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Two-dimensional electron-hole systems in a strong magnetic field: Composite fermion picture for multicomponent plasmas

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Electron-hole systems on a Haldane sphere are studied by exact numerical diagonalization. Low-lying states contain one or more types of bound charged excitonic complexes X_k^-, interacting through appropriate pseudopotentials. Incompressible ground states of such multicomponent plasmas are found. A generalized multicomponent Laughlin wave function and composite fermion picture are shown to predict the low-lying states of an electron-hole gas at any value of the magnetic field. [S0163-1829(99)51540-3]

I. INTRODUCTION

Recently there has been considerable interest in two-dimensional systems containing both electrons and holes in the presence of a strong magnetic field.[1–8] In such systems, neutral (X^0) and charged excitons (X^-) and larger exciton complexes (X_k^-, k neutral X^0's bound to an electron) can occur. The excitonic ions X_k^- are long-lived fermions,[6,7] whose energy spectra contain Landau-level structure.[4,7] In this paper we investigate by exact numerical diagonalization small systems containing N_e electrons and N_h holes ($N_e \geqslant N_h$), confined to the surface of a Haldane sphere.[9] For $N_h=1$ these systems serve as simple guides to understanding photoluminescence.[1–5] For $N_h>1$ it is possible to form a multi-component plasma containing electrons and X_k^- ions.[7] We propose a model[10] for determining the incompressible quantum fluid states[11] of such plasmas and confirm its validity by numerical calculations. In addition, we introduce a generalized composite fermion (CF) picture[12] for the multi-component plasma[13] and use it to predict the low-lying bands of angular momentum multiplets for any value of the magnetic field.

II. BOUND STATES

In a sufficiently strong magnetic field, the only bound electron-hole complexes are the neutral exciton X^0 and the spin-polarized charged excitonic ions X_k^- (electron $e^- \equiv X_0^-$, charged exciton $X^- \equiv X_1^-$, charged biexciton X_2^-, etc.).[6,7] All other complexes found at weaker magnetic fields (e.g., spin-singlet charged exciton[1] or spin-singlet biexciton) unbind.[8] The angular momenta of complexes X^0 and X_k^- on a Haldane sphere[9] with monopole strength $2S$ are $l_{X^0}=0$ and $l_{X_k^-}=|S|-k$.[7] The binding energies of an exciton $\varepsilon_0=-E_{X^0}$ and of excitonic ions $\varepsilon_k=E_{X_{k-1}^-}+E_{X^0}-E_{X_k^-}$ (E_A is the energy of complex A) are listed in Table I for several different values of $2S$. It is apparent that $\varepsilon_0>\varepsilon_1>\varepsilon_2>\varepsilon_3$. Depending on the ratio $N_e:N_h$, we expect to find different combinations of complexes that have the largest total binding energy. When $N_e=N_h$, we expect N_h neutral excitons X^0 to form. When $N_e \geqslant 2N_h$, the low-lying states will contain N_h charged excitons X^- and N_e-2N_h free electrons e^-. For $N_h<N_e<2N_h$ we expect to find larger charged exciton complexes.

III. PSEUDOPOTENTIALS

Whether the states with largest binding energy form the lowest energy band of the electron-hole system depends on the interaction between charged complexes X_k^-. The interaction of a pair of charged particles A and B of angular momentum l_A and l_B can be described by a pseudopotential $V_{AB}(L)$ where $\hat{L}=\hat{l}_A+\hat{l}_B$ is the total pair angular momentum.[14] It is convenient to plot pseudopotentials as a function of the relative angular momentum $\mathcal{R}=l_A+l_B-L$.[15] Figure 1 shows $V_{AB}(\mathcal{R})$ for the pairs e^-e^-, e^-X^-, X^-X^-, and $e^-X_2^-$ at the monopole strength $2S=17$. Roughly, the pseudopotential parameters $V_{AB}(\mathcal{R})$ calculated for different pairs AB and for a given $2S$ lie on the same curve. Small differences between energies V_{AB} calculated for different pairs at the same $\mathcal{R}$ are due to different values of l_A and l_B

TABLE I. Binding energies ε_0, ε_1, ε_2, and ε_3 of X^0, X^-, X_2^-, and X_3^-, respectively, in the units of e^2/λ.

$2S$	ε_0	ε_1	ε_2	ε_3
10	1.329 504 3	0.072 835 7	0.041 106 9	0.025 226 8
15	1.304 567 9	0.067 710 8	0.039 528 2	0.026 292 7
20	1.291 931 3	0.064 788 6	0.038 132 4	0.026 032 8

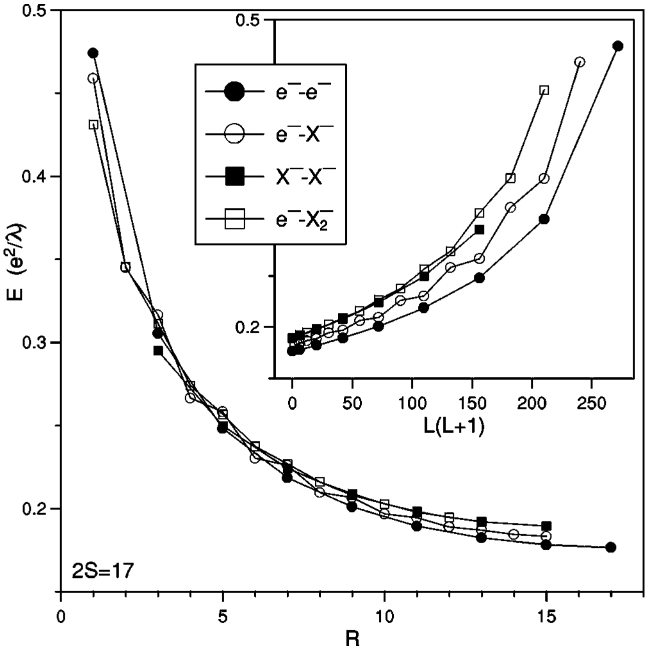

FIG. 1. Pseudopotentials $V_{e^-e^-}$ (filled circles), $V_{e^-X^-}$ (open circles), $V_{X^-X^-}$ (filled squares), and $V_{e^-X_2^-}$ (open squares) on a Haldane sphere with $2S=17$, as a function of $\mathcal{R}$ (main frame) and $L(L+1)$ (inset).

and to the finite size and polarization of composite particles. Only the latter effect, important at small $\mathcal{R}$, persists for $2S\to\infty$, i.e., in the planar geometry.

The major and critical difference between four plotted pseudopotentials lies in the allowed values of $\mathcal{R}$. If all A and B were point charges, the allowed pair angular momenta for two identical fermions ($A=B$) would be $L=2l_A-j$, where j is an odd integer, i.e., $\mathcal{R}=1,3,\ldots$ and $\mathcal{R}\leqslant 2l_A$. For two distinguishable particles ($A\neq B$), the values of L would satisfy $|l_A-l_B|\leqslant L\leqslant l_A+l_B$, i.e., $\mathcal{R}=0,1,2,\ldots$ and $\mathcal{R}\leqslant 2\min(l_A,l_B)$. However, if A or B is a composite particle, one or more pair states with largest L (smallest $\mathcal{R}$) are forbidden, and the corresponding pseudopotential parameters are effectively infinite (AB repulsion has a hard core). For $A=X^-_{k_A}$ and $B=X^-_{k_B}$, the smallest allowed $\mathcal{R}$ can be deduced from the mapping[8] between the electron-hole and two-spin systems,

$$\mathcal{R}_{AB}^{\min}=2\min(k_A,k_B)+1. \tag{1}$$

Thus, in Fig. 1, $\mathcal{R}_{e^-X^-}\geqslant 1$, $\mathcal{R}_{X^-X^-}\geqslant 3$, etc.

Low-lying states of an N_e electron and N_h hole system can contain a number of charged complexes X_k^- (X^- and possibly larger ones) interacting with one another and with electrons through appropriate pseudopotentials. It has been shown[15] that the Laughlin $\nu=1/m$ state occurs in the gas of (identical) fermions if the pseudopotential increases faster than linearly as a function of $L(L+1)$ in the vicinity of $\mathcal{R}=m$. As seen in the inset in Fig. 1, this is true for both $V_{e^-e^-}$ and $V_{X^-X^-}$, and also (at even $\mathcal{R}$) for $V_{e^-X^-}$ and $V_{e^-X_2^-}$. In Ref. 7 we found Laughlin states of one-component X^- gas formed at $N_e=2N_h$. In the present paper we concentrate on a more general situation, where more than one kind of charged particles occur in an electron-hole system, and find incompressible fluid states of such multi-component plasma.

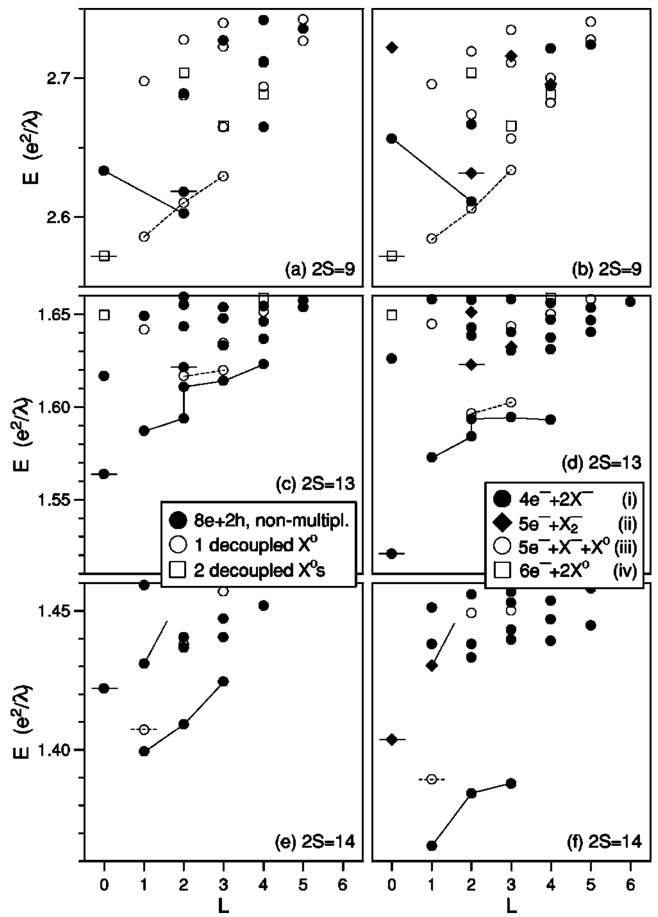

FIG. 2. Left: low energy spectra of the $8e+2h$ system on a Haldane sphere at $2S=9$ (a), $2S=13$ (c), and $2S=14$ (e). Right: approximate spectra calculated for all possible groupings containing excitons (charged composite particles interacting through pseudopotentials as in Fig. 1). Lines connect corresponding states in left and right frames.

IV. NUMERICAL RESULTS

As an illustration, we present first the results of exact diagonalization performed for the system with $N_e=8$ and $N_h=2$. We expect low-lying bands of states containing the following combinations of complexes: (i) $4e^-+2X^-$, (ii) $5e^-+X_2^-$, (iii) $5e^-+X^-+X^0$, and (iv) $6e^-+2X^0$. All groupings (i)–(iv) contain an equal number $N=N_e-N_h$ of singly charged complexes; however, both the angular momenta of involved complexes and the relevant hard cores are different. The total binding energies are $\varepsilon_{\rm i}=2\varepsilon_0+2\varepsilon_1$, $\varepsilon_{\rm ii}=2\varepsilon_0+\varepsilon_1+\varepsilon_2$, $\varepsilon_{\rm iii}=2\varepsilon_0+\varepsilon_1$, and $\varepsilon_{\rm iv}=2\varepsilon_0$. Clearly, $\varepsilon_{\rm i}>\varepsilon_{\rm ii}>\varepsilon_{\rm iii}>\varepsilon_{\rm iv}$. However, which of the groupings contains the ground state depends also on the interactions between all the charged particles.

In Fig. 2, we show the low energy spectra obtained by diagonalizing the ten-particle ($8e+2h$) system at $2S=9$ (a), $2S=13$ (c), and $2S=14$ (e). Filled circles mark the nonmultiplicative states[7,8] (i.e., states containing no decoupled neutral excitons), and the open circles and squares mark the multiplicative states with one and two decoupled excitons,

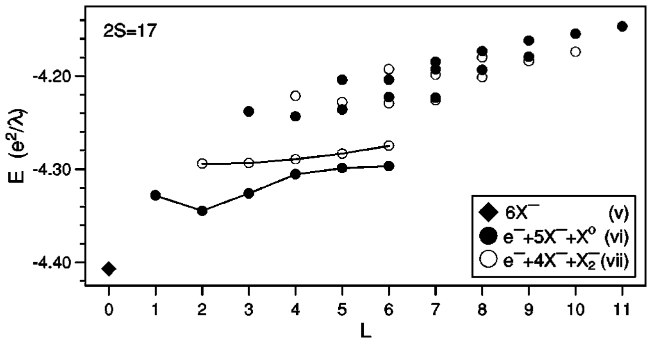

FIG. 3. Low energy spectra of different charge configurations of the $12e+6h$ system on a Haldane sphere at $2S=17$: $6X^-$ (diamonds), $e^-+5X^-+X^0$ (filled circles), and $e^-+4X^-+X_2^-$ (open circles).

respectively. In frames (b), (d), and (f) we plot the low energy spectra of different charge complexes interacting through appropriate pseudopotentials (see Fig. 1), corresponding to four possible groupings (i)–(iv). By comparing left and right frames, we can identify low-lying states of type (i)–(iv) in the electron-hole spectra.

In general, energies calculated from pseudopotentials V_{AB} in Fig. 2 underestimate energies of the corresponding electron-hole system if N and $2S$ are large. This can be partially understood in terms of polarization effects in the two-particle pseudopotentials. Better approximate pseudopotentials for the $8e+2h$ system are close to those of a pair of point charges with appropriate angular momenta l_A and l_B, except for the hard cores.[7]

It is unlikely that a system containing a large number of different species (e.g., e^-, X^-, X_2^-, etc.) will form the absolute ground state of the electron-hole system. However, different charge configurations can form low-lying excited bands. An interesting example is the $12e+6h$ system at $2S=17$. The $6X^-$ grouping (v) has the maximum total binding energy $\varepsilon_{\rm v}=6\varepsilon_0+6\varepsilon_1$. Other expected low-lying bands correspond to the following groupings: (vi) $e^-+5X^-+X^0$ with $\varepsilon_{\rm vi}=6\varepsilon_0+5\varepsilon_1$ and (vii) $e^-+4X^-+X_2^-$ with $\varepsilon_{\rm vii}=6\varepsilon_0+5\varepsilon_1+\varepsilon_2$. We have checked other groupings that have large binding energies and find that they all have higher total energies than (v)–(vii).

Although we are unable to perform an exact diagonalization for the $12e+6h$ system in terms of individual electrons and holes, we can use appropriate pseudopotentials and binding energies of groupings (v)–(vii) to obtain the low-lying states in the spectrum. The results are presented in Fig. 3. There is only one $6X^-$ state (the $L=0$ Laughlin $\nu_{X^-}=1/3$ state[7]) and two bands of the states in each of groupings (vi) and (vii). A gap of $0.0626e^2/\lambda$ separates the ground state from the lowest excited state.

V. GENERALIZED LAUGHLIN WAVE FUNCTION

It is known that if the pseudopotential $V(\mathcal{R})$ decreases quickly with increasing $\mathcal{R}$, the low-lying multiplets avoid (strongly repulsive) pair states with one or more of the smallest values of $\mathcal{R}$.[15,16] For the (one-component) electron gas on a plane, avoiding pair states with $\mathcal{R}<m$ is achieved with the factor $\Pi_{i<j}(x_i-x_j)^m$ in the Laughlin $\nu=1/m$ wave function. For a system containing a number of distinguishable types of fermions interacting through Coulomb-like pseudopotentials, the appropriate generalization of the Laughlin wave function will contain a factor $\Pi(x_i^{(a)}-x_j^{(b)})^{m_{ab}}$, where $x_i^{(a)}$ is the complex coordinate for the position of the ith particle of type a, and the product is taken over all pairs. For each type of particle, one power of $(x_i^{(a)}-x_j^{(a)})$ results from the antisymmetrization required for indistinguishable fermions and the other factors describe Jastrow-type correlations between the interacting particles. Such a wave function guarantees that $\mathcal{R}_{ab}\geqslant m_{ab}$, for all pairings of various types of particles, thereby avoiding large pair repulsion.[10,14] Fermi statistics of particles of each type requires that all m_{aa} are odd, and the hard cores defined by Eq. (1) require that $m_{ab}\geqslant\mathcal{R}_{ab}^{\rm min}$ for all pairs.

VI. GENERALIZED COMPOSITE FERMION PICTURE

In order to understand the numerical results obtained in the spherical geometry (Figs. 2 and 3), it is useful to introduce a generalized CF picture by attaching to each particle fictitious flux tubes carrying an integral number of flux quanta ϕ_0. In the multicomponent system, each a particle carries flux $(m_{aa}-1)\phi_0$ that couples only to charges on all other a particles and fluxes $m_{ab}\phi_0$ that couple only to charges on all b particles, where a and b are any of the types of fermions. The effective monopole strength[3,12,15,17] seen by a CF of type a (CF-a) is

$$2S_a^*=2S-\sum_b (m_{ab}-\delta_{ab})(N_b-\delta_{ab}). \tag{2}$$

For different multicomponent systems, we expect generalized Laughlin incompressible states (for two components denoted as $[m_{AA},m_{BB},m_{AB}]$) when all the hard-core pseudopotentials are avoided and CF's of each kind fill completely an integral number of their CF shells (e.g., $N_a=2l_a^*+1$ for the lowest shell). In other cases, the low-lying multiplets are expected to contain different kinds of quasiparticles (QP-A,QP-B, . . .) or quasiholes (QH-A,QH-B, . . .) in the neighboring incompressible state.

Our multicomponent CF picture can be applied to the system of excitonic ions, where the CF angular momenta are given by $l_{X_k^-}^*=|S_{X_k^-}^*|-k$. As an example, let us first analyze the low-lying $8e+2h$ states in Fig. 2. At $2S=9$, for $m_{e^-e^-}=m_{X^-X^-}=3$ and $m_{e^-X^-}=1$, we predict the following low-lying multiplets in each grouping: (i) $2S_{e^-}^*=1$ and $2S_{X^-}^*=3$ gives $l_{e^-}^*=l_{X^-}^*=1/2$. Two CF-X^-'s fill their lowest shell ($L_{X^-}=0$), and we have two QP-e^-'s in their first excited shell, each with angular momentum $l_{e^-}^*+1=3/2$ ($L_{e^-}=0$ and 2). Addition of L_{e^-} and L_{X^-} gives total angular momenta $L=0$ and 2. We interpret these states as those of two QP-e's in the incompressible [331] state. Similarly, for other groupings we obtain: (ii) $L=2$; (iii) $L=1$, 2, and 3; and (iv) $L=0$ ($\nu=2/3$ state of six electrons).

At $2S=13$ and 14, we set $m_{e^-e^-}=m_{X^-X^-}=3$ and $m_{e^-X^-}=2$ and obtain the following predictions. First, at $2S=13$: (i) The ground state is the incompressible [332] state at $L=0$; the first excited band should therefore contain states with one QP-QH pair of either kind. For the e^- excitations, the QP-e^- and QH-e^- angular momenta are $l^*_{e^-}=3/2$ and $l^*_{e^-}+1=5/2$, respectively, and the allowed pair states have $L_{e^-}=1$, 2, 3, and 4. However, the $L_{e^-}=1$ state has to be discarded, as it is known to have high energy in the one-component (four-electron) spectrum.[17] For the X^- excitations, we have $l^*_{X^-}=1/2$, and pair states can have $L_{X^-}=1$ or 2. The first excited band is therefore expected to contain multiplets at $L=1$, 2^2, 3, and 4. The low-lying multiplets for other groupings are expected at: (ii) $2S^*_{X_2^-}=3$ gives no bound X_2^- state; setting $m_{e^-X^-}=1$ we obtain $L=2$; and (iii) $L=2$ and 3; and (iv) $L=0$, 2, and 4. Finally, at $2S=14$ we obtain: (i) $L=1$, 2, and 3; (ii) incompressible [3*2] state at $L=0$ ($m_{X^-X^-}$ is irrelevant for one X^-) and the first excited band at $L=1$, 2, 3, 4, and 5; (iii) $L=1$; and (iv) $L=3$.

For the $12e+6h$ spectrum in Fig. 3, the following CF predictions are obtained: (v) For $m_{X^-X^-}=3$ we obtain the Laughlin $\nu=1/3$ state with $L=0$. Because of the hard core of $V_{X^-X^-}$, this is the only state of this grouping. (vi) We set $m_{X^-X^-}=3$ and $m_{e^-X^-}=1$, 2, and 3. For $m_{e^-X^-}=1$ we obtain $L=1$, 2, 3^2, 4^2, 5^3, 6^3, 7^3, 8^2, 9^2, 10, and 11. For $m_{e^-X^-}=2$ we obtain $L=1$, 2, 3, 4, 5, and 6. For $m_{e^-X^-}=3$ we obtain $L=1$. (vii) We set $m_{X^-X^-}=3$, $m_{e^-X_2^-}=1$, $m_{X^-X_2^-}=3$, and $m_{e^-X^-}=1$, 2, or 3. For $m_{e^-X^-}=1$ we obtain $L=2$, 3, 4^2, 5^2, 6^3, 7^2, 8^2, 9, and 10. For $m_{e^-X^-}=2$ we obtain $L=2$, 3, 4, 5, and 6. For $m_{e^-X^-}=3$ we obtain $L=2$. In groupings (vi) and (vii), the sets of multiplets obtained for higher values of $m_{e^-X^-}$ are subsets of the sets obtained for lower values, and we would expect them to form lower energy bands since they avoid additional small values of $\mathcal{R}_{e^-X^-}$. However, note that the (vi) and (vii) states predicted for $m_{e^-X^-}=3$ (at $L=1$ and 2, respectively) do not form separate bands in Fig. 3. This is because $V_{e^-X^-}(L)$ increases more slowly than linearly as a function of $L(L+1)$ in the vicinity of $\mathcal{R}_{e^-X^-}=3$ (see Fig. 1). In such case the CF picture fails.[15]

The agreement of our CF predictions with the data in Figs. 2 and 3 (marked with lines) is really quite remarkable and strongly indicates that our multicomponent CF picture is correct. We were indeed able to confirm predicted Jastrow-type correlations in the low-lying states by calculating their coefficients of fractional parentage.[15,18] We also verified the CF predictions for other systems that we were able to treat numerically. If exponents m_{ab} are chosen correctly, the CF picture works well in all cases.

VII. SUMMARY

Charged excitons and excitonic complexes play an important role in determining the low energy spectra of electron-hole systems in a strong magnetic field. We have introduced general Laughlin-type correlations into the wave functions and proposed a generalized CF picture to elucidate the angular momentum multiplets forming the lowest energy bands for different charge configurations occurring in the electron-hole system. We have found Laughlin incompressible fluid states of multicomponent plasmas at particular values of the magnetic field and the lowest bands of multiplets for various charge configurations at any value of the magnetic field. It is noteworthy that the fictitious Chern-Simons fluxes and charges of different types or colors are needed in the generalized CF model. This strongly supports the view that the effective magnetic field seen by the CF's is simply a mathematical construct, not a ''real'' physical magnetic field.

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