In-plane magnetic-field effect on transport properties of the chiral edge state in a quasi-three-dimensional quantum well structure

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The transport properties of a quasi-three-dimensional, 200-layer quantum-well structure are investigated at integer filling in the quantum Hall state, concomitant with the chiral edge state condition. We find that the transverse magnetoresistance R_{xx}, the Hall resistance R_{xy}, and the vertical resistance R_{zz} all follow a similar behavior with *both* temperature and in-plane magnetic field. A general characteristic of the influence of increasing in-plane field B_{in} is that the quantization condition first improves, but above a critical value B_{in}^{C}, the quantization is systematically removed. We consider the interplay of the chiral edge state transport and the bulk (quantum Hall) transport properties. This mechanism may arise from the competition of the cyclotron energy with the superlattice band-structure energies. A comparison of the results with existing theories of the chiral edge state transport with in-plane field is also discussed. [S0163-1829(99)10735-5]

INTRODUCTION

The integer quantum Hall effect has been observed in many quasi-three-dimensional (Q3D) structures,[1–5] where the interlayer tunneling bandwidth is much smaller than the 2D quantum Hall gap $E_g=\hbar\omega_c$. In such materials, in a quantum Hall (QH) state, the edge of the sample is enveloped by a sheath of current-carrying chiral edge states, which is the 2D extension of the 1D states at the edge of a single layer QH fluid. These chiral edge states have been predicted theoretically,[6,7] and confirmed experimentally.[2] A curious property of these Q3D systems at integer filling is the observation of a very similar temperature dependence of the in-plane resistance at the center of the QH state (R_{xx}^{min}) and the inverse of the vertical transport resistance ($G_{zz}\sim 1/R_{zz}$). In particular, with magnetic field perpendicular to the layers, both follow an activated behavior (with a very small gap) at higher temperatures, typically above 0.3 K, and a Coulomb gap behavior at lower temperatures where both R_{xx}^{min} and G_{zz} approach some asymptotic (residual) value. This is the case for both the systems discussed here,[8] and also in an independent study.[2] The motivation for the present work has been to try to understand the origin of this universal temperature dependence, and to consider the influence of finite in-plane field on the properties of the transport tensor. This second point is particularly important since there are theoretical predictions[9] for the in-plane field dependence of the chiral surface transport.

BRIEF REVIEW OF OUR RESULTS

The main results of the present work are the observation that in tilted magnetic fields, the quantization first improves at integer filling (as seen by the reduction in dissipation in R_{xx}, the enhancement of R_{zz}, and the broadening of the R_{xy} Hall plateau), followed by the gradual disappearance of the quantum Hall state above a characteristic in-plane magnetic field B_{in}^{C}. We further note that the temperature dependence of all measured components of the transport tensor also follow the same behavior. The results suggest an interplay between the bulk quantum Hall state and the chiral edge state systems, and provide a test of the theoretical models for the transport properties of the chiral edge state.

BRIEF DESCRIPTION OF OUR EXPERIMENT

To explore both the bulk quantum Hall and chiral edge state behavior in a quasi-three-dimensional system, we employed a 200-layer GaAs/$Al_xGa_{1-x}As$ quantum-well structure. The samples used were prepared by molecular-beam epitaxy, and involved 200 periods of 19-nm GaAs quantum wells separated by Si δ-center-doped ($n_d=4.0\times10^{11}\,\mathrm{cm}^{-2}$) $Al_{0.1}Ga_{0.9}As$ barriers of thickness 4 nm. To

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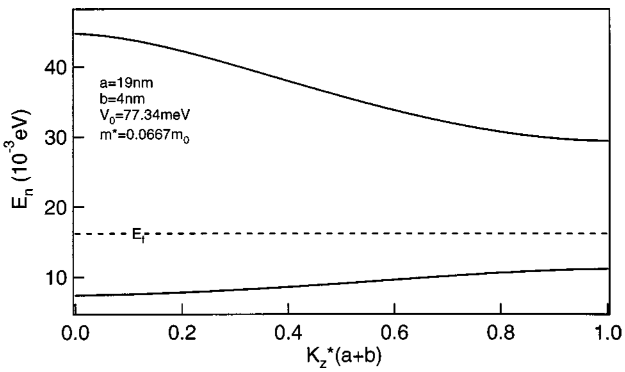

FIG. 1. Kronig-Penny model for the band structure of the 200-layer quantum-well system.

prevent layer depletion and to offset the surface pinning potential, a cap layer and a sequence of $Al_{0.1}Ga_{0.9}As$ doped layers were grown.[10] The relevant parameters for this structure are the interlayer spacing $c=a+b=23$ nm, where $a=19$ nm and $b=4$ nm are the widths of the well and barrier, respectively, barrier height $V_0=77$ meV calculated using the Schrödinger equation with doping density of Possion distribution; and low-field mobility $\mu_H=6562\ \mathrm{cm}^2/\mathrm{V\,s}$. The Kronig-Penny model band structure is shown in Fig. 1.

A conventional Hall bar configuration was used in measuring the in-plane magnetoresistance R_{xx} and R_{xy}, as shown in Figs. 2 and 3. To measure the vertical R_{zz} transport, sections from the same sample were processed by a vertical etching process[10] to provide a mesalike structure as shown in Fig. 4. We note that R_{zz} was a *four-terminal* measurement, and that independent two-terminal measurements[11,12] did not show any mixing of R_{xx} and R_{xy} into the R_{zz} signal. Measurements were carried out with standard ac lock-in methods with a current of 50 nA/layer. No evidence for heating or hot-electron gas effects were observed. For all measurements shown here, a rotating platform immersed in the mixture of a dilution refrigerator, associated with a superconducing magnet was employed.

The evidence that the Q3D sample exhibited complete quantization is shown in Fig. 3. At 8.7 T, concomitant with R_{xx} minima (R_{xx}^{min}) in Fig. 2, and the R_{zz} maximum (R_{zz}^{max}) there is a well-developed Hall plateau in R_{xy}, which saturates at $64.45\pm0.010\ \Omega$. This value corresponds to a quantization of $R_{xy}=\frac{1}{200}(\hbar/2e^2)$ where *all* 200 layers participate, and demonstrates a spin-unpolarized integer QH effect at the filling factor $\nu=2$. This demonstrates that for the Q3D sample studied here, with the field perpendicular to the layers, the Fermi energy lies in a gap, and that none of the layers are depleted.

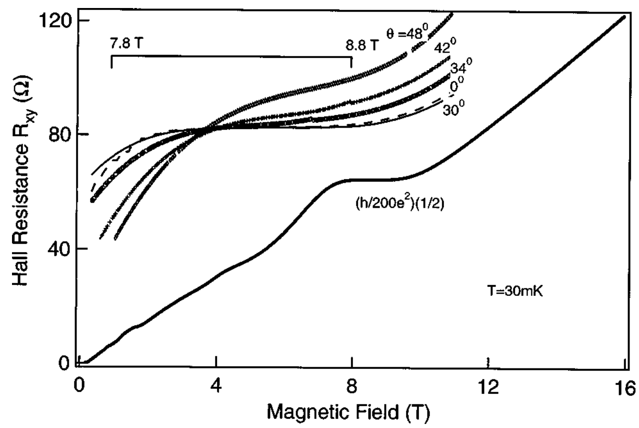

FIG. 3. Field dependence of Hall resistance R_{xy} at 30 mK with zero titled angle. The inset shows the R_{xy} $\nu=2$ plateaus at different angles.

RESULTS

The behavior of the Q3D studied at low temperatures (30 mK) is shown in Figs. 2–4 for the three transport tensor components R_{xx}, R_{xy}, and R_{zz}, respectively. The full curves represent the cases where the magnetic field was perpendicular to the layers ($\theta=0$). The inset curves in Figs. 2–4 represent the details of the angular dependence of the transport components in the vicinity of integer filling $\nu=2$. Here the angle represents the angle between the magnetic field and the normal to the conducting layers. Starting with Fig. 2 we see that at $\nu=2$ there is a minimum in the dissipation near 30°, i.e., R_{xx}^{min} approaches the lowest value at this angle, and at higher angles the dissipation increases. In Fig. 3 the width of the $\nu=2$ Hall plateau is largest at 30° then rapidly decreases with increasing angle. And finally, in Fig. 4 the maximum of

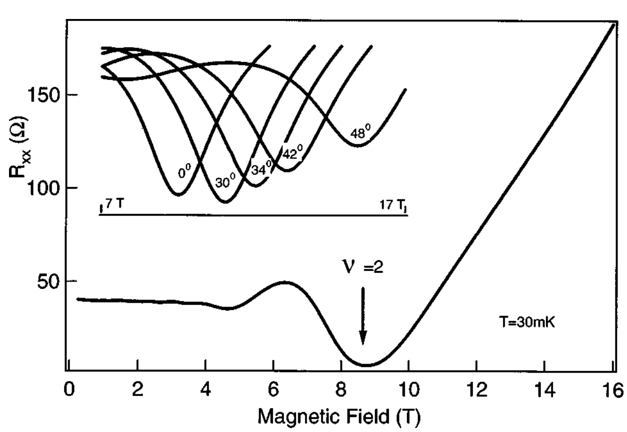

FIG. 2. Field dependence of magnetoresistance R_{xx} at 30 mK with zero titled angle. The inset shows the R_{xx} $\nu=2$ minimum at different angles.

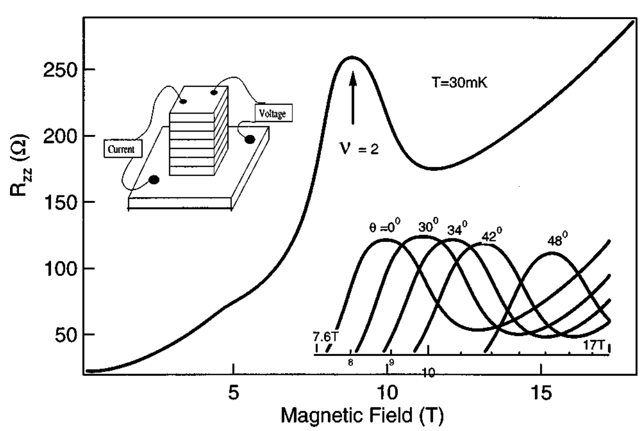

FIG. 4. Field dependence of vertical magnetoresistance R_{zz} at 30 mK with zero titled angle. The inset figure illustrates the R_{zz} $\nu=2$ peaks at different angles. The mesa configuration is also indicated.

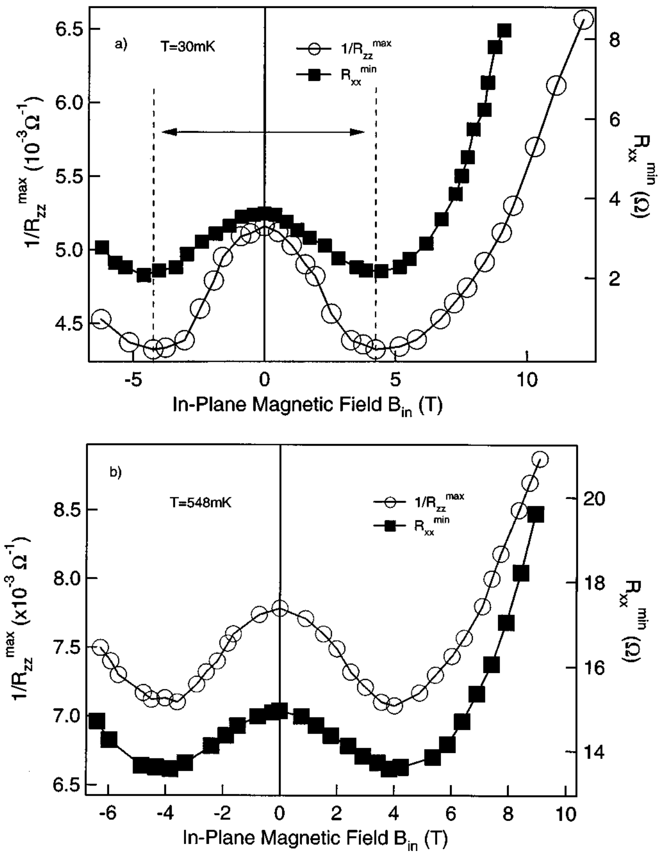

FIG. 5. (a) In-plane magnetic field dependence of $1/R_{zz}^{\max}$ and $R_{xx}^{\min}$ at 30 mK. There is an optimal value of the in-plane field $B_{\parallel}$. Here, $1/R_{zz}^{\max}$ and $R_{xx}^{\min}$ are plotted on different scales. With the in-plane field increasing from zero to 8 T, $R_{xx}^{\min}$ changes by almost an order of magnitude, but the change in $1/R_{zz}^{\max}$ is very small by comparison. (b) In-plane magnetic field dependence of $1/R_{zz}^{\max}$ and $R_{xx}^{\min}$ at 548 mK.

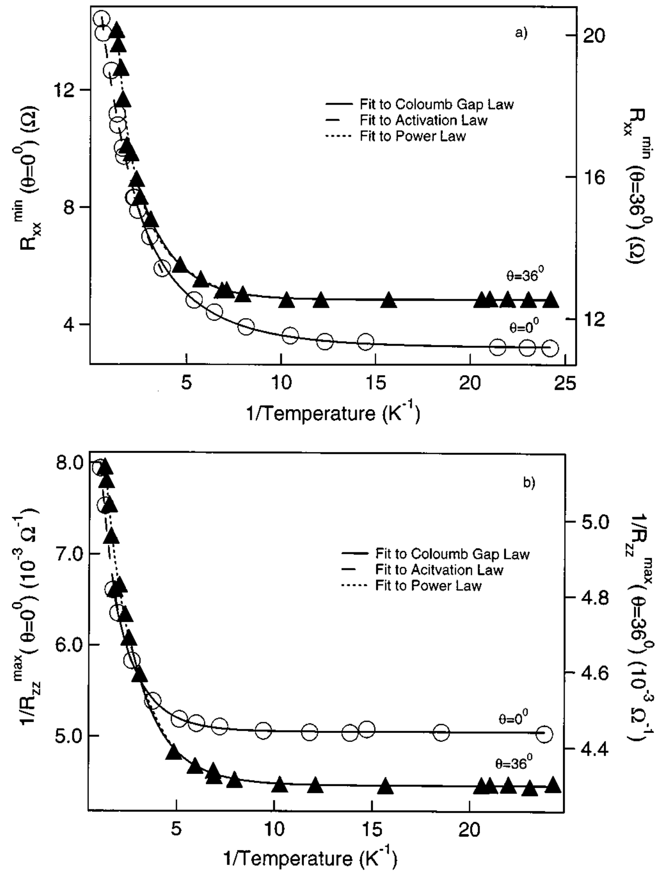

FIG. 6. (a) Temperature dependence of $R_{xx}^{\min}$ at different titled angles. (▲, $\theta=0°$, left-hand scale; ○, $\theta=36°$, right-hand scale). The solid lines are low-temperature (below 0.3 K) fits to the Coulomb gap form. Above 0.3 K, the short dashed line for $\theta=0°$ is the fit to the activated form, and the long dashed line for $\theta=36°$ is the fit to the power-law form. The parameters of the fits are as follows: Coulomb gap, $T_c=1.25$ K, $R_0=29.1$Y Ω, and $R_{\rm res}=2.28$ Ω at $\theta=0°$ and $T_c=4.82$ K, $R_0=62.2$ Ω, $R_{\rm res}=12.5$ Ω for $\theta=36°$; activation, $\Delta=0.73$ K, $R_0=15.44$ Ω at $\theta=0°$; power law, $R_0=12.3$ Ω, $R_1=6.07$ Ω at $\theta=36°$. (b) Temperature dependence of $1/R_{zz}^{\max}$ at different titled angles. Symbols and lines have the same representation as in (a). The parameters of the fits are as follows: Coulomb gap, $T_c=16$ K, $\sigma_0=0.004\ \Omega^{-1}$ and $\sigma_{\rm res}=3.7\times10^{-5}\ \Omega^{-1}$ at $\theta=0°$ and $\alpha=0.42$, $T_c=42.5$ K, $\sigma_0=0.085\ \Omega^{-1}$ and $\sigma_{\rm res}=0.0043\ \Omega^{-1}$ for $\theta=36°$; activation, $\Delta/K=2.21$, $\sigma_0=0.004\ \Omega^{-1}$ at $\theta=0°$; power law, $\sigma_0=0.0039\ \Omega^{-1}$, $\sigma_1=6.0\times10^{-5}\ \Omega^{-1}$ at $\theta=36°$.

the vertical resistance $R_{zz}^{\max}$ has a slight extremum at 30°, followed by a reduction at higher angles. In all three cases the effect of increasing angle corresponds to an initial improvement of the quantization condition, followed by a rapid removal of quantization above an optimum angle. In Fig. 5 a more detailed study of these trends is presented in the form $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$. For the case of no in-plane field, there is a clear relationship between the measured transverse resistance R_{zz} and the conductivity, $G_{zz}\sim 1/R_{zz}$. In this paper, we take the viewpoint that a variable titled field at integer filling is simply the case where the transverse field is constant, and the in-plane field varies. Hence we retain the definition that $G_{zz}\sim 1/R_{zz}$ (at $\nu=2$ filling in this case) with increasing in-plane field. Since the total field **B** must be increased to maintain the $\nu=2$ filling with increasing angle, the optimum angle (more accurately, 27°) corresponds to a perpendicular field of $\mathbf{B}_\perp=\mathbf{B}\cos(\theta)=8.7$ T and an in-plane field $\mathbf{B}_{\rm in}=\mathbf{B}\sin(\theta)=4.4$ T. We will return to the influence of the in-plane magnetic field in the Discussion.

Another striking feature of the transport properties is their temperature dependence, as is shown in Fig. 6. Here we show $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$ ($\sim G_{zz}$) temperature for $\theta=0°$, and also for $\theta=36°$, which is well above the optimum angle where the quantization is reduced. There are two temperature ranges of interest. First, at high temperatures (above 0.3 K) and $\theta=0°$, $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$ both show activated behavior ($R=R_0e^{-(\Delta/2KT)}$ and $G=G_0e^{-\Delta/2KT}$, respectively), and at low temperatures (below 0.3 K) both exhibit Coulomb-gap-like behavior ($R=R_{\rm res}+R_0e^{-\sqrt{T_c/T}}$ and $G=G_{\rm res}+G_0e^{-\sqrt{T_c/T}}$, respectively). In titled magnetic field, and above the optimum angle, both exhibit power-law (nonactivated) behavior at high temperatures ($R=R_0+R_1T^{\alpha}$ and $G=G_0+G_1T^{\alpha}$, respectively). However, the low-temperature behavior remains Coulomb-gap-like. (The parameter values are defined in the caption of Fig. 6.) The nearly identical temperature dependence of $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$ is further shown in Fig. 7(a) where they are plotted against each other for the different temperature ranges and angles. The crossover from

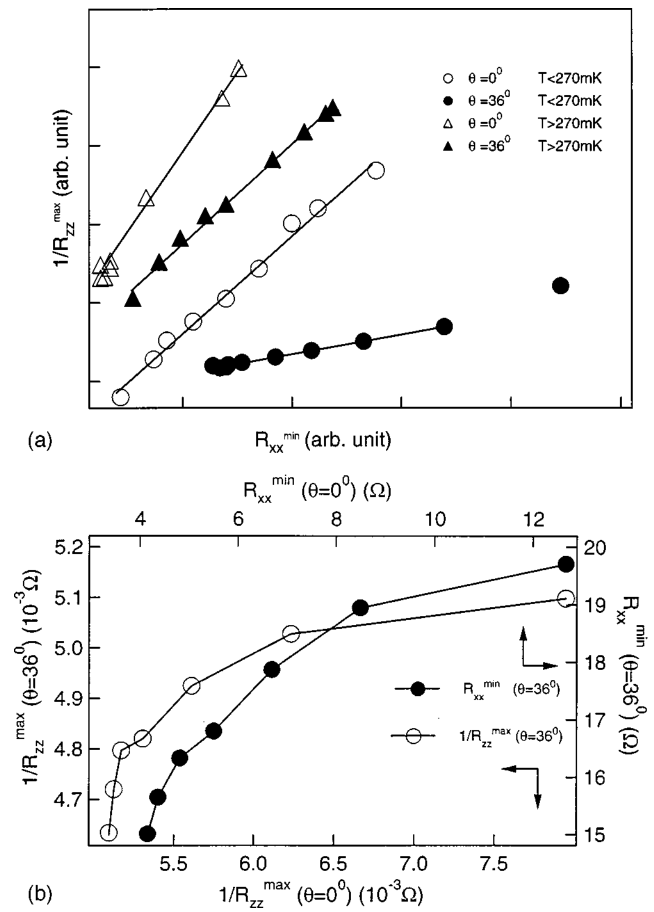

FIG. 7. (a) Demonstration of the uniform relationship between $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$. (b) Demonstration of difference in functional dependence between $\theta=0°$ activated and $\theta=36°$ power-law behavior at higher temperatures.

activated to power-law dependence with angle is more clearly demonstrated in Fig. 7(b), where, for each tensor parameter, the zero angle and tilted values are plotted. We note that in our four terminal measurements, we find no systematic evidence for universal conductance fluctuations, although extensive efforts were made to measure these effects, as reported in Ref. 2.

DISCUSSION

The results discussed above indicate an unusual coupling of the bulk quantum well transport (R_{xx} and R_{xy}) at integer filling with the corresponding behavior of the chiral edge state transport ($1/R_{zz}$). This is true both in terms of the temperature dependence, and in the angular (in-plane field) dependence. At zero angle, both $R_{xx}^{\min}$ and $1/R_{zz}^{\max}$ remain finite in the low-temperature limit, indicating the presence of dissipation in both the bulk quantum Hall state and in the chiral edge state. And, in finite in-plane field, the angular dependence of these two tensor components consistently track each other, first as the quantization improves, and then as it is removed. Indeed, the two states appear to be interconnected. The crossover from activated to power-law behavior above the optimum angle also appears in both parameters. Although the theoretically predicted geometrical relationship[6] of $G_{zz}^{\max}\approx C/L$ (where C and L are the height and circumference of a mesa-type structure, respectively) has been demonstrated in experiment,[2] the anomalous low-temperature dissipation is observed in all reported measurements of $G_{zz}^{\max}$. We note that a finite value of $G_{zz}^{\max}$ is consistent with the theory[9] (see discussion below). Given the apparent coupling of the bulk and edge state transport properties, it is not clear that the surface transport is truly decoupled from the bulk.

We next turn to a discussion of the effects of the in-plane field. Chalker and Sondhi have treated the chiral edge state conductivity $\sigma(B_\perp)$ with in-plane field[9] $B_\perp$. Their results show that R_{zz} should exhibit positive magnetoresistance, following a Drude formula, with a field scaling field $B_0=\Phi_0/al_{\rm el}$, where Φ_0 is the flux quanta, a is the lattice spacing, and $l_{\rm el}$ is the elastic length according to the relation $\sigma(B_\perp)=\sigma(B_\perp=0)/[1+(B_\perp/B_0)^2]$. To test this prediction, we have treated the data *below* the optimum angle shown in Fig. 5. We find that this description only fits a short range of in-plane field <4 T, with the fitting parameters $\sigma(B_\perp=0)=5.15\times10^{-3}\ \Omega^{-1}$, $B_0=7.15$ T, $l_{\rm el}=252$ Å. These parameters indicate, assuming that the theoretical relationship is valid in this range, that the elastic scattering length is comparable to the interlayer spacing of 230 Å, and also the magnetic length $l_{\rm el}=252$ Å. The values of these parameters are clearly outside the limits of applicability of the theory,[9] which requires $l_{\rm el}\gg l_B^2/a$. The absence of macroscopic conductance fluctuation in the present case may be an additional indication that our system is outside the limits of the theory. This would also be consistent with our lack of observation of conductance fluctuations, due to the small length scales involved.

We may further consider the behavior of the transport tensor components above the optimum angle, where the quantization is removed with increasing in-plane field. There are several treatments of multiple well structures in titled magnetic fields. Marlow and co-workers have studied cyclotron resonance in a coupled two-layer quantum well in titled magnetic fields.[13] They find wave-function hybridization and subband energy splitting, which result from the in-plane magnetic field. Although a detailed description of the removal of the quantization must be worked out theoretically, a general argument for the mechanism may be made. In reference to Fig. 1, we note that the condition for quantization, with the Fermi level in a gap, will change with increasing in-plane field due to fact that the eigenvalues of the Hamiltonian will change when the in-plane field is added. It would appear then, from our experiments, that the band structure as given in Fig. 1 starts to change significantly above an in-plane field of 4 T, and it is no longer possible to maintain the quantization condition with the Fermi level in a gap. Although more work is needed, this effect may put severe limitations on the ability to study the chiral state with large in-plane fields.

SUMMARY

In summary, we have studied the transport properties associated with the integer quantum Hall effect in a 200-layer quantum-well superlattice in the Hall-bar configuration, and also the vertical transport associated with the chiral edge state in the mesa configuration, both on the same material.

These transport properties have also been studied in tilted magnetic fields at the position of the $\nu=2$ filling has been maintained for increasing in-plane field. We find that there is a direct correlation between the in-plane magnetoresistance R_{xx} and the vertical conductivity $1/R_{zz}$, both in terms of the temperature dependence and in the angular dependence. For in-plane fields up to 4 T, the quantization improves, but for larger in-plane fields, the quantization is removed. These results suggest a strong interplay between the bulk quantum Hall states and the chiral edge states. If we assume that the chiral edge state is present, then its properties are substantially different from theoretical expectations, since there appears to be considerable dissipation that assumes a residual value in the low-temperature limit. Finally, we find that the quantization is removed at tilt angles, an effect that is most likely due to the modification of the subband structure at high in-plane magnetic fields.

ACKNOWLEDGMENTS

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