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Fluctuation broadening of the plasma resonance line in the vortex liquid state of layered superconductors

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The Josephson plasma resonance (JPR) provides a sensitive probe of vortex states in layered superconductors. We demonstrate that in the case of weak damping in the liquid phase, broadening of the JPR line is caused mainly by random Josephson coupling arising from the density fluctuations of pancake vortices. In this case the JPR line has the universal shape, which is determined only by parameters of the superconductors and temperature. This mechanism gives a natural explanation for the experimentally observed asymmetric lineshape. The tail at high frequencies arises due to mixing of the propagating plasma modes by random Josephson coupling, while the tail at small frequencies is caused by the localized plasma modes originating from a rare fluctuation suppression of the Josephson coupling in large areas. [S0163-1829(99)51130-2]

Observation of the Josephson plasma resonance (JPR) in the mixed state of layered high temperature and organic superconductors[1–4] allows the study of Josephson coupling of layers over a wide range of fields and temperatures. The Josephson coupling in the mixed state is mainly determined by correlations between arrangements of pancake vortices in neighboring layers and therefore it can serve as a probe of vortex state. The resonance absorption is determined by the spatial and temporal behavior of the ''local coherence parameter'' $\mathcal{C}_n(\mathbf{r},t)\equiv\cos\varphi_{n,n+1}(\mathbf{r},t)$, with $\varphi_{n,n+1}=\varphi_{n+1}-\varphi_n-(2\pi s/\Phi_0)A_z$ being the gauge-invariant phase difference between the layers n and $n+1$. Here $\mathbf{r}=(x,y)$ and z are the coordinates in the ab plane and along the c axis, and s is the interlayer spacing. Nonzero $\varphi_{n,n+1}$ arises due to the misalignment of pancake vortices in neighboring layers. Time variations of $\varphi_{n,n+1}$ caused by pancake motion are usually much slower than the plasma oscillations. In this case the plasma mode probes a snapshot of the instantaneous phase distribution. In the liquid phase at high fields, $B\gg B_J$, correlations between pancake positions in neighboring layers are almost absent. Here $B_J=\Phi_0/\lambda_J^2$, $\lambda_J=\gamma s$ is the Josephson length, and γ is the anisotropy factor. In this state $\mathcal{C}_n(\mathbf{r})$ rapidly oscillates in space so that its variations are much larger than average. The resonance in such a situation occurs because small phase oscillations induced by the external electric field change slowly in space and average out these rapid variations. We will estimate that the resonance is formed at the typical scale λ_J^2/a, with a being the intervortex spacing. This large scale averaging leads to a fairly sharp resonance, with the resonance frequency squared ω_p^2 being roughly proportional to the averaged cosine factor $\mathcal{C}\equiv\mathcal{C}(\mathbf{B},T)$ (see, e.g., Refs. 5 and 6)

$$\omega_p^2(B,T)\approx\omega_0^2(T)\mathcal{C},\quad \mathcal{C}\equiv\langle\cos\varphi_{n,n+1}(\mathbf{r})\rangle, \tag{1}$$

where $\omega_0^2(T)=c^2/\epsilon_c\lambda_c^2(T)$, λ_c is the c component of the London penetration depth, ϵ_c is the dielectric constant, and $\langle\cdots\rangle$ denotes the thermodynamic average. However, the averaging by the smooth oscillating phase is not complete. Large scale fluctuations of $\mathcal{C}_n(\mathbf{r})$, induced by pancake density fluctuations, lead to inhomogeneous broadening of the JPR line. This mechanism was proposed in Ref. 5 to explain line broadening in the vortex glass state. In this paper we analyze broadening of the JPR line in the vortex liquid due to the random Josephson coupling. An attractive feature of the line broadening in the liquid phase is that it is an intrinsic property of the material caused by thermal fluctuations. The plasma resonance line has a very peculiar asymmetric shape with the long tail in the high-field part.[1–3] This shape can be naturally explained by the proposed mechanism. The high-frequency/high-field tail of the line probes mixing of the propagating plasma modes by the random Josephson coupling, while the low-frequency/low-field tail probes the localized plasma modes originating due to rare fluctuation suppression of the coupling. We will analytically calculate the resonant absorption in both regions.

The external alternating electric field applied along the c axis $\mathcal{D}_z=\mathcal{D}_{z0}\exp(i\omega t)$ induces small oscillations of the interlayer phase difference $\varphi'_{n,n+1}$. In general, charging of layers[7] and deviations of the quasiparticle distribution function from equilibrium[8] should be accounted for in the time-dependent equation for the phase difference. To illustrate the physics of the inhomogeneous line broadening, we will consider a simplified equation, in which we do not account for these effects[9,5]

$$\left(\frac{\omega^2}{\omega_0^2}+\lambda_J^2\hat{L}\nabla^2-\mathcal{C}_n(\mathbf{r})\right)\varphi'_{n,n+1}=-\frac{i\omega\mathcal{D}_z}{4\pi J_0}, \tag{2}$$

where $J_0=c\Phi_0/8\pi^2\lambda_c^2 s$ is the Josephson current. The second term in the brackets accounts for the inductive interaction between the junctions. The operator $\hat{L}$ acts on the layer index n and is defined as $\hat{L}A_n=\Sigma_m L_{nm}A_m$ with $L_{nm}=\int_0^{2\pi}(dq/2\pi)L(q)\cos(n-m)q$, and $L(q)=[2(1-\cos q)+s^2/\lambda_{ab}^2]^{-1}$, with λ_{ab} being the ab components of the London penetration depth. We neglect in Eq. (2) time variations of $\mathcal{C}_n(\mathbf{r},t)$ assuming them to be small during the time $1/\omega$. If we neglect the charging effects,[7] then the oscillating internal electric field is connected with the solution of Eq. (2) by the Josephson relation $E_z \approx (i\omega\Phi_0/2\pi cs)\varphi'_{n,n+1}$. The resonant absorption is given by the imaginary part of the inverse dielectric constant $1/\varepsilon_c(\omega)=E_z/\mathcal{D}_z$. We split $\mathcal{C}_n(\mathbf{r})$ into the average and fluctuating parts, $\mathcal{C}_n(\mathbf{r})=\mathcal{C}+u_n(\mathbf{r})$. The correlation function of $u_n(\mathbf{r})$ is given by

$$\langle u_n(\mathbf{r})u_{n'}(\mathbf{r}')\rangle=\langle \cos\varphi_{n,n+1}(\mathbf{r})\cos\varphi_{n',n'+1}(\mathbf{r}')\rangle-\mathcal{C}^2.$$

This correlation function depends weakly on E_J. Taking the limit $E_J\to 0$, we obtain

$$\langle u_n(\mathbf{r})u_{n'}(\mathbf{r}')\rangle \approx \frac{1}{2}S(\mathbf{r}-\mathbf{r}')\delta_{nn'},$$

where $S(\mathbf{r})=\langle\cos[\varphi_{n,n+1}(\mathbf{r})-\varphi_{n,n+1}(0)]\rangle$ is the static phase correlation function at $E_J=0$. Static configurations $\varphi_{n,n+1}(\mathbf{r})$ are mainly determined by the thermal fluctuations of pancake vortices, and $S(\mathbf{r})$ drops at distances of the order of the intervortex spacing a.

An important observation is that in spite of the rapid variations of $\mathcal{C}_n(\mathbf{r})$ in Eq. (2), the solution $\varphi'\equiv\varphi'_{n,n+1}(\mathbf{r})$ varies smoothly in space. At $\omega\approx\omega_p$ the typical length scale L_φ of phase variations can be estimated by balancing the typical kinetic energy of supercurrents, $E_0(\varphi')^2/L_\varphi^2$, with the typical fluctuation of the random Josephson energy, $E_J(\varphi')^2 a/L_\varphi$. Here $E_0=s\Phi_0^2/16\pi^3\lambda_{ab}^2$ is the in-plane phase stiffness and $E_J=E_0/\lambda_J^2$ is the Josephson energy per unit area. This gives $L_\varphi=\lambda_J^2/a\gg a$. Smoothly varying φ' effectively averages the rapid variation of $\mathcal{C}_n(\mathbf{r})$ and the plasma frequency is simply determined by $\mathcal{C}(\mathbf{B},T)$. Fluctuations of $\mathcal{C}_n(\mathbf{r})$ smoothed over the area L_φ^2, $\mathcal{C}_{L_\varphi}=L_\varphi^{-2}\int_{r<L_\varphi}d\mathbf{r}\mathcal{C}_n(\mathbf{r})$, produce inhomogeneous broadening of the JPR line. Calculating the mean squared fluctuation of $\mathcal{C}_{L_\varphi}$, $\langle(\mathcal{C}_{L_\phi}-\mathcal{C})^2\rangle \approx a^2/L_\phi^2=a^4/\lambda_J^4$, we obtain the estimate for the inhomogeneous linewidth

$$\omega_b^2\approx\omega_0^2(T)B_J/B. \tag{3}$$

Inhomogeneous line broadening is determined by the amplitude of fluctuations of $u_n(\mathbf{r})$, $S_0=\int d\mathbf{r}S(\mathbf{r})\approx a^2$, i.e., $\omega_b^2 \propto S_0$. On the other hand, according to the high-temperature expansion,[6,10] the average cosine is also determined by S_0:

$$\mathcal{C}(\mathbf{B},T)\approx(E_J/2T)S_0. \tag{4}$$

This relation allows us to connect the linewidth with the resonance frequency. In Ref. 10, S_0 and $\mathcal{C}(\mathbf{B},T)$ were calculated taking into account both the vortex and regular phase fluctuations. At low temperatures, $T\ll 2\pi E_0$, the result can be written as

$$\mathcal{C}(\mathbf{B},T)\approx\frac{f_s E_0 B_J}{2TB}\left[1+\frac{T}{2\pi E_0}\ln\left(\frac{B}{B_J}\right)\right], \tag{5}$$

where $f_s(T)$ is the universal function of T/E_0. The logarithmic correction appears due to the regular phase fluctuations. Comparing Eqs. (1) and (5) with Eq. (3), we obtain an estimate for the relative linewidth in the liquid state due to the inhomogeneous broadening, $\omega_b/\omega_p\approx T/E_0\ll 1$.

Consider now the problem quantitatively. Equation (2) is similar to the Schrödinger equation for a particle in a random potential and the problem is mapped onto the problem of the density of states in such a system. Then one can use the same techniques. The problem of density of states does not have an analytical solution over the whole energy range. Only asymptotics for positive and negative energies can be calculated.

At frequencies well above the resonance frequency one can treat the random coupling $u_n(\mathbf{r})$ as a perturbation. A perturbative analysis of Eq.(2) can be conveniently performed using the Green's function formalism. We define the Green's function $\mathcal{G}_{nn'}(\mathbf{r},\mathbf{r}';E)$ as a solution of the equation,

$$[E+\lambda_J^2\hat{L}\nabla^2-u_n(\mathbf{r})]\mathcal{G}_{nn'}(\mathbf{r},\mathbf{r}';E)=\delta_{nn'}\delta(\mathbf{r}-\mathbf{r}').$$

Here we introduced the dimensionless "energy" $E=\omega^2/\omega_0^2-\mathcal{C}$. Knowledge of $\mathcal{G}_{nn'}(\mathbf{r},\mathbf{r}';E)$ allows one to calculate the phase responses to arbitrary external perturbations. In particular, the response to the homogeneous external field is given by the averaged Fourier transform of $\mathcal{G}_{nn'}(\mathbf{r},\mathbf{r}';E)$,

$$\mathcal{G}(\mathbf{k},q;E)=\sum_n\int d\mathbf{r}\langle\mathcal{G}_{n0}(\mathbf{r},0;E)\rangle\exp(-i\mathbf{kr}-iqn),$$

at $\mathbf{k},q=0$. The dielectric constant $\varepsilon_c(\omega)$ is connected with $\mathcal{G}(\mathbf{k},q;E)$ as $\varepsilon_c/\varepsilon_c(\omega)=(\omega^2/\omega_0^2)\mathcal{G}(0;\omega^2/\omega_0^2-\mathcal{C})$. The JPR line, given by the imaginary part of $1/\varepsilon_c(\omega)$, can be connected with the spectral density $A_0(E)$, $A_0(E)=\mathrm{Im}[\langle\mathcal{G}(0;E-i\delta)\rangle]/\pi$

$$p(\omega)\equiv\mathrm{Im}\left[\frac{\varepsilon_c}{\varepsilon_c(\omega)}\right]=\frac{\pi\omega^2}{\omega_0^2}A_0\left(\frac{\omega^2}{\omega_0^2}-\mathcal{C}\right). \tag{6}$$

The perturbative expansion of $\mathcal{G}(\mathbf{k},q;E)$ with respect to $u_n(\mathbf{r})$ can be performed using the diagrammatic technique and $\mathcal{G}(\mathbf{k},q;E)$ is represented as

$$\mathcal{G}(\mathbf{k},q;E)=\frac{1}{E-L(q)\lambda_J^2k^2-\Sigma(\mathbf{k},q;E)}, \tag{7}$$

where $\Sigma(\mathbf{k},q;E)$ is the self-energy function. It, in turn, can be represented as the perturbation series with respect to $u_n(\mathbf{r})$. In the lowest order with respect to $u_n(\mathbf{r})$, which is equivalent to the Born approximation for scattering, $\Sigma(\mathbf{k},q;E)$, is given by

$$\Sigma(\mathbf{k},q;E)=\frac{1}{2}\int\frac{d\mathbf{k}'dq'}{(2\pi)^3}\mathcal{G}(\mathbf{k}',q';E)S(\mathbf{k}-\mathbf{k}'). \tag{8}$$

The imaginary part of $\Sigma(\mathbf{k},q;E)$, which we denote by Σ_2, determines the line broadening, while the real part, Σ_1, determines the shift of the resonance frequency due to inhomogeneities. The high-frequency asymptotics of $\Sigma(\mathbf{k},q;E)$ can be obtained by replacing $\mathcal{G}(\mathbf{k},q;E)$ with its bare value $\mathcal{G}_0(\mathbf{k},q;E)=[E-L(q)\lambda_J^2k^2-i\delta]^{-1}$. The main contribution to Σ_2 comes from the region $k\sim\sqrt{E}/\lambda_J\ll 1/a$. In this region one can neglect the k dependence of $S(\mathbf{k})$ and replace it by S_0. The same replacement is possible for almost all terms in the perturbation series. The only exception is the Born integral for Σ_1 logarithmically diverging at $k \gg \sqrt{E}/\lambda_J$. This divergency is cut by the k dependence of $S(\mathbf{k})$ at $k \approx 1/a$. Using the replacement $S(\mathbf{k}) \rightarrow S_0 \approx (2T/E_J)\mathcal{C}$ and performing integrations with respect to $\mathbf{k}$ and q we obtain the following expressions for the imaginary and real parts of the self-energy function at large ''energies''

$$\Sigma_2\Big|_{E\rightarrow\infty} \equiv \Sigma_\infty = \frac{T\mathcal{C}}{2E_0}, \quad \Sigma_1\Big|_{E\rightarrow\infty} \approx -\frac{\Sigma_\infty}{\pi}\ln\frac{\lambda_J^2}{a^2 E}. \tag{9}$$

$\Sigma_\infty \approx a^2/\lambda_J^2$ provides the typical ''energy'' scale for the broadening. In Σ_1 we separate the logarithmic contribution, $-(2\Sigma_\infty/\pi)\ln(\lambda_J^2/a^2)$, coming from large k, and the remaining part which is determined by S_0. Further analysis of the perturbation expansion shows that the expansion parameter at large E is $\Sigma_\infty/[E+(2\Sigma_\infty/\pi)\ln(\lambda_J^2/a^2)]$. Therefore the components of the self energy can be represented in the following scaling form:

$$\frac{\Sigma_2}{\Sigma_\infty} = s_2(\zeta), \quad \frac{\Sigma_1}{\Sigma_\infty} = -\frac{2}{\pi}\ln\frac{\lambda_J^2}{a^2} - s_1(\zeta) \tag{10}$$

with $\zeta = E/\Sigma_\infty + (2/\pi)\ln(\lambda_J^2/a^2)$. $s_1(\zeta)$ and $s_2(\zeta)$ are the dimensionless functions with the asymptotics, $s_2 \rightarrow 1$ and $s_1 \rightarrow (1/\pi)\ln(1/\zeta)$ at $\zeta \rightarrow \infty$. Using these expressions we can represent the JPR absorption (6) in scaling form

$$p(\omega)\omega_b^2/\omega^2 = f(\zeta), \quad \zeta = (\omega^2 - \tilde{\omega}_p^2)/\omega_b^2, \tag{11}$$

$$f(\zeta) = \frac{s_2(\zeta)}{[\zeta + s_1(\zeta)]^2 + s_2^2(\zeta)}, \tag{12}$$

where ω_b is the resonance width, $\omega_b^2 = (T/2E_0)\omega_p^2$, in agreement with the estimate (3). We assume this width to be larger than the linewidth due to the quasiparticle dissipation. $\tilde{\omega}_p$ is the resonance frequency shifted by the random Josephson coupling, $\tilde{\omega}_p^2 = \omega_p^2[1-(T/\pi E_0)\ln(B/B_J)]$. This negative shift is due to the second-order perturbative correction to the ground-state ''energy,'' similar to the well-known result of quantum mechanical perturbation theory. Note that the inhomogeneous correction is two times larger than the correction due to the regular phase fluctuations [see Eq. (5)] and has the opposite sign. The high-frequency tail of the resonant absorption, $\omega^2 - \omega_p^2 \gg \omega_b^2$, is given by $p(\omega) \approx \omega^2\omega_b^2/(\omega^2 - \tilde{\omega}_p^2)^2$. The whole shape of the line is determined by the single dimensionless parameter $\mu = T/E_0$. In particular, the width of the line is $\mu\omega_p$, the maximum absorption scales as $1/\mu$, and the absorption at $\omega^2 - \omega_p^2 \approx \omega_p^2$ scales as μ. Using the field dependence of the plasma frequency given by Eqs. (1) and (5) we can also present the scaling parameter ζ as a function of the magnetic field

$$\zeta = (B - B_r)/B_b + (1/\pi)\ln(B/B_J), \tag{13}$$

with $B_r = f_s\omega_0^2 E_J \Phi_0/(2T\omega^2)$ and $B_b \approx (f_s\omega_0^2/4\omega^2)B_J \approx (T/2E_0)B_r$. This allows us to obtain from Eqs. (11) and (12) the field dependence of the resonant absorption, which is usually probed in the JPR experiments.

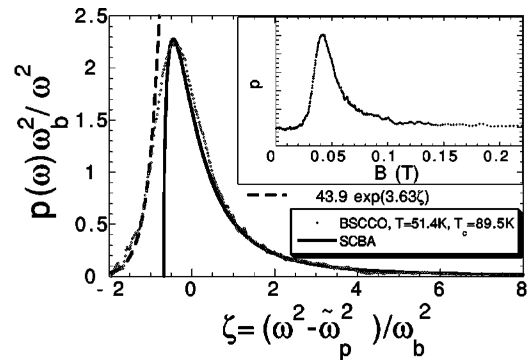

FIG. 1. Plot of the reduced JPR lineshape $p(\omega)\omega_b^2/\omega^2$ vs reduced frequency ζ obtained within SCBA [Eqs. (11) and (14)]. For comparison we also show the experimental JPR line for BSCCO plotted as the function of ζ from Eq. (13) (courtesy of M. Gaifullin and Y. Matsuda, the inset shows raw data). The parameters B_r and B_b in Eq. (13) are chosen to make the experimental and theoretical lines match at large ζ giving $B_r = 0.053$ T and $B_b = 0.016$ T. The left-hand side of the line is fitted to exponent for comparison with Eq. (21)

The problem is now reduced to a calculation of the dimensionless functions s_1 and s_2. These functions can be calculated approximately if we keep only the first term in the expansion of Σ given by Eq. (8) [self-consistent Born approximation (SCBA)]. This leads to the following equations:

$$s_2 = \frac{1}{2} + \frac{1}{\pi}\arctan\frac{\zeta + s_1}{s_2},$$

$$s_1 = -\frac{1}{2\pi}\ln[(\zeta + s_1)^2 + s_2^2], \tag{14}$$

which we solve numerically. Figure 1 shows the dependence of the reduced JPR absorption $p(\omega)\omega_b^2/\omega^2$ on the reduced frequency ζ. Note that the SCBA actually breaks down at $\zeta \lesssim 1$, i.e., it describes quantitatively only the right-hand side of the line. For comparison we took the experimental JPR line for optimally doped $Bi_2Sr_2CaCu_2O_8$ (BSCCO) crystal with $T_c = 89.5$ K (Ref. 11) at $\omega/2\pi = 45$ GHz and $T = 51.4$ K. We replotted this line as the function of the scaling parameter ζ from Eq. (13), where the parameters B_r and B_b are chosen to make the experimental and theoretical lines match at large ζ giving $B_r = 0.053$ T and $B_b = 0.016$ T. The obtained ratio $B_b/B_r \approx 0.3$ is somewhat larger then the theoretical estimate $T/2E_0 \approx 0.17$, which we obtain taking $\lambda_{ab} = 200$ nm$/\sqrt{1-(T/T_c)^2}$.

The SCBA approach gives the JPR line terminating at some finite frequency. In reality a long absorption tail exists on the low-frequency side of the line due to the rare fluctuation configurations corresponding to suppression of $\mathcal{C}_n(\mathbf{r})$ over large areas. This is very similar to the localization tail below the bottom of the conduction band in disordered semiconductors (Lifshitz tail, see, e.g., Ref. 12). An important simplification is that due to the large scale of spatial changes of $\varphi'_{n,n+1}(\mathbf{r})$, the random ''potential'' $u_n(\mathbf{r})$ can be treated as a short ranged Gaussian random variable with the probability distribution

$$P[u_n(\mathbf{r})]\propto \exp[-\mathcal{H}\{u_n\}], \quad \mathcal{H}\{u_n\}=\sum_n \int d\mathbf{r}\,\frac{[u_n(\mathbf{r})]^2}{S_0}.$$

Following a standard line of reasoning,[12] we estimate the spectral density $A_0(E)$ with exponential accuracy as $A_0(E) \propto \exp[-\Phi(E)]$ with

$$\Phi(E)=\min_u \mathcal{H}\{u_n(\mathbf{r})\}|_{\mathcal{E}_0\{u\}=E}, \tag{15}$$

where $\mathcal{E}_0\{u\}$ is the ground-state energy for a given potential fluctuation $u_n(\mathbf{r})$,

$$\mathcal{E}_0\{u\}=\min_\Psi\left[H_0\{\Psi\}+\sum_n \int d\mathbf{r} u_n(\mathbf{r})\Psi_n^2(\mathbf{r})\right], \tag{16}$$

with $H_0\{\Psi\}=\lambda_J^2\Sigma_{n,m}\int d\mathbf{r}L_{nm}\nabla\Psi_n\nabla\Psi_m$ and normalization $\Sigma_n\int d\mathbf{r}\Psi_n^2=1$. A conditional minimization in Eq. (15) can be performed using the Lagrange technique

$$\Phi(E)=-\beta E+\min_{\Psi,u}\left[\beta H_0\{\Psi\}+\mathcal{H}\{u_n(\mathbf{r})\} +\beta\sum_n \int d\mathbf{r} u_n(\mathbf{r})\Psi_n^2(\mathbf{r})\right], \tag{17}$$

where β is the Lagrange factor, which has to be found from the relation $\mathcal{E}_0\{u\}=E$. Optimization with respect to $u_n(\mathbf{r})$ gives $u_n(\mathbf{r})=-S_0\beta\Psi_n^2(\mathbf{r})/2$. Substituting this expression into Eq. (17) and variating it with respect to Ψ_n, we obtain the nonlinear eigenvalue problem

$$-\lambda_J^2\hat{L}\nabla^2\Psi_n(\mathbf{r})-(\beta S_0/2)\Psi_n^3(\mathbf{r})=E\Psi_n(\mathbf{r}), \tag{18}$$

which determines the optimum Ψ_n and β. To simplify this equation, we (i) apply the operator $\hat{L}^{-1}=s^2/\lambda_{ab}^2+\nabla_n^2$ on both sides and neglect small term s^2/λ_{ab}^2 in the resulting equation (here ∇_n^2 is defined as $\nabla_n^2\alpha_n\equiv\alpha_{n+1}+\alpha_{n-1}-2\alpha_n$), and (ii) introduce dimensionless variables $\psi_n(\mathbf{r})=\Psi_n(\mathbf{r})/\Psi_0(0)$, $\tilde{\mathbf{r}}=\mathbf{r}\sqrt{\beta S_0\Psi_0^2(0)/2}/\lambda_J$, and $\epsilon_o=2E/[\beta S_0\Psi_0^2(0)]$. These transformations lead to the dimensionless nonlinear eigenvalue problem

$$-\tilde{\nabla}^2\psi_n+\nabla_n^2\psi_n^3=-\epsilon_o\nabla_n^2\psi_n(\tilde{\mathbf{r}}), \tag{19}$$

which has to be solved with the condition $\psi_0(0)=1$ and with the dipolelike asymptotics at large distances

$$\psi_n(\tilde{\mathbf{r}})=-a\nabla_n^2(|\epsilon_o|\tilde{r}^2+n^2)^{-1/2}, \quad \tilde{r},n\gg 1.$$

The spectral density $A_0(E)$ can be expressed through ϵ_o and $\psi_n(\tilde{\mathbf{r}})$ as

$$A_0(E)\propto\exp\left(-\frac{\psi_4\lambda_J^2E}{|\epsilon_o|S_0}\right), \quad \psi_4=\sum_n \int d\tilde{\mathbf{r}}\psi_n^4. \tag{20}$$

A numerical solution of Eq. (19) gives $\epsilon_o=-0.164$, $\psi_4=2.418$, and $a=0.224$. From Eq. (20) we now obtain $A_0(E)\propto\exp(-14.72\lambda_J^2E/S_0)$, which corresponds to the exponential tail of the JPR absorption

$$p(\omega)\propto\exp(3.18\zeta)=\exp\left[-\frac{7.36E_0(\tilde{\omega}_p^2-\omega^2)}{T\omega_p^2}\right]. \tag{21}$$

An exponential fit of the left-hand side of the experimental line in Fig. 1 gives $p\propto\exp(3.63\zeta)$, consistent with above result.

In conclusion, we studied the universal shape of the JPR line due to thermal fluctuations of pancakes in the vortex liquid. We showed that the line broadening is strongly asymmetric, in agreement with experimental observations. We found that the relative linewidth is given by the parameter T/E_0, which also determines the strength of thermal fluctuations at $B=0$ and is directly related to the temperature dependent λ_{ab}.

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