Dissipative tunneling in a bath of two-level systems

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The dynamical behavior of a two-level system coupled linearly to a dissipative bath of two-level systems whose tunneling splittings and coupling parameters are distributed according to an Ohmic spectrum is studied using numerical path integral methods as well as the noninteracting blip approximation for an equivalent harmonic environment characterized by a scaled spectral density. Comparisons with the behaviors of the conventional spin-boson model of similar coupling strength reveal a weaker quenching of coherence in the present case. Finally, the boundary in parameter space between coherent and incoherent dynamics is mapped. [S0163-1829(99)02426-1]

I. INTRODUCTION

The model of a low-dimensional system coupled to a bath of harmonic oscillators has been used extensively as the paradigm for dissipation in condensed phase dynamics. In the classical limit, the harmonic bath leads[1] to the well-known generalized Langevin equation of motion where the conventional force exerted on the system is supplemented by a ''random'' component as well as a frictional term. By contrast, the fully quantum mechanical picture is much more complicated and leads to a variety of phenomena governed by the interplay among coherence, tunneling, and dephasing. The so-called spin-boson model, comprised of a two-level system interacting with a harmonic bath, is deceptively simple and at the same time extraordinarily rich in dynamical behavior. Its early understanding emerged from variational treatments[2,3] and the noninteracting blip approximation,[4] while more recent studies have employed numerical path integral techniques based on Monte Carlo integration[5] or iterative procedures.[6]

The present article explores the tunneling dynamics of a two-level system (TLS) in contact with a bath of two-level systems as described by the Hamiltonian

$$H=-\hbar\Omega\sigma_x^0-\sum_{i=1}^{n}\frac{1}{2}\hbar\omega_i\sigma_x^i-\sigma_z^0\sum_{i=1}^{n}c_i\sqrt{\frac{\hbar}{2\omega_i}}\sigma_z^i$$
$$\equiv H_0+H_b+H_{\rm int}. \tag{1.1}$$

Here σ_x and σ_z are the usual Pauli spin matrices, the tunneling splittings are $2\hbar\Omega$ and $2\hbar\omega_i$ for the bare system and the bath spins, respectively, and the parameters of the bath are specified from the spectral density function

$$J(\omega)=\frac{\pi}{2}\sum_{i=1}^{n}\frac{c_i^2}{\omega_i}\delta(\omega-\omega_i). \tag{1.2}$$

Throughout this paper we assume that the interaction between system and bath is turned on at $t=0$, such that the initial density matrix factorizes into its system and bath components, and that the bath is initially at thermal equilibrium:

$$\rho(0)=\tilde{\rho}(0)e^{-\beta H_b}. \tag{1.3}$$

In the last equation $\tilde{\rho}(0)$ is the initial density operator of the system spin. We are interested in the effects of the TLS bath on the dynamics of the observable system in the thermodynamic limit where $n\to\infty$.

While the Hamiltonian of Eq. (1.1) is of interest in the study of phenomena observed in spin glasses, its similarity with the spin-boson Hamiltonian allows more general questions to be addressed, such as the differences between the dissipative role of harmonic spectrum vs two-level baths. Caldeira *et al.*[7] have treated the TLS bath via second order perturbation theory and obtained an influence functional that is related to that arising from a harmonic bath via a temperature-dependent factor. In a recent article,[8] Makri employed a cumulant expansion of the influence functional to demonstrate that separable baths of the present type can (in the $n\to\infty$ limit) be mapped on effective harmonic baths *exactly*, i.e., the linear response approximation becomes exact in this case. As a consequence, the perturbative treatment of Caldeira *et al.* is exact for *any* value of the system-bath coupling strength, reducing the problem to the better understood case of the spin-boson Hamiltonian.

The present paper uses numerical path integral methods to evaluate the influence functional from a TLS bath and the ensuing tunneling dynamics of the system. The numerical calculations, along with the mapping to an effective harmonic bath and use of the noninteracting blip approximation,[4] are employed to characterize the dynamics of Eq. (1.1) in most regimes of parameter space and to establish the boundary between coherent and incoherent behavior.

Section II discusses the mapping of the TLS environment on an effective bath of harmonic oscillators and the expressions for the system dynamics obtained from the noninteracting blip approximation. In Sec. III we describe the discretized path integral formalism and the numerical evaluation of the influence functional from a TLS bath. Section IV describes the results and contrasts the behavior of the present Hamiltonian to that of the spin-boson model. Finally, Sec. V concludes.

II. THEORETICAL ANALYSIS

Recent work[8] showed the influence functional from a general anharmonic bath of independent degrees of freedom is *identical* to that from a harmonic bath of fictitious degrees of freedom bilinearly coupled to the system

$$H_{b,\mathrm{eff}}=\sum_i \frac{1}{2}p_i^2+\frac{1}{2}\omega_i^2x_i^2, \quad H_{\mathrm{int,eff}}=-\sigma_z\sum_i c_ix_i, \tag{2.1}$$

whose spectral density is specified by the relation

$$J_{\mathrm{eff}}(\omega,\beta)=\frac{2}{\hbar}\tanh\left(\frac{1}{2}\hbar\omega\beta\right)\int_0^\infty \mathrm{Re}\, C_\beta(t)\cos\omega t\, dt. \tag{2.2}$$

Here $C_\beta(t)$ is the (quantum) force autocorrelation function of the actual anharmonic bath at the given temperature. This mapping of a nonlinear medium onto a Gaussian bath is exact for baths of the type described by Eq. (1.1), irrespective of the magnitude of the overall coupling strength. The above equivalence, whose classical analog is often justified in the spirit of the central limit theorem,[9–12] is meaningful *only* in the context of modulating the dynamics of the observable system. Notice that the parameters of the equivalent effective bath are, in general, temperature dependent.

In the particular case where the bath is composed of independent two-level systems, the force autocorrelation function is equal to

$$\begin{aligned}C_\beta(t)&=\sum_i \frac{\hbar c_i^2}{2\omega_i}\langle\sigma_z^i(t)\sigma_z^i(0)\rangle_\beta\\ &=\frac{\hbar}{2}\sum_i \frac{c_i^2}{\omega_i}\left[\cos\omega_it-i\tanh\left(\frac{1}{2}\hbar\omega_i\beta\right)\sin\omega_it\right].\end{aligned} \tag{2.3}$$

Use of Eq. (2.2) directly leads to the relation

$$\begin{aligned}J_{\mathrm{eff}}(\omega,\beta)&=\frac{\pi}{2}\tanh\left(\frac{1}{2}\hbar\omega\beta\right)\sum_i \frac{c_i^2}{\omega_i}\delta(\omega-\omega_i)\\ &=\tanh\left(\frac{1}{2}\hbar\omega\beta\right)J(\omega).\end{aligned} \tag{2.4}$$

This is the result obtained by Caldeira *et al.*[7] using second order perturbation theory. According to the analysis presented in Ref. 8 the above result is exact, i.e., the effects of the TLS bath on the dynamics of the observable system are identical to those induced by an effective harmonic bath whose spectral density is given by Eq. (2.4), even in the strong coupling regime.

Assuming that the frequencies ω_i are distributed in a continuous interval that includes the tunneling splitting of the system spin, one expects dephasing effects to damp the coherent tunneling oscillations of the bare system and, at strong coupling, completely quench coherence. The above mapping on a harmonic bath invites use of the noninteracting blip approximation (NIBA) to study the tunneling of a spin coupled to a reservoir of two-level systems. Of particular interest is the transition from coherent to incoherent dynamics following preparation of the system in the ''right'' state. The average position of the system spin is given in terms of the time-evolved density operator by the expression

$$P(t)=\mathrm{Tr}(\rho(t)\sigma_z^0), \tag{2.5}$$

whose Laplace transform

$$\tilde{P}(\lambda)=\int_0^\infty P(t)e^{-\lambda t}\,dt \tag{2.6}$$

is given within the NIBA by the relation

$$\tilde{P}(\lambda)\approx(\lambda+f(\lambda))^{-1}, \tag{2.7}$$

where

$$f(\lambda)=4\Omega^2\int_0^\infty \cos\left(\frac{4}{\pi\hbar}Q_1(t)\right)\exp\left(-\lambda t-\frac{4}{\pi\hbar}Q_2(t)\right)dt \tag{2.8}$$

and the functions Q_1 and Q_2 are given by the relations

$$\begin{aligned}Q_1(t)&=\int_0^\infty \frac{J_{\mathrm{eff}}(\omega)}{\omega^2}\sin\omega t\, d\omega,\\ Q_2(t)&=\int_0^\infty \frac{J_{\mathrm{eff}}(\omega)}{\omega^2}(1-\cos\omega t)\coth\left(\frac{1}{2}\hbar\omega\beta\right)d\omega.\end{aligned} \tag{2.9}$$

In the case of a harmonic bath with Ohmic spectral density and for weak coupling and low temperature, $\tilde{P}(\lambda)$ possesses a pair of complex conjugate poles with negative real part which lead to underdamped dynamics.[4] The transition from coherent to incoherent dynamics is signified by the coalesc-

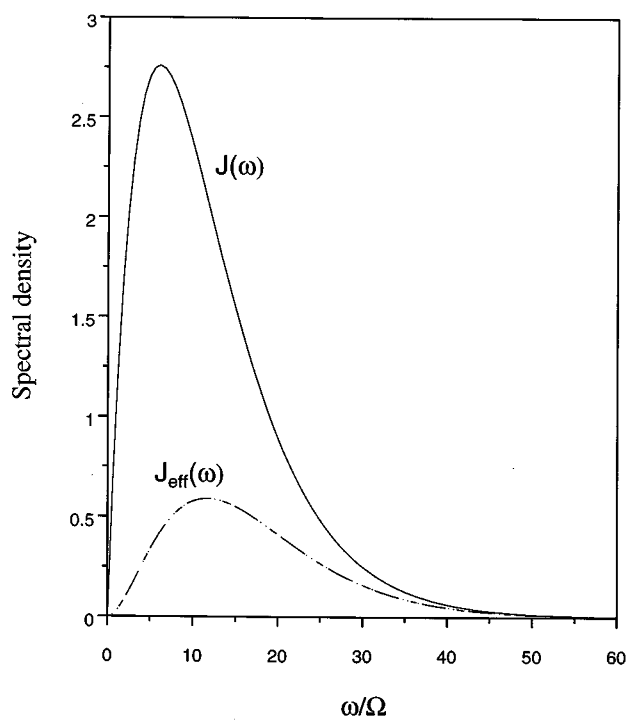

FIG. 1. The spectral densities discussed in Sec. IV. Solid line: Ohmic spectral density describing the TLS bath. Chain-dotted line: spectral density of the effective harmonic bath at a high temperature ($\hbar\Omega\beta=0.05$). The effective spectral density at very low temperatures is indistinguishable from the solid line.

ing of these poles on the negative real axis, and incoherent behavior is characterized by distinct poles that lie on the real axis and which are responsible for the observed exponential decay.

To map the coherent-incoherent boundary in the case of the TLS bath, we locate numerically the poles of $\tilde{P}(\lambda)$. Specifically, for a given temperature we decrease the Kondo parameter gradually until the zeros of this function on the negative real axis coalesce and subsequently acquire an imaginary part. We have also inverted the Laplace transform to obtain the average system position:

$$P(t)=\frac{1}{2\pi i}\int_C e^{\lambda t}\tilde{P}(\lambda), \tag{2.10}$$

where the contour C runs from $-i\infty$ to $+i\infty$ lying entirely to the right of all singularities of $\tilde{P}(\lambda)$. The results of this treatment are presented in Sec. IV.

III. NUMERICAL PATH INTEGRAL TREATMENT

Tracing out the bath first brings Eq. (2.5) into the form

$$P(t)=\mathrm{Tr}_{\mathrm{sys}}(\tilde{\rho}(t)\sigma_z^0), \tag{3.1}$$

where

$$\tilde{\rho}(t)=\mathrm{Tr}_b(e^{-iHt/\hbar}\rho(0)e^{iHt/\hbar}) \tag{3.2a}$$

is the reduced density operator of the system. It is convenient to evaluate the latter in the left-right basis,

$$\tilde{\rho}(s'',s';t)=\mathrm{Tr}_b\langle s''|e^{-iHt/\hbar}\rho(0)e^{iHt/\hbar}|s'\rangle, \tag{3.2b}$$

where $s',s''=\pm 1$ are the values of the system spin and the trace is evaluated with respect to the TLS bath. Employing the discretized path integral representation of the time evolution operators in Eq. (3.1), the reduced density matrix takes the form

$$\tilde{\rho}(s_N^+,s_N^-;t)=\sum_{s_0^\pm=\pm1}\sum_{s_1^\pm=\pm1}\cdots\sum_{s_{N-1}^\pm=\pm1}\sum_{\mathbf{q}_0^\pm=\pm1}\sum_{\mathbf{q}_1^\pm=\pm1}\cdots\sum_{\mathbf{q}_{N-1}=\pm1}\langle s_N^+\mathbf{q}_N|e^{-iH\Delta t/\hbar}|\mathbf{q}_{N-1}^+s_{N-1}^+\rangle\cdots\langle s_1^+\mathbf{q}_1^+|e^{-iH\Delta t/\hbar}|\mathbf{q}_0^+s_0^+\rangle$$
$$\times\langle s_0^+\mathbf{q}_0^+|\rho(0)|\mathbf{q}_0^-s_0^-\rangle\langle s_0^-\mathbf{q}_0^-|e^{iH\Delta t/\hbar}|\mathbf{q}_1^-s_1^-\rangle\cdots\langle s_{N-1}^-\mathbf{q}_{N-1}^-|e^{iH\Delta t/\hbar}|\mathbf{q}_N s_N^-\rangle. \tag{3.3}$$

Here the vectors $\mathbf{q}_k^\pm$ contain the coordinates of the bath spins along the given forward or backward path defined by the coordinates $\{\mathbf{q}_0^\pm,\ldots,\mathbf{q}_{N-1}^\pm,\mathbf{q}_N\}$ and $\Delta t=t/N$ is the time step employed in the discretization of the path integral. Next, we employ a symmetric splitting of the time evolution operator

$$e^{-iH\Delta t/\hbar}\approx e^{-i(H_b+H_{\mathrm{int}})\Delta t/2\hbar}e^{-iH_0\Delta t/\hbar}e^{-i(H_b+H_{\mathrm{int}})\Delta t/2\hbar}, \tag{3.4}$$

which becomes exact as the time step Δt approaches zero. Performing the sum over the bath spins for each realization of the system coordinates, the reduced density matrix takes the form

$$\tilde{\rho}(s_N^+,s_N^-;t)=\sum_{k_0^\pm=\pm1}\sum_{k_1^\pm=\pm1}\cdots\sum_{k_{N-1}^\pm=\pm1}\langle s_N^+|e^{-iH_0\Delta t/\hbar}|s_{N-1}^+\rangle\cdots\langle s_1^+|e^{-iH_0\Delta t/\hbar}|s_0^+\rangle\langle s_0^+|e^{-\beta H_0}|s_0^-\rangle$$
$$\times\langle s_0^-|e^{iH_0\Delta t/\hbar}|s_1^-\rangle\cdots\langle s_{N-1}^-|e^{iH_0\Delta t/\hbar}|s_N^-\rangle F(s_0^+,s_1^+,\ldots,s_N^+,s_0^-,\ldots,s_N^-;\Delta t), \tag{3.5}$$

where

$$F(s_0^\pm,s_1^\pm,\ldots s_N^\pm;\Delta t)=\sum_{\mathbf{q}_0^\pm=\pm1}\sum_{\mathbf{q}_1^\pm=\pm1}\cdots\sum_{\mathbf{q}_{N-1}^\pm=\pm1}\sum_{\mathbf{q}_N=\pm1}\langle\mathbf{q}_N|e^{-i[H_b+H_{\mathrm{int}}(s_N^+)]\Delta t/2\hbar}|\mathbf{q}_{N-1}^+\rangle\langle\mathbf{q}_{N-1}^+|e^{-i[H_b+H_{\mathrm{int}}(s_{N-1}^+)]\Delta t/2\hbar}$$
$$\times e^{-i[H_b+H_{\mathrm{int}}(s_{N-2}^+)]\Delta t/2\hbar}|\mathbf{q}_{N-2}^-\rangle\cdots\langle\mathbf{q}_1^+|e^{-i[H_b+H_{\mathrm{int}}(s_1^+)]\Delta t/2\hbar}e^{-i[H_b+H_{\mathrm{int}}(s_0^+)]\Delta t/2\hbar}|\mathbf{q}_0^+\rangle\langle\mathbf{q}_0^+|e^{-\beta H_b}|\mathbf{q}_0^-\rangle$$
$$\times\langle\mathbf{q}_0^-|e^{i[H_b+H_{\mathrm{int}}(s_0^-)]\Delta t/2\hbar}e^{-i[H_b+H_{\mathrm{int}}(s_1^-)]\Delta t/2\hbar}|\mathbf{q}_1^-\rangle\cdots\langle\mathbf{q}_{N-1}^-|e^{i[H_b+H_{\mathrm{int}}(s_N^-)]\Delta t/2\hbar}|\mathbf{q}_N\rangle. \tag{3.6}$$

Notice that the operators in the last equation are sums of independent bath spins, and thus the influence functional becomes a product of one-dimensional factors. Exploiting this feature, we evaluate the influence functional numerically by using the matrix multiplication method.[13–15]

IV. RESULTS AND DISCUSSION

The results presented below were performed with a discrete bath whose parameters correspond to the conventional Ohmic spectral density with exponential cutoff,

$$J(\omega)=\frac{1}{2}\pi\hbar\xi\omega e^{-\omega/\omega_c}, \tag{4.1}$$

where the Kondo parameter ξ is a measure of the system-bath coupling strength. This parameter is proportional to the friction coefficient in the classical Langevin description and provides a measure of the dissipation strength. This spectral density is depicted in Fig. 1, along with that of the effective harmonic bath at two temperatures. Note that the presence of

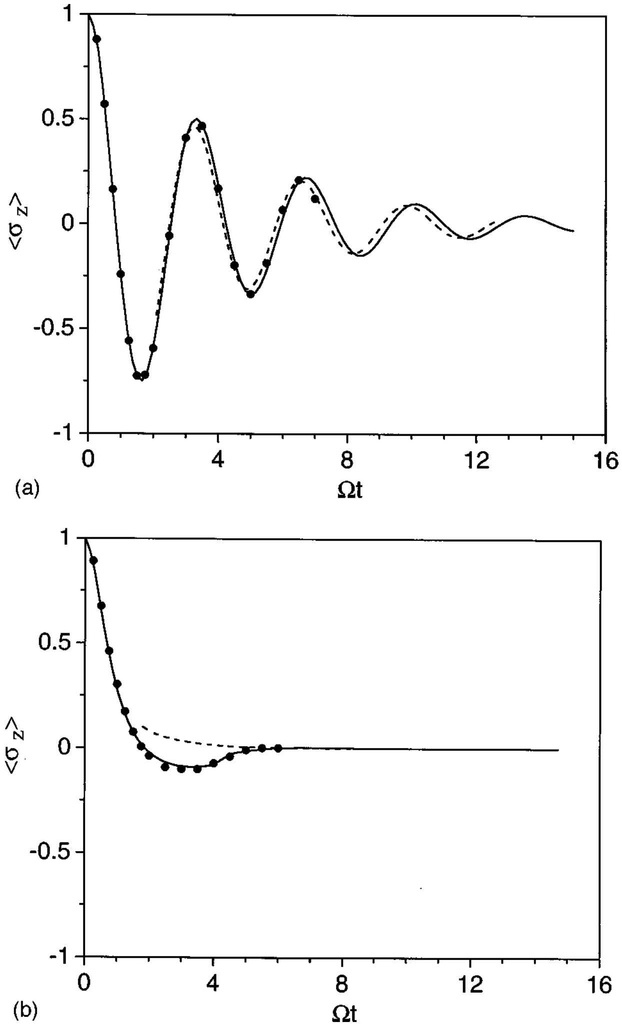

FIG. 2. Expectation value of the system position, Eq. (2.5), for $\hbar\Omega\beta=2$. Solid circles: numerical path integral results for the TLS bath. Solid line: numerical path integral results for the effective harmonic bath characterized by a temperature-dependent spectral density. Dashed line: bath of harmonic oscillators. (a) $\xi=0.1$. (b) $\xi=0.5$.

the hyperbolic tangent factor decreases appreciably the magnitude of the effective bath spectral density at high temperatures and thus the resulting damping is expected to be weaker compared to that observed in the case of the spin-boson model.

For the purpose of performing the numerical path integral calculation the bath is discretized into n discrete modes. The bath parameters are chosen such that the integrated density of states up to a frequency ω_k equals the number k of bath modes, while the couplings are determined from Eq. (4.1).[8] This particular discretization of the bath leads to rapid convergence with respect to the number of discrete spins employed. The results presented below were obtained with 50 bath degrees of freedom distributed in a frequency interval that extends up to $10\omega_c$, but we have verified that the observed behaviors are insensitive to the particular type of bath discretization. The cutoff frequency is chosen as $\omega_c=6\,\Omega$.

Figures 2–4 present results for the given bath of two-level systems, a harmonic bath of the same spectral density $J(\omega)$,

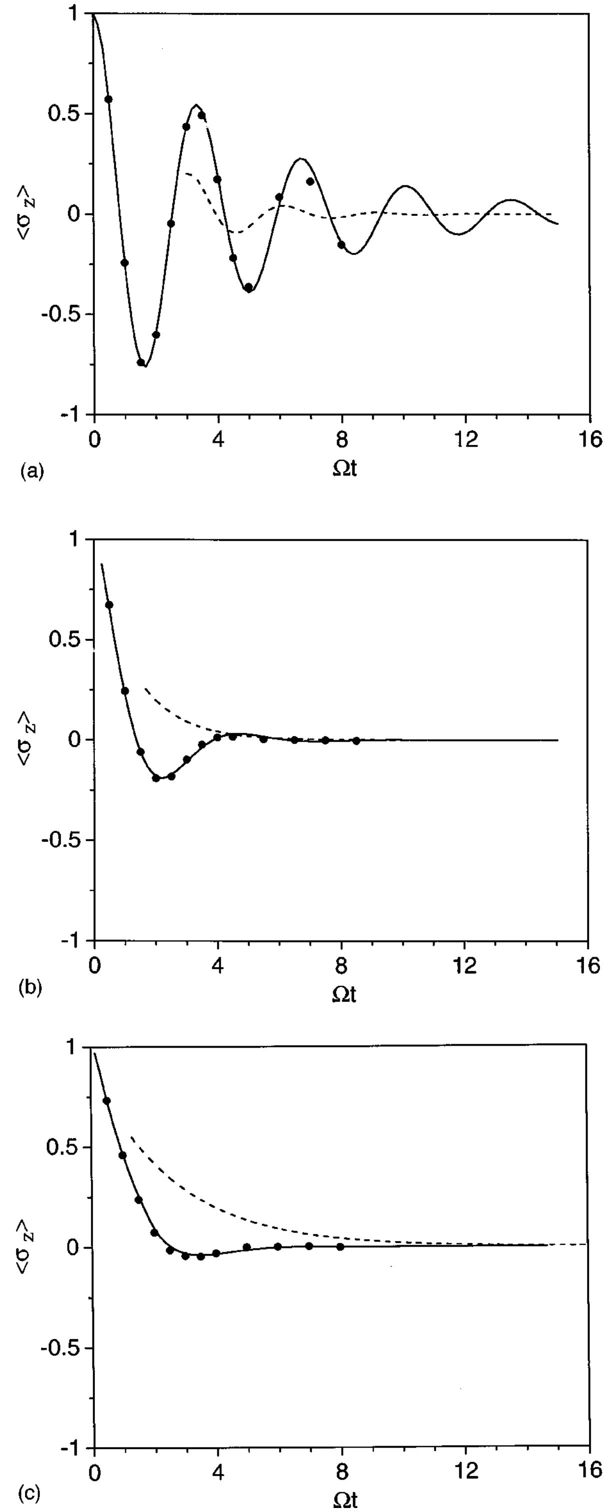

FIG. 3. Expectation value of the system position, Eq. (2.5), for $\hbar\Omega\beta=0.5$. Solid circles: numerical path integral results for the TLS bath. Solid line: numerical path integral results for the effective harmonic bath characterized by a temperature-dependent spectral density. Dashed line: bath of harmonic oscillators. (a) $\xi=0.1$. (b) $\xi=0.5$. (c) $\xi=0.8$.

and an effective harmonic bath whose spectral density $J_{\text{eff}}(\omega)$ is given by Eq. (2.4). The harmonic bath calculations were performed using a combination of full spin summation and the tensor multiplication procedure developed earlier in our

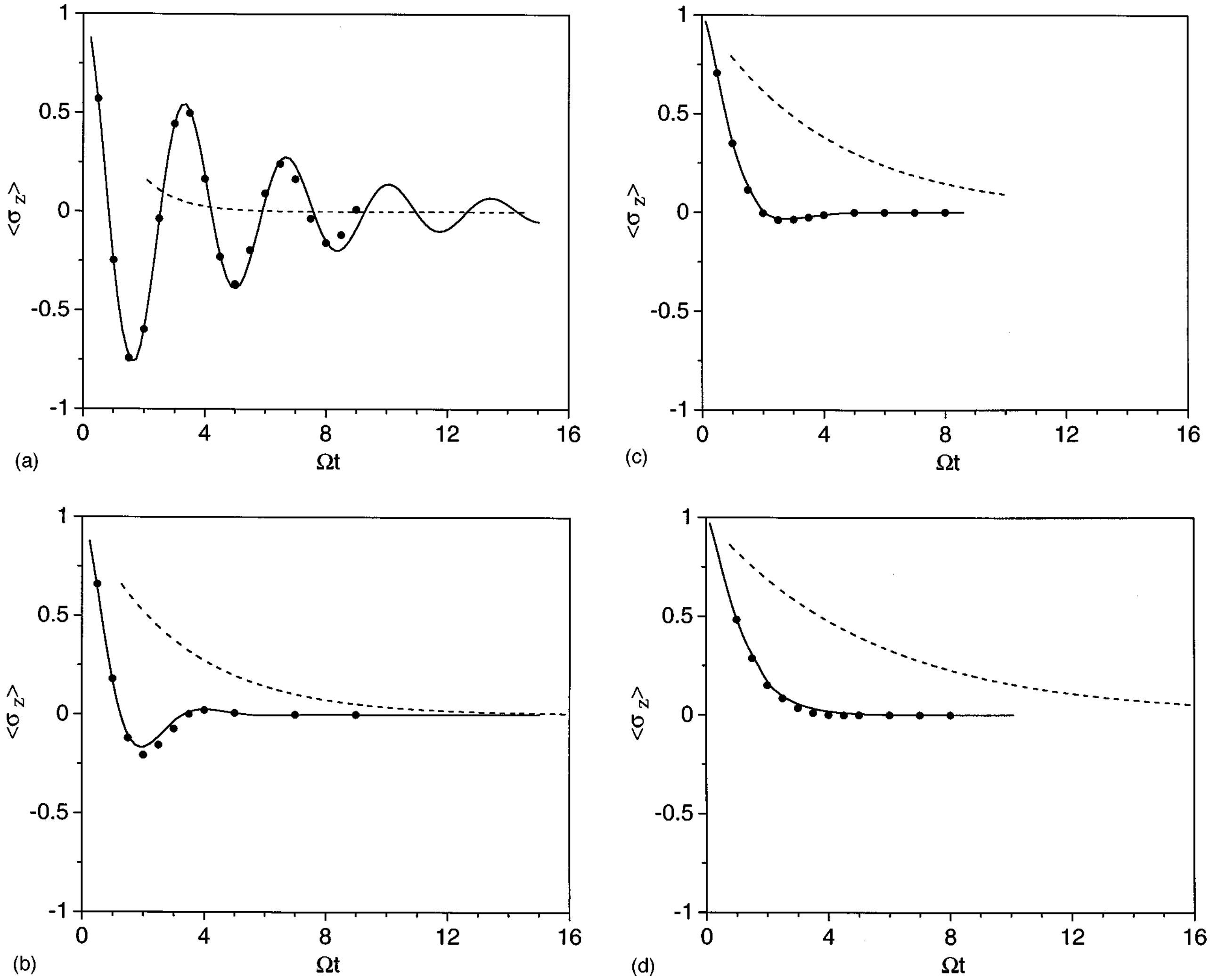

FIG. 4. Expectation value of the system position, Eq. (2.5), for $\hbar\Omega\beta=0.05$. Solid circles: numerical path integral results for the TLS bath. Solid line: numerical path integral results for the effective harmonic bath characterized by a temperature-dependent spectral density. Dashed line: bath of harmonic oscillators. (a) $\xi=0.1$. (b) $\xi=0.5$. (c) $\xi=0.8$. (d) $\xi=1.2$.

group.[16] The agreement between the numerical results obtained from a TLS bath and those of the effective harmonic one is generally excellent. The small discrepancies observed at some of the longer time points are due to inadequate convergence of the full path sum due to the use of time steps that are too large. We have verified that the TLS influence functional obtained via the numerical matrix multiplication method is indeed identical to the harmonic one with the effective spectral density of Eq. (2.2).

At zero temperature, $\tanh(\frac{1}{2}\hbar\omega\beta)\rightarrow 1$ (see also Fig. 1) and the dynamics in the presence of a TLS bath are expected to agree well with those of the conventional spin-boson model. At low but finite temperature the effective spectral density also reverts to the Ohmic form, except for low frequencies. Behavior distinct from the spin-boson model arises when the temperature-dependent factor differs appreciably from unity at frequencies around the tunneling splitting of the system, i.e., when $\hbar\Omega\beta\sim 1$. Expansion of the hyperbolic tangent at these low frequencies produces an additional power of ω in the spectral density, which leads to coherent behavior that persists up to values of the Kondo parameter that exceed the Ohmic spin-boson critical value $\xi=\frac{1}{2}$. Specifically, the TLS bath leads to underdamped oscillations of the tunneling system even at $\xi=0.8$, which are replaced by incoherent decay at larger values of the dissipation parameter. As the temperature is raised (see Figs. 3 and 4) the effective harmonic spectral density begins to deviate more from that of the TLS bath, resulting in more coherent dynamics compared to that of the spin-boson Hamiltonian of Ohmic spectral density. Specifically, the latter is known to display incoherent dynamics at all temperatures when the dimensionless dissipation parameter exceeds $\frac{1}{2}$, and the boundary is shifted to much weaker friction at higher temperatures; by contrast, coherences are seen to persist well beyond that value in the present case where the bath is composed of two-level systems. Finally, at high temperatures the TLS and bosonic baths lead to markedly different behaviors, dominated in the present case by underdamped oscillations at weak or moderate friction which are absent from the spin-boson dynamics. These findings are in qualitative agreement with the conclusions reached by Shao and Hänggi in an article that appeared after the present paper was submitted.[17] The calculations presented here em-

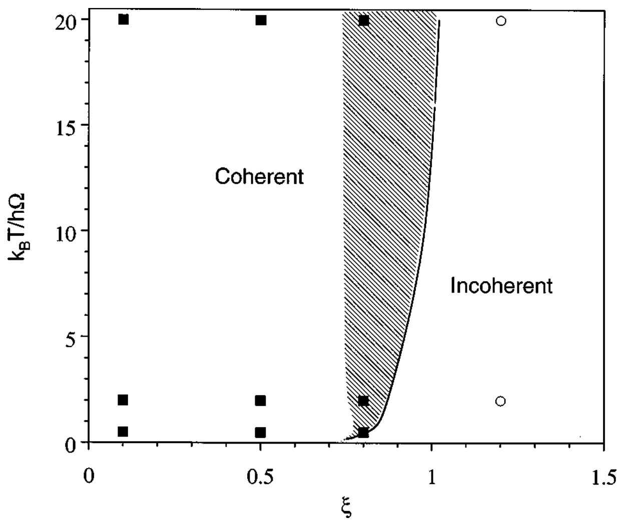

FIG. 5. Phase diagram for the tunneling system in contact with a TLS bath. The solid squares and hollow circles indicate parameters at which the numerical path integral results exhibit underdamped oscillations and incoherent decay, respectively. The solid line shows the coherent-incoherent boundary predicted by the NIBA for an effective harmonic bath of temperature-dependent spectral density for $\omega_c = 20\Omega$. The shaded area indicates the region of parameter space in which $P(t)$ has a small negative lobe that does not fall below -0.01 (see the text).

phasize the very gradual nature of the transition in the case of a TLS bath, showing that some coherence persists even if the Kondo parameter significantly exceeds the $\xi = \frac{1}{2}$ boundary established by these authors. The weaker ability of the TLS bath to damp the tunneling oscillations of the system is due to the availability of only two energy levels for each bath degree of freedom, in contrast to harmonic baths where an infinite number of levels can be populated.

In order to gain further insight into the tunneling dynamics of a spin coupled to a TLS bath we examine the behavior of the average system position as given by the NIBA. Figure 5 shows the onset of purely exponential decay, corresponding to the relation between temperature and friction strength for which $P(t)$, obtained numerically from Eqs. (2.7)–(2.10), decays to zero without turning negative. These results are shown for $\omega_c = 20\Omega$, since NIBA is not reliable for small values of the cutoff frequency. The coherent-incoherent boundary shifts to higher friction if the cutoff frequency is lowered. Examination of the Laplace transform $\tilde{P}(\lambda)$ reveals multiple pairs of complex conjugate poles in the complex plane (with negative real parts) which persist even to the right of the boundary deduced from the behavior of $P(t)$. Although such poles are generally associated with oscillatory behavior of the system position, interference among residues and/or dominance of poles with very small imaginary parts leads to a decay of $P(t)$ which is for all practical purposes exponential for parameters in the incoherent regime shown in Fig. 5. Eventually, as the friction is increased further at any given temperature, the poles of $\tilde{P}(\lambda)$ move to the negative real axis. Our simulations, as well as solution of the NIBA equations, indicate that the coherent-incoherent boundary shifts to stronger friction as the temperature is raised, in contrast to the known behavior of an Ohmic bath of bosons. Also shown as a shaded area in Fig. 5 is the region of parameter space in which the decay is almost incoherent, corresponding to parameters for which the minimum value reached by $P(t)$ lies between 0 and -0.01. The width of this area is considerable at moderate or high temperatures, indicating that the transition from coherent to incoherent dynamics is much more gradual than in the spin-boson case. Lastly, we note that the decay rate in the incoherent regime is an increasing function of temperature.

V. SUMMARY

We have explored the influence of a reservoir of two-level systems on the tunneling dynamics of a single TLS using numerical path integral methods and the noninteracting blip approximation. The main conclusions of our study are summarized below.

(1) In the macroscopic limit where the number of bath degrees of freedom approaches infinity the effects of a chosen TLS bath are equivalent to those of a harmonic one whose spectral density is modified by a temperature-dependent factor.

(2) The average position of the system undergoes underdamped tunneling oscillations at small to moderate dissipation strengths. Coherent effects generally persist at much higher friction than in the case of a bosonic bath. At large values of the Kondo parameter the system exhibits incoherent decay.

(3) The coherent-incoherent boundary is diffuse. It exhibits a non-negligible dependence on the cut-off frequency and a weak dependence on temperature. In particular, the onset of incoherent dynamics shifts to stronger friction as the temperature is raised, in contrast to the behavior induced by bosonic environments.

(4) The decay rate in the incoherent regime increases with increasing temperature.

(5) At very low temperatures the TLS bath reverts to the conventional harmonic bath of the same spectral density.

We stress that the above findings apply only to a bath of independent two-level systems whose coupling strength depends on the frequency alone. In most physical models of spin glasses the coupling strength is a function of distance as well. Such models have been studied recently by Orth *et al.*[18] It would be highly desirable to extend the above studies by including interactions among the two-level systems comprising the bath in order to mimic an Ising model of large scale lattices of spin glasses.[19] However, we are presently impeded by the nature of the available path-integral methodology, as its feasibility hinges on the separability of the bath. In light of the unusual dynamics identified in this article which dealt with the simplest of these baths, the above more complicated models are expected to reveal interesting behaviors and will undoubtedly continue to attract much theoretical attention.

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