}c$ for InSb), m_* is the effective mass of the electrons at the bottom of the conduction band, and E_g is the width of the forbidden gap separating valence and conduction bands. In response to a laser pulse, due to the velocity-dependent mass the conduction electrons can simulate the dynamics of a relativistic plasma, at the pulse intensity which is a tiny fraction of the intensity required to create similar conditions in a normal gaseous plasma. This similarity has been widely exploited in the past. In particular, using methodologies of relativistic plasmas, the parametric amplification of electromagnetic waves as well as the parametric excitation of density waves have been studied.[4] Tzoar and Gersten demonstrated that a weak nonparabolicity ($p^2/m_*^2c_*^2 \ll 1$) in InSb can lead to the self-focusing of the CO_2 laser beam.[5]

These investigations were carried out two decades ago, and consequently dynamical properties of the nonlinear interaction of a laser beam with semiconductors were studied mainly on nanosecond time scales. We would like to emphasize that the principal difference between most semiconductor plasmas and the usual gaseous one concerns the collisionality. The comparatively small collision time of carriers in semiconductors, typically $\tau_R \approx 10^{-12}$–10^{-13} sec, hinders the excitation of collective modes.[6] On the other hand, for nanosecond laser pulses, the field intensity must be much less than 10^7 W/cm^2 to avoid the breakdown of the semiconductor. Consequently, the ''relativistic'' nonparabolicity factor $p^2/m_*^2c_*^2 \sim e^2E^2/m_*^2c_*^2\omega^2$ had to be kept much lower than unity (E and ω are the electric field and frequency of the laser radiation, respectively). Due to these limitations the expected effects appeared to be small, and, therefore, the research in this direction was soon abandoned.

Recent achievements in short-pulse generation, however, have motivated studies of short-pulse propagation in narrow gap *n*-doped semiconductors when the nonparabolicity of the conduction band is the dominant effect responsible for the nonlinear refraction of the laser pulse.[7] Readily available picosecond (femtosecond) intense pulses with wavelengths ranging from the ultraviolet to the midinfrared can be used to observe the collisionless collective phenomena in semiconductor plasma. For instance, in the case of InSb, one must use a CO_2 laser with a wavelength $\lambda = 10.8\mu$ to avoid one- and two-photon absorption. Although the production of ultrashort pulses in infrared ranges with $\lambda > 10\mu$ is currently based on challenging optical parametric down-conversion processes, midinfrared pulses ($\lambda \approx 10\mu$) as short as 130 fs were created a decade ago[8] using semiconductor switching. In Ref. 9 the modulation instability in narrow-gap semiconductors is suggested to produce subpicosecond midinfrared pulses. The applied intensities of such short pulses can be as high as few GW/cm^2. Provided the pump fluence is kept below 0.5 J/cm^2, irreversible damage of semiconductor samples does not take place,[10] allowing us to consider effects with a finite nonparabolicity factor $p^2/m_*^2c_*^2(\approx 1)$. Finite nonparabolicity naturally induces a saturation of the nonlinear, intensity-dependent refractive index of semiconductors.

In what follows we consider the dynamics of short laser pulses in the plasma of narrow-gap semiconductors. The main effort is devoted to studies of the generation of two-

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dimensional stable solitonic channels (filaments) that can be developed due to the self-focusing of a laser pulse in a bulk semiconductor which exhibits the above-mentioned nonlinearity saturation. It is shown that saturation of the nonlinear response stabilizes the process of self-focusing, and allows the formation of spatial solitons.

The dynamics of short laser pulses in the semiconductor collisionless plasma can be described by Maxwell and quasi-hydrodynamic equations.[4,5] The validity of the hydrodynamic approach for the semiconductor plasma requires that both the Fermi energy (E_F) and the temperatures are low ($E_g \gg E_F, k_B T$). It can be shown that the equation for the vector potential **A** of the field reads

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{c^2}{\epsilon_0}\Delta \mathbf{A} + \omega_e^2 \mathbf{A}\left(1 + \frac{e^2 \mathbf{A}^2}{m_*^2 c_*^2 c^2}\right)^{-1/2} = 0, \qquad (2)$$

where $\omega_e = (4\pi e^2 n_0 / m_* \epsilon_0)^{1/2}$ is the effective plasma frequency, n_o is electron density in the conduction band, and ϵ_0 is the dielectric constant of the lattice (for details, see Refs. 7 and 9). For the vector potential a Coulomb gauge is assumed ($\nabla \cdot \mathbf{A} = 0$). The contribution of the magnetic part of the Lorentz force causing a nonlinear variation of the plasma density is negligibly small since $c_*/c \ll 1$. Consequently, effects related to the density variation of electrons are neglected in Eq. (2).

A propagating circularly polarized laser pulse, with a carrying frequency ω and wave number k, is given as

$$\mathbf{A} = \frac{1}{2}(\mathbf{x} + i\mathbf{y})A(\mathbf{r},t)\exp(-i\omega t + ikz) + \text{c.c.}, \qquad (3)$$

where $A(\mathbf{r},t)$ is slowly varying field envelope ($\omega \gg \partial/\partial t, k \gg \boldsymbol{\nabla}$), and ω and k satisfy the dispersion relation $\omega^2 = \omega_e^2 + k^2 c^2/\epsilon_0$. Note that for circularly polarized waves, the electron energy does not depend on the fast time, and consequently there is no harmonics generation.

From Eq. (2) one can obtain a nonlinear Schrodinger equation (NSE) with saturating nonlinearity using variables $z' = z$, $\tau = t - z/v_g$ ($v_g = \epsilon_0^{-1/2} k c^2/\omega$ is a group velocity)

$$i\frac{\partial A}{\partial z} + \Delta_\perp A + \left(1 - \frac{1}{(1+|A|^2)^{1/2}}\right)A = 0. \qquad (4)$$

Here $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is a two-dimensional Laplacian describing pulse diffraction. In Eq. (4) the following normalizations are used: t/T, z/Z, $r_\perp/R$, and A/A_s where $T = \omega^{-1}$, $Z = 2\epsilon_0^{-1/2} c\omega/\omega_e^2$, $R = \epsilon_0^{-1/2} c/\omega_e$ and $A_s = m_* c c_*/e$. Deriving Eq. (4), we assumed that the semiconductor sample is moderately doped ($n_0 \approx 10^{16}$–10^{17} cm^{-3}), and that the corresponding plasma is transparent for CO_2 laser pulses ($\omega \gg \omega_e$). As a consequence, the term related to the temporal spreading of the pulse $[\sim(\omega_e^2/\omega^2)\partial^2 A/\partial \tau^2]$ is neglected in Eq. (4). However, effects related to the temporal reshaping of the pulse should be important for heavily doped samples, and can lead to the formation of temporal solitonic structures.[9]

Equation (4) admits a stationary, nondiffracting axially symmetric solution of the form $A = U(r)\exp(i\lambda z)$, where $r = (x^2+y^2)^{1/2}$, and λ is the nonlinear wave-vector shift. The radially dependent envelope $U(r)$ obeys an ordinary nonlinear differential equation

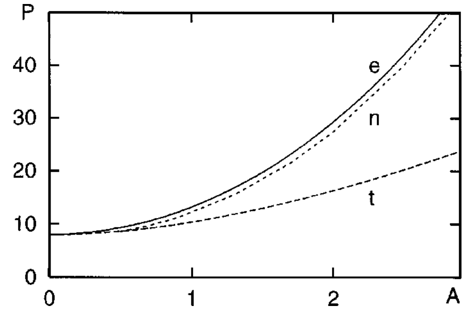

FIG. 1. Equilibrium power as a function of the amplitude. The equilibrium curve *e* together with the numerically obtained *n* curve and trapping curve *t*.

$$\frac{d^2 U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \lambda U + \left(1 - \frac{1}{(1+U^2)^{1/2}}\right)U = 0. \qquad (5)$$

This equation corresponds to a boundary value problem assuming that U has its maximum at $r=0$, and imposing the boundary conditions $U_r, U_{rr} \to 0$ as $r \to \infty$. One can map Eq. (5) in the (U, U_r) plane (phase plane), and compare the resulting equation with a nonconservative motion to show (as Vakhitov and Kolokolov did for another saturating nonlinearity[11]) that provided the eigenvalue λ satisfies the condition $0 < \lambda < 1$, Eq. (5) admits an infinity of discrete bound states $U_j(r)$ ($j = 0,1,2 \ldots$), characterized by n zeros at finite r. In what follows we consider only the lowest-order nodeless solution of Eq. (5), i.e., a ''ground state'' that is positive and monotonically decreasing with increasing r. Numerical simulations show that the amplitude of the ground-state solution $[U_0(r=0,\lambda)]$ is a growing function of λ. The beam width becomes wider at low and high amplitudes, having its smallest size at some intermediate amplitude.

The stability of the ground-state solution can be analyzed using the well-known stability criterion of Vakhitov and Kolokolov.[11] According to this criterion the ground-state solution is stable against a small perturbation if $\partial P/\partial \lambda > 0$, where P represents the integrated intensity (power) of the trapped mode:

$$P(\lambda) = 2\int_0^\infty U_0(r,\lambda) r\, dr. \qquad (6)$$

Such a stable nondiffracting solution (also see Ref. 13) can be called a soliton according to the modern terminology. Curve n in Fig. 1 corresponds to the power as a function of the amplitude A obtained using the numerical solution of Eq. (5). Since this curve has a positive slope and λ is a growing function of the amplitude A, the derivative of the power is positive $[P'(\lambda) = P'(A)A'(\lambda) > 0]$, and the corresponding ground-state solution is stable for any amplitude. The numerical simulations of Eq. (4) confirm the stability of the ground-state soliton solution. Even if a small initial perturbation is imposed, the soliton propagates without distortion for a long distance.

The complex dynamics of a beam governed by a NSE with saturating nonlinearity can be analyzed by the variational approach.[12] This approach determines the relations between the characteristic parameters of the localized solution approximated by a trial function. The variational method gives qualitatively good results, provided the beam does not undergo structural changes during its evolution. The first standard step is to construct the Lagrangian

$$L=|\nabla_\perp A|+\frac{i}{2}(A\partial_z A^*-\text{c.c.})-F(|A|^2), \tag{7}$$

where the asterisk denotes complex conjugation, and $F=|A|^2+2[1-(1+|A|^2)^{1/2}]$ is the nonlinear term. Appropriate variation of the Lagrangian yields Eq. (4) as the Euler-Lagrange equation. In the optimization procedure, the first variation of the variational function must vanish on a suitably chosen trial function. As a trial function, we will use the Gaussian-shaped beam,

$$A=A_0(z)\exp\left[-\frac{x^2}{2a_x^2(z)}-\frac{y^2}{2a_y^2(z)}+i\phi(z)\right], \tag{8}$$

where $\phi=x^2b_x(z)+y^2b_y(z)+\phi_0(z)$. The evolution of the laser field is parametrized by z-dependent amplitude A_0, spatial widths a_x and a_y, and phase ϕ_0. The parameters b_x and b_y are the wave-front curvatures. Substituting expression (8) into Eq. (7), and demanding that the variation of the spatially averaged Lagrangian with respect to each of these parameters is zero, we obtain the corresponding set of Euler-Lagrange equations,

$$\frac{d^2\mathbf{R}_\perp}{dz^2}=-2\frac{\partial}{\partial\mathbf{R}_\perp}V(\mathbf{R}_\perp), \tag{9}$$

where the vector $\mathbf{R}_\perp=(a_x,a_y)$ is the width. The effective potential V has the form

$$V=\frac{1}{a_x^2}+\frac{1}{a_y^2}-\frac{K(A_0^2)}{A_0^2}, \tag{10}$$

with the nonlinearity function

$$K(u)=2u+8[1-(1+u)^{1/2}]+8\ln\{0.5[1+(1+u)^{1/2}]\}. \tag{11}$$

During the field evolution, the power is conserved, $P=A_0^2(z)a_x(z)a_y(z)=\text{const}$. Thus Eqs. (9) and (10) are analogous to those describing the dynamics of a particle in a two-dimensional potential. Straightforward analysis shows that if the power of the pulse is less than a critical one, $P<P_c=8$, the potential has a negative slope $\partial V/\partial\mathbf{R}_\perp<0$, and consequently the force pushes an effective particle in the positive $\mathbf{R}_\perp$ direction. In other words, the effective width of the beam increases continuously, leading to its diffraction. However, for $P>P_c$ the potential becomes a two-dimensional well where the effective particle is trapped and oscillates, bouncing elastically from the walls. If the beam initially has a plane front [i.e., $b_x(0)=b_y(0)=0$], the trapping condition is $a_{0x}^{-2}+a_{0y}^{-2}<A_{0m}^{-2}K(A_{0m}^2)$, where A_{0m}, a_{0x}, and a_{0y} are the initial amplitude and transverse dimensions of the beam, respectively. The generation of a two-dimensional oscillating waveguide takes place. The equilibrium parameters of the beam that correspond to the effective particle settled in the bottom of the well (obtained by using relation $\partial V/\partial\mathbf{R}_\perp=0$) are $a_{eq}=a_{ex,ey}=2^{1/2}[K'(A_{eq}^2)-K(A_{eq}^2)/A_{eq}^2]^{-1/2}$, where a_{eq} and A_{eq} are the equilibrium width and amplitude of the beam, respectively. One can see in Fig. 1 that the equilibrium curve e obtained by the variational approach closely follows the n curve which corresponds to the exact numerical solution of Eq. (5). The beam above the critical power ($P>P_c$) is trapped, provided it is in the trapping region, and its parameters will oscillate around the equilibrium ones. In the region above the t curve (in Fig. 1) the trapping condition is satisfied for a symmetric beam ($a_x=a_y$). As a consequence, the beam with initial parameters situated on the left-hand side of the equilibrium curve will focus initially, while for the state on the right of this curve it will first be defocused. Note that in the case of weak nonparabolicity ($|A|^2\ll 1$), Eq. (4) reduces to a NSE with cubic nonlinearity, and a beam with above critical power will collapse after a finite distance of propagation.

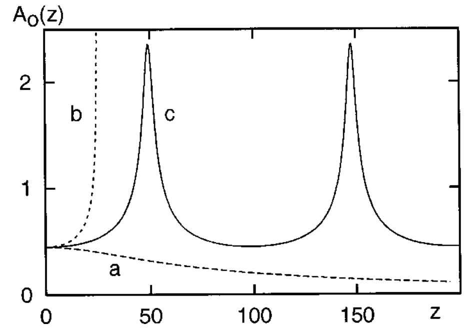

FIG. 2. The amplitude A_0 as a function of propagation z obtained by variational method. The a curve corresponds to the diffraction. The Kerr nonlinearity is given by the b curve. The c curve exhibits oscillations around the equilibrium.

In Fig. 2 we plot the evolution of the beam amplitude obtained by numerical simulations of Eq. (9). The initial Gaussian beam is assumed to be symmetric, $a_{0x}=a_{0y}=10$, and to have the power $P=20$. The a curve corresponds to the case when the nonlinearity is neglected. The beam amplitude is reduced by half at the distance $z_d\approx 60$. The b curve is associated with the Kerr-type nonlinearity ($\sim|A|^2$). As one might expect, the beam collapses at a distance shorter than the diffraction length z_d. The saturation nonlinearity, related to the finite nonparobolicity of the band structure, prevents the infinite growth of the field amplitude, resulting in an oscillatory wave guide (c curve). A similar behavior can be obtained for an asymmetric beam ($a_{0x}\neq a_{0y}$), provided the trapping condition is satisfied.

Results of the variational approach can be used to understand the main features of the beam dynamics. For instance, the predicted equilibrium curve lies reasonably close to the numerically obtained one. Although this approach describes the beam dynamics qualitatively well, it is unable to account for structural changes of the beam shape. The exact dynamics of the beam is obtained by numerical simulations of Eq. (4). The typical behavior of the trapped beam is shown in Fig. 3, where the field amplitude of the initially symmetric Gaussian beam $|A|=A_{0m}\exp[-(x^2+y^2)/2a_0^2]$ is plotted versus the radius r and the propagation coordinate z. The beam power is $P=A_{0m}^2a_0^2=20$, while its width is $a_0=10$. As has been predicted by the variational approach, the beam is self-

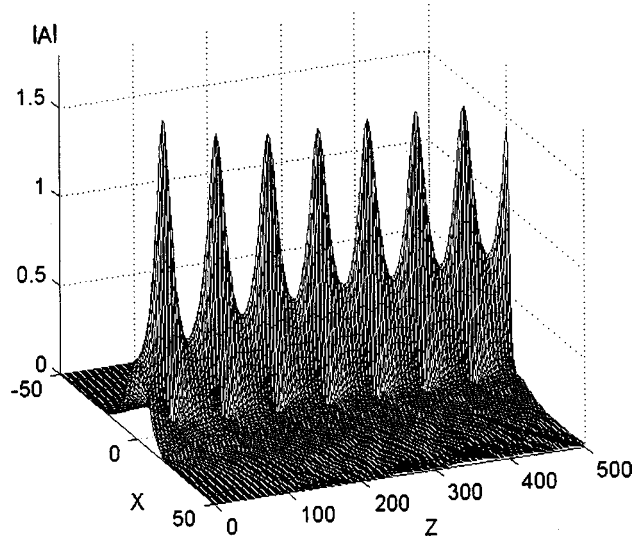

FIG. 3. The numerically obtained beam evolution exhibiting damped oscillations toward equilibrium.

trapped, and its parameters oscillate near the equilibrium state, corresponding to the power $P=20$. Note, however, that due to the appearance of the radiation spectrum the amplitude of these oscillations is monotonically decreasing with increasing z (see also Refs. 12 and 14). For larger z the oscillations are damped out, and the formation of a ground solitonic state takes place. If the initial profile of the beam is close to the equilibrium one, then the beam quickly reaches the profile of ground-state equilibrium, and propagates for a long distance without distortion of its shape. The initial beam in the trapping region, even quite far from equilibrium, will either focus or defocus to the ground state, exhibiting damped oscillations around it. Essentially the same damped behavior, leading to the formation of a symmetric ground-state soliton, can be obtained for an initially asymmetric beam. Consequently, the ground-state equilibrium seems to be an attractor.

Such an evolution scenario can be altered due to the modulation instability (MI), and under certain conditions it will result in the beam breakup. Indeed, MI can take place for laser beams with a power much higher than the critical one. If such a beam is sufficiently wide, it may be unstable with respect to small perturbations of its spatial structure. Rigorous treatment of MI is beyond the scope of this paper. However, to obtain some insight into this problem, we analyze the stability of a constant amplitude field with respect to small perturbations. Assuming that $A=A_0+\epsilon\exp(\chi z+i\mathbf{k}_\perp\mathbf{r}_\perp)$, where $\epsilon\ll|A_0|(=\text{const})$, from Eq. (4) we find that the region of instability is determined by the condition $|k_\perp|<k_m=|A_0|/(1+|A_0|^2)^{3/4}$, while the growth rate χ reaches its maximum $\chi_m=k_m^2/2$ for perturbations with a transverse spatial period $L_\perp=2^{3/2}\pi/k_m$.

We carried out numerical simulations of nonlinear dynamics of MI for a wide, high-power Gaussian beam with an initial amplitude $A_{0m}=1$ and widths $a_{0x}=a_{0y}=100$. For the amplitude $\epsilon=0.1$ of imposed perturbation, the wave vectors $k_x=k_y=0.2$ are chosen to be in the domain of instability. The intensity pattern at three different propagation distances z in Fig. 4 demonstrates the breakup of the beam into higher-intensity filaments. Each filament has the tendency to evolve

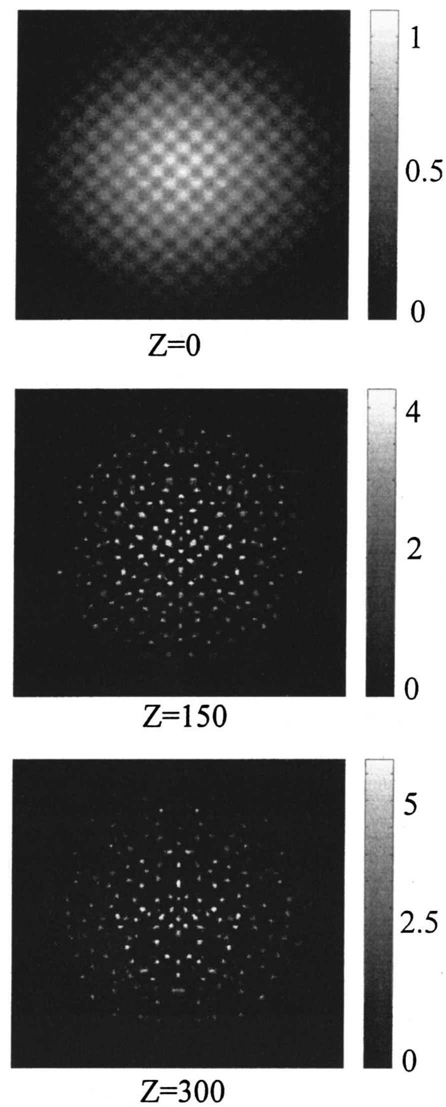

FIG. 4. The initial beam intensity pattern followed by beam breakup patterns of interacting filaments at propagation distances $z=150$ and 300.

toward its own equilibrium state, corresponding to the power it carries. However, due to the mutual interactions of filaments, further dynamics is difficult to interpret. Therefore, we cannot conclude that at the final state of their evolution those filaments form a stable solitonic structure. MI simulations for low-power beams ($P_c<P<60$) have been made, but the instability was not observed even in the case of wide beams. We believe that for such a beam ($A_{0m}\ll 1$) the growth rate of the instability ($\chi\sim A_{0m}^2$) becomes negligibly small, and the beam reaches its equilibrium state before MI develops.

In dimensional units the intensity and the power of the laser beam are defined as $I=\epsilon_0^{1/2}c|E|^2/4\pi$ and $P=\pi a_0^2 I$, respectively, where a_0 is the spot size of the beam. The critical power is expressed as

$$P_c=2\epsilon_0^{-1/2}c^3\frac{m_*^2c_*^2}{e^2}\left(\frac{\omega}{\omega_e}\right)^2. \tag{12}$$

In the case of an InSb semiconductor, the relevant parameters are $T=77$ K, $m_*=m_e/74$, $c_*=c/253$, and $\epsilon_0=16$. The critical power is $P_c=12(\omega/\omega_e)^2$ W. For a CO_2 laser beam with a frequency $\omega=1.74\times10^{14}$ rad/sec propagating in InSb with a carrier density $n_0=2\times10^{16}$ cm^{-3}, the critical power for the self-trapping is $P_c\approx1.3$ kW. The real numbers for the normalization introduced in Eq. (4) are $Z=89$ μm, $T=57\times10^{-16}$ s, $R=4{,}4$ μm, and $E_s=\omega A_s/c=1.6\times10^5$ V/cm. Using these parameters, the simulated dynamics in Fig. 3 corresponds to a laser beam of power 31 kW, spot size (i.e., the beam width at half maximum) 73 μm, and initial intensity 0.05 GW/cm^2. The first focus, i.e., the first maximum of the field intensity corresponding to 0.86 GW/cm^2, appears at a distance $z_f=4$ mm. Notice that the nonparabolicity factor is 1 at the intensity $I_s=0.27$ GW/cm^2. For the chosen laser frequency the condition $2\hbar\omega<E_g$ is satisfied and, consequently, the effects related to the multiphoton absorption are negligibly small. The relaxation time of free carriers is assumed to be $\tau_{0R}=5\times10^{-13}$ sec using data on the Hall mobility. Therefore, the absorption length related to the scattering of carriers appears to be $l_{\rm abs}\approx(\omega/\omega_e)^2c\tau_{0R}=10$ mm. However, a strong laser field itself contributes to increase the relaxation time. Indeed, if in the semiconductors the momentum losses of carriers occur predominantly through the scattering on ionized impurities (that is likely to be case for InSb at temperature 77 K) it is generally assumed that the relaxation time is given by a power law $\tau_R=\tau_{0R}(\overline{e_E}/\overline{e_0})^{3/2}$, where $\overline{e_E}$ and $\overline{e_0}$ are the total average energy of the carriers in the presence and absence of the field, respectively. Considering the parameters presented above, one obtains $\tau_R\sim10^2\tau_{0R}$. As a consequence, the nonlinear dynamics of the laser field develops at a distance shorter than the absorption length. However, if the absorption losses are determined by carrier–optical phonon scattering (for instance, at room temperature) the relaxation time turns out to be smaller than τ_{0R} and in samples of a few millimeters the field will be strongly attenuated before the nonlinear processes manifest themselves significantly. Thus the relaxation time is very much case dependent, and for each particular semiconductor sample should be evaluated carefully. To insure the generation of a solitonic structure in a few of microns, the laser field parameters have to be close to the ground-state equilibrium.

In conclusion, we show that ultrashort intense laser pulses propagating in the semiconductor plasma can be trapped in a self-generated guide. After the emission of a certain amount of radiation, the trapped beam converges toward a stable equilibrium, with the eventual formation of stable spatial solitons. We believe that narrow-band-gap semiconductors are good candidates for applications in optical information processing systems based on soliton interactions at midinfrared wavelengths.

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