Comparison of coherent and weakly incoherent transport models for the interlayer magnetoresistance of layered Fermi liquids

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(Received 7 December 1998; revised manuscript received 19 March 1999)

The interlayer magnetoresistance of layered metals in a tilted magnetic field is calculated for two distinct models for the interlayer transport. The first model involves coherent interlayer transport, and makes use of results of semiclassical or Bloch-Boltzmann transport theory. The second model involves weakly incoherent interlayer transport where the electron is scattered many times within a layer before tunneling into the next layer. The results are relevant to the interpretation of experiments on angular-dependent magnetoresistance oscillations (AMRO) in quasi-one- and quasi-two-dimensional organic metals. We find that the dependence of the magnetoresistance on the direction of the magnetic field is identical for both models except when the field is almost parallel to the layers. An important implication of this result is that a three-dimensional Fermi surface is not necessary for the observation of the Yamaji and Danner oscillations seen in quasi-two- and quasi-one-dimensional metals, respectively. A universal expression is given for the dependence of the resistance at AMRO maxima and minima on the magnetic field and scattering time (and thus the temperature). We point out three distinctive features of coherent interlayer transport: (i) a beat frequency in the magnetic oscillations of quasi-two-dimensional systems, (ii) a peak in the angular-dependent magnetoresistance when the field is sufficiently large and parallel to the layers, and (iii) a crossover from a linear to a quadratic field dependence for the magnetoresistance when the field is parallel to the layers. Properties (i) and (ii) are compared with published experimental data for a range of quasi-two-dimensional organic metals. [S0163-1829(99)02236-5]

I. INTRODUCTION

One of the most fundamental concepts in electronic transport theory for elemental metals and semiconductors is that electronic transport involves the coherent motion of electrons in band or Bloch states associated with well-defined wave vectors and group velocities.[1] An important issue is whether this concept is applicable to interlayer transport in strongly correlated electron systems such as high-T_c superconductors,[2–4] organic conductors,[5] and layered manganite compounds with colossal magnetoresistance.[6] If the interlayer transport is incoherent the motion between layers is diffusive and it is not possible to define band states extending over many layers and a Fermi velocity perpendicular to the layers. In that case a three-dimensional Fermi surface cannot be defined, and Bloch-Boltzmann transport theory cannot describe the interlayer transport.

Extensive experimental studies have been made of the angular-dependent magnetoresistance oscillations (AMRO's)[7] which occur in layered organic conductors[8] when the direction of the magnetic field is varied. The theoretical interpretation of these oscillations often involves a three-dimensional Fermi surface, and their observation is sometimes interpreted as evidence of the existence of a three-dimensional Fermi surface. In quasi-one-dimensional metals these effects are are known as Danner,[9] Lebed (or magic angle),[10–12] and third angular effects,[13] depending on whether the magnetic field is rotated in the $\mathbf{a}-\mathbf{c}$, $\mathbf{b}-\mathbf{c}$, or $\mathbf{a}-\mathbf{b}$ plane, respectively. (The most- and least-conducting directions are the $\mathbf{a}$ and $\mathbf{c}$ axes, respectively). In quasi-two-dimensional systems, the effects observed include the Yamaji[14] oscillations and the anomalous AMRO's in the low-temperature phase of α-(BEDT-TTF)$_2M$Hg(SCN)$_4[M=$K,Rb,Tl].[7,15]

The explaination of the Lebed effect is controversial and a number of different theories have been proposed.[5,12,16] It is not clear that coherent transport models can explain the angle-dependent magnetoresistance in the quasi-one-dimensional (TMTSF)$_2$PF$_6$ at pressures of about 10 kbar,[5,10,11,17,18] or the anomalous AMRO's.[19] Consequently, we focus on the Danner and Yamaji oscillations here because their explanation in terms of a three-dimensional Fermi surface has generally been accepted. The resistance perpendicular to the layers is a maximum when the field direction is such that the electron velocity (perpendicular to the layers) averaged over its trajectories on the three-dimensional Fermi surface is zero.[9,20]

Several different models for incoherent interlayer transport have been considered previously. We shall distinguish between what we shall refer to as *weakly* and *strongly* incoherent interlayer transport. The former occurs when there is direct transfer of the electron from one layer to another, and the intralayer momentum is conserved in the process. Consequently, interference between wave functions on adjacent layers is possible. However, the transport can be incoherent in the sense that tunneling events are uncorrelated because the electron is scattered many times within the layer between tunneling events. This model has been used to describe interlayer transport in the cuprates[21–23] and organics.[24] In contrast, strongly incoherent transport occurs if the the intralayer momentum is not conserved by tunneling and there is no interference between wave functions on adjacent layers. This can occur because the tunneling is associated with elastic scattering,[25] because of inelastic processes such as coupling to a bath of phonons,[23] or because of non-Fermi-liquid ef-

TABLE I. For a range of quasi-two-dimensional materials, we list whether or not beats in magnetic oscillations and a peak in the angular dependent magnetoresistance at 90° has been observed. For coherent interlayer transport both these features should be present, provided a wide enough range of magnetic fields is explored. A question mark indicates that the measurement has not been made.

	Beats	Peak at 90°
α-(BEDT-TTF)$_2$NH$_4$Hg(SCN)$_4$	no (Ref. 54)	no (Ref. 54)
α-(BEDT-TTF)$_2$KHg(SeCN)$_4$	no (Ref. 28)	no (Ref. 28)
α-(BEDT-TTF)$_2$KHg(SCN)$_4$ above 20 T	no (Ref. 55)	no (Ref. 56)
α-(BEDT-TTF)$_2$TlHg(SeCN)$_4$	no (Refs. 57 and 58)	?
α-Et$_2$Me$_2$N[Ni(dmit)$_2$]$_2$	?	yes (Ref. 59)
α-(BEDT-TSF)$_2$KHg(SCN)$_4$ above 6 kbar	yes (Ref. 60)	?
β_H-(BEDT-TTF)$_2$I$_3$	yes (Refs. 61 and 62)	yes (Ref. 31)
β-(BEDT-TTF)$_2$IBr$_2$	yes (Refs. 51 and 63)	yes (Ref. 63)
κ-(BEDT-TTF)$_2$I$_3$	no (Ref. 64)	yes (Ref. 65)
κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ at 7 kbar	?	yes (Ref. 66)
κ-(BEDT-TTF)$_2$Cu(SCN)$_2$	no (Ref. 67)	? (Ref. 68)
θ-(BEDT-TTF)$_2$I$_3$	no (Ref. 69)	yes (Ref. 70)
Sr$_2$RuO$_4$	yes (Ref. 71)	yes (Ref. 72)

fects such as spin-charge separation.[5,26] For both weakly and strongly incoherent transport, the interlayer conductivity of a bulk sample is determined by the tunneling rate between two adjacent layers. The interlayer resistance is then equal to the number of layers in the sample times the resistance between two layers.

In this paper we present details of calculations of the angular dependence of the interlayer magnetoresistance for both coherent and weakly incoherent interlayer transport when there is a Fermi liquid within each layer. Our main result is that coherent interlayer transport is not *necessary* to explain the Yamaji and Danner oscillations. Hence their observation is *not* evidence of the existence of a three-dimensional Fermi surface. In contrast, we point out three properties of the interlayer magnetoresistance which occur only if the interlayer transport is coherent: (i) a beat frequency in the magnetic oscillations of quasi-two-dimensional systems, (ii) a peak in the angle-dependent magnetoresistance when the field is parallel to the layers for sufficiently high fields, and (iii) a crossover from linear to quadratic field dependence when the field is parallel to the layers. A brief report of some of the results presented here appeared previously.[27]

In Sec. II we present our main result [Eq. (1)], an analytical expression for the interlayer conductivity in the presence of a magnetic field which is tilted at an angle θ relative to the normal to the layers. This result is valid for incoherent transport for all field directions and for coherent transport, provided the field is not almost parallel to the layers. We then use this expression to explain the basic features of the Danner and Yamaji oscillations. Simple expressions are described for the dependence of the interlayer resistance on the magnitude of the magnetic field and the scattering rate when the angle θ is at an AMRO maxima or minima. In Sec. III we derive Eq. (1) for the case of coherent interlayer transport for both quasi-two- and quasi-one-dimensional systems. This involves evaluating Chambers' formula, a result of Bloch-Boltzmann transport theory. In Sec. IV we derive Eq. (1) for weakly incoherent transport in both quasi-two- and quasi-one-dimensional systems. In Sec. V we consider unambigu-

ous signatures of coherent interlayer transport and compare these signatures to published experimental data for a range of quasi-two-dimensional metals.

II. ANGULAR-DEPENDENT MAGNETORESISTANCE OSCILLATIONS

We assume that each layer of the metal is a Fermi liquid whose elementary excitations are fermions with a wave vector (k_x, k_y) and with a dispersion relation of the form $\epsilon(k_x, k_y)$. We consider the simplest posssible dispersion relations for quasi-one- and quasi-two-dimensional systems. (For a summary of our notation, see Table I in Ref. 27). The interlayer conductivity in the absence of a magnetic field is denoted σ_{zz}^0. In this paper we will show that in a tilted magnetic field the interlayer conductivity, for both coherent interlayer transport (except for fields very close to the layers) and weakly incoherent interlayer transport (for all field directions), is

$$\sigma_{zz}(\theta) = \sigma_{zz}^0 \left[J_0(\gamma \tan \theta)^2 + 2 \sum_{\nu=1}^{\infty} \frac{J_\nu(\gamma \tan \theta)^2}{1 + (\nu \omega_0 \tau \cos \theta)^2} \right],$$

(1)

where $J_\nu(x)$ is the νth-order Bessel function, ω_0 is the oscillation frequency associated with the magnetic field, and γ is a constant that depends on the geometry of the Fermi surface.[27] The scattering time τ is assumed to be independent of the momentum of the electron, but can vary with temperature.

If the field is sufficiently large and the temperature sufficiently low that $\omega_0 \tau \gg 1$, then the first term in Eq. (1) is dominant. However, if $\gamma \tan \theta$ equals a zero of the zeroth-order Bessel function, then at that angle σ_{zz} will be a minimum and the interlayer resistivity will be a maximum. If $\gamma \tan \theta \gg 1$, then the zeros occur at angles θ_n given by

$$\gamma \tan \theta_n = \pi(n - \tfrac{1}{4}) \quad (n = 1, 2, 3, \dots).$$

(2)

This condition was first derived for the quasi-two-dimensional case by Yamaji,[14] and for the quasi-one-
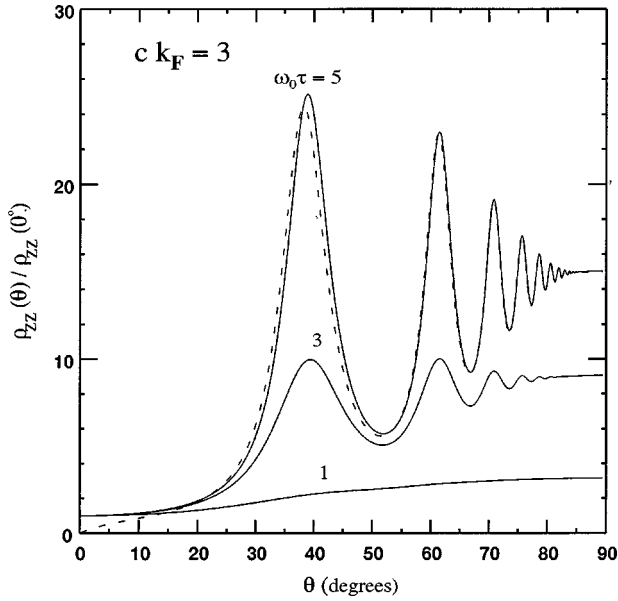
FIG. 1. Angular-dependent magnetoresistance oscillations. The dependence of the interlayer resistance for a typical quasi-two-dimensional system on the direction of the magnetic field is shown for a range of magnetic fields. θ is the angle between the field and the normal to the layers. τ is the scattering time within the layers, and ω_0 is the cyclotron frequency when the field is perpendicular to the layers. The curves shown are plots of Eq. (1), which is valid for all θ for incoherent interlayer transport and for all θ except close to 90° for coherent interlayer transport. Note that the location of the maxima and minima is independent of the field and the scattering time. The dashed curve is a plot of the asymptotic expression (6), which can be seen to be a very good approximation for $\theta > 20°$.

dimensional case by Danner, Kang, and Chaikin.[9] Determination of these angles experimentally provides a value for γ, and thus information about the intralayer Fermi surface. The values of the Fermi surface area of quasi-two-dimensional systems determined from AMRO's are in good agreement with the Fermi surface areas determined from the frequency of magneto-oscillations.[7] Furthermore, AMRO's can be used to map out the actual shape of the Fermi surface within the layer (see, for example, Refs. 20, 28, and 29).

The angular dependence of the interlayer resistivity $\rho_{zz} \simeq 1/\sigma_{zz}$,[30] given by Eq. (1), for parameter values relevant for typical quasi-two-dimensional systems is shown in Fig. 1. Figure 2 shows the angular dependence of ρ_{zz} for parameter values relevant to $(TMTSF)_2ClO_4$. The results are similar to the experimental results in Ref. 9, except near 90°. Both figures are very similar to the results of numerical integration of Chambers' formula for coherent transport [Eq. (17)], except near 90°. For coherent transport there is a small peak in $\rho_{zz}(\theta)$ at $\theta = 90°$.[9,31] This is due to the existence of closed orbits on the Fermi surface when the field lies close to the plane of the layers. For incoherent transport these orbits do not exist, and so the associated magnetoresistance is not present. Since Eq. (1) is also valid for incoherent interlayer transport, the Danner and Yamaji oscillations can be explained equally well in terms of weakly incoherent transport.

A. Asymptotic form

We want to find an expression for $\sigma_{zz}(\theta)$ as $\theta \rightarrow \pi/2$. Using the asymptotic form[32]
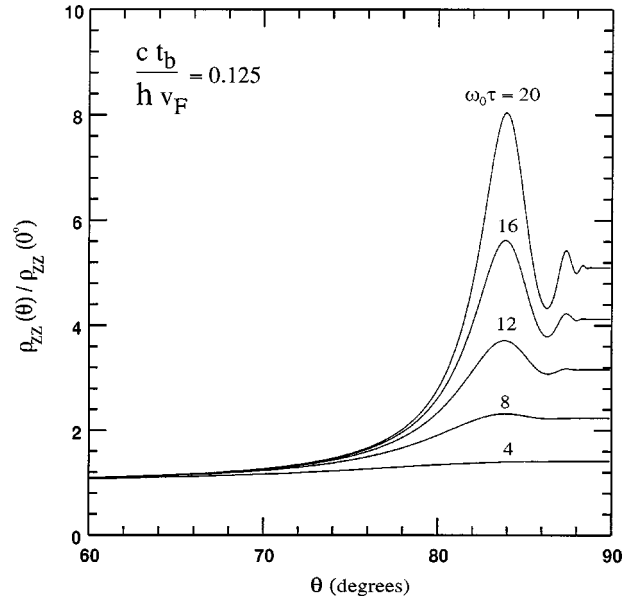

FIG. 2. Dependence of the interlayer resistance of a quasi-one-dimensional system on the direction of the magnetic field. θ is the angle between the magnetic field and the least conducting direction, with the field in the same plane as the most conducting direction. The parameter which defines the anisotropy of the intralayer hopping $\gamma \equiv 2ct_c/\hbar v_F = 0.25$. τ is the intralayer scattering time, and ω_0 is the frequency at which the electrons oscillate between the chains when the field is perpendicular to the layers. Except very close to 90° this figure is similar to the experimental data on $(TMTSF)_2ClO_4$ in Ref. 9.

$$J_n(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{n\pi}{2} - \frac{\pi}{4} \right), \qquad (3)$$

which is valid for $z \gg n^2$, we can simplify Eq. (1) for the conductivity. We rewrite it as

$$\sigma_{zz} = \sigma_{zz}^0 \left[J_0(\mu)^2 + 2 \sum_{\nu=1}^{\infty} \frac{J_\nu(\mu)^2}{1 + (\nu x)^2} \right], \qquad (4)$$

where $\mu = \gamma \tan\theta$ and $x = \omega_0 \tau \cos\theta$. We can substitute Eq. (3) for $J_n(\mu)$ provided that $x > 1$, so that the sum in Eq. (4) converges rapidly. Separating the sum into the sum of even and odd terms, and using the fact that $2\cos^2[\mu - (n\pi/2) - (\pi/4)] = 1 + \sin(2\mu)$ for even n and $1 - \sin(2\mu)$ for odd n, gives

$$\sigma_{zz} = \frac{2\sigma_{zz}^0}{\pi\mu} \left[(1 + \sin 2\mu) \left(\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{1 + (2nx)^2} \right) \right.$$
$$\left. + (1 - \sin 2\mu) \left(\sum_{n=0}^{\infty} \frac{1}{1 + (2n+1)^2 x^2} \right) \right]. \qquad (5)$$

These series can be evaluated using the residue theorem[33] to give

$$\frac{\sigma_{zz}}{\sigma_{zz}^0} = \frac{1}{\mu x} \left[\coth\left(\frac{\pi}{x} \right) + \frac{\sin(2\mu)}{\sinh\left(\frac{\pi}{x} \right)} \right]. \qquad (6)$$
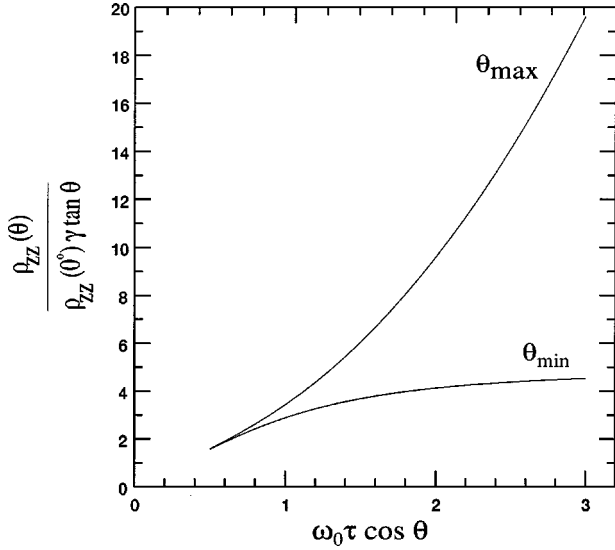
FIG. 3. Universal dependence of the interlayer resistivity on the magnetic field and scattering time when the field is tilted at an angle corresponding to an AMRO minimum (θ_{min}) and an AMRO maximum (θ_{max}). For high fields the resistivity at the minima becomes independent of field, and has the same temperature dependence as the zero-field resistivity. For high fields the resistivity at the maxima increases quadratically with field, and has the same temperature dependence as the inverse of the zero-field resistivity. The curves are not plotted for small $\omega_0 \tau \cos \theta$ because the results derived in the text are not valid in that regime.

For $\omega_0 \tau > 1$, Eq. (6) actually turns out to be a good approximation for $\gamma \tan \theta > 1$ (see Fig. 1). It will now be used to analyze the field and temperature dependence of the AMRO maxima and minima.

B. Field and temperature dependence of the resistivity at critical angles

Resistivity maxima: For the resistivity ρ_{zz} to be a maximum, $\theta = \theta_{max}^n$, where

$$\gamma \tan \theta_{max}^n = \left(n - \frac{1}{4}\right)\pi. \qquad (7)$$

From this we can simplify $\sin(2\mu)$ in Eq. (6), giving

$$\sin(2\mu) = \sin(2\gamma \tan \theta) = -\cos(2n\pi) = -1 \qquad (8)$$

for all n. The resistivity is then written as

$$\frac{\rho_{zz}(\theta_{max}^n, B)}{\rho_{zz}(B=0)\left(n - \frac{1}{4}\right)\pi} = \frac{\omega_0 \tau \cos \theta_{max}}{\tanh\left(\dfrac{\pi}{2\omega_0 \tau \cos \theta_{max}}\right)}. \qquad (9)$$

This expression is plotted in Fig. 3. Now if the field is sufficiently high and the temperature sufficiently low that $\omega_0 \tau \cos \theta_{max}^n \gg 1$, then the resistivity becomes

$$\frac{\rho_{zz}(\theta_{max}^n)}{\rho_{zz}(B=0)} = \frac{\gamma}{\pi}(\omega_0 \tau)^2 \sin(2\theta_{max}^n). \qquad (10)$$

Hence, at a fixed field the resistivity at the AMRO maxima will have the same temperature dependence as the scattering time, which is inversely proportional to the zero-field resistivity.

Resistivity minima: Similar arguments will show that for the interlayer resistivity to be a minimum $\theta = \theta_{min}^n$, where

$$\gamma \tan \theta_{min}^n = \left(n + \frac{1}{4}\right)\pi, \qquad (11)$$

and the resistivity then becomes

$$\frac{\rho_{zz}(\theta_{min}^n, B)}{\rho_{zz}(B=0)\left(n + \frac{1}{4}\right)\pi} = \omega_0 \tau \cos \theta_{min}^n \tanh\left(\frac{\pi}{2\omega_0 \tau \cos \theta_{min}^n}\right). \qquad (12)$$

This is plotted in Fig. 3. When $\omega_0 \tau \cos \theta_{min} \gg 1$, then

$$\frac{\rho_{zz}(\theta_{min})}{\rho_{zz}(B=0)} = \frac{\pi^2}{2}\left(n + \frac{1}{4}\right), \qquad (13)$$

which is independent of the field and scattering rate. Thus, at the AMRO minima the resistance will have the same temperature dependence as the zero-field resistance.

Field in the layers: Now as $\theta \to \pi/2$, $x\mu \to \gamma\omega_0\tau$, and $x \to 0$, taking these limits in (6) gives

$$\sigma_{zz}\left(\theta = \frac{\pi}{2}\right) = \frac{\sigma_{zz}^0}{\gamma\omega_0\tau}. \qquad (14)$$

The resistivity is linear in field at moderately high fields. However, caution is in order because in deriving Eq. (6) above, we required that $x > 1$. That this is more restrictive than need be is suggested by the fact that Fig. 1 shows that Eq. (6) remains valid near 90°. Indeed, Eq. (14) is valid: it agrees with the calculations of other authors for both coherent[34] and incoherent transport[35] in a quasi-two-dimensional system, provided the field is not too large (see Secs. IV B and V C). Such a linear interlayer magnetoresistance has been observed in Sr_2RuO_4 (Ref. 36) and $(TMTSF)_2ClO_4$ (Ref. 37). However, for coherent transport the dependence on field becomes quadratic for sufficiently high fields[34] (see Sec. V). Note that Eq. (14) is actually independent of the scattering time τ. This means that for moderate fields the interlayer resistivity will only depend weakly on temperature. This was observed in $(TMTSF)_2ClO_4$: when the temperature was increased from 0.9 to 8 K, the zero-field resistivity (which is proportional to the scattering rate) increased by a factor of about 6, but the resistance at 12 T increased by less than 10% (Ref. 37, p. 64).

III. COHERENT INTERLAYER TRANSPORT

If the interlayer transport is coherent, one can define a wave vector k_z perpendicular to the layers and a three-dimensional dispersion relation $\epsilon_{3D}(\vec{k})$ of the form

$$\epsilon_{3D}(\vec{k}) = \epsilon(k_x, k_y) - 2t_c \cos(k_z c), \qquad (15)$$

where t_c is the interlayer hopping integral, c is the layer separation, and $\epsilon(k_x, k_y)$ is the intralayer dispersion relation,

simple examples of which are given below. The electronic group velocity perpendicular to the layers is

$$v_z = \frac{1}{\hbar} \frac{\partial \epsilon(\vec{k})}{\partial k_z} = \frac{2ct_c}{\hbar} \sin(ck_z). \qquad (16)$$

We calculate the interlayer conductivity by solving the Boltzmann equation in the relaxation time approximation, which leads to Chambers' formula[1]

$$\sigma_{zz} = \frac{e^2 \tau}{4\pi^3} \int v_z(\vec{k}) \bar{v}_z(\vec{k}) \left(-\frac{\partial f(E)}{\partial E} \right) d^3\vec{k}, \qquad (17)$$

where $f(E)$ is the Fermi function and τ is the scattering time which is assumed to be the same at all points on the Fermi surface. $\bar{v}_z(\vec{k})$ is the electron velocity perpendicular to the layers, and is averaged over its trajectories on the Fermi surface,

$$\bar{v}_z(\vec{k}) = \frac{1}{\tau} \int_{-\infty}^{0} \exp\left(\frac{t}{\tau} \right) v_z[\vec{k}(t)] dt, \qquad (18)$$

where $\vec{k}(0) = \vec{k}$. The time dependence of the wave vector $\vec{k}(t)$ is found by integrating the semiclassical equation of motion

$$\frac{d\vec{k}}{dt} = -\frac{e}{\hbar^2} \vec{\nabla}_k \epsilon_{3D} \times \vec{B}. \qquad (19)$$

Now if the temperature is sufficiently low that $T \ll E_F$, then $\partial f/\partial E$ in Eq. (17) can be replaced by a δ function at the Fermi energy, and Eq. (17) becomes

$$\sigma_{zz} = \frac{e^2 \tau}{4\pi^3} \int v_z(\vec{k}) \bar{v}_z(\vec{k}) \delta[E_F - \epsilon_{3D}(\vec{k})] d^3\vec{k}. \qquad (20)$$

A. Quasi-two-dimensional case

Here we consider a quasi-two-dimensional system with the energy dispersion relation

$$\epsilon_{3D}(\vec{k}) = \frac{\hbar^2}{2m^\star}(k_x^2 + k_y^2) - 2t_c \cos(k_z c), \qquad (21)$$

where $m^\star$ is the effective mass of the electron. We assume the interlayer hopping is sufficiently small that $t_c \ll E_F$. The Fermi surface is then a warped cylinder (see Ref. 27). Substituting the energy dispersion relation from Eq. (21) we obtain the components of the group velocity

$$\vec{v}(\vec{k}) = \frac{1}{\hbar} \vec{\nabla}_k \epsilon_{3D} = \left(\frac{\hbar k_x}{m^\star}, \frac{\hbar k_y}{m^\star}, \frac{2ct_c \sin(ck_z)}{\hbar} \right). \qquad (22)$$

In order to calculate the time dependence of k_z we must integrate Eq. (19), which can be written in the form

$$\frac{d\vec{k}}{dt} = \frac{e}{m^\star}(-k_y B \cos\theta, k_x B \cos\theta, k_y B \sin\theta). \qquad (23)$$

Terms of order t_c have been neglected once we assume $t_c \tan\theta \ll \hbar^2 k_F/m^*c$, where k_F is the Fermi wave vector, defined by $E_F = \hbar^2 k_F^2/2m^*$. Differentiating the x and y components of Eq. (23) with respect to time, we obtain a second-order differential equation whose solution gives $k_y(t) = k_F \cos(\omega_c t)$ and $k_x(t) = k_F \sin(\omega_c t)$, and

$$\omega_c = \frac{eB \cos\theta}{m^\star} \qquad (24)$$

is the cyclotron frequency. Substitution of this into the z component of Eq. (23), and integrating, gives

$$k_z(t) = k_z(0) + k_F \tan\theta \sin(\omega_c t). \qquad (25)$$

In order to calculate the z component of the group velocity, we substitute the expression for $k_z(t)$ into the z component of Eq. (22), giving

$$v_z[k_z(0), \phi] = C \sin[\mu \sin\phi + k_z(0)c], \qquad (26)$$

where

$$\mu = ck_F \tan\theta, \qquad (27)$$

ϕ is the angle around the orbit, and $C = 2ct_c/\hbar$. Integrating the velocity in Eq. (26) over a period gives us the average velocity, which can be written as

$$\langle v_z \rangle \propto \int_0^{2\pi} \sin(\mu \sin\phi) d\phi \sim J_0(\gamma \tan\theta), \qquad (28)$$

and in the absence of scattering this average velocity is equal to zero when $\gamma \tan\theta$ equals a zero of J_0. These particular values of θ correspond to the peaks in the resistivity.

We can write Eq. (17) in a slightly simplified form in order to highlight the fact that the integral is a surface integral. If the warping of the Fermi surface is small, we can parametrize the surface using k_z and ϕ, where $k_x = k_F \cos\phi$ and $k_y = k_F \sin\phi$, giving

$$\sigma_{zz} = \frac{e^2 \tau m^*}{4\pi^3 \hbar^2} \int_{FS} dS \, v_z(\vec{k}) \bar{v}_z(\vec{k})$$

$$= \frac{e^2 \tau m^*}{4\pi^3 \hbar^2} \int_{-\pi/c}^{\pi/c} dk_z \int_0^{2\pi} d\phi \, v_z(\vec{k}) \bar{v}_z(\vec{k}). \qquad (29)$$

Here $\bar{v}_z(\vec{k})$ is defined in Eq. (18), and the prefactor $m^*/\hbar^2$ arises from the δ function. In terms of the parametrized surface, we have

$$\bar{v}_z(\vec{k}) = \int_{-\infty}^{0} \frac{d\phi'}{\tau\omega_c} \exp(\phi'/\tau\omega_c) v_z[k_z(0), \phi - \phi'], \qquad (30)$$

where we used the fact that $\phi' = \omega_c t$ and $v_z[\vec{k}(t)] = v_z[k_z(0), \phi - \phi']$. For closed electron orbits the electron group velocities are periodic functions of ϕ and ϕ'. Thus the range of integration of ϕ' can be cut up into segments each having length 2π. The conductivity is then

$$\sigma_{zz} = \frac{e^2 m^*}{4\pi^3 \hbar^2} \int_{-\pi/c}^{\pi/c} dk_z(0) \frac{1}{1 - \exp(-2\pi/\tau\omega_c)}$$

$$\times \int_0^{2\pi} d\phi \, v_z[k_z(0), \phi] \int_{-2\pi}^{0} \frac{d\phi'}{\omega_c}$$

$$\times v_z[k_z(0), \phi - \phi'] \exp(\phi'/\tau\omega_c). \qquad (31)$$

We use trigonometric identities to expand Eq. (26), and substitute the Bessel generating functions[32] to obtain

$$v_z[k_z(0), \phi - \phi']$$

$$= C \sin[k_z(0)c] \left[J_0(\mu) + 2 \sum_{k=1}^{\infty} J_{2k}(\mu) \right.$$

$$\times \cos[(2k)(\phi - \phi')] \right] + C \cos[k_z(0)c]$$

$$\times \left[2 \sum_{k=0}^{\infty} J_{2k+1}(\mu) \sin[(2k+1)(\phi - \phi')] \right]. \quad (32)$$

If we substitute this into Eq. (30), we obtain

$$\bar{v}_z(\phi) = C \int_{-\infty}^{0} \frac{d\phi'}{\tau \omega_c} \left(\sin[k_z(0)c] \left[J_0(\mu) \right. \right.$$

$$+ 2 \sum_{k=1}^{\infty} J_{2k}(\mu) \cos[(2k)(\phi - \phi')] \right] + \cos[k_z(0)c]$$

$$\times \left[2 \sum_{k=0}^{\infty} J_{2k+1}(\mu) \sin[(2k+1)(\phi - \phi')] \right] \right)$$

$$\times \exp(\phi'/\tau \omega_c). \quad (33)$$

Substituting equations for $v_z[k_z(0), \phi - \phi']$ and $v_z[k_z(0), \phi]$ into Eq. (31), we note terms that survive is when $k = l$, since integrals such as $\int_0^{2\pi} d\phi \cos(2k\phi)\cos(2l\phi) = \pi \delta_{kl}$, where δ_{kl} is the Kronecker delta, thus giving

$$\sigma_{zz} = \frac{e^2(2ct_c)^2 m^*}{4\pi^3 \hbar^4 \omega_c} \int_{-\pi/c}^{\pi/c} dk_z(0)$$

$$\times \left[2\pi \tau \omega_c \sin[k_z(0)c]^2 J_0(\mu)^2 \right.$$

$$+ \frac{4\pi}{\omega_c \tau} \left(\sin[k_z(0)c]^2 \sum_{k=1}^{\infty} \frac{J_{2k}(\mu)^2}{(2k)^2 + (1/\omega_c \tau)^2} \right.$$

$$+ \cos[k_z(0)c]^2 \sum_{k=0}^{\infty} \frac{J_{2k+1}(\mu)^2}{(2k+1)^2 + (1/\omega_c \tau)^2} \right) \right]. \quad (34)$$

Performing the integral over $k_z(0)$ yields the final expression for the conductivity, which is of the form of Eq. (1). This result was previously given by Yagi et al.[38]

B. Quasi-one-dimensional case

For the quasi-one-dimensional case we begin with the dispersion relation[9]

$$\epsilon_{3D}(\vec{k}) = \hbar v_F(|k_x| - k_F) - 2t_b \cos(k_y b) - 2t_c \cos(k_z c) \quad (35)$$

where v_F is the Fermi velocity and t_b is the intrachain hopping within the layers. The Fermi surface consists of two sheets at $k_x \cong \pm k_F$. By proceeding as for the quasi-two-

dimensional case, the rate of change of the wave vector [and defining the magnetic field $\vec{B} = (B \sin\theta, 0, B \cos\theta)$] is given by

$$(36a)$$

$$\frac{d\vec{k}}{dt} = \frac{1}{\hbar^2} \begin{pmatrix} -2bB \cos\theta e t_b \sin(bk_y) \\ eB\hbar \cos\theta v_F \\ 2bB \sin\theta e t_b \sin(bk_y) \end{pmatrix}, \quad (36b)$$

$$(36c)$$

where we neglect terms involving t_c by assuming that $v_F \gg v_z \tan\theta$, i.e., the warping of the Fermi surface is small and the magnetic field is not too close to the layers. Integrating the second equation gives

$$k_y(t) = \frac{\omega_B}{b} t + k_y(0), \quad (37)$$

where

$$\omega_B \equiv \frac{eBb \cos\theta v_F}{\hbar} \quad (38)$$

is the speed at which the wave vector traverses the Fermi surface. To obtain $k_z(t)$ we substitute Eq. (37) into Eq. (36c), and integrate to obtain

$$k_z(t) = k_z(0) - \frac{2ebt_b B \sin\theta}{\hbar^2 \omega_B} \cos[\omega_B t + bk_y(0)]. \quad (39)$$

Substitution into the z component of the velocity yields

$$v_z[k_z(0), \phi - \phi'] = \frac{2ct_c}{\hbar} \sin[ck_z(0) - \gamma \tan\theta \cos(\phi - \phi')], \quad (40)$$

where $\phi' = -\omega_B t$, $\phi = bk_y(0)$, and $\gamma = 2ct_b/v_F \hbar$. This is similar in form to the z component of the velocity for the quasi-two-dimensional case [compare with Eq. (26)]. The interlayer conductivity can then be written as

$$\sigma_{zz} = \frac{e^2}{4\pi^3 b \hbar v_F} \int_{-\pi/c}^{\pi/c} dk_z(0) \int_{-\pi/b}^{\pi/b} d\phi \, v_z[k_z(0), \phi]$$

$$\times \int_{-2\pi}^{0} \frac{d\phi'}{\omega_B} \frac{e^{\phi'/\tau \omega_B}}{[1 - \exp(-2\pi/\tau \omega_B)]}$$

$$\times v_z[k_z(0), \phi - \phi']. \quad (41)$$

The integral from -2π to 0 over ϕ' is obtained by noting that the electron group velocity is a periodic function of ϕ', we can cut the range of ϕ' into segments having length 2π. The factor $(1 - e^{-2\pi/\tau \omega_B})$ is a consequence of this. Proceeding as for the quasi-two-dimensional case leads to a result of the form of Eq. (1). As far as we are aware, this expression has not been derived previously for quasi-one-dimensional systems.

IV. WEAKLY INCOHERENT INTERLAYER TRANSPORT

Suppose that the coupling between the layers is sufficiently weak that the time it takes an electron to hop between the layers (approximately $\hbar/t_c$) is much longer than the scattering time. This means that the intralayer scattering rate is

much larger than the interlayer hopping integral

$$\frac{\hbar}{\tau} \gg t_c, \tag{42}$$

and the mean free path perpendicular to the layers is much smaller than the interlayer spacing. If this condition holds then the interlayer transport will be incoherent in the sense that successive interlayer tunneling events are uncorrelated.[21] Previous estimates of t_c and τ in various layered organic metals[7,9,39] suggest these quantities may be comparable at low temperatures. Furthermore, the scattering rate usually increases quadratically with temperature,[40] and at temperatures of the order of tens of K this condition will almost certainly be satisfied.[41] The interlayer conductivity is then proportional to the tunneling rate between just two adjacent layers. If we assume that the intralayer momentum is conserved, the tunneling rate can be calculated using standard formalisms for tunneling in metal-insulator-metal junctions.[42] Modeling the interlayer transport in this way is reasonable, because many organic conductors consist of conducting layers separated by insulating layers of anions that are several Å thick. Furthermore, intrinsic Josephson type effects have been observed in the superconducting state of κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ (Ref. 43).

We consider the simplest possible model for the tunneling between layers, direct transfer described by the Hamiltonian

$$H_{12} = -t_c \int d^2\vec{r} [c_1(\vec{r})^\dagger c_2(\vec{r}) + c_2(\vec{r})^\dagger c_1(\vec{r})], \tag{43}$$

where $c_1(\vec{r})^\dagger$ creates an electron in layer 1 at $\vec{r}$. Note that the interlayer transport is coherent in the sense that during the tunneling process the phase information in the electron's wave function is not completely lost. However, it is incoherent in the sense that due to the large intralayer scattering rate the interlayer transport cannot be described by Bloch states extending over many layers. If we consider a sequence of tunneling events, they are uncorrelated because after a tunneling event an electron is scattered many times before it tunnels to the next layer.

The interlayer current I associated with H_{12} and produced by a voltage V can be calculated using the formalism developed for metal-insulator-metal junctions.[42] The result for the interlayer conductivity is

$$\sigma_{zz} = \frac{c}{L_x L_y} \frac{dI}{dV}\bigg|_{V=0} = \frac{2e^2 t_c^2 c}{\hbar L_x L_y} \int d^2r \int d^2r' \int \frac{dE}{2\pi} A_1(\vec{r}, \vec{r}', E)$$

$$\times A_2(-\vec{r}, \vec{r}', E) \frac{\partial f(E)}{\partial E}, \tag{44}$$

where A_1 and A_2 are the spectral functions for layers 1 and 2, and L_x and L_y are the dimensions of the layers. It will be seen below that in the presence of a tilted magnetic field A_1 and A_2 are not identical. The zero-field limit (for which $A_1 = A_2$) of this expression has been used in treatments of incoherent interlayer transport in the cuprate superconductors.[22,23,26]

If we assume $T \ll E_F$, then $\partial f(E)/\partial E$ can be replaced with a δ function to give

$$\sigma_{zz} = \frac{e^2 t_c^2 c}{\hbar \pi L_x L_y} \int d^2r \int d^2r' A_1(\vec{r}, \vec{r}', E_F) A_2(\vec{r}', \vec{r}, E_F). \tag{45}$$

This can be rewritten by noting that the spectral function can be written as $A_{1,2}(\vec{r}', \vec{r}, E_F) = -1/i[G_{1,2}^+(\vec{r}, \vec{r}', E_F) - G_{1,2}^-(\vec{r}', \vec{r}, E_F)]$, leading to

$$\sigma_{zz}(\vec{r}', \vec{r}, E_F) = \frac{e^2 t_c^2 c}{\hbar \pi L_x L_y} \int d^2r \int d^2r'$$

$$\times [G_1^+(\vec{r}, \vec{r}', E_F) G_2^-(\vec{r}', \vec{r}, E_F)$$

$$+ G_1^-(\vec{r}', \vec{r}, E_F) G_2^+(\vec{r}, \vec{r}', E_F)]. \tag{46}$$

In the Landau gauge, the vector potential $\vec{A}$, for the magnetic field $\vec{B} = (B_x, 0, B_z) = (B \sin\theta, 0, B \cos\theta)$, is

$$\vec{A} = (0, xB_z - zB_x, 0), \tag{47}$$

where $\vec{B}$ and $\vec{A}$ are related by $\vec{B} = \vec{\nabla} \times \vec{A}$. The vector potential in the two layers (see Ref. 27) are not equal, and differ by a gauge transformation $\vec{A}_2 = \vec{A}_1 + \vec{\nabla}\Lambda$ where

$$\vec{\nabla}\Lambda = \vec{A}_1 - \vec{A}_2 = (0, -cB \sin\theta, 0). \tag{48}$$

The Green's functions in the two layers are not equal. This reflects the fact that even though the magnetic field is invariant under a gauge transformation the Green's function is not. The Green's function for layer 1 is thus multiplied by a phase factor $\exp\{(i/\hbar)e[\Lambda(\vec{r}) - \Lambda(\vec{r}')]\}$, giving

$$G_2^+(\vec{r}, \vec{r}') = \exp\left\{\frac{i}{\hbar} e\Lambda(\vec{r})\right\} G_1^+(\vec{r}, \vec{r}') \exp\left\{\frac{-i}{\hbar} e\Lambda(\vec{r}')\right\}. \tag{49}$$

Making use of this relationship, we have

$$\sigma_{zz} = \frac{2e^2 t_c^2 c}{\hbar \pi} \int d^2r |G_1^+(\vec{r}, 0, E_F)|^2 \cos\left(\frac{ecB}{\hbar} \sin\theta y\right), \tag{50}$$

where we have used the fact that $|G_1^+(\vec{r}, 0, E_F)|^2$ is translation invariant.

Note that Eq. (50) is a very general expression which holds provided that intralayer momentum is conserved and the interactions between the layers can be neglected. It is valid in the presence of interactions within the layers and for a non-Fermi liquid.[44] Second, this expression shows that for weakly incoherent interlayer transport the interlayer conductivity is completely determined by the *one*-electron Green's function, whereas the intralayer conductivity is determined by *two*-electron Green's functions.[35]

It is the averaging of the phase factor over the spatial integral in Eq. (50) that gives rise to the AMRO effect. The length scale associated with the magnetic field for the quasi-two-dimensional system is the cyclotron length R, which at the Fermi energy is $R = \hbar k_F/(eB \cos\theta)$. For the quasi-one-dimensional case the length scale associated with oscillations perpendicular to the chains is $R = 2t_b/(e v_F B \cos\theta)$.[45] At this length scale the phase difference between the wave function of adjacent layers is $\Lambda(R) = eB \sin\theta cR = \gamma \tan\theta$. Naively,

we might expect maximum resistivity when this phase difference is an odd multiple of π, leading to a condition different from Eq. (2). However, one must take into account averaging of the electron position over the perpendicular direction.

We now proceed to evaluate Eq. (50) for the simplest possible situation, where there is a Fermi liquid within each layer and the magnetic field is small enough that we can take the semiclassical limit of the Green's functions.

A. Quasi-two-dimensional case

The Green's functions for layer 1, in the absence of scattering, can be written[46]

$$G(\vec{r},\vec{r}',t)_1 = \frac{m^*}{2\pi i\hbar t}\frac{\omega_c t/2}{\sin(\omega_c t/2)}\exp\left(\frac{i\omega_c}{\hbar}L\right) \quad (51)$$

and

$$L = \frac{m^*}{2}\left[\frac{|\vec{r}-\vec{r}'|^2}{2}\cot\left(\frac{\omega_c t}{2}\right)+(x+x')(y'-y)\right], \quad (52)$$

where $\omega_c = \omega_0\cos\theta = eB\cos\theta/m^*$ is the cyclotron frequency. In order to calculate the conductivity we follow the same approach that Hackenbroich and von Oppen[47] used to study Shubnikov–de Haas oscillations in two-dimensional electron systems. In the presence of scattering, the energy-dependent Green's function is

$$G^+(\vec{r},\vec{r}',E) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\,\exp\left(\frac{i}{\hbar}(E+i\Gamma)t\right)G(\vec{r},\vec{r}',t), \quad (53)$$

where $\Gamma = \hbar/2\tau$ is the scattering rate. The retarded Green's functon is obtained using $G^-(\vec{r}',\vec{r},E) = [G^+(\vec{r},\vec{r}',E)]^*$. We perform the integral in Eq. (53) by the stationary phase method, which is valid in the semiclassical limit ($\hbar\to0$). The stationary phase condition gives

$$E = \frac{m^*\omega_c^2}{8}\left(\frac{|\vec{r}-\vec{r}'|}{\sin(\omega_c t/2)}\right)^2. \quad (54)$$

This shows that if the cycloron radius is R_c then $G^+(\vec{r},\vec{r}',E)$ vanishes for $|\vec{r}-\vec{r}'|>2R_c$; however, for $|\vec{r}-\vec{r}'|<2R_c$ there exists two different cyclotron orbits and one finds an infinite set of stationary times given by[47]

$$T_{n,q} = \frac{2\pi n}{\omega_c}+t_q, \quad (55)$$

where n determines the number of revolutions the electron makes to get from $\vec{r}$ to $\vec{r}'$ and $q=S$ or L denoting the two different paths it can take (see Fig. 4). The times to traverse these paths are calculated using the the stationary phase condition and $E=m^*R_c^2\omega_c^2/2$, to give

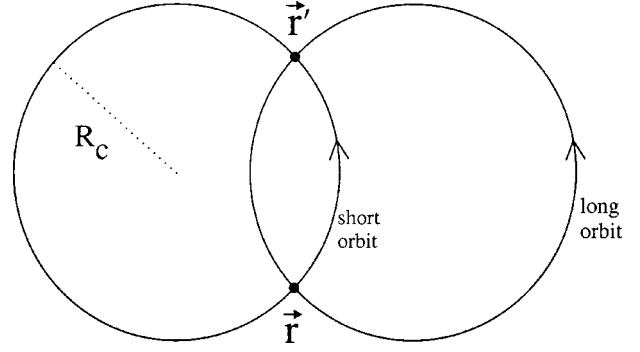$$t_S = \frac{2}{\omega_c}\arcsin\left(\frac{|\vec{r}-\vec{r}'|}{2R_c}\right),$$

FIG. 4. Short and long semiclassical orbits joining two points within a layer of a quasi-two-dimensional system in a magnetic field perpendicular to the layers. R_c is the radius of the cyclotron orbit.

$$t_L = \frac{2}{\omega_c}\left[\pi - \arcsin\left(\frac{|\vec{r}-\vec{r}'|}{2R_c}\right)\right]. \quad (56)$$

Putting all this together and performing the integrals, we obtain

$$G_1^+(\vec{r},\vec{r}',E) = \frac{m^*}{2i\hbar}\sum_{n=0}^{\infty}\sum_{q=S,L}\left(\frac{\omega_c}{\pi i\hbar E\sin(\omega_c T_{n,q})}\right)^{1/2}$$
$$\times\exp\left(-\frac{T_{n,q}}{2\tau}\right)\exp\left(\frac{i}{\hbar}S_{n,q}-\frac{i\pi}{2}\eta_{n,q}\right), \quad (57)$$

where

$$S_{n,q} = ET_{n,q}+\frac{m^*\omega_c}{2}\left[\frac{|\vec{r}-\vec{r}'|^2}{2}\cot\left(\frac{\omega_c T_{n,q}}{2}\right)\right.$$
$$\left.+(x+x')(y'-y)\right], \quad (58)$$

and η is the Maslov or Morse index [the number of conjugate points along the orbit (Ref. 46, p. 223)], $\eta_{n,S}=2n$, and $\eta_{n,L}=2n+1$. Equation (50) can be written

$$\sigma_{zz} = \frac{2e^2t_c^2c}{\hbar\pi}\int d^2|\vec{r}-\vec{r}'||G_1^+(\vec{r},\vec{r}',E_F)|^2$$
$$\times\cos\left(\frac{ec}{\hbar}B\sin\theta\left|\vec{r}-\vec{r}'\right|\sin\phi\right), \quad (59)$$

where ϕ is the angle between the vector $|\vec{r}-\vec{r}'|$ and the x axis. Substituting the semiclassical expressions for the Green's functions into the above equation and changing the integrals over r and r' to polar coordinates, one obtains a double sum [denoted by the subscripts (1 and 2) of n and q] over the classical trajectories:

$$\sigma_{zz} = \left(\frac{m^* e t_c}{\hbar^2 \pi}\right)^2 \left(\frac{\omega_c c}{2E_F}\right) \sum_{n_1,n_2=0}^{\infty} \sum_{q_1,q_2=S,L} \int_0^{2\pi} d\phi \int_0^{2R_c} |\vec{r}-\vec{r}'| d|\vec{r}-\vec{r}'|$$

$$\times \exp\left(\frac{-[T_{n_1,q_1} + T_{n_2,q_2}]}{2\tau}\right) \frac{1}{\sqrt{\sin(\omega_c T_{n_1,q_1})\sin(\omega_c T_{n_2,q_2})}} \exp\left(\frac{i}{\hbar}[S_{n_2,q_2} - S_{n_1,q_1}]\right.$$

$$\left. -\frac{i\pi}{2}[\eta_{n_2,q_2} - \eta_{n_1,q_1}]\right) \cos\left(\frac{ecB}{\hbar}\sin\theta|\vec{r}-\vec{r}'|\sin\phi\right). \tag{60}$$

We make the simplification that $q_1 = q_2$, for when $q_1 \neq q_2$ the integrand oscillates; that is, as $\hbar \to 0$ the oscillations cancel each other and therefore do not contribute. This gives

$$\sigma_{zz} = \left(\frac{m^* e t_c}{\hbar^2 \pi}\right)^2 \left(\frac{\omega_c c}{2E_F}\right) \int_0^{2\pi} d\phi \int_0^{2R_c} |\vec{r}-\vec{r}'| d|\vec{r}-\vec{r}'| \sum_{n_1,n_2=0}^{\infty} \exp\left(-\frac{\pi}{\omega_c \tau}(n_1+n_2)\right)$$

$$\times \left[\frac{\exp(-t_S/\tau)}{\sin(\omega_c t_S)} + \frac{\exp(-t_L/\tau)}{\sin(\omega_c t_L)}\right] \cos\left(2\pi\left[\frac{E_F}{\hbar\omega_c} - \frac{1}{2}\right](n_2-n_1)\right) \cos\left(\frac{ecB}{\hbar}\sin\theta|\vec{r}-\vec{r}'|\sin\phi\right). \tag{61}$$

Terms with $n_1 \neq n_2$ correspond to the Shubnikov–deHaas oscillations. We neglect these by setting $n_1 = n_2$, since they will be smaller that the leading order terms by a factor of order $\exp(-\pi/\omega_c \tau)$, and thus we have

$$\sigma_{zz} = A \int_0^{2\pi} d\phi \sum_{n=0}^{\infty} \exp\left(-\frac{2\pi n}{\omega_c \tau}\right)\left\{\int_0^{\pi/\omega_c} \exp\left(-\frac{t_S}{\tau}\right)\cos\left[\eta\sin\left(\frac{\omega_c t_S}{2}\right)\right]dt_S + \int_{\pi/\omega_c}^{2\pi/\omega_c} \exp\left(-\frac{t_L}{\tau}\right)\cos\left[\eta\sin\left(\frac{\omega_c t_L}{2}\right)\right]dt_L\right\}, \tag{62}$$

where $A = e^2 t_c^2 m^* c/\pi^2 \hbar^4$ and $\eta = (2ec/\hbar)B\sin\theta R_c \sin\phi = 2ck_F \tan\theta \sin\phi = 2\gamma \tan\theta \sin\phi$. Combining the integrations over t_S and t_L, and performing the summation over n, one obtains

$$\sigma_{zz} = \frac{A}{(1-e^{-2\pi/\omega_c \tau})}\int_0^{2\pi} d\phi$$

$$\times\left\{\int_0^{2\pi/\omega_c} \exp(-t/\tau)\cos\left[\eta\sin\left(\frac{\omega_c t}{2}\right)\right]dt\right\}. \tag{63}$$

To evaluate the integral over t, we make use of the identity[32]

$$\cos(\eta\sin\beta) = J_0(\eta) + 2\sum_{k=1}^{\infty} J_{2k}(\eta)\cos(2k\beta). \tag{64}$$

The conductivity then simplifies to

$$\sigma_{zz} = A\tau\int_0^{2\pi}\left[J_0(\eta) + 2\sum_{k=1}^{\infty} \frac{J_{2k}(\eta)}{1+(k\tau\omega_c)^2}\right]d\phi. \tag{65}$$

This integral is of the form $\int_0^{2\pi} J_{2k}(z\sin\phi)d\phi$, where $z = 2\gamma\tan\theta$, which can be evaluated using the relation[32]

$$\int_0^{2\pi} J_{2k}(z\sin\phi)d\phi = 2\pi J_k(z/2)^2. \tag{66}$$

We then obtain an expression for the conductivity which is of the form Eq. (1). Previously, Yoshioka calculated the interlayer tunneling of a quasi-two-dimensional system in the absence of scattering.[48]

B. Quasi-one-dimensional case

The Hamiltonian within a layer in a magnetic field is

$$H = \alpha i\hbar v_F \frac{\partial}{\partial x} - 2t_b \cos\left[b\left(\frac{1}{i}\frac{\partial}{\partial y} - exB\cos\theta\right)\right], \tag{67}$$

where $\alpha = \pm 1$ denotes which sheet of the Fermi surface the electron is on. The wave function within a layer is given by[49]

$$\psi_{k_x,k_y,\alpha}(x,N,t) = \exp\left\{i\left[-\frac{\epsilon t}{\hbar} + k_x x + bk_y N\right.\right.$$

$$\left.\left. -\alpha\lambda\sin(k_y b - qx)\right]\right\}, \tag{68}$$

where N denotes the number of the chain, and x is the distance along the chain; the dispersion relations are

$$\epsilon_\alpha(k_x,k_y) = \alpha\hbar k_x v_F \tag{69}$$

and

$$q = \frac{ebB\cos\theta}{\hbar} = \frac{\omega_B}{v_F}, \tag{70}$$

where ω_B is the oscillation frequency given by Eq. (38), and

$$\lambda = \frac{2t_b}{ebv_FB\cos\theta}. \tag{71}$$

The transverse motion of the electrons due to the field is approximately λb.[45] The one-electron advanced Green's function in the absence of scattering is

$G^+(x,x',N,N',t,0)$

$$= \sum_{k_x,k_y,\alpha} \psi^*_{k_x,k_y,\alpha}(x',N,t)\psi_{k_x,k_y,\alpha}(x,N,0) \quad (72)$$

for $t>0$. Taking the Fourier transform (with respect to time) of this and including a scattering rate $\Gamma=\hbar/2\tau$,

$G^+(x,x',N,N',E)$

$$= \frac{1}{i\hbar} \int_0^\infty dt \exp\left\{\frac{i}{\hbar}(E+i\Gamma)t\right\} G^+(x,x',N,N',t,0). \quad (73)$$

After performing the integral in t and k_x

$$G^+ = -\frac{i}{\hbar v_F} \sum_{k_y,\alpha} \frac{1}{\alpha} \exp\{i[bk_y(N-N')+\alpha\lambda L]\}$$

$$\times \exp\left\{\frac{i|x-x'|}{\hbar v_F}\left(E+\frac{i\hbar}{2\tau}\right)\right\}, \quad (74)$$

where

$$L= \sin(k_y b-qx')- \sin(k_y b-qx). \quad (75)$$

This is similar to the quasiclassical Green's function given by Gorkov and Lebed.[50]

Conductivity (46) then becomes

$$\sigma_{zz}=\frac{e^2t_c^2c}{\hbar^3 v_F^2 \pi L_x L_y} \sum_{N,N'} \sum_{k_{y1},k_{y2},\alpha} \int dx \int dx' \left[\exp\left\{i(N-N')\left(k_{y1}b-k_{y2}b-\frac{ebcB\sin\theta}{\hbar}\right)\right\}\right.$$

$$\times \exp\{i\alpha\lambda S_1\}\exp\left\{-\frac{|x-x'|}{v_F\tau}\right\}+\text{c.c.}\right], \quad (76)$$

where

$$S_1=L_1-L_2= \sin(k_{y1}b-qx')- \sin(k_{y1}b-qx)$$
$$- \sin(k_{y2}b-qx')+ \sin(k_{y2}b-qx). \quad (77)$$

If we now let $M_\pm=N\pm N'$ then the sum over M_- gives a δ function $2\pi\delta[k_{y1}b,k_{y2}b-(ebcB\sin\theta/\hbar)]$. Replacing $k_{y1}b$ with $k_y b$, the conductivity then simplifies to

$$\sigma_{zz}=\frac{e^2t_c^2c}{\hbar^3 \pi v_F^2 L_x} \sum_{k_y,\alpha} \int dx \int dx'$$

$$\times \left[\exp\{i\alpha\lambda S_2\}\exp\left\{-\frac{|x-x'|}{v_F\tau}\right\}+\text{c.c.}\right], \quad (78)$$

where

$$S_2= \sin(k_y b-qx')- \sin(k_y b-qx)$$
$$- \sin\left(k_y b-\frac{ebcB\sin\theta}{\hbar}-qx'\right)$$
$$+ \sin\left(k_y b-\frac{ebcB\sin\theta}{\hbar}-qx\right). \quad (79)$$

Now shift $k_y b$ to $k_y b+qx'$ and make the substitution $\Delta =ebcB\sin\theta/\hbar$, thus S_2 becomes

$$S_2= \sin[k_y b]- \sin[k_y b-q(x-x')]- \sin(k_y b-\Delta)$$
$$+ \sin[k_y b-\Delta-q(x-x')]. \quad (80)$$

We now let $x_\pm=(x\pm x')/2$ and $x_-=v_F t$, and perform the integral over x_+. This simplifies the conductivity, giving

$$\sigma_{zz}=\frac{2e^2t_c^2c}{\hbar^3 \pi v_F} \sum_{k_y,\alpha} \int_0^\infty dt \exp\{i\alpha\lambda S_3\}\exp\left\{-\frac{2t}{\tau}\right\}+\text{c.c.}, \quad (81)$$

where S_3 is given by

$$S_3= \sin(k_y b)- \sin(k_y b-2\omega_B t)- \sin(k_y b-\Delta)$$
$$+ \sin(k_y b-\Delta-2\omega_B t), \quad (82)$$

and $\omega_B=qv_F$. This can be separated into two parts and simplified using the appropriate trigonometric identities

$$S_3\equiv\mu-\beta$$

$$\equiv 2\cos\left(k_y b-\frac{\Delta}{2}\right)\sin\left(\frac{\Delta}{2}\right)$$

$$-2\cos\left(k_y b-\frac{\Delta}{2}-2\omega_B t\right)\sin\left(\frac{\Delta}{2}\right)$$

$$=\Delta\cos\left(k_y b-\frac{\Delta}{2}\right)-\Delta\cos\left(k_y b-\frac{\Delta}{2}-2\omega_B t\right), \quad (83)$$

where we have taken $\Delta\ll 1$. We can justify this by considering the dimensions of the unit cell and the magnetic flux passing through the area of the cell. If the magnetic field $B\sim 10$ T, and the area $bc\sim 10^{-18}$ m^2, then the flux $\Phi=Bbc$ will be small and thus $\Delta=\Phi/\Phi_0\ll 1$, where $\Phi_0=\hbar/e$ is a flux quantum. Rewriting the conductivity, we obtain

$$\sigma_{zz}=\frac{2e^2t_c^2c}{\hbar^3 \pi v_F} \sum_\alpha \int_0^\infty dt \exp\left\{-\frac{2t}{\tau}\right\} \int_{-\pi/b}^{\pi/b} \frac{dk_y}{2\pi}$$

$$\times \exp\{-i\alpha\lambda\beta\}\exp\{i\alpha\lambda\mu\}+\text{c.c.} \quad (84)$$

Using the identity[32]

$$\exp\left[\frac{z}{2}\left(h-\frac{1}{h}\right)\right]=\sum_{n=-\infty}^\infty h^n J_n(z), \quad (85)$$

the exponentials in μ and β become

$$\exp\{-i\alpha\lambda\beta\} = \sum_{n=-\infty}^{\infty} -i^n J_n(\alpha\lambda\Delta)$$
$$\times \exp\left\{in\left[k_y b - \frac{\Delta}{2} - 2\omega_B t\right]\right\},$$
$$(86)$$
$$\exp\{i\alpha\lambda\mu\} = \sum_{n'=-\infty}^{\infty} i^{n'} J_{n'}(\alpha\lambda\Delta)\exp\left\{in'\left[k_y b - \frac{\Delta}{2}\right]\right\}.$$

Substitution of these into Eq. (84) and performing the integral in t gives

$$\sigma_{zz} = \frac{e^2 t_c^2 \tau c}{2\hbar^3 \pi^2 v_F} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} -i^n i^{n'} J_n(\alpha\lambda\Delta)J_{n'}(\alpha\lambda\Delta)$$
$$\times \left[\frac{1}{1+in\omega_B\tau}\right]\int_{-\pi/b}^{\pi/b} dk_y \exp\left\{i\left(k_y b - \frac{\Delta}{2}\right)[n'+n]\right\}$$
$$+ \text{c.c.} \qquad (87)$$

This integral is zero unless $n = -n'$; thus

$$\sigma_{zz} = \frac{e^2 t_c^2 \tau c}{b\hbar^3 \pi v_F} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \left[\frac{J_n(\alpha z)^2}{1+in\omega_B\tau}\right] + \text{c.c.}, \qquad (88)$$

where $z = \lambda\Delta/\hbar = (2ct_b/\hbar v_F)(B\sin\theta/B\cos\theta) = \gamma\tan\theta$, and γ is the same as for the coherent case. The summation over α is performed by noting that $J_n(z)^2 = J_n(-z)^2$ for all n. Finally we include the complex conjugate to obtain an expression which can be written as Eq. (1).

C. Magnetic field parallel to the layers

We consider here the field range over which result (14) holds for incoherent interlayer transport. We define the magnetic field as $\vec{B} = (B_x, 0, 0)$ and the vector potential as $\vec{A} = (0, -zB_x, 0)$. It is easiest to work with spectral functions in the momentum representation; the interlayer conductivity is then given by

$$\sigma_{zz} = \frac{e^2 t_c^2 c}{\hbar\pi} \sum_k A_1(\vec{k}, E_F)A_2(\vec{k}, E_F), \qquad (89)$$

where A_1 and A_2 are the spectral functions for the two layers. Due to momentum conversation, we can then write the spectral function for layer two in terms of layer one as

$$A_1(\vec{k}, E_F) = A_2\left(\vec{k} - \frac{e}{\hbar}\vec{A}, E_F\right) = A(\vec{k}, E_F), \qquad (90)$$

where

$$A(\vec{k}, E_F) = \frac{2\Gamma}{[E_F - \epsilon(\vec{k})]^2 + \Gamma^2}, \qquad (91)$$

and $\Gamma = \hbar/2\tau$ and $\epsilon(\vec{k})$ is the dispersion within the layer.

We now specialize to the quasi-two-dimensional case. Substituting Eqs. (91) and (90) into the conductivity gives

$$\sigma_{zz} = \frac{e^2 t_c^2 c}{\hbar\pi^3}\int dk_x dk_y \frac{\Gamma}{\left[E_F - \frac{\hbar^2}{2m^*}(k_x^2+k_y^2)\right]^2 + \Gamma^2} \frac{\Gamma}{\left\{E_F - \frac{\hbar^2}{2m^*}\left[k_x^2 + \left(k_y + \frac{e}{\hbar}cB\right)^2\right]\right\}^2 + \Gamma^2}. \qquad (92)$$

Now, introduce polar coordinates (k, ϕ), so $k_x^2 + k_y^2 = k^2$ and $k_y = k\cos\phi$, and define $\Delta \equiv (ecB/\hbar)^2$ so that Eq. (92) becomes

$$\sigma_{zz} = \frac{e^2 t_c^2 c}{\hbar\pi^3}\int_0^\infty k\,dk \int_0^{2\pi} d\phi \frac{\Gamma}{\left[\frac{\hbar^2}{2m^*}k_F^2 - \frac{\hbar^2}{2m^*}k^2\right]^2 + \Gamma^2} \frac{\Gamma}{\left[\frac{\hbar^2}{2m^*}k_F^2 - \frac{\hbar^2}{2m^*}\left(k^2 + \Delta + \frac{2e}{\hbar}cBk\cos\phi\right)\right]^2 + \Gamma^2}. \qquad (93)$$

Suppose that the field is sufficiently large that $\Gamma \ll \hbar eBk_F c/m^*$ (which corresponds to $\omega_0\tau\gamma \gg 1$); then the first spectral function has a sharp peak near $k = k_F$, whereas near that peak the second term varies slowly. Hence we set $k = k_F$ in the second term, and then integrate over k to give

$$\sigma_{zz} = \frac{e^2 t_c^2 c\Gamma m^*}{\pi^2\hbar^3}\int_0^{2\pi} d\phi \frac{1}{\left[\frac{\hbar^2}{2m^*}\left(\Delta + \frac{2e}{\hbar}cBk_F\cos\phi\right)\right]^2 + \Gamma^2}. \qquad (94)$$

When $\Delta \ll ecBk_F/\hbar$, this integral will be dominated by the behavior near the two zeros of $\cos\phi$, so we can write the integral as

$$\sigma_{zz} = \frac{2e^2 t_c^2 c\Gamma m^*}{\pi^2\hbar^2}\int_{-\infty}^\infty d\phi \frac{1}{\left[\frac{\hbar}{m^*}ecBk_F\phi\right]^2 + \Gamma^2}. \qquad (95)$$

Performing the integral gives Eq. (14), resulting in a magnetoresistance which is linear in field.

When $\Delta \sim ecBk_F/\hbar$, that is, $ecB/\hbar k_F \sim 1$, deviations from this linear in field behavior will occur. If $c \sim 10$ Å, $ck_F \sim 3$, and then $B \approx 2000$ T. Similar arguments apply to the quasi-one-dimensional case. It will be shown in Sec. V that for coherent interlayer transport, the deviations from linear dependence can occur at much lower fields.

V. DEFINITIVE TESTS FOR COHERENT INTERLAYER TRANSPORT

We have shown that the Yamaji and Danner oscillations exist for both coherent and weakly incoherent interlayer transport and so cannot be used to establish that the Fermi surface is three dimensional. We now consider three properties which are different for coherent and incoherent interlayer transport.

A. Beats in magnetic oscillations

For quasi-two-dimensional systems, definitive evidence of the existence of a three-dimensional Fermi surface is the observation of a beat frequency in de Haas–van Alphen and Shubnikov–de Haas oscillations.[27] The frequency F of these oscillations is determined by extremal areas A of the Fermi surface, $F = \hbar A/(2\pi e)$ (Ref. 7). For the warped cylindrical Fermi surface (see Fig. 1 in Ref. 27) there are two extremal areas, corresponding to the "neck" and "belly" orbits. The small difference between the two areas leads to a beating of the corresponding frequencies F_1 and F_2. In a tilted magnetic field the frequency difference is

$$\frac{F_1 - F_2}{F_1} = \frac{4t_c}{E_F} J_0(k_F c \tan \theta). \tag{96}$$

Table I lists several materials in which such beat frequencies have been observed. In β-(BEDT-TTF)$_2$I$_3$ and β-(BEDT-TTF)$_2$IBr$_2$ the angular dependence of the beat frequency is consistent with Eq. (96) and $t_c/E_F \simeq 1/175$ and 1/280, respectively.[51]

However, Table I indicates that in many other quasi-two-dimensional organic metals no beat frequency has been observed. This could be because the interlayer transport is incoherent or because the interlayer hopping t_c is so small that the beats cannot be resolved experimentally. Suppose that oscillations but no beats are seen in the field range from B_{min} to B_{max}. This means that $\cos[2\pi(F_1-F_2)/B]$ has no zeros in this field range, implying that

$$F_1 - F_2 < \frac{B_{min}B_{max}}{B_{max} - B_{min}}. \tag{97}$$

This together with Eq. (96) can be used to establish an upper bound for t_c/E_F. For κ-(BEDT-TTF)$_2$I$_3$ the absence of beating has been used to establish $t_c/E_F < 1/3000$.[7,39] This implies a resistivity anisotropy $\rho_{xx}/\rho_{zz} \sim (t_c/E_F)^2 < 10^{-7}$. However, the observed[52] anisotropy in the κ-(BEDT-TTF)$_2$X materials is about 10^{-3}. This inconsistency suggests that the interlayer transport may be incoherent in κ-(BEDT-TTF)$_2$I$_3$. However, it could be that the measured value of 10^{-3} is too large because resistivity anisotropy is too large because the measurement of ρ_{xx} involves some component of ρ_{zz} due to an imhomogeneous current distribution or the current path being changed by sample defects.

B. Peak in the angle-dependent magnetoresistance at 90°

Numerical solutions of Chambers' formula (20) for coherent interlayer transport show that for both quasi-one-dimensional[9] and quasi-two-dimensional[31] materi-

als, at sufficiently high fields, the angle-dependent magnetoresistance has a peak as the field direction approaches the layers (i.e., at $\theta = 90°$). This peak is absent for incoherent interlayer transport (see Figs. 1 and 2). Hanasaki et al.[31] identified the peak as being due to closed orbits which occur when the field is parallel to the layers. These orbits are associated with the cyclotron frequency

$$\Omega = \omega_0\left(\frac{2t_c m c^2}{\hbar^2}\right)^{1/2} = \omega_0 \gamma \left(\frac{t_c}{E_F}\right)^{1/2}, \tag{98}$$

and so will only be important when the field is sufficiently large that $\Omega\tau > 1$.

Table I lists whether or not the peak has been observed for a range of quasi-two-dimensional metals. Note that the presence (absence) of the peak is not always consistent with the observed presence (absence) of beats. This can be because the two sets of measurements were done on different samples of different purity (and thus had different values of τ) or because the field was not large enough to observe the peak. The presence of a peak at 90° in the AMRO data[9] for (TMTSF)$_2$ClO$_4$ suggests that it has coherent interlayer transport.

C. Crossover from linear to quadratic field dependence for a magnetic field parallel to the layer

Schofield, Wheatley, and Cooper[34] considered the interlayer magnetoresistance for quasi-two-dimensional systems with coherent interlayer transport and a magnetic field parallel to the layer. Equation (25) of Ref. 34 gives an expression for the interlayer conductivity for all values of the magnetic field. They showed that when $\Omega\tau \ll 1$ the magnetoresistance increases linearly with field, as in Eq. (14). However, for $\Omega\tau \gg 1$ the field dependence becomes quadratic and is given by

$$\frac{\sigma_{zz}(B)}{\sigma_{zz}(0)} = \frac{1.96}{(\gamma\omega_0\tau)^2}\left(\frac{E_F}{t_c}\right)^{1/2}. \tag{99}$$

The deviations from linear behavior will occur when $\Omega\tau > 1$, i.e, $\omega_0\tau > (1/\gamma)(E_F/t_c)^{1/2}$. For typical organic samples this will happen in the field range of 10–100 T. In contrast for incoherent interlayer transport, it was shown in Sec. IV C that the deviation from the linear field dependence would not occur until about 2000 T. We are unaware of any material in which a search for this linear to quadratic crossover has been made. This field dependence is to be contrasted to that at angles slightly different from 90°, which will be given by Eq. (14). The ratio of these two expressions provides a means to determine t_c/E_F since γ and $\omega_0\tau$ can be deduced from AMRO data.

VI. CONCLUSIONS

We have presented detailed calculations of the interlayer magnetoresistance of quasi-one- and quasi-two-dimensional Fermi liquids in a tilted magnetic field. Two distinct models were used for the interlayer transport. The first involved coherent interlayer transport and made use of results of semiclassical or Bloch-Boltzmann transport theory. The second model involved weakly incoherent interlayer transport where

the electron is scattered many times within a layer before coherently tunneling into the next layer. We found that the dependence of the interlayer magneoresistance on the direction of the magnetic field is identical for both models except when the field is almost parallel to the layers. An important implication of this result is that coherent transport is not necessary for the observation of the Yamaji and Danner oscillations. Hence observation of one of these effects in a particular material cannot be interpreted as evidence that the material has a three-dimensional Fermi surface. Instead, we propose three unambiguous tests for coherent interlayer transport: (i) a beat frequency in the magnetic oscillations in quasi-two-dimensional systems, (ii) a peak in the angular-dependent magnetoresistance when the field is parallel to the layers, and (iii) a crossover from a linear to a quadratic field dependence for the interlayer magnetoresistance when the field is parallel to the layers. A survey of published experimental data on a wide range of quasi-two-dimensional organic metals suggests that some have properties (i) and (ii) others do not.

In future publications we will examine the frequency dependent interlayer conductivity and the Lebed and third angular effects in quasi-one-dimensional systems. A much greater challenge is to explain the AMRO observed in $(TMTSF)_2PF_6$ at pressures of about 10 kbar,[10,18] and in the low-temperature phase of α-$(BEDT\text{-}TTF)_2M Hg(SCN)_4[M$ = K,Rb,Tl]. The angular dependence of the latter is inverted compared to that of the Yamaji effect. In particular, the magnetoresistance is smallest when the field is in the layers, the opposite of what one expects based on the simple Lorentz force arguments relevant to semiclassical magnetoresistance. Understanding this may require knowledge of the effect of an orbital magnetic field on a strongly correlated electron system. Little is known about this problem except in the extreme quantum limit of a partially filled lowest Landau level,[53] which is far from the situation considered here where usually of the order of tens of Landau levels are filled.

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