Reply to "Comment on 'Single-particle Green functions in exactly solvable models of Bose and Fermi liquids' "

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It is shown that the sea-boson model given in G.S. Setlur and Y.C. Chang, Phys. Rev. B **57**, 15 144 (1998), is capable of reproducing the four-point correlation functions of fermion operators within the random-phase approximation, although an explicit expression for the sea-boson operator is still lacking.
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We concede that the definition for the sea-boson operator given in Eq. (14) of our paper[1] is not adequate. We are grateful to Cune and Apostol[2] for pointing out this inadequacy. However, we would like to point out that the lack of an explicit expression for the sea-boson operator does not upset the entire scheme of our model. The sea-boson model as defined in our paper[1] [with the assumption that $a_{\mathbf{k}}(\mathbf{q})$ behave like bosonic operators in the random-phase approximation (RPA) limit] is still a useful scheme for obtaining many physical results. Since we are unable to find an explicit expression for the sea-boson operator in terms of the fermion operators, we can only claim that the sea-boson model presented in our paper[1] is an approximation to the interacting Fermi gas system in the RPA limit. The validity of this approximation can be checked by comparing the physical results obtained by the sea-boson model with those obtained via the conventional method. In this reply, we show that various relevant quantities are reproduced correctly by our sea-boson model.

Table I displays the correspondence between the fermion language and the sea-boson language as adopted in our model. We will show that various physical quantities in the fermion language are reproduced in the sea-boson language based on this table. It is clear at the outset that $\langle FS|c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}|FS\rangle=\theta(k_f-|\mathbf{k}|)$ is correctly reproduced in the sea-boson language, since $\langle FS|n^{\beta}(\mathbf{k})|FS\rangle=\theta(k_f-|\mathbf{k}|)$. Next we show that the off-diagonal Fermi bilinear is consistent with the diagonal one. To this end we examine the commutator (in the fermion language),

$$[c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2},c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}]=(\delta_{\mathbf{p},\mathbf{k}-\mathbf{q}/2}-\delta_{\mathbf{p},\mathbf{k}+\mathbf{q}/2})c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2}. \tag{1}$$

In the sea-boson language, we have

$$\Big(\Lambda_{\mathbf{k}}(\mathbf{q})a_{\mathbf{k}}(-\mathbf{q})+\Lambda_{\mathbf{k}}(-\mathbf{q})a^{\dagger}_{\mathbf{k}}(\mathbf{q}),n^{\beta}(\mathbf{p}) + \sum_{\mathbf{q}} a^{\dagger}_{\mathbf{p}-\mathbf{q}/2}(\mathbf{q})a_{\mathbf{p}-\mathbf{q}/2}(\mathbf{q})-\sum_{\mathbf{q}} a^{\dagger}_{\mathbf{p}+\mathbf{q}/2}(\mathbf{q})a_{\mathbf{p}+\mathbf{q}/2}(\mathbf{q})\Big)$$
$$=\Lambda_{\mathbf{k}}(\mathbf{q})\delta_{\mathbf{k},\mathbf{p}+\mathbf{q}/2}a_{\mathbf{k}}(-\mathbf{q})-\Lambda_{\mathbf{k}}(\mathbf{q})\delta_{\mathbf{k},\mathbf{p}-\mathbf{q}/2}a_{\mathbf{k}}(-\mathbf{q}) +\Lambda_{\mathbf{k}}(-\mathbf{q})\delta_{\mathbf{k},\mathbf{p}+\mathbf{q}/2}a^{\dagger}_{\mathbf{k}}(\mathbf{q})-\Lambda_{\mathbf{k}}(-\mathbf{q})\delta_{\mathbf{k},\mathbf{p}-\mathbf{q}/2}a^{\dagger}_{\mathbf{k}}(\mathbf{q})$$
$$=(\delta_{\mathbf{p},\mathbf{k}-\mathbf{q}/2}-\delta_{\mathbf{p},\mathbf{k}+\mathbf{q}/2})c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2} \tag{2}$$

as required. Let us now compute the four-point function,

$$\langle FS|c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2}c^{\dagger}_{\mathbf{k}'+\mathbf{q}'/2}c_{\mathbf{k}'-\mathbf{q}'/2}|FS\rangle =\delta_{\mathbf{k}+\mathbf{q}/2,\mathbf{k}'-\mathbf{q}'/2}\delta_{\mathbf{k}-\mathbf{q}/2,\mathbf{k}'+\mathbf{q}'/2}\theta(k_f-|\mathbf{k}+\mathbf{q}/2|) \times[1-\theta(k_f-|\mathbf{k}-\mathbf{q}/2|)]. \tag{3}$$

In the sea-boson language, the same quantity is given by

$$\langle FS|[\Lambda_{\mathbf{k}}(\mathbf{q})a_{\mathbf{k}}(-\mathbf{q})+\Lambda_{\mathbf{k}}(-\mathbf{q})a^{\dagger}_{\mathbf{k}}(\mathbf{q})][\Lambda_{\mathbf{k}'}(\mathbf{q}')a_{\mathbf{k}'}(-\mathbf{q}') +\Lambda_{\mathbf{k}'}(-\mathbf{q}')a^{\dagger}_{\mathbf{k}'}(\mathbf{q}')]|FS\rangle$$
$$=\langle FS|\Lambda^{2}_{\mathbf{k}}(\mathbf{q})|FS\rangle\delta_{\mathbf{k},\mathbf{k}'}\delta_{\mathbf{q},\mathbf{q}'}$$
$$=\theta(k_f-|\mathbf{k}+\mathbf{q}/2|)[1-\theta(k_f-|\mathbf{k}-\mathbf{q}/2|)]\delta_{\mathbf{k},\mathbf{k}'}\delta_{\mathbf{q},-\mathbf{q}'}. \tag{4}$$

Therefore, we see that this correspondence is self-consistent. In light of these computations it is perhaps not an exaggera-

TABLE I. Correspondence between the fermion language and sea-boson language.

Fermion language	Sea-boson language
$c_{\mathbf{k}}$, $c^{\dagger}_{\mathbf{k}}$ are fermions	$a_{\mathbf{k}}(\mathbf{q})$ and $a^{\dagger}_{\mathbf{k}}(\mathbf{q})$ are bosons
$\{c_{\mathbf{k}},c_{\mathbf{k}'}\}=0$; $\{c_{\mathbf{k}},c^{\dagger}_{\mathbf{k}'}\}=\delta_{\mathbf{k},\mathbf{k}'}$	$[a_{\mathbf{k}}(\mathbf{q}),a_{\mathbf{k}'}(\mathbf{q}')]=0$; $[a_{\mathbf{k}}(\mathbf{q}),a^{\dagger}_{\mathbf{k}'}(\mathbf{q}')]=\delta_{\mathbf{k},\mathbf{k}'}\delta_{\mathbf{q},\mathbf{q}'}$
$c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2}$	$\Lambda_{\mathbf{k}}(\mathbf{q})a_{\mathbf{k}}(-\mathbf{q})+\Lambda_{\mathbf{k}}(-\mathbf{q})a^{\dagger}_{\mathbf{k}}(\mathbf{q})$
$c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}$	$n^{\beta}(\mathbf{k})+\Sigma_{\mathbf{q}}a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q})a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q})$ $-\Sigma_{\mathbf{q}}a^{\dagger}_{\mathbf{k}+\mathbf{q}/2}(\mathbf{q})a_{\mathbf{k}+\mathbf{q}/2}(\mathbf{q})$
$c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}|FS\rangle=\theta(k_F-|\mathbf{k}|)|FS\rangle$	$a_{\mathbf{k}}(\mathbf{q})|FS\rangle=0$

tion to claim that the interacting quantities in our paper[1] are reproduced as well as the noninteracting ones, which are reproduced very well indeed. It is also worth pointing out that in the above calculations $|\mathbf{k}|$ is not restricted to be equal to the Fermi momentum, neither is $\mathbf{q}$ assumed to be small. Thus up to four-point functions at least, the correlation functions and the relevant commutation rules are reproduced with the correct short-wavelength behavior. Therefore, the criticism that our claims regarding short-wavelength behavior are exaggerated is not true. The only problem is that the commutation rule between two off-diagonal Fermi bilinears is recovered only up to the RPA terms. To see this we write, in the fermion language,

$$[c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2},c^{\dagger}_{\mathbf{k}'+\mathbf{q}'/2}c_{\mathbf{k}'-\mathbf{q}'/2}] = c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}'-\mathbf{q}'/2}\delta_{\mathbf{k}-\mathbf{q}/2,\mathbf{k}'+\mathbf{q}'/2} - c^{\dagger}_{\mathbf{k}'+\mathbf{q}'/2}c_{\mathbf{k}-\mathbf{q}/2}\delta_{\mathbf{k}+\mathbf{q}/2,\mathbf{k}'-\mathbf{q}'/2}. \tag{5}$$

In order to conform to the spirit of the random-phase approximation, we are required to take the c-number expectation value of whatever occurs on the right side of the commutation rules whenever it is quadratic in the sea bosons. Therefore, in the RPA sense we may rewrite the above commutation rule as follows :

$$[c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2},c^{\dagger}_{\mathbf{k}'+\mathbf{q}'/2}c_{\mathbf{k}'-\mathbf{q}'/2}] \approx (\overline{n}_{\mathbf{k}+\mathbf{q}/2}-\overline{n}_{\mathbf{k}-\mathbf{q}/2})\,\delta_{\mathbf{k},\mathbf{k}'}\,\delta_{\mathbf{q},-\mathbf{q}'}. \tag{6}$$

We may interpret the expectation value as being with respect to the full interacting ground state. When we do this we are actually dealing with the generalized RPA. In this sea-boson language, the same is reproduced as we shall see below,

$$[c^{\dagger}_{\mathbf{k}+\mathbf{q}/2}c_{\mathbf{k}-\mathbf{q}/2},c^{\dagger}_{\mathbf{k}'+\mathbf{q}'/2}c_{\mathbf{k}'-\mathbf{q}'/2}] = [\Lambda_{\mathbf{k}}(\mathbf{q})a_{\mathbf{k}}(-\mathbf{q})+\Lambda_{\mathbf{k}}(-\mathbf{q})a^{\dagger}_{\mathbf{k}}(\mathbf{q}),\Lambda_{\mathbf{k}'}(\mathbf{q}')a_{\mathbf{k}'}(-\mathbf{q}') + \Lambda_{\mathbf{k}'}(-\mathbf{q}')a^{\dagger}_{\mathbf{k}'}(\mathbf{q}')] = (\overline{n}_{\mathbf{k}+\mathbf{q}/2}-\overline{n}_{\mathbf{k}-\mathbf{q}/2})\,\delta_{\mathbf{k},\mathbf{k}'}\,\delta_{\mathbf{q},-\mathbf{q}'}. \tag{7}$$

Therefore, while the formula for $c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}$ is exact, the corresponding formula for the off-diagonal bilinear is valid only in the RPA sense. Taken together, these formulas are valid only in the RPA sense. Thus, our approach, although lacking an explicit expression for $a_{\mathbf{k}}(\mathbf{q})$, is nevertheless quite robust from a practical standpoint, which is all that matters in the end.

[1] G.S. Setlur and Y.C. Chang, Phys. Rev. B **57**, 15 144 (1998).

[2] L.C. Cune and M. Apostol, preceding comment, Phys. Rev. B **60**, 8388 (1999).