# Theory of dephasing by external perturbation in open quantum dots

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We propose a random matrix theory describing the influence of a time-dependent external field on the average magnetoresistance of open quantum dots. The effect of the external field is taken into account to all orders of perturbation theory, and the result is applicable to both weak and strong fields.
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The anomalous magnetoresistance of bulk disordered systems is governed by weak localization (WL).[1–3] Being an interference phenomenon, it is extremely sensitive to inelastic processes commonly referred to as dephasing.

Recently, another object for the study of quantum effects appeared—ballistic quantum dots.[4] In the absence of inelastic processes, the transport properties of the dots are well described within the random matrix theory (RMT).[5] The magnetoresistance within this theory manifests itself as a crossover between two universal ensembles (orthogonal and unitary), and the strength of the magnetic field defines the position on this crossover. This picture per se did not include dephasing, and the dephasing processes were considered on a phenomenological basis.[6] The relation of this phenomenological description used to fit the data of Ref. 7 to the microscopic mechanisms of dephasing is still an open question.

In this paper, we propose a random matrixlike theory of magnetoresistance affected by an external time-dependent perturbation. We will find both the amplitude and frequency dependence of the magnetoresistance using only one unknown parameter. This parameter can be related to the correlator of the level velocities due to the same perturbation at zero frequency and, thus, in principle, can be determined by an independent experiment. After the strength of the potential is normalized by this parameter, all results become universal. As for the experimental realization, we imply changing the shape of a quantum dot by applying an external ac bias to the dot forming leads.[4]

Before we proceed, let us mention that the effect studied in the present paper is similar to that of Refs. 8 and 9, where it was shown that a uniform ac-electric field suppresses the weak localization correction to the conductivity of a disordered wire. The results of Ref. 8 are not directly applicable to quantum dots with size $L$ so small that the Thouless energy $E_T \sim \hbar/\tau_{erg}$ is much greater than all other energy scales of the problem (here $\tau_{erg}$ is the characteristic time for a classical particle to cover all of the available phase space).

On the other hand, in this limit, one can use the RMT to study the conductance of the system, see Ref. 10. All corrections to the RMT are as small as $N_{ch}/g_{dot}$; $g_{dot}=E_T/\delta_1$ and $\delta_1$ is the mean level spacing. We consider the WL correction to the conductance of quantum dots with a large number of open channels $N_{ch}$. In this approximation, we neglect the effects of the electron-electron interaction on the conductance which are as small as $1/N_{ch}^2$,[11] while the weak localization correction to the conductance is proportional to $1/N_{ch}$. The same condition also allows us to use a conventional diagrammatic technique[12] to take the ensemble average.

The Hamiltonian of the system is[10]

$$\hat{H}=\hat{H}_D+\hat{H}_L+\hat{H}_{LD}, \tag{1}$$

where $\hat{H}_D$ is the Hamiltonian of the electrons in the dot, which is determined by the $M\times M$ matrix $H_{nm}$:

$$\hat{H}_D=\sum_{n,m=1}^{M}\psi_n^\dagger H_{nm}\psi_m, \tag{2}$$

where the thermodynamic limit $M\to\infty$ is assumed. We consider the case, where $H_{nm}$ is a time-dependent random matrix in the form

$$H_{nm}(t)=\mathcal{H}_{nm}+V_{nm}\varphi(t). \tag{3}$$

Here, the time-independent part of the Hamiltonian $\mathcal{H}_{nm}$ is a random realization of the $M\times M$ matrix, which obeys the correlation function

$$\langle \mathcal{H}_{nm}\mathcal{H}^*_{n'm'}\rangle=\lambda\,\delta_{nn'}\delta_{mm'}+\lambda'\,\delta_{mn'}\delta_{nm'}, \tag{4}$$

where $\lambda=M(\delta_1/\pi)^2$ and $\lambda'=\lambda(1-g_h/4M)$, and $g_h$ defines the crossover from orthogonal ($g_h=0$) to unitary ($g_h=4M$) ensembles. The parameter $g_h$ has the meaning of the dephasing rate due to an external magnetic field in the units of the level spacing $\delta_1$.[3,10] It can be estimated as $g_h \simeq g_{dot}(\Phi/\Phi_0)^2$, where $\Phi$ is the magnetic flux through the dot and $\Phi_0=hc/2e$ is the flux quantum. The time-dependent perturbation is described by the symmetric $M\times M$ matrices $V_{nm}$ and the function of time $\varphi(t)$.

The coupling between the dot and the leads is

$$\hat{H}_{LD}=\sum_{\alpha,n,k}[W_{n\alpha}\psi_\alpha^\dagger(k)\psi_n+\text{H.c.}], \tag{5}$$

where $\psi_n$ corresponds to the states of the dot, $\psi_\alpha(k)$ denotes different electron states in the leads, and momentum $k$ labels

 

continuous states in each channel $\alpha$. For a dot connected with two leads by $N_l$ and $N_r$ channels, respectively, we denote the left lead channels by $1 \leqslant \alpha \leqslant N_l$ and the right channels by $N_l+1 \leqslant \alpha \leqslant N_{ch}$, where $N_{ch}=N_l+N_r$. The electron spectrum in the leads near the Fermi surface can be linearized:

$$\hat{H}_L = v_F \sum_{\alpha,k} k \psi_\alpha^\dagger(k) \psi_\alpha(k), \tag{6}$$

where $v_F = 1/2\pi\nu$ is the Fermi velocity, and $\nu$ is the density of states at the Fermi surface.

The coupling constants $W_{n\alpha}$ in Eq. (5) are[10]

$$W_{n\alpha} = \left( \frac{M\delta_1}{\pi^2 \nu} \right)^{1/2} \begin{cases} t_\alpha & \text{if } n=\alpha \leqslant N_{ch}, \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

where $t_\alpha$ determines the dimensionless conductance of each lead (in units of $2e^2/h$) according to

$$g_l = \sum_{\alpha=1}^{N_l} \frac{4 t_\alpha t_\alpha^*}{(1+t_\alpha t_\alpha^*)^2}, \quad g_r = \sum_{\alpha=N_l+1}^{N_{ch}} \frac{4 t_\alpha t_\alpha^*}{(1+t_\alpha t_\alpha^*)^2} \tag{8}$$

and $|t_\alpha| \leqslant 1$. The factor in Eq. (7) is chosen so that the ensemble average scattering matrix $\mathcal{S}_{\alpha\beta}$ of a dot with fully open channels ($t_\alpha = 1$) is zero. A more complicated structure of $\hat{W}$ can always be reduced to the form (7) by suitable rotations.

The scattering matrix of the system, $\hat{\mathcal{S}}$, has the form

$$\mathcal{S}_{\alpha\beta}(t,t') = \delta_{\alpha\beta}\delta(t-t') - 2\pi i \nu W_{\alpha n}^\dagger G_{nm}(t,t') W_{m\beta}, \tag{9}$$

and the Green's function $G_{nm}(t,t')$ is the solution to

$$\left( i\frac{\partial}{\partial t} - \hat{H}(t) + i\pi\nu \hat{W}\hat{W}^\dagger \right) \hat{G}(t,t') = \delta(t-t'), \tag{10}$$

where the matrices $\hat{H}$ and $\hat{W}$ are comprised by their elements (3) and (7), respectively.

The average dimensionless dc conductance of the dot at temperature $T$ in terms of the scattering matrix of the system in linear response theory is[13]

$$g = \left\langle \int_{-\infty}^{+\infty} dt_1 dt_2 \, \mathrm{tr}\left[ \hat{\tau}_l \mathcal{S}(t,t_1) \hat{\tau}_r \mathcal{S}^\dagger(t_2,t) \right] \right\rangle f(t_1-t_2), \tag{11}$$

$$f(t) = \int_{-\infty}^{+\infty} d\omega \, e^{i\omega t} \frac{\partial}{\partial \omega} \frac{1}{e^{\omega/T}+1} = \frac{\pi T t}{\sinh \pi T t}, \tag{12}$$

where $\langle \cdots \rangle$ stands for both ensemble and time $t$ averages. We also introduce a notation for the projector on the left lead, $\hat{\tau}_l$, which is a diagonal $N_{ch} \times N_{ch}$ matrix with the first $N_l$ diagonal elements equal to unity and the other diagonal elements equal to zero, $\hat{\tau}_r = \hat{I} - \hat{\tau}_l$.

The ensemble average $\langle \mathcal{S}(t,t_1)\mathcal{S}^\dagger(t_2,t) \rangle \sim \delta(t_1-t_2)$. This property allows us to eliminate one of the time integrals in Eq. (11) and the thermal function $f(t)$: $f(0)=1$. Therefore, the result does not depend on the electron temperature, similarly to the conductivity of bulk metal in the weak localization regime without interaction.

We perform calculations of the average conductance keeping the leading terms in $1/M$. The diagrammatic technique is somewhat similar to that developed for bulk metals,[12] where the small parameter is $1/\epsilon_F \tau_{imp}$, with $\epsilon_F$ being the Fermi energy and $\tau_{imp}$ being the elastic mean free time.

First we find the ensemble average Green's function $\langle G^{(R)} \rangle$. One can see that $\langle G^{(R)} \rangle$ is diagonal, $\langle G^{(R)}_{nm}(\epsilon) \rangle = \delta_{nm} G^{(R)}_n(\epsilon)$. Using the self-consistency equation for the Green's function, Fig. 1(a), we find

$$G_n^{(R)}(\epsilon) = \frac{1}{i\sqrt{\lambda M}} \begin{cases} \dfrac{1}{1+t_n t_n^*}, & n \leqslant N_{ch}, \\ \dfrac{\displaystyle\sum_{\alpha=1}^{N_{ch}} \frac{2 t_\alpha t_\alpha^*}{1+t_\alpha t_\alpha^*} + i\epsilon}{1 + \dfrac{\displaystyle\sum_{\alpha=1}^{N_{ch}} \frac{2 t_\alpha t_\alpha^*}{1+t_\alpha t_\alpha^*} + i\epsilon}{4M}}, & n > N_{ch}. \end{cases} \tag{13}$$

Above we introduced the dimensionless energy $\epsilon$ measured in the units of $\sqrt{\lambda/4M} = \delta_1/2\pi$. We expand these Green's functions in $\epsilon/M$ and $g_{l,r}/M$, since only those terms survive the thermodynamic limit $M \to \infty$. For the same reason, in the expression for $G_n^{(R)}$ with $n \leqslant N_{ch}$ we neglect such terms since the contribution of these elements to the final result is already of the order of $N_{ch}/M$.

We rearrange Eq. (11) as the following:

$$g = \frac{N_r g_l^2 + N_l g_r^2}{(g_l+g_r)^2} + \int dt_1 dt_2 \, \mathrm{tr} \langle \hat{J}_l \mathcal{S}(t,t_1) \hat{J}_r \mathcal{S}^\dagger(t_2,t) \rangle, \tag{14}$$

$$\hat{J}_{l,r} = \hat{\tau}_{l,r} - \frac{g_{l,r}}{g_l+g_r}\hat{I}. \tag{15}$$

Equation (14) immediately follows from Eq. (11) and from the unitarity of the $\mathcal{S}$ matrix $\mathcal{S}\mathcal{S}^\dagger = \hat{I}$. The calculations of the conductance in the form of Eq. (14) are significantly simpler since the vertices (15) are not dressed by the dashed lines, see Fig. 1(b). This trick is similar to the calculation of the conductivity of disordered bulk systems in terms of the current-current, rather than density-density, correlation function, Refs. 2 and 3.

Now, we substitute the scattering matrix defined by Eq. (9) to Eq. (14). One can independently average to the leading order in $1/(g_l+g_r)$ with the help of Eq. (13) and obtain the classical conductance

$$g_{cl} = \frac{g_l g_r}{g_l+g_r}. \tag{16}$$

Particularly, for the dot with fully open channels ($t_\alpha = 1$), the average $\mathcal{S}$ matrix vanishes, and the first term of Eq. (14) gives $g_{cl} = N_l N_r / N_{ch}$, since in this case $g_{l,r} = N_{l,r}$.

The first order correction in $1/(g_l+g_r)$ to Eq. (16) is given by the diagram in Fig. 2(a). It represents the WL correction to the conductance and has the expression

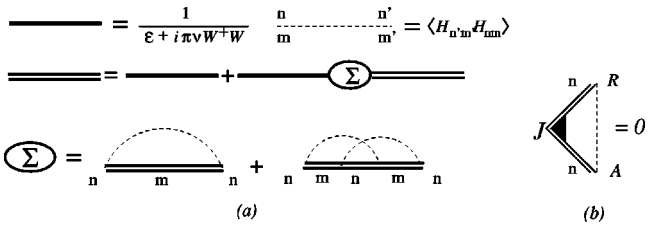


FIG. 1. (a) Diagrams for the ensemble average Green's function. The second term in the self-energy includes an intersection of dashed lines and is as small as $1/M$. (b) The representation of the conductance in the form of Eq. (14) forbids renormalization of the vertices $I$ from Eq. (15) by disorder.

$$\Delta g_{wl} = -\frac{f_l g_r^2 + f_r g_l^2}{(g_l+g_r)^2} \int_0^{2\pi/\omega} \frac{\omega\, dT}{2\pi} \int_0^{\infty} 2\, d\tau\, \mathcal{C}(T,\tau,-\tau); \tag{17}$$

where the form factors $f_{l,r}$ are given by

$$f_l = \sum_{\alpha=1}^{N_l} \frac{16(t_\alpha t_\alpha^*)^2}{(1+t_\alpha t_\alpha^*)^4}, \quad f_r = \sum_{\alpha=N_l+1}^{N_{ch}} \frac{16(t_\alpha t_\alpha^*)^2}{(1+t_\alpha t_\alpha^*)^4}. \tag{18}$$

The Cooperon $\mathcal{C}$ is defined in Fig. 2(b):

$$\left(\frac{\partial}{\partial\tau} + \mathcal{K}(T,\tau)\right)\mathcal{C}(T,\tau,\tau') = \delta(\tau-\tau'), \tag{19}$$

where time is measured in the units of inverse level spacing $2\pi/\delta_1$ and the ''Hamiltonian'' for the Cooperon is

$$\mathcal{K}(T,\tau) = g_* + \pi^2 C_0[\varphi(T+\tau/2) - \varphi(T-\tau/2)]^2. \tag{20}$$

Here $g_*$ characterizes the total dephasing due to the escape as well as the magnetic field,

$$g_* = g_l + g_r + g_h,$$

and we chose $\varphi(t) = \cos\omega t$ to describe the time dependence of the perturbation.

The only unknown parameter $C_0$ in Eq. (20) depends on the strength of the perturbation. In terms of the original Hamiltonian (3), it is defined as

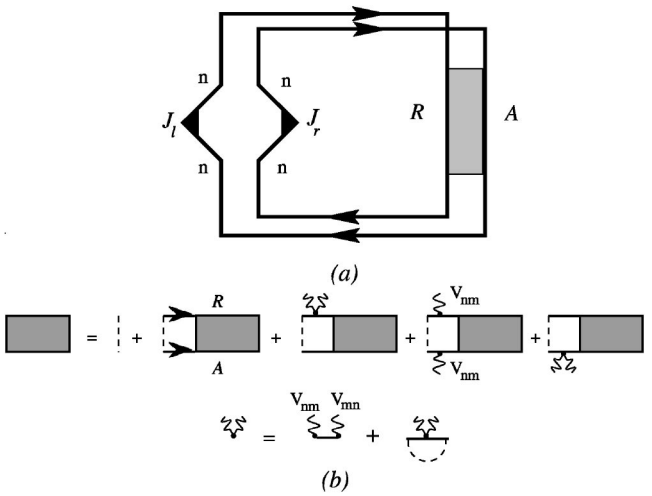


FIG. 2. (a) The diagram for the WL correction to the conductance. (b) The diagram equation for the Cooperon.

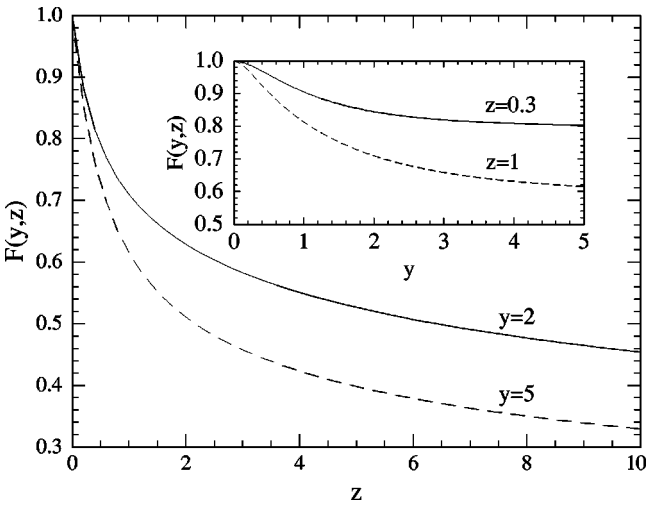


FIG. 3. Representative curves of $F(y,z)$ as a function of $z$ for two values of $y$. $F(y,z)$ decreases linearly with $z$ at small values of $z$. The inset shows the $y$ dependence of the function $F(y,z)$ for two values of $z$. $F(y,z)$ decreases quadratically in $y$ at small values of $y$ and saturates at larger $y$.

$$C_0 = \frac{2}{\pi^2 M\lambda} \sum_{nm} V_{nm}^2, \tag{21}$$

where we used the fact that the matrix $\hat{V}$ is symmetric. This parameter is related to the typical value of the level velocities, which characterizes the evolution of an energy level $\epsilon_\nu(X)$ under the external perturbation $X\hat{V}$,[14]

$$\delta_1^2 C_0 = \left\langle \left(\frac{\partial\epsilon_\nu}{\partial X}\right)^2 \right\rangle - \left\langle \frac{\partial\epsilon_\nu}{\partial X} \right\rangle^2. \tag{22}$$

Since all other responses (e.g., parametric dependence of the conductance of the dot) are expressed in terms of the universal functions of the same parameter $C_0$,[14] it can be found from independent measurements. For the rather unrealistic case of homogeneous electric field $E$ introduced into a dot of linear size $L$, one can estimate $C_0 \simeq (eEL)^2/(E_T\delta_1)$. It is important to emphasize that a homogeneous shift of all levels does not affect the magnetoresistance and that is why the average level velocity $\langle \partial\epsilon_\nu/\partial X\rangle$ is irrelevant.

In the absence of time dependent perturbation $\varphi \equiv 0$,[10,15] one obtains from Eqs. (17)–(20):

$$\Delta g_{wl}^{(0)} = -\frac{f_l g_r^2 + f_r g_l^2}{(g_l+g_r)^2 g_*}. \tag{23}$$

The solution to Eq. (19) gives the weak localization correction to the conductance $\Delta g_{wl}$ in the presence of the time dependent field. It can be expressed in terms of the unperturbed correction (23) as

$$\frac{\Delta g_{wl}}{\Delta g_{wl}^{(0)}} = F(y,z), \quad y = \frac{\pi\omega}{g_*\delta_1}, \quad z = \frac{\pi^2 C_0}{g_*}, \tag{24}$$

where the dimensionless function $F(y,z)$ is given by

$$F(y,z) = \int_0^{\infty} dx\, e^{-x-z\phi} I_0[z\phi], \quad \phi = x - \frac{\sin xy}{y}. \tag{25}$$

Here $I_0(\xi)$ is the modified Bessel function. Some curves for this function are plotted in Fig. 3.

Equations (24) and (25) are the main results of our paper. They give a universal description of the effect of an external field on the weak localization correction. Below we will discuss different asymptotic regimes and compare them with the results for bulk systems.[3]

For a weak external field $z \ll \max(1, y^{-2})$ we find

$$\frac{\Delta g_{wl}}{\Delta g_{wl}^{(0)}} = 1 - \frac{\pi^2 C_0}{g_*} \frac{\pi^2 \omega^2}{\pi^2 \omega^2 + \delta_1^2 g_*^2}. \tag{26}$$

In this regime the correction is quadratic in frequency for a slowly oscillating field at $\omega$ smaller than the dephasing rate $1/\tau_\phi$. However, the frequency dependence saturates at large frequency. It is different from the result for bulk systems, where a characteristic spatial scale shrinks as $1/\sqrt{\omega}$, whereas in our case it is determined by the size of the dot.

In the opposite limit of a strong external field, $z \gg \max(1, y^{-2})$, we have to consider separately the cases of fast, $y \gg 1$, and slow, $y \ll 1$ field oscillations. In the first case we find

$$\frac{\Delta g_{wl}}{\Delta g_{wl}^{(0)}} = \left( \frac{g_*}{2\pi^2 C_0} \right)^{1/2}. \tag{27}$$

The linear dependence of the quantum correction on $1/\sqrt{C_0}$ is similar to that for bulk systems. Contrary to the situation in bulk systems, the result does not depend on the frequency $\omega$ for reasons we have already discussed.

In the case of a slowly varying field, $y \ll 1$, but still $zy^2 \gg 1$ (strong field) we obtain

$$\frac{\Delta g_{wl}}{\Delta g_{wl}^{(0)}} = \frac{\Gamma(1/6)}{\pi \Gamma(5/6)} \left( \frac{2\delta_1^2 g_*^3}{9 C_0 \omega^2} \right)^{1/3}, \tag{28}$$

i.e., the dependences on both the amplitude and the frequency are different from the bulk case.

In conclusion, we propose a random matrix theory describing the influence of a time-dependent external field on the average magnetoresistance of open quantum dots. This dependence can be recast in the form of the universal function Eq. (25) of one fitting parameter Eq. (22) which can be fixed by an independent experiment. The results cannot be described by a simple replacement $g_* \to g_* + \gamma_\phi$. Finally, we note that thermal fluctuations of the gate potentials may induce dephasing by virtue of the mechanism considered here. However, the spectral density of such fluctuations is model dependent and thus not universal.

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