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# Experimental observation of the de Haas–van Alphen effect in a multiband quantum-well sample

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We report measurements of magnetic quantum oscillations (the de Haas–van Alphen effect) in a quantum well containing more than one subband. The Fourier transform of the magnetization oscillations shows the expected frequencies proportional to the occupancies of each subband, but additional frequency components, in accordance with theoretical predictions of Alexandrov and Bratkovsky, are also detected. These components relate to the sums and differences of the individual subband frequencies, and are a consequence of coupling of the individual subband occupancies due to oscillation of the electrochemical potential with magnetic field in a canonical Fermi system. This is an experimental verification of the Alexandrov and Bratkovsky theory in a multisubband system. [S0163-1829(99)51340-4]

## I. INTRODUCTION

The de Haas–van Alphen[1] (dHvA) effect, or oscillatory magnetization as a function of magnetic field, is well established as an informative and detailed probe of the Fermi surface properties of three-dimensional systems. These oscillations are a consequence of Landau quantization of the fermion energy levels in a magnetic field. In recent years, there has been increased interest in its application to two-dimensional systems (2DS) formed in semiconductor heterojunctions and quantum wells, because it can yield direct information about the nature of the electronic density of states in a magnetic field. Quantum oscillations of the magnetoresistance, the Shubnikov–de Haas (SdH) effect, although much more easily measured and of similar physical origin, are more complicated to analyze since they depend on the details of electron scattering and in general distinguish between localized and extended states. To date however, the published experimental work[2] on the dHvA effect in 2DS has concentrated on samples where there is only one occupied sub-band within a quantum well, or at a semiconductor heterojunction.

Recent theoretical work by Alexandrov and Bratkovsky[3] (AB) on the dHvA effect in samples with more than one occupied subband suggests that there should be qualitative differences in the frequency components of the magnetization oscillations in a 2DS compared with the 3D case. This is because in a 2DS with fixed fermion density (canonical ensemble, CE) oscillations with field occur in the electrochemical potential $\mu$ or ''Fermi energy,'' which in the ideal case is pinned to the uppermost fractionally occupied Landau level (LL). In a 3D system by contrast, even with fixed fermion density, $\mu$ is essentially constant because the overall density of states is comparatively weakly perturbed by Landau quantization of the transverse-field motion. Bratkovsky[4] has further argued, if a 2DS is electrically contacted (as for an SdH measurement) electrons can pass to or from the 2DS, and make fixed $\mu$ (grand canonical ensemble, GCE) the appropriate constraint.

In this paper, we present an experimental study of the dHvA effect in an uncontacted 2DS with multiple subbands, and show results consistent with the AB predictions. We also compare these results with conventional SdH measurements in the same material.

## II. EXPERIMENT

The experiments were performed using a high-sensitivity torque magnetometer system that has been described previously.[5] Its basic form is a moveable electrode (or rotor) with a sample attached, suspended by a fine phosphor-bronze fiber in front of two fixed plates (or stators). The sample normal is set at a small angle, typically 20°, to the applied magnetic field direction. Thus, interaction between the applied field and magnetic moment of the sample produces a torque and thereby slight rotation of the rotor with respect to the stators. This rotation is detected capacitatively using an ac bridge, producing an output proportional to the torque on the sample. Magnetic moments of less than $10^{-12}$ Am$^2$ can be detected.

Measurements are presented here on an $In_xGa_{1-x}As$ single quantum well structure, grown by molecular beam epitaxy, lattice matched to InP. The sample consists of a 10-nm undoped $In_{0.53}Ga_{0.47}As$ cap, 20-nm undoped $In_{0.52}Al_{0.48}As$, a 5-nm $In_{0.52}Al_{0.48}As$ spacer, 50-nm undoped $In_{0.53}Ga_{0.47}As$, a 400-nm undoped $In_{0.52}Al_{0.48}As$ buffer on an Fe-doped InP substrate. An *n*-type Si delta layer is grown between the two $In_xAl_{1-x}As$ layers with a doping concentration of $6\times10^{12}$ cm$^{-2}$. Modelling of the layer structure using a self-consistent Poisson-Schrödinger solver (Fig. 1) indicates three occupied subbands, having energies of 141.4 V, 33.6 and 11.7 meV below the Fermi level.

A Hall bar was processed from the wafer for conventional magnetoresistance measurements.

 

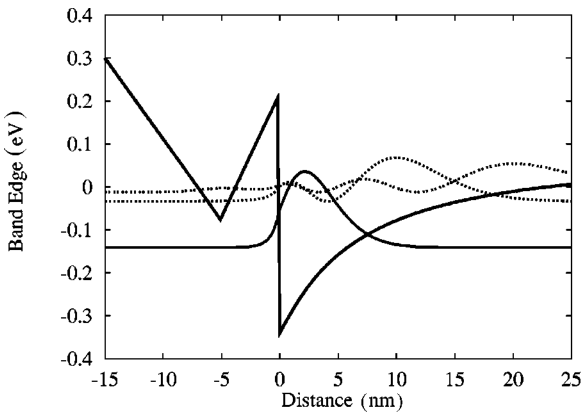


FIG. 1. Calculated conduction band profile for a 50-nm $In_{0.53}Ga_{0.47}As$ quantum well grown in $In_{0.52}Al_{0.48}As$ on InP. Carriers are provided by Si delta doping 5 nm from the well with density $6\times10^{12}$ $cm^{-2}$. The zero of energy is chosen at the Fermi level. Three subbands are occupied with energies $-141.5$, $-33.6$, and $-11.7$ meV. Wave functions are superimposed on the diagram, offset vertically for clarity by their corresponding subband energies.

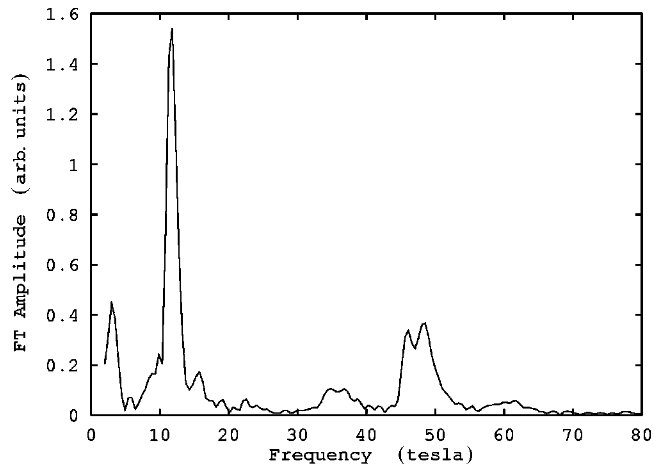


FIG. 3. The Fourier transform of the SdH data indicates three subbands with frequencies of 3.1, 11.8, and 48.4 tesla. These frequencies are converted to number densities by multiplying by 2e/h.

## III. RESULTS

Figure 2 shows SdH oscillations (after removal of a slowly varying background) taken at a temperature of 4.2 K and the fast Fourier transform (FT) in Fig. 3 demonstrates the presence of these three subbands with carrier densities of $2.3\times10^{16}$, $5.7\times10^{15}$, and $1.5\times10^{15}$ $m^{-2}$, respectively, in good agreement (within 10 percent) with the above modelling. The other peaks more weakly visible in the FT are discussed shortly. All the dHvA data presented here were taken at the system base temperature of 1.26 K, to maximize the strength of the oscillations, although data were taken up to 2 K. The measurements were made on an electrically isolated sample of about 1-cm square cut from the wafer and mounted on the magnetometer rotor using vacuum grease. A typical trace from our magnetometer system consists of two components—a paramagnetic background due to impurities in the rotor and sample substrate, and the oscillatory dHvA component from the electron gas. The background is removed by fitting a polynomial function to the data and then subtracting this function numerically. The data are then Fourier transformed to obtain the frequency spectrum. Figure 4 shows the magnetization data from the sample after removal of the background component, and its corresponding Fourier transform is shown in Fig. 5. The frequency spectrum shows three peaks corresponding to the three occupied subbands with peaks at $f_0=49.6$, $f_1=12.2$, and $f_2=3.3$ T corresponding to carrier densities of $2.40\times10^{16}$, $5.90\times10^{15}$ and $1.6\times10^{15}$ $m^{-2}$, respectively. These agree well with the SdH values and indicate that the wafer is fairly uniform. Significantly, however, there are several additional frequency components, for example around 15.7, 38.4, and 63.2 T.

## IV. DISCUSSION

To understand this observation of additional frequency components it is first useful to summarize the AB theoretical predictions for a 2DS. They consider the case of two or three subbands, and include collision broadening of the ideal (delta function) LL density of states to a Lorentzian lineshape. If the electron density is constant, and the broadening of the LLs is negligible compared with their energy separation, then $\mu$ be pinned to the uppermost occupied Landau level and therefore oscillates with the strength of the applied field.

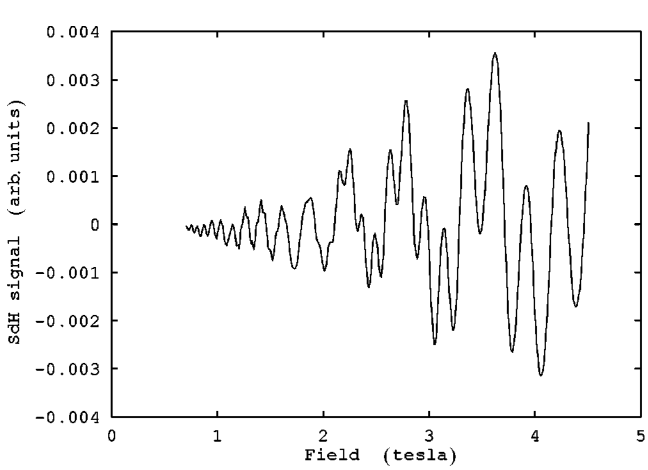


FIG. 2. SdH data taken at 4.2 K on a Hall bar fabricated by photolithography.

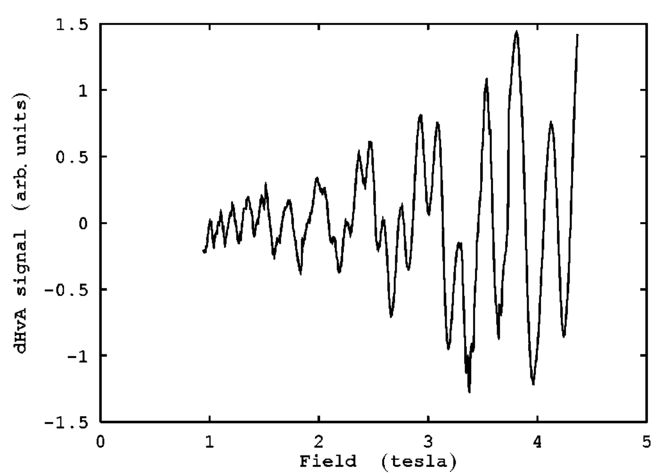


FIG. 4. dHvA data taken at 1.26 K on a 1-cm square piece of wafer. The Fourier transform of the data is shown in Fig. 5.

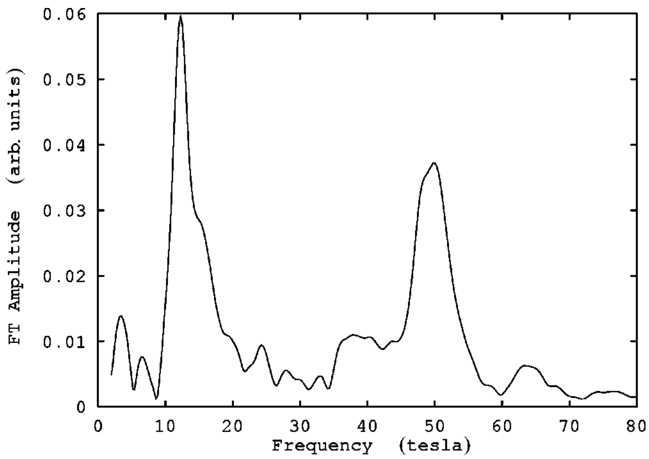


FIG. 5. The Fourier transform of the data of Fig. 4. This indicates that three subbands are occupied with frequencies of 3.3, 12.2 and 49.6 tesla.

Theoretical magnetization curves at zero temperature show Fourier spectra with peaks in addition to the individual subband frequencies and their harmonics. These additional peaks occur at frequencies corresponding to the sums and differences of subband frequencies (and their harmonics). It is easy to understand the origin of the additional frequency components: when more than one subband is occupied, oscillations of $\mu$ depend on the highest LL of all the subbands. This effectively produces a nonlinear coupling of the subbands, leading to distortion of the dHvA waveform. (Although the magnetic field has a component parallel to the 2D subbands in the dHvA measurements, the small diamagnetic shift produced is small compared with both the subband energies and the cyclotron energy, and would not produce an oscillatory coupling between the subbands.)

For fixed $\mu$ in contrast, the number density in the subbands, as well as the total number density, oscillates with magnetic field. In this instance, AB find that the additional structures in the Fourier spectra are either absent or strongly suppressed compared with constant number density. If $\mu$ is fixed, the individual subbands should each provide just their own frequency, independent of the other subbands. Our observations of additional structure in the FT spectra from contactless dHvA measurements thus represent experimental verification of the predictions[3] by AB for the canonical ensemble (Table I).

It is of interest to compare these results with conventional SdH measurements. Even here, the Fourier spectrum of the SdH data (Fig. 2) shows frequency components (the temperature is higher, which may account for the weakness of these components in SdH) that can be assigned to combinations of the subband harmonics (Table II). Suppression of the mixing frequencies might be expected[4] in terms of the AB predictions,[3] if attaching contacts and connecting wires to the sample should provides a sufficient charge reservoir, making the 2D system open (GCE) and maintaining $\mu$ fixed in the 2D electron layer. However, there are reasons why this is not necessarily the case. Firstly, merely contacting the sample may not in itself maintain fixed $\mu$, because the electrostatics of the band bending need to be taken into account. Gating the sample would be more physically relevant. Secondly, it may be that in a real sample, the distinction between the CE and GCE is not always a sharp one. It has been suggested[6] that even when $\mu$ can be taken as constant across the sample, it may nevertheless depend on field, measured with respect to the subband energies. This is because the 2D density of states oscillates with field. Changes in the number of electrons in the 2DS will cause a degree of band bending in the vicinity of the 2D layer, which oscillates with field. Although a small effect (especially at the high 2D densities and relatively low fields appropriate to our samples), it could in principle give rise to harmonic distortion. It is also possible[6] that $\mu$ is not a constant across the sample at low temperatures as the magnetic field is varied if the 2D electron gas is not generally in thermodynamic equilibrium with the spatially remote donor region. In the extreme case, the electron gas could be effectively isolated from the rest of the sample as in the canonical ensemble. A range of samples with differing spacer widths could help test this hypothesis. An extension of the AB model to include a self-consistent calculation (via Poisson's and Schrödinger's equations) of the band profile of the semiconductor structure would be valuable in quantifying this effect.

Another explanation of additional frequencies in SdH to consider is an oscillatory intersubband scattering rate,[7] which could lead to an additional nonlinear coupling between the SdH contributions from the occupied subbands. We are planning to extend measurements to lower temperatures, which should eliminate acoustic phonon scattering, to investigate this further. Against this, however, measurements of dHvA at 2.0 K showed no significant change in harmonic content. Experiments by Coleridge[8] also call this interpretation into doubt, and are better explained by an oscillatory Fermi level.

TABLE I. dHvA frequencies and their interpretation.

| Frequency/tesla | Assignment | Comment |
|---|---|---|
| 3.3 | $f_2$ | Strong |
| 6.5 | $2f_2$ | |
| 12.2 | $f_1$ | Strong |
| 15.7 | $f_1+f_2$ | Shoulder |
| 19.6 | $f_1+2f_2$ | Weak shoulder |
| 24.3 | $2f_1$ | Strong |
| 28.0 | $2f_1+f_2$ | Weak |
| 33.0 | $2f_1+2f_2$ | Weak |
| 38.4 | $f_0-f_1$ | Broad peak |
| 49.6 | $f_0$ | Double peak? |
| 63.2 | $f_0+f_1$ | |
| 75.4 | $f_0+2f_1$ | Weak, broad peak |

TABLE II. SdH frequencies and their interpretation.

| Frequency/tesla | Assignment | Comment |
|---|---|---|
| 3.3 | $f_2$ | Strong |
| 11.6 | $f_1$ | Strong |
| 15.6 | $f_1+f_2$ | |
| 35.9 | $f_0-f_1$ | Broad peak |
| 47 | $f_0$ | Double peak |
| 60.7 | $f_0+f_1$ | Weak, broad peak |

A desirable experimental test would be to make a direct comparison between dHvA measurements in a sample with and without connecting wires. Although this was attempted, it has not been possible to do so without disturbing the operation of our magnetometer. Interestingly, dHvA on a sample on which small ohmic contacts had been made at the corners (but without connecting wires) did show some reduction of one $(f_0+f_1)$ of the combination frequencies. We are currently pursuing both these lines of research.

In summary, we have presented an experimental study of the de Haas–van Alphen effect in a multisubband 2D electron system. Our results are consistent with the theory of Alexandrov and Bratkovsky which predicts the presence of additional components in the dHvA frequency spectrum, corresponding to sums and differences of the individual subband frequencies when the Fermi level $\mu$ oscillates with field. Similar effects observed (albeit weakly) in conventional SdH oscillations on the same material may be produced for the same reason, but are less amenable to quantitative theoretical analysis. Detailed theoretical modeling of our results is desirable. Experimental work on contacted samples is being pursued and may provide a further test of their theory.

## ACKNOWLEDGMENTS

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