RAPID COMMUNICATIONS

Magnetic-field dependence of dynamical vortex response in two-dimensional Josephson junction arrays and superconducting films

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The dynamical vortex response of a two-dimensional array of the resistively shunted Josephson junctions in a perpendicular magnetic field is inferred from simulations. It is found that, as the magnetic field is increased at a fixed temperature, the response crosses over from normal to anomalous, and that this crossover can be characterized by a single dimensionless parameter. It is described how this crossover should be reflected in measurements of the complex impedance for Josephson junction arrays and superconducting films.
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Two-dimensional (2D) vortex physics is strongly reflected in the properties of Josephson junction arrays (JJA) and superconducting films, as well as high-T_c superconductors.[1,2] In the absence of a magnetic field, the phase transition is of the Kosterlitz-Thouless (KT) type and is driven by the unbinding of thermally created vortex-antivortex pairs. In the presence of a perpendicular magnetic field, ordered flux lattices are formed which melt at high enough temperatures. Although these thermodynamic aspects are well known,[1,2] the dynamical response properties, which contain key information of the vortex physics and can be extracted from measurements of the complex impedance and the flux noise,[1–3] are much less well understood.[1,2] In the present paper we focus on the temperature region above the 2D flux lattice melting and infer that the dynamical response as a function of the perpendicular magnetic field has intriguing characteristics.

The features of vortex dynamics are contained in the complex impedance $Z(\omega)$ or equivalently the conductivity $\sigma(\omega)=1/Z(\omega)$, which can be expressed as $\sigma(\omega)=1/R_N - 1/i\omega L_k\epsilon(\omega)$ with the normal-state resistance R_N, the kinetic inductance L_k, and the vortex dielectric function $1/\epsilon(\omega)$ describing the effect of the vortices on the dynamics.[1,4,5] In the present work we obtain $1/\epsilon(\omega)$ and the resistance $R=\mathrm{Re}[\sigma^{-1}(\omega=0)]$ from simulations and characterize the vortex dynamics in terms of these quantities.

We use the 2D resistively shunted junction (RSJ) model on an $L\times L$ triangular lattice with periodic boundary conditions (see Fig. 1). The equations of motion in the presence of an external magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ follow from the local current conservation

$$\dot{\theta}_i=-\sum_j G_{ij}\sum_k{}' \,[\sin(\theta_j-\theta_k-A_{jk})+\eta_{jk}], \tag{1}$$

where θ_i is the phase of the complex order parameter at site i, G_{ij} is the lattice Green's function, and the primed summation is over six nearest neighbors of j. The thermal noise current η_{jk} in units of the single junction critical current I_c satisfies $\langle\eta_{ij}(t)\rangle=0$ and $\langle\eta_{ij}(t)\eta_{kl}(0)\rangle=2T(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk})\delta(t)$, where $\langle\cdots\rangle$ is the ensemble average and the temperature T is in units of the Josephson coupling strength $J\equiv\hbar I_c/2e$. The magnetic bond angle $A_{ij}\equiv(2\pi/\Phi_0)\int_i^j\mathbf{A}\cdot d\mathbf{l}$ with the flux quantum Φ_0 and the magnetic vector potential $\mathbf{A}=Bx\hat{\mathbf{y}}$ satisfies the plaquette sum $\Sigma_p A_{ij}=2\pi f$ with the frustration f, corresponding to the number of flux quanta per elementary triangle. The time t is in units of $\hbar/2eR_NI_c$ with the shunt resistance R_N, and in the following we simplify the unit system to $R_N=I_c=\hbar/2e=1$. We use a second-order algorithm for numerical integration with the time step $\Delta t=0.05$ and the system size $L=64$. The key quantity is the time correlation function $G(t)$, which is related with the dielectric function $1/\epsilon(\omega)$ by

$$\frac{1}{i\omega}\left[\frac{1}{\epsilon(\omega)}-\frac{1}{\epsilon(0)}\right]=-\int_0^\infty dt e^{i\omega t}G(t), \tag{2}$$

and is determined from $G(t)=(2\rho_0T/\sqrt{3}L^2)\langle F(t)F(0)\rangle$,[6–8] where $\rho_0=\sqrt{3}\langle\cos(\theta_i-\theta_j-A_{ij})\rangle$ is the superfluid density,[8,9] and $F(t)=\Sigma_{\langle ij\rangle}\sin(\theta_i-\theta_j-A_{ij})\mathbf{e}_{ij}\cdot\hat{\mathbf{x}}$, with the sum over all links and $\mathbf{e}_{ij}$ being the unit vector from i to j. In our unit system ρ_0 is the same as the kinetic inductance $1/L_k$ so that the relation between $\sigma(\omega)$ and $1/\epsilon(\omega)$ reduces to

$$\sigma(\omega)=1-\frac{\rho_0}{i\omega\epsilon(\omega)}. \tag{3}$$

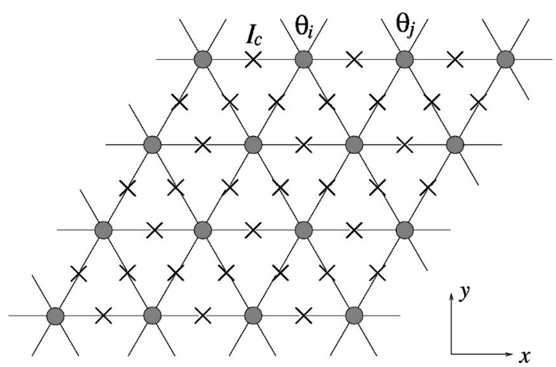

FIG. 1. Triangular array of Josephson junctions. The superconducting island at site i is associated with an order parameter phase angle θ_i and the Josephson junction between adjacent islands with a critical current I_c.

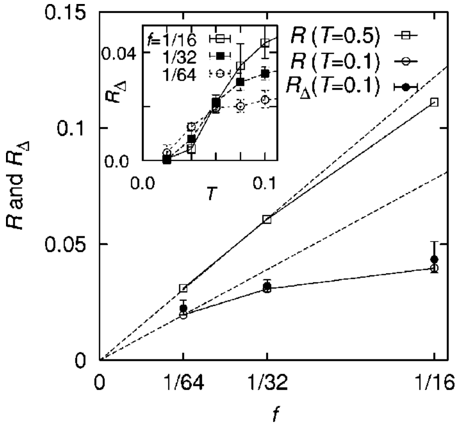

FIG. 2. Resistance R as a function of frustration f at $T=0.5$ (squares) and 0.1 (circles). The empty symbols represent R obtained as described in text, and the filled circles as obtained by the method described in Ref. 5 (denoted by R_Δ). The dashed lines through the origin correspond to expectation from free vortex diffusion. The full drawn lines are guides to the eyes. The inset shows that the onset of resistance starts well below $T=0.1$.

Figure 2 shows the resistance R as a function of frustration f at $T=0.1$ and 0.5 obtained from $G(t)$ using Eqs. (2) and (3) together with $R=\mathrm{Re}[1/\sigma(\omega=0)]$. In order to check the present method we have also calculated the resistance in a completely independent way using the fluctuating twist boundary conditions described in Ref. 5. As seen in Fig. 2 both determinations give consistent results but the present method appears to converge much better. The inset in Fig. 2 shows R versus T and verifies that $T=0.1$ and 0.5 are both well above the resistive transition temperature. The naive expectation is that R is proportional to the density of free vortices which in turn is proportional to f.[1] This corresponds to a situation where all vortices effectively move diffusively as independent particles. As seen in Fig. 2 this expectation is borne out for $T=0.5$ for the two lowest frustrations $f=1/64$ and 1/32 (these data points fall on a straight line through the origin). However, the highest frustration $f=1/16$ falls slightly below this line indicating a suppression of the free vortex diffusion. This effect is much more pronounced for the lower temperature $T=0.1$, where both resistances for $f=1/16$ and $f=1/32$ fall well below the straight line through the origin. Since the vortex pinning caused by a triangular array is very weak,[10] the essential part of the suppression is caused by some vortex correlation mechanism and not by a lattice pinning effect.

Figure 3 shows the real and imaginary part of $1/\epsilon(\omega)$ obtained from $G(t)$ by aid of Eq. (2). One immediately notes a qualitative difference; in Figs. 3(a) and (b) the maximum of $|\mathrm{Im}[1/\epsilon(\omega)]$ is very close to the crossing point with $\mathrm{Re}[1/\epsilon(\omega)]$, whereas in Fig. 3(c) this maximum is displaced to the right. In order to analyze these differences in a systematic way we parametrize the curves by an interpolation between two response forms. The first is the Drude form which corresponds to vortices effectively diffusing as independent particles. This form is hence expected to apply to the region where the resistance is linear in f (as for $f=1/64$ and 1/32 at $T=0.5$ in Fig. 2). The time correlation function in this case is of the exponential form $G^{\mathrm{Drude}}(t)=(2/\tilde{\epsilon}\pi)e^{-t\sigma_0}$ and the corresponding response function is given by

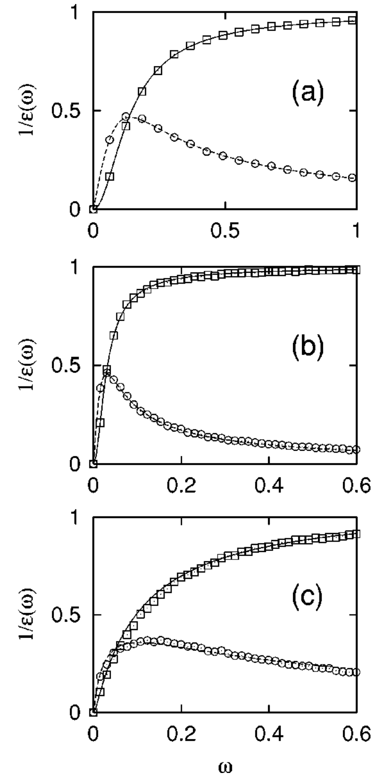

FIG. 3. The response function $1/\epsilon(\omega)$ for (a) $T=0.5$ and $f=1/16$, (b) $T=0.1$ and $f=1/64$, and (c) $T=0.1$ and $f=1/16$. The full and dashed lines are the fits to the functional form described in text. For each pair (T,f) this essentially gives a one-parameter characterization.

$$\mathrm{Re}\left[\frac{1}{\epsilon(\omega)}\right]=\frac{1}{\tilde{\epsilon}}\frac{\omega^2}{\omega^2+\sigma_0^2},$$

where $\tilde{\epsilon}$ describes an effective static polarization of the vortex interaction due to vortex-antivortex pairs,[1] and $\mathrm{Re}[1/\epsilon(\omega)]\propto\omega^2$ for small ω. The second response form describes the dynamical response of *bound* vortex-antivortex pairs whose phenomenological form has been proposed in Ref. 1 [Minnhagen phenomenology (MP)] and is given by Ref. 5: $G^{\mathrm{MP}}(t)=(2/\tilde{\epsilon}\pi)[\mathrm{Ci}(\omega_0 t)\sin\omega_0 t-\mathrm{si}(\omega_0 t)\cos\omega_0 t]$, where $\mathrm{Ci}(x)\equiv-\int_x^\infty dt\cos t/t$, $\mathrm{si}(x)\equiv-\int_x^\infty dt\sin t/t$, and $G^{\mathrm{MP}}\propto 1/t$ for large times. The corresponding response function is given by

$$\mathrm{Re}\left[\frac{1}{\epsilon(\omega)}-\frac{1}{\epsilon(0)}\right]=\frac{1}{\tilde{\epsilon}}\frac{\omega}{\omega+\omega_0},$$

which goes to zero linearly with ω and gives a vanishing resistance. It is sometimes referred to as an anomalous vortex dynamics since it exhibits a nonanalytic divergence of the conductivity for small ω: $\sigma(\omega)\propto-\ln\omega$.[11] This form has been shown, from simulations of various models of 2D vortex physics in the absence of magnetic field, to describe the response of the low-temperature phase,[5,7,12] where the dynamics is dominated by bound vortex-antivortex pairs.[1]

In general both bound pairs and free vortices can exist at the same time, e.g., for $f=0$ above the KT transition and for a finite f above the resistive transition, and the dynamical response is expected to contain contributions from both of

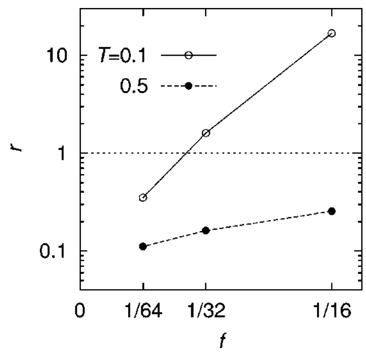

FIG. 4. The parameter r characterizing the response as a function of f. At $T=0.5$, r increases slightly with increasing f but remains much smaller than 1, characterizing a Drude-like response. At $T=0.1$, r changes from $r \ll 1$ to $r \gg 1$ with increasing f, signifying a crossover from Drude-like to MP-like response.

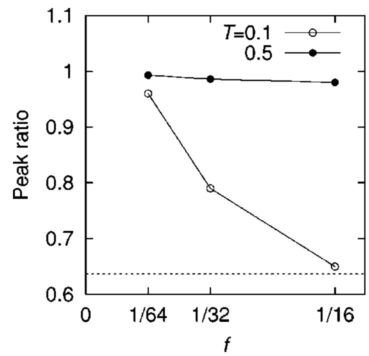

FIG. 5. Peak ratio as a function of frustration f. At $T=0.5$ the peak ratio decreases slightly with increasing f but remains close to the Drude value 1. At $T=0.1$ the peak ratio crosses over from 1 to the MP value $2/\pi$.

the response types.[5,7] In the present work we use the interpolation introduced in Ref. 5: $G^{\mathrm{int}}(t)=\tilde{\epsilon}G^{\mathrm{Drude}}(t)G^{\mathrm{MP}}(t)$, which contains three parameters, $\tilde{\epsilon}$, σ_0, and ω_0, or equivalently $\tilde{\epsilon}$, $r=\omega_0/\sigma_0$, and R. Here r is the important parameter which changes the response from pure Drude for $r=0$ to pure MP for $r=\infty$, and R satisfies $\sigma_0=R[\rho_0(\pi+r\ln r)]/(1-R)[\pi\tilde{\epsilon}(1+r^2)]$ from G^{int} together with Eqs. (2) and (3). Since R can be independently determined without curve fitting as in Fig. 2, we are left with only two fitting parameters $\tilde{\epsilon}$ and r. Furthermore, for a finite f above the resistive transition there are few bound vortex-antivortex pairs present in the static limit so that $\tilde{\epsilon}\approx 1$,[1] and the dynamical response is essentially characterized by the single parameter r. In the following analysis we use R in Fig. 2 and fit data to the dynamical response function $1/\epsilon(\omega)$ with two free parameters $\tilde{\epsilon}$ and r. Figure 3 shows three typical examples; the fits are very good and the obtained values $\tilde{\epsilon}$ are in all cases close to 1[$\tilde{\epsilon}\approx 1.0, 1.0$, and 0.93 for Figs. 3(a), (b), and (c), respectively].

Figure 4 shows the characterization of the dynamical response in terms of the parameter r. The response for $T=0.5$ corresponds to a small r and hence Drude-like behavior. This is consistent with independent diffusion of *free* vortices and with the linear relation between R and f found in Fig. 2. However, as seen in Fig. 2, r increases somewhat with f, which suggests an increase of ‘‘vortex-pair-like’’ correlations. This seems consistent with the small suppression of R with increasing f for $T=0.5$ in Fig. 2. The same feature is much more dramatic for $T=0.1$: r increases from a small Drude-like value to a large MP-like value with f, which suggests that the response for the largest f is dominated by ‘‘vortex-pair-like’’ correlations, and is consistent with the large suppression of R at large f for $T=0.1$ in Fig. 2.

An alternative way to analyze the crossover from Drude to MP response is to focus on the peak ratio, defined as the ratio $|\mathrm{Re}[1/\epsilon(\omega)]/\mathrm{Im}[1/\epsilon(\omega)]|$ at the maximum of $|\mathrm{Im}[1/\epsilon(\omega)]|$. For the pure Drude response this ratio is unity whereas for the pure MP response it is $2/\pi\approx 0.64$ [see Fig. 3 where (a) and (b) have peak ratios close to the Drude value whereas the peak ratio in (c) is close to the MP value]. Figure 5 displays the peak ratios obtained directly from the data (by obtaining a sufficient number of data points close to the maximum) and shows that the crossover from Drude-like to MP-like behavior at $T=0.1$ with increasing f is clearly signaled as a decrease in the peak ratio.

Our simulations show that in a certain parameter range the dynamical response as a function of frustration crosses over from a free-particle-like response to a response which has the same form as the response of bound vortex-antivortex pairs and that this crossover is linked to a suppression of the resistance. In Ref. 8 the same type of crossover from Drude to MP response with increasing frustration was found for a 2D XY model with time-dependent Ginzburg-Landau (TDGL) dynamics. In Ref. 11 it was found that a triangular JJA with a small finite frustration shows a MP-like response well below the zero-frustration KT transition and in Ref. 7 it was suggested, based on the peak ratio, that the same is true for the superconducting film measured in Ref. 13.

Several theoretical attempts have been made to understand the origin of the anomalous dynamics:[14,15] Ref. 14 ascribes the effect to a coupling between spin waves and vortices, suggesting that there should be anomalous dynamics for TDGL but not for RSJ, in contrast to what has been demonstrated in the present work. Reference 15 attributes the effect to vortex correlations which should apply to both the TDGL and RSJ. However, none of the theoretical formulations yields a crossover to the MP form and a shift of the peak ratio from 1 to $2/\pi$ with *increasing magnetic field* and hence with an *increasing resistance*. Thus an adequate theoretical explanation is still largely lacking. In Ref. 8 it was found from simulations that an increase of the magnetic field also causes an increased density of antivortices suggesting that vortex-antivortex correlations may play a role with a possible connection to the fact that the MP response also describes the response of the low-temperature phase for $f=0$.

Our analyzing method makes it possible to assess to what extent the crossover from normal to anomalous response described here shows up in measurements on real systems.

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