# Thermal and tunneling pair creation of quasiparticles in quantum Hall systems

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We make a semiclassical analysis of thermal pair creations of quasiparticles at various filling factors in quantum Hall systems. It is argued that the gap energy is reduced considerably by the Coulomb potential made by impurities. It is also shown that a tunneling process becomes important at low temperature and at strong magnetic field. We fit typical experimental data excellently based on our semiclassical results of the gap energy. [S0163-1829(99)10535-6]

## I. INTRODUCTION

The quantum Hall (QH) effect[1,2] has attracted much attention from various points of view. It is characterized by the appearance of Hall plateaux and minima in the longitudinal resistivity. Observation of a zero-resistance state implies the existence of a gap in the excitation spectrum leading to the incompressibility of the system. A Hall plateau develops when quasiparticles are pinned by impurities. Quasiparticles are vortices[3] and skyrmions.[4] The aim of this paper is to investigate semiclassically the mechanism of thermal creations of quasiparticle pairs in the presence of impurities. It is pointed out that a quantum-mechanical tunneling plays an important role in this process.

Quasiparticles are activated thermally at finite temperature $T$, and contribute to the longitudinal current. It is experimentally known that the longitudinal resistivity exhibits a behavior of the Arrhenius type

$$\rho_{xx} \propto \exp\left(-\frac{\Delta_{\text{gap}}}{2k_B T}\right), \tag{1.1}$$

with $k_B$ the Boltzmann constant. The gap energy $\Delta_{\text{gap}}$ is expected to be given by the excitation energy of a pair of quasihole ($\Delta_{qh}$) and quasielectron ($\Delta_{qe}$). However, the gap energy experimentally observed is much smaller than the theoretical value even if an effect of finite layer thickness is taken into account. Phenomenologically it is well given by

$$\Delta_{\text{gap}} = \Delta_{qh} + \Delta_{qe} - \Gamma_{\text{offset}}, \tag{1.2}$$

with a sample-dependent offset $\Gamma_{\text{offset}}$.

The offset may be dominated by a Landau-level broadening due to impurities.[5,6] They are mainly provided by the donors in the bulk situated several hundreds of angstroms away from the electron layer. The Hamiltonian includes the impurity term $H_{\text{imp}}$ given by

$$H_{\text{imp}} = e\int d^2x\,\rho(\boldsymbol{x})V_{\text{imp}}(\boldsymbol{x}), \tag{1.3}$$

where $V_{\text{imp}}(\boldsymbol{x})$ is the Coulomb potential made by impurities. For a single impurity it may be approximated by

$$V_{\text{imp}}(\boldsymbol{x}) = \pm\frac{Ze}{4\pi\varepsilon}\frac{1}{\sqrt{|\boldsymbol{x}|^2 + d_{\text{imp}}^2}}, \tag{1.4}$$

where $\pm Ze$ is the impurity charge, $\varepsilon$ is the dielectric constant ($4\pi\varepsilon \simeq 12.9$), and $d_{\text{imp}}$ is the distance from the layer to the impurity in the bulk.

MacDonald *et al.*[6] derived qualitatively the behavior (1.2) by studying an impurity effect on the activation energy of magnetorotons[7] in a perturbation theory, though their predicted value for $\Delta_{\text{gap}}$ becomes negative and is physically unacceptable. Furthermore, it is not clear how magnetorotons (electrically neutral objects) would explain magnetotransport experiments. See also Ref. 8 for a related analysis based on magnetorotons.

We present a simple semiclassical picture for a pair creation of quasihole and quasielectron. Arguing that it occurs to minimize the impurity term (1.3), we derive the formula (1.2) with

$$\Gamma_{\text{offset}} \simeq e^*|V_{\text{imp}}(0)|, \tag{1.5}$$

where a quasiparticle is assumed to be pointlike. Here, $e^*$ is the electric charge of quasiholes, $e^* = e/m$ at the filling factor $\nu = n/m$ with odd $m$ ($m = 1,3,5,\ldots$). We also argue that thermal activation is aided by a tunneling process at sufficiently low temperature and at strong magnetic field. The Arrhenius formula (1.1) is generalized as

$$\rho_{xx} \propto \exp\left(-\frac{\Delta_{\text{gap}}}{2k_B T}\right)\left[1 + e^{-S_{\text{tunnel}}/\hbar}\exp\left(\frac{A^*}{k_B T}\right)\right]^{1/2}. \tag{1.6}$$

This formula contains two energy scales $\Delta_{\text{gap}}$ and $A^*$, and $S_{\text{tunnel}}$ expresses all the effects due to the tunneling process.

This paper is composed as follows. In Sec. II, we summarize theoretical values of gap energies at various filling factors. We then compare them with typical experimental data based on the formula (1.2). In Sec. III, we discuss semiclassically the dispersion relation of a neutral excitation mode made of a quasihole-quasielectron pair. In Sec. IV, analyzing thermal creations of quasiparticle pairs, we derive the Arrhenius formula (1.1) and the generalized formula (1.6) together with Eqs. (1.2) and (1.5). We show that the generalized formula gives an excellent fitting of the resistivity $\rho_{xx}$ for typical data.

 

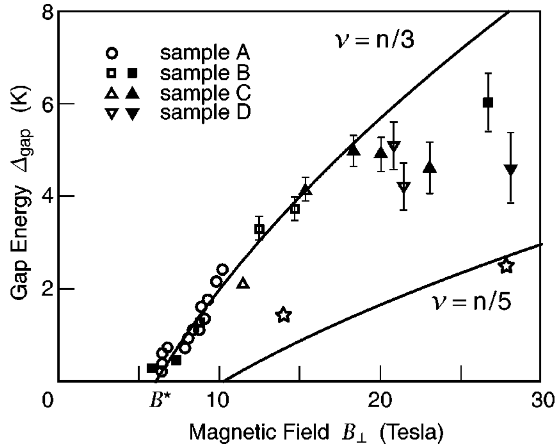


FIG. 1. A theoretical result is compared with experimental data for the activation energy at 1/3, 2/3, 4/3, 5/3, and 2/5, 3/5 (star symbols). The data are taken from Boebinger *et al.* (Ref. 16). Theoretical curves are based on vortex-excitation formulas (2.3). The impurity potential $V_{\rm imp}(0)$ is taken common for all samples. See also Fig. 6 for the data at $B_\perp = 8.9$ T and $B_\perp = 20.9$ T.

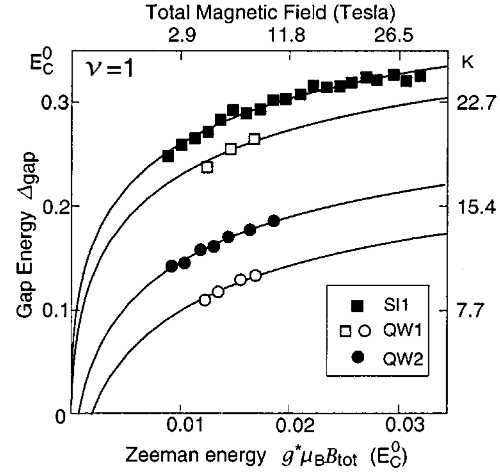


FIG. 2. A theoretical result is compared with experimental data at $\nu=1$. The data are taken from Schmeller *et al.* (Ref. 18). The theoretical curve is based on the skyrmion-excitation formula (2.7). The offset $\Gamma_{\rm offset}$ increases as the mobility decreases. There are two curves for one sample (QW1) but with different mobilities. The mobility changes when electrons are pushed against the wall by a bias voltage, as will result in the increase of the Coulomb energy $\Gamma_{\rm offset}$ made by impurities.

## II. GAP ENERGIES

Vortices are quasiparticles[3] in fractional QH states. They have electric charges $\pm e^*$ at $\nu = n/m$, where $e^* = e/m$. The excitation energy of a vortex pair is solely made of the Coulomb energy

$$\Delta_{qh} + \Delta_{qe} = \alpha_{\rm pair}^{1/m} E_C^0, \tag{2.1}$$

where $E_C^0 = e^2/4\pi\varepsilon l_B$ is the energy unit. It is expected that

$$\alpha_{\rm pair}^{1/m} = \frac{1}{m^2}\alpha_{\rm pair}. \tag{2.2}$$

There are several independent estimations on the numerical parameter $\alpha_{\rm pair}^{1/3}$: $\alpha_{\rm pair}^{1/3} \simeq 0.056$ according to Laughlin,[9] $\alpha_{\rm pair}^{1/3} \simeq 0.053$ according to Chakraborty,[10] $\alpha_{\rm pair}^{1/3} \simeq 0.094$ according to Morf and Halperin,[11] $\alpha_{\rm pair}^{1/3} \simeq 0.105$ according to Haldane and Rezayi,[12] $\alpha_{\rm pair}^{1/3} \simeq 0.106$ according to Girvin, MacDonald, and Platzman,[13] $\alpha_{\rm pair}^{1/3} \simeq 0.065$ according to our semiclassical analysis.[14] Actual samples have finite layer widths, which may decrease considerably the Coulomb energies.[15] We treat $\alpha_{\rm pair}^{1/m}$ as a phenomenological parameter to analyze experimental data. As we derive in Sec. IV, the gap energies at $\nu = n/3$ and $\nu = n/5$ are given by

$$\Delta_{\rm gap}^{1/3} = \alpha_{\rm pair}^{1/3} E_C^0 - \frac{e}{3}|V_{\rm imp}(0)|, \tag{2.3a}$$

$$\Delta_{\rm gap}^{1/5} = \alpha_{\rm pair}^{1/5} E_C^0 - \frac{e}{5}|V_{\rm imp}(0)|. \tag{2.3b}$$

We have fitted typical data due to Boebinger *et al.*[16] based on these formulas in Fig. 1. We have used $\alpha_{\rm pair}^{1/3} = 0.50/3^2 \simeq 0.056$ and $\alpha_{\rm pair}^{1/5} = 0.64/5^2 \simeq 0.026$, where the relation (2.2) holds approximately. We have taken the impurity potential $V_{\rm imp}(0)$ common to all samples, $e|V_{\rm imp}(0)| = 20.4$ K. It would imply $Z/d_{\rm imp} \simeq 1/650$ (Å) if the impurity potential (1.4) is assumed.

The $\nu=1$ QH state is a QH ferromagnet, where skyrmions[4] are excited. The excitation energy consists of the exchange energy $E_{\rm ex}$, the Coulomb self-energy $E_C$, and the Zeeman energy $E_Z$,

$$E_{\rm ex} = \sqrt{\frac{\pi}{32}} E_C^0, \tag{2.4}$$

$$E_c = \frac{\beta}{2\kappa} E_C^0, \tag{2.5}$$

$$E_Z = 2\tilde{g}\kappa^2 \ln\left(\frac{\sqrt{2\pi}}{32\tilde{g}} + 1\right) E_C^0, \tag{2.6}$$

where $\tilde{g} = g^*\mu_B B/E_C^0$. The skyrmion size $\kappa$ is determined to minimize the total energy. The resulting gap energy[14] is

$$\Delta_{\rm gap}^1 \simeq 2\left(\sqrt{\frac{\pi}{32}} + \frac{3\beta}{4\kappa}\right) E_C^0 - e|V_{\rm imp}(0)|, \tag{2.7}$$

with the skyrmion size

$$\kappa = \frac{1}{2}\beta^{1/3}\left\{\tilde{g}\ln\left(\frac{\sqrt{2\pi}}{32\tilde{g}} + 1\right)\right\}^{-1/3}. \tag{2.8}$$

The parameter $\beta$ measures the strength of the Coulomb energy, and we have $\beta = 3\pi^2/64$ for a large skyrmion. However, an actual skyrmion size is small, $\kappa \simeq 1$. Furthermore, there will be a modification due to a finite thickness of the layer.[17] We treat $\beta$ as a phenomenological parameter. We have used $\beta = 0.24$ to fit typical data[18] in Fig. 2. The potential $V_{\rm imp}(0)$ is taken phenomenologically as $e|V_{\rm imp}(0)| \simeq 40-50$ K. It would imply $Z/d_{\rm imp} \simeq 2.5/650$ (Å) in Eq. (1.4).

Electrons are excited to a higher Landau level and spins are flipped at $\nu = 2,4,\ldots$ . The gap energy is

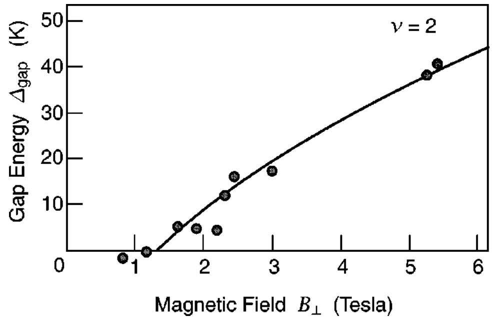


FIG. 3. A theoretical result is compared with experimental data at $\nu=2$. The data are taken from Usher *et al.* (Ref. 20). The points plotted are those obtained by subtracting $\hbar\omega_c - g^*\mu_B B$ from the observed ones. The theoretical curve is just for the Coulomb-energy part and the impurity term in the electron-excitation formula (2.9). The impurity potential $V_{imp}(0)$ is taken common for all samples.

$$\Delta^\nu_{gap} = \hbar\omega_c + \alpha^\nu_{pair} E^0_C - g^*\mu_B B - e|V_{imp}(0)|, \tag{2.9}$$

where $\alpha^\nu_{pair}$ is the Coulomb energy associated with the electron-quasihole excitation. It has been estimated that $\alpha^2_{pair} = \sqrt{\pi/8} \simeq 0.63$ by Kallin and Halperin.[19] We have used $\alpha^2_{pair} = 0.65$ to fit typical data due to Usher *et al.*[20] in Fig. 3. The potential $V_{imp}(0)$ is taken phenomenologically as $e|V_{imp}(0)| \simeq 37.7$ K. It would imply $Z/d_{imp} \simeq 2/650$ (Å) in Eq. (1.4).

## III. DISPERSION RELATION

Thermal fluctuation activates a quasielectron out of the ground state, leaving behind a quasihole. They are created as electrically neutral objects. Having charges $\pm e^*$ in the magnetic field $B_\perp$, with $e^* = e/m$ at $\nu = n/m$, they feel the Coulomb attractive force as well as the Lorentz force. We examine semiclassically the condition that these two forces are balanced.[19,21] Let $V_{pair}(r)$ be the potential energy of the quasiparticle pair with a separation $r$: The attractive force is $\partial V_{pair}(r)/\partial r$. The Lorentz force is $e^* v B$ when the pair moves parallel to the $x$ axis with velocity $v$. They are balanced when

$$\frac{\partial V_{pair}(r)}{\partial r} = e^* v B. \tag{3.1}$$

On the other hand the velocity is given by

$$v = \frac{1}{\hbar}\frac{\partial E_{pair}(\boldsymbol{k})}{\partial k} \tag{3.2}$$

in terms of the dispersion relation $E_{pair}(\boldsymbol{k})$ with $\boldsymbol{k} = (k,0)$. The total energy $E_{pair}$ is different from the potential energy $V_{pair}$ by the kinetic energy, but it is quenched by the lowest Landau level projection.[7] Then, we may equate

$$E_{pair} = V_{pair}. \tag{3.3}$$

It follows from Eqs. (3.1), (3.2), and (3.3) that

$$r = mkl^2_B. \tag{3.4}$$

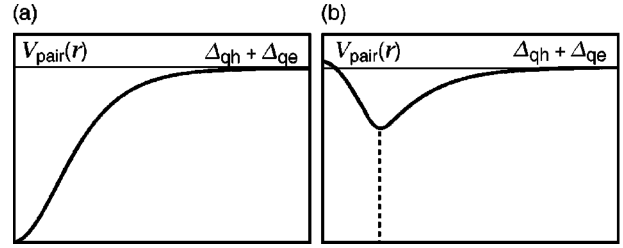


FIG. 4. The potential energy $V_{pair}(r)$ of a quasihole-quasielectron pair is illustrated. It may be regarded as the dispersion relation of a neutral excitation with use of $r = mkl^2_B$, where the wave vector is given by $(k,0)$. (a) The dispersion relation has a gapless mode, which is the case in QH ferromagnets. (b) The dispersion relation may have a minimum point (at $r = r_m$) describing a magnetoroton, as occurs when a short range repulsive interaction acts between a quasihole and a quasielectron.

The dispersion relation $E_{pair}(\boldsymbol{k})$ of a neutral excitation is obtainable from the potential energy $V_{pair}(r)$ with use of this relation. This semiclassical picture is easily justified by a quantum-mechanical analysis.

The potential energy $V_{pair}(r)$ may be approximated by

$$V_{pair}(r) \simeq \Delta_{qh} + \Delta_{qe} - \frac{e^{*2}}{4\pi\varepsilon r}, \tag{3.5}$$

for $r \gg l_B$. However, this is a poor approximation for small separation. Indeed, according to this formula, $V_{pair}(r)$ becomes negative for sufficiently small $r$. It is necessary to take into account an overlap of quasiparticles. Quasiparticles are extended objects, vortices, and skyrmions, described by classical fields. We place a quasihole at the origin $(\boldsymbol{x}=0)$ and a quasielectron at the point $(\boldsymbol{x}=\boldsymbol{r})$. The density modulation is $\varrho_{pair}(\boldsymbol{x};\boldsymbol{r}) = \varrho_{qh}(\boldsymbol{x}) + \varrho_{qe}(\boldsymbol{x}-\boldsymbol{r})$. The Coulomb energy is

$$V_{pair}(r) = \frac{1}{2}\frac{e^2}{4\pi\varepsilon}\int d^2x d^2x' \frac{\varrho_{pair}(\boldsymbol{x};\boldsymbol{r})\varrho_{pair}(\boldsymbol{x}';\boldsymbol{r})}{|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{3.6}$$

It depends only on the distance $r$ between two quasiparticles provided they have cylindrical symmetric configurations. This is the excitation energy of a quasiparticle pair apart from a possible Zeeman energy. It is reduced to Eq. (3.5) when two quasiparticles are sufficiently apart. It is a dynamical problem how $\varrho_{pair}(\boldsymbol{x};\boldsymbol{r})$ behaves as $\boldsymbol{r}\to 0$. We have $V_{pair}(0) = 0$ if the quasihole density is precisely cancelled by the quasielectron density, $\varrho_{qh}(\boldsymbol{x}) = -\varrho_{qe}(\boldsymbol{x})$, as illustrated in Fig. 4(a). It implies the existence of a gapless mode in the dispersion relation $E_{pair}(k)$ via the relations (3.3) and (3.4).

If the spin degree of freedom is frozen, there exists no cancellation since the QH state is incompressible. Otherwise, a gapless mode which can only exist in the density fluctuation would lead to compressibility. Hence, it must be that $\varrho_{pair}(\boldsymbol{x};\boldsymbol{r}) \neq 0$ at $\boldsymbol{r}=0$. When there exists a short-range repulsive interaction between a vortex and an antivortex, the energy $E_{pair}(r)$ may have a minimum describing a magnetoroton at $r = r_m \simeq l_B$ as in Fig. 4(b).

In QH ferromagnets, on the contrary, the cancellation occurs because the dispersion relation contains a gapless mode,[22] as illustrated in Fig. 4(a). A gapless mode develops in the spin fluctuation, and hence QH ferromagnets are incompressible in spite of the existence of a gapless mode. By neglecting the Zeeman energy, the perturbative dispersion relation is given by[23,24]

$$E_{\text{pair}}(\boldsymbol{k}) = \frac{2\rho_s}{\rho_0}\boldsymbol{k}^2, \tag{3.7}$$

as implies

$$V_{\text{pair}}(r) = \frac{2\rho_s}{m^2\rho_0 l_B^4} r^2 \quad \text{at} \quad r \simeq 0, \tag{3.8}$$

where $\rho_s$ is the spin stiffness $\rho_s = \nu e^2/16\sqrt{2\pi}(4\pi\varepsilon) l_B$.

## IV. THERMAL ACTIVATION

We study thermal creations of quasiparticle pairs in QH ferromagnets with a gapless dispersion relation [Fig. 4(a)]. We consider two cases. First we analyze a purely thermal process. We then include a tunneling process. As we shall see, it is obvious that our analysis is applicable also to the system where quasiparticles are vortices without gapless modes [Fig. 4(b)]. It is applicable also to certain integer QH systems, say at $\nu = 2,4,\ldots$, where there are no quasielectrons: Here, electrons are activated with quasiholes left behind.

### A. Thermal process

At finite temperature $T$, thermal spin fluctuation occurs with the rate proportional to the Boltzmann factor $\exp[-E_{\text{pair}}(\boldsymbol{k})/k_B T]$ with Eq. (3.7). A well-separated quasiparticle pair $(r \to \infty)$ is created with rate $\exp[-(\Delta_{qh} + \Delta_{qe})/k_B T]$, where use was made of Eq. (3.5).

Thermal activation of quasiparticles is greatly enhanced in the presence of impurities bearing electric charges (Fig. 5). An impurity creates a Coulomb potential around it. For definiteness we assume that it has a positive charge. As we have seen in Sec. III, thermal spin fluctuation is regarded as a creation of a quasihole-quasielectron pair. The pair may be broken near an impurity because a quasielectron is attracted by the Coulomb force due to the impurity and a quasihole is expelled by it. The activation energy is given by Eq. (1.2), where $\Gamma_{\text{offset}}$ is the energy gain (1.5) when the quasiparticle is trapped by a charged impurity [Fig. 5(b)]. When a quasielectron is trapped by an impurity, only a quasihole moves and contributes to an Ohmic current [Fig. 5(a)].

We estimate the number density of quasiparticles in thermal equilibrium at temperature $T$. On one hand, activated from the ground state near an impurity, a quasiparticle is transferred to the center of the impurity [Fig. 5(b)]. The height of the potential barrier to jump over is $A^* + \Delta_{\text{gap}}$. The transition rate is

$$R_{\uparrow} = c\rho_0 \exp\left(-\frac{A^* + \Delta_{\text{gap}}}{k_B T}\right), \tag{4.1}$$

where $c$ is a constant depending on the density of impurities. On the other hand, recombined with a quasihole, a quasielectron is transferred back to the ground state. The height of the potential barrier to jump over is $A^*$. The transition rate is

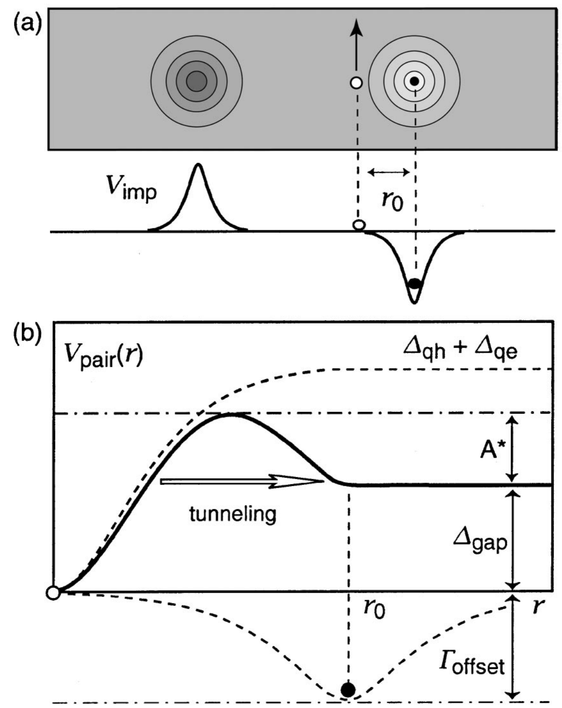


FIG. 5. (a) An impurity creates a Coulomb potential around it. It enhances thermal activation of a quasiparticle-quasihole pair. A quasielectron is attracted and trapped by the Coulomb potential due to a positive impurity charge, while a quasihole is expelled by it. A quasihole contributes to an Ohmic current. (b) The creation energy $E_{\text{pair}}(r)$ of a quasiparticle pair is considerably reduced by the Coulomb potential due to an impurity charge. The gap energy of one pair is given by $\Delta_{\text{gap}} \simeq \Delta_{qh} + \Delta_{qe} - \Gamma_{\text{offset}}$, where $\Gamma_{\text{offset}}$ is the energy gain. The effective range of an impurity is denoted by $r_0$.

$$R_{\downarrow} = n_{qh} n_{qe} \sigma_{\text{pair}} \exp\left(-\frac{A^*}{k_B T}\right), \tag{4.2}$$

where $n_{qh}$ and $n_{qe}$ are the number densities of quasiholes and quasielectrons, $\sigma_{\text{pair}}$ is a certain cross section. When the system is at thermal equilibrium there exists a detailed balance between these two transitions, $R_{\uparrow} = R_{\downarrow}$, from which we derive

$$n_{qh} n_{qe} = \frac{c\rho_0}{\sigma_{\text{pair}}} \exp\left(-\frac{\Delta_{\text{gap}}}{k_B T}\right). \tag{4.3}$$

Since quasiholes and quasiparticles are activated in pairs, we find

$$n_{qh} = n_{qe} = n_0 \exp\left(-\frac{\Delta_{\text{gap}}}{2k_B T}\right), \tag{4.4}$$

at the center of the plateau, where $n_0 = \sqrt{c\rho_0/\sigma_{\text{pair}}}$. The Ohmic current is given by the formula (1.1) with (1.2) since it is proportional to the number density of quasiparticles.

The QH system is unstable when the gap energy $\Delta_{\text{gap}}$ becomes negative. QH states break down when

$$\Delta_{qh} + \Delta_{qe} < \Gamma_{\text{offset}}. \tag{4.5}$$

The excitation energy of the pair decreases as the magnetic field decreases. The critical magnetic field is derived from Eq. (4.5),

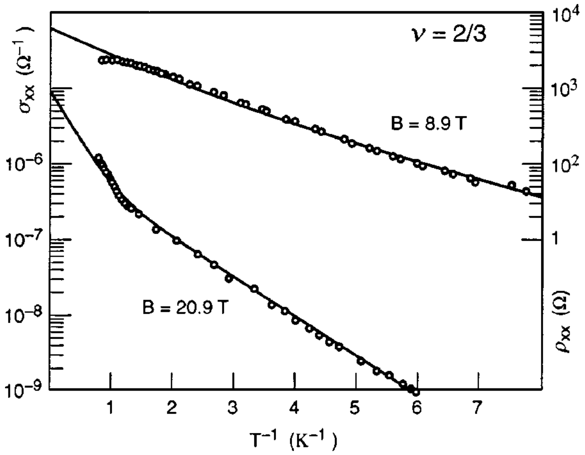


FIG. 6. Temperature dependence of the minimum of the longitudinal conductance $\sigma_{xx}$ and resistance $\rho_{xx}$ at $\nu=2/3$. The data are taken from Boebinger *et al.* (Ref. 16). Theoretical curves are given by the generalized formula (4.9) with vortex excitation (2.3a). See also Fig. 1.

$$B_{\perp}^{*}=m^{2}\frac{16\pi^{2}\varepsilon^{2}\hbar}{e^{3}\alpha_{\rm pair}^{1/m}}|V_{\rm imp}(0)|^{2}, \tag{4.6}$$

for vortex activation (2.1) at $\nu=n/m$. QH states do not exist for $B<B_{\perp}^{*}$. This is consistent with typical data (Fig. 1).

### B. Tunneling process

We have so far considered a purely thermal process of pair creation. However, a tunneling process enhances thermal activation at sufficiently low temperature. When a pair of quasiparticles acquires an energy $\Delta_{\rm gap}$ thermally, it can tunnel across the potential barrier with height $A^*$ as in Fig. 5. The transition rate is

$$R_{\uparrow}^{\rm tunnel}=c\rho_{0}e^{-S_{\rm tunnel}/\hbar}\exp\left(-\frac{\Delta_{\rm gap}}{k_{B}T}\right), \tag{4.7}$$

where $S_{\rm tunnel}$ expresses all the effects due to the tunneling process. It depends on the height $A^*$ and the range $r_0$. It is obvious the transition rate (4.7) dominates the rate (4.1) as $T\to 0$. The rate of recombination process is still given by Eq. (4.2), because of the plateau in the potential for $r>r_0$ in Fig. 5. The detailed balance implies

$$R_{\uparrow}+R_{\uparrow}^{\rm tunnel}=R_{\downarrow}, \tag{4.8}$$

with Eqs. (4.1), (4.2), and (4.7), from which we obtain

$$n_{qh}=n_{qe}=n_{0}\exp\left(-\frac{\Delta_{\rm gap}}{2k_{B}T}\right)\left[1+e^{-S_{\rm tunnel}/\hbar}\exp\left(\frac{A^{*}}{k_{B}T}\right)\right]^{1/2}. \tag{4.9}$$

This formula contains two energy scales $\Delta_{\rm gap}$ and $A^*$. We have fitted typical data due to Boebinger *et al.*[16] in Fig. 6. In so doing we have determined $\Delta_{\rm gap}$ by our theoretical formula (2.3a), $\Delta_{\rm gap}=\Delta_{\rm gap}^{1/3}$ with use of $\Gamma_{\rm offset}=\frac{1}{3}e|V_{\rm imp}(0)|=6.8$ K. The theoretical curve (for $B=8.9$ T) is obtained by using $\Delta_{\rm gap}\simeq 1.7$ K, $A^*\simeq 0.69$ K, and $S_{\rm tunnel}/\hbar\simeq 2.0$. The theoretical curve (for $B=20.9$ T) is obtained by using $\Delta_{\rm gap}\simeq 6.1$ K, $A^*\simeq 3.7$ K, and $S_{\rm tunnel}/\hbar\simeq 4.0$. The tunneling process makes an important contribution at strong magnetic field because $A^*$ becomes larger. They explain quite well the temperature dependence of the minimum of the longitudinal resistance $\rho_{xx}$.

## V. DISCUSSIONS

We have analyzed semiclassically a mechanism of thermal and tunneling pair creations of quasiparticles in the presence of impurities. Our formulas (1.2) with (1.5) account for experimental data quite well. The impurity effect is summarized into the parameter $Z/d_{\rm imp}$ in Eq. (1.4). We list characteristic features at various filling factors.

(A) Experimental data by Boebinger *et al.*[16] at fractional filling factors are explained by excitation of vortices with $Z/d_{\rm imp}\simeq 1/650$ (Å). Activation energy is rather insensitive to samples.

(B) Experimental data by Usher *et al.*[20] at $\nu=2$ are explained by excitation of electrons into higher Landau level with $Z/d_{\rm imp}\simeq 2/650$ (Å). Activation energy is rather insensitive to samples.

(C) Experimental data by Schmeller *et al.*[18] at $\nu=1$ are explained by excitation of skyrmions with $Z/d_{\rm imp}\simeq 2.5/650$ (Å). Activation energy is sensitive to sample movilities.

These numbers ($Z=1\sim 3$ and $d_{\rm imp}\simeq 650$ Å) are quite reasonable. If we take the results seriously, it seems that only skyrmions are sensible to sample movilities. This might be related to the fact that the skyrmion has no intrinsic size. However, we wish to urge caution. First of all, our semiclassical analysis is the first order approximation to the problem, and further improvements will be necessary. For instance, we have assumed that quasiparticles are pointlike objects to derive the gap-energy formula (1.2) with (1.5). It is clear in Fig. 5 that the formula should be modified when the overlap of quasihole and quasielectron is not negligible at their dissociation range $r_0$. Second, experimental data are taken from different sources at different dates. It is necessary to make careful experiments by using a single sample to determine the parameter $Z/d_{\rm imp}$ at various filling factors. We wish to propose such experiments.

We have pointed out the importance of tunneling process in thermal activation at sufficiently small temperature and at strong magnetic field. It is remarkable that the temperature dependence of the minimum of the longitudinal resistance $\rho_{xx}$ is fitted excellently by our formula (1.6) over a wide range of temperature. This formula is very different from any of the previously proposed ones.[25] QH systems may acquire additional interest from the importance of tunneling process. We would like to make a quantitative analysis of this tunneling process in a future report.

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