

# Pairing symmetry and long-range pair potential in a weak-coupling theory of superconductivity

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We study the superconducting phase with two-component order-parameter scenario, such as  $d_{x^2-y^2} + e^{i\theta}s_\alpha$ , where  $\alpha = xy, x^2 + y^2$ . We show that in absence of orthorhombicity, the usual  $d_{x^2-y^2}$  does not mix with usual  $s_{x^2+y^2}$  symmetry gap in an anisotropic band structure. But the  $s_{xy}$  symmetry *does* mix with the usual  $d$ -wave symmetry for  $\theta=0$ . The  $d$ -wave symmetry with higher harmonics present in it also mixes with higher-order extended  $s$ -wave symmetry. The required pair potential to obtain *higher anisotropic*  $d_{x^2-y^2}$  and extended  $s$ -wave symmetries is derived by considering longer ranged two-body attractive potential in the spirit of tight-binding lattice. We demonstrate that the dominant pairing symmetry changes drastically from  $d$ - to  $s$ -like as the attractive pair potential is obtained from longer ranged interaction. More specifically, a typical length scale of interaction  $\xi$ , which could be even/odd multiples of lattice spacing, leads to predominant  $s/d$ -wave symmetry. The role of long-range interaction on pairing symmetry has further been emphasized by studying the typical interplay in the temperature dependencies of these higher-order  $d$ - and  $s$ -wave pairing symmetries. [S0163-1829(99)12333-6]

## I. INTRODUCTION

Many experiments were performed to find clues regarding the mechanism of high- $T_c$  superconductivity and the nature of the superconducting pair wave function. Notwithstanding this effort, the nature of the orbital symmetry of the order parameter is not yet known completely after a decade of its discovery although strong evidence of a major  $d_{x^2-y^2}$  symmetry exists.<sup>1-3</sup> Phase- and node-sensitive experiments also reported a sign reversal of the order parameter supporting  $d$ -wave symmetry.<sup>4</sup> The most current scenario that appears from various experiments and theory shows that the pairing symmetry of this family could be a mixed one like  $d_{x^2-y^2} + e^{i\theta}\alpha$  where  $\alpha$  could be something in the  $s$ -wave family or  $d_{xy}$ . The electron doped  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$  superconductors are, however, pure  $s$ -wave-like.<sup>5</sup>

Tunneling experiments had questioned the pure  $d$ -wave symmetry<sup>6</sup> as the data were interpreted as an admixture of  $d$ - and  $s$ -wave components due to orthorhombicity in YBCO.<sup>7,8</sup> Possibility of a minor but finite  $id_{xy}$  symmetry along with the predominant  $d_{x^2-y^2}$  has also been suggested<sup>10</sup> in connection with magnetic defects or small fractions of a flux quantum  $\Phi_0 = hc/2e$  in YBCO powders. Similar proposals came from various other authors in the context of magnetic field, magnetic impurity, interface effect, etc.<sup>11-13</sup> These proposals got the correct momentum when experimental data on longitudinal thermal conductivity by Krishana *et al.*<sup>12</sup> of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  compounds and that by Movshovich *et al.*<sup>12</sup> showed supportive indication to such proposals. There are experimental results related to interface effects as well as in the bulk that indicates mixed pairing symmetry (with dominant  $d$ -wave symmetry),<sup>13</sup> thus providing a strong threat to the pure  $d$ -wave models.

In this paper our main aim is to study the possibility of a mixed pairing symmetry state with  $\Delta(k) = \Delta_{d_{x^2-y^2}} + e^{i\theta}s_\alpha$  where  $\alpha = xy, x^2 + y^2$  for  $\theta=0, \pi/2$  with both  $d$  and  $s$  on an equal footing. We show that  $d_{x^2-y^2}$  can mix with  $s_{xy}$  in the

tetragonal group for  $\theta=0$  but not for  $\theta=\pi/2$ . The phase of the second condensate state is thus extremely important. We then show that even though the *lowest-order*  $d_{x^2-y^2}$  cannot mix with  $s_{x^2+y^2}$ , the corresponding *higher-order* symmetries can mix freely with each other. By *lowest-order* we mean the usual  $d$ -wave (i.e., simple  $\cos k_x - \cos k_y$  form), extended  $s$ -wave (i.e., simple  $\cos k_x + \cos k_y$  form), and so on. By *higher-order* we mean such symmetries with higher harmonics present in it, like the  $\cos \xi k_x \pm \cos \xi k_y$  form where  $\xi = na$  ( $n=1,2,3, \dots$ ) or even more complicated, like  $\cos 2k_x \cos k_y \pm \cos k_x \cos 2k_y$ , and so on. This will be clearer as we proceed. Now, in order to obtain such pairing symmetry in the respective channels one needs effective attractive pairing potential  $V(\xi k, \xi k')$ . We derive, in the spirit of tight-binding longer range attraction than the usual nearest (or next-nearest) neighbor one such interaction potential. The potential  $V(\xi k, \xi k')$  therefore changes the position of its minimum from that of the usual  $d$ - or  $s$ -wave cases for  $n > 1$ . We show, depending on the position of the pair potential or, in other words, longer ranged attractions  $\xi = 2a, 3a, 4a$ , etc. the dominant symmetry changes from  $d_{x^2-y^2}$  for  $\xi=a$ - to  $s$ -like otherwise.

This study can particularly be justified based on the following grounds. (i) On general grounds, long-range interactions arise from a decrease in screening as one approaches the insulator. In specific models of superconductivity like the spin-fluctuation mediated models, an increase in the antiferromagnetic correlation length occurs with underdoping. (ii) One of the potential theories of high-temperature superconductivity that favors  $d$ -wave symmetry is the spin-fluctuation theory.<sup>14</sup> The gap symmetry of the spin-fluctuation theory is, however, not the *simplest*  $d$ -wave but *higher-order*  $d$ -wave, approximately of the form  $(\cos k_x - \cos k_y)(\cos k_x + \cos k_y)^N$ .<sup>15</sup> Explicit  $k$  anisotropy of the gap in spin-fluctuation mediated superconductivity was obtained by Lenck and Carbotte<sup>16,8</sup> in BCS theory with the phenomenological spin susceptibility as pairing interaction using fast-Fourier-transform technique, without any prior assumption about the symmetry of the gap.

They concluded that the gap, although having nodal lines along  $k_x=k_y$ , does not have the simplest  $d$ -wave symmetry but rather *higher-order*  $d$ -wave symmetry with higher harmonics present in it. Therefore this work provides a real-space derivation of a pair potential that produces higher-order  $d$ -wave symmetry similar to that present in the spin-fluctuation theory. (iii) In the magnetic scenario of the cuprates,<sup>17</sup> one can set  $\xi$  equal to the magnetic coherence length which is larger than the lattice spacing.<sup>3</sup> The coherence length in the superconducting state which is different for different materials may be because a short-range interaction requires larger densities than a long-range one in order to produce coherent motion that leads to superconductivity. (The  $T_c-x$  relationship is not unique in all high- $T_c$  systems; some start to superconduct with very small doping  $x$  whereas some systems require larger  $x$ .) (iv) The high- $T_c$  systems are in very complicated circuits and the electronic correlation effects may not be adequately accounted unless one considers next-nearest or further-neighbor repulsion. Therefore, in the spirit of tight-binding lattice, the effective attraction may only arise with more distant attractive interaction. (v) In a most recent angle-resolved photoemission (ARPES) experiment by a well-known group,<sup>18</sup> such requirement of long-range interaction was realized. One of their essential findings is that, as the doping decreases, the maximum gap increases, but the slope of the gap near the nodes decreases. This particular feature, although consistent with  $d$  wave, cannot be fit by simple  $\cos(2\phi)$  but requires a finite mixing of  $\cos(6\phi)$  as well, where  $\phi$  is the angle between  $k_x, k_y$  given as  $\tan^{-1}(k_y/k_x)$ . The  $\cos(6\phi)$  contains higher harmonics than simple  $(\cos k_x - \cos k_y)$ . The rest of the layout of the paper is as follows: In Sec. II, we derive the pair potential required for higher anisotropic  $d$ - and extended  $s$ -wave symmetries. We also provide a brief prescription of finding coupled gap equations for the amplitudes of such higher anisotropic symmetries. In Sec. III, we present and discuss in detail all of the numerical results providing strong signature of change in dominant pairing symmetry with range of interaction. Finally, we conclude in Sec. IV.

## II. MODEL CALCULATION

Let us consider that the overlap of orbitals in different unit cells is small compared to the diagonal overlap. Then, in the spirit of tight-binding lattice description, the matrix element of the pair potential may be obtained as

$$\begin{aligned}
V(\vec{q}) = & \sum_{\vec{\delta}} V_{\vec{\delta}} e^{i\vec{q}\vec{R}_{\vec{\delta}}} = V_0^r + V_1 f^d(k) f^d(k') + V_1 g(k) g(k') \\
& + V_2 f^{d_{xy}}(k) f^{d_{xy}}(k') + V_2 f^{s_{xy}}(k) f^{s_{xy}}(k') \\
& + V_3 f^d(2k) f^d(2k') + V_3 g(2k) g(2k') \\
& + 2V_4 \tilde{f}_1^d(2k) \tilde{f}_1^d(2k') + 2V_4 \tilde{f}_2^d(2k) \tilde{f}_2^d(2k') \\
& + 2V_4 \tilde{g}_1(2k) \tilde{g}_1(2k') + 2V_4 \tilde{g}_2(2k) \tilde{g}_2(2k') \\
& + V_5 f^{d_{xy}}(2k) f^{d_{xy}}(2k') + V_5 f^{s_{xy}}(2k) f^{s_{xy}}(2k') \\
& + V_6 f^d(3k) f^d(3k') + V_6 g(3k) g(3k'), \quad (1)
\end{aligned}$$

where in the first result of Eq. (1)  $\vec{R}_{\vec{\delta}}$  locates nearest-neighbor and further neighbors,  $\vec{\delta}$  labels and  $V_n$ ,  $n = 1, \dots, 6$  represents strength of attraction between the respective neighbor interaction. The first term in the above equation  $V_0^r$  refers to the on-site interaction which is considered as repulsive but can be attractive as well giving rise to isotropic  $s$  waves. In this paper, we shall not consider the isotropic  $s$  wave for a mixed symmetry with  $d$  wave (cf. Ref. 19). The form factors of the potential are obtained as

$$\begin{aligned}
f^d(nk) &= \cos(nk_x a) - \cos(nk_y a), \\
g(nk) &= \cos(nk_x a) + \cos(nk_y a), \\
f^{d_{xy}}(nk) &= 2 \sin(nk_x a) \sin(nk_y a), \\
f^{s_{xy}}(nk) &= 2 \cos(nk_x a) \cos(nk_y a), \\
\tilde{f}_1^d(2k) &= \cos(2k_x a) \cos(k_y a) - \cos(k_x a) \cos(2k_y a), \\
\tilde{f}_2^d(2k) &= \sin(2k_x a) \sin(k_y a) - \sin(k_x a) \sin(2k_y a), \\
\tilde{g}_1(2k) &= \cos(2k_x a) \cos(k_y a) + \cos(k_x a) \cos(2k_y a), \\
\tilde{g}_2(2k) &= \sin(2k_x a) \sin(k_y a) + \sin(k_x a) \sin(2k_y a), \quad (2)
\end{aligned}$$

where  $f^d(nk), g(nk)$  leads to usual  $d_{x^2-y^2}, s_{x^2+y^2}$  pairing symmetry for  $n=1$  and unusual or higher-order  $d_{x^2-y^2}, s_{x^2+y^2}$  pairing symmetry respectively, which results from interactions along the  $x$  and  $y$  axes (i.e., first-, third-, sixth-neighbor interaction). While the usual and higher-order  $d_{xy}, s_{xy}$  pairing symmetry results from  $f^{d_{xy}}(nk), f^{s_{xy}}(nk)$ , the fourth-neighbor interaction gives rise to unconventional  $d$ - and extended  $s$ -wave pairing symmetry through  $\tilde{f}_n^d(2k)$  and  $\tilde{g}_n(2k)$  given in Eq. (2). In deriving Eqs. (1) and (2), terms responsible for triplet pairing which are not important for high- $T_c$  systems are neglected. We shall discuss now the mixed phase symmetry of  $d_{x^2-y^2}$  with other symmetries taking two of the potential terms at a time, namely, a combination of potential terms in Eq. (1) (2nd,3rd), (6th,7th), (14th,15th) gives rise to pairing symmetry  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0) f^d(\xi k) + e^{i\theta} \Delta_{s_{x^2+y^2}}(0) g(\xi k)$  where  $\xi = na$ ,  $a$  is the lattice constant and will be taken as unity. Similarly, a combination of (2nd,4th), (6th,12th), and so on, will give rise to pairing symmetry  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0) f^d(\xi k) + e^{i\theta} \Delta_{d_{xy}}(0) f^{d_{xy}}(\xi k)$ , etc.

Free energy of a superconductor with arbitrary pairing symmetry may be written as

$$F_{k,k'} = -\frac{1}{\beta} \sum_{k,p=\pm} \ln(1 + e^{-p\beta E_k}) + \frac{|\Delta_k|^2}{V_{kk'}}, \quad (3)$$

where  $E_k = \sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2}$  are the energy eigenvalues of a Hamiltonian that describes superconductivity. We minimize the free energy, Eq. (3) i.e.,  $\partial F / \partial |\Delta| = 0$ , to get the gap equation as

$$\Delta_k = \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \tanh\left(\frac{\beta E_{k'}}{2}\right), \quad (4)$$

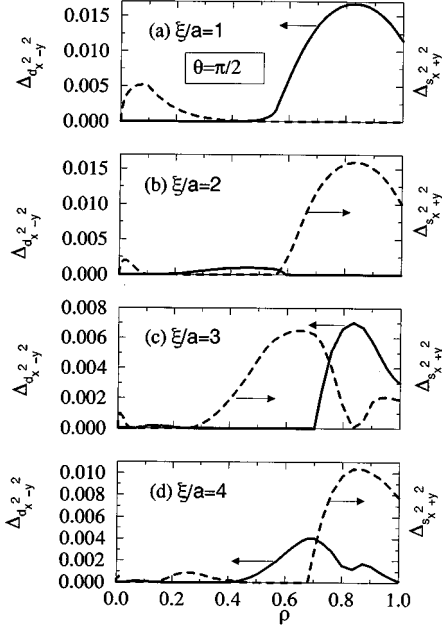


FIG. 1. Amplitudes of the  $\Delta_{d_{x^2-y^2}}$  (solid lines) and  $\Delta_{s_{x^2+y^2}}$  (dashed lines) as a function of band filling  $\rho$  for  $\theta = \pi/2$  (i.e.,  $d_{x^2-y^2} + is_{x^2+y^2}$ ) phase in various values of  $\xi/a$ . While the usual  $d_{x^2-y^2}$  does not mix with usual  $s_{x^2+y^2}$  (a), higher component  $d_{x^2-y^2}$  and  $s_{x^2+y^2}$  (c) and (d) can mix with each other freely even in the absence of orthorhombicity. It is worth noticing the change in the dominant pairing symmetry with  $\xi/a$  (e.g., for  $\xi/a=2$  the only dominant symmetry is  $s$ -wave-like).

where  $\epsilon_k$  is the dispersion relation taken from the ARPES data<sup>9</sup> and  $\mu$  the chemical potential will control band filling through a number-conserving equation given below. For two-component order-parameter symmetries, as mentioned above, we substitute the required form of the potential and the corresponding gap structure into either side of Eq. (4) which gives us an identity equation. Then separating the real and imaginary parts together with comparing the momentum dependences on either side of it we get gap equations for the amplitudes in different channels as

$$\Delta_j = \sum_k V_j \frac{\Delta_j f_k^{j2}}{2E_k} \tanh\left(\frac{\beta E_k}{2}\right), \quad j=1,2. \quad (5)$$

Considering mixed symmetry of the form  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0)f^d(nk) + \Delta_{s_{x^2+y^2}}(0)g(nk)$  one identifies  $\Delta_1 = \Delta_{d_{x^2-y^2}}(0)$ ,  $\Delta_2 = \Delta_{s_{x^2+y^2}}(0)$  and  $f_k^1 = f^d(nk)$ ,  $f_k^2 = g(nk)$ , and, similarly, for mixed symmetries of the form  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0)f^d(nk) + \Delta_{\alpha_{xy}}(0)f^{\alpha_{xy}}(nk)$  where  $\alpha \equiv s, d$ ,  $\Delta_2 = \Delta_{\alpha_{xy}}(0)$  and  $f_k^2 = f^{\alpha_{xy}}(nk)$ , and so on. The potential required to get such pairing symmetries is discussed in Eq. (1).

The number-conserving equation that controls the band filling through chemical potential  $\mu$  is given by

$$\rho(\mu, T) = \sum_k \left( 1 - \frac{(\epsilon_k - \mu)}{E_k} \tanh\left(\frac{\beta E_k}{2}\right) \right). \quad (6)$$

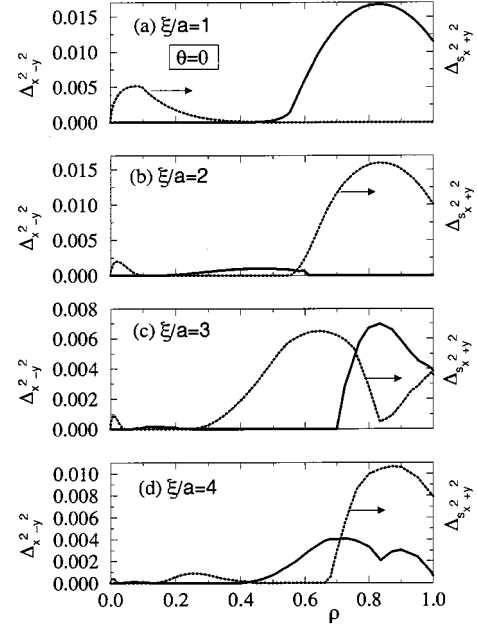


FIG. 2. Same as that of Fig. 1 except  $\theta=0$  (i.e.,  $d_{x^2-y^2} + s_{x^2+y^2}$  symmetry). The predominant symmetry always tries to expel (minimize) occurrence of the other symmetry at its optimum doping.

We solve self-consistently the above three equations [Eqs. (5) and (6)] in order to study the phase diagram of a mixed order-parameter superconducting phase. The numerical results obtained for the gap amplitudes through Eqs. (5) and (6) will be compared with free energy minimizations via Eq. (3) to get the phase diagrams.

### III. RESULTS AND DISCUSSIONS

We present in this section our numerical results for a set of fixed parameters, e.g., a cutoff energy  $\Omega_c = 500$  K around the Fermi level above which superconducting condensate does not exist; a fixed ratio  $V_1/V_2 = 0.71$  in Eq. (5) between the strengths of pairing interaction channels throughout. In Figs. 1 and 2 we present results for  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0)f^d(\xi k) + e^{i\theta}\Delta_{s_{x^2+y^2}}(0)g(\xi k)$  symmetries for  $\theta = \pi/2$  and  $\theta = 0$ , respectively. Such symmetries would arise from a combination of two-component pair potentials (2nd,3rd),(6th,7th),(14th,15th), and so on. We shall discuss only the results of  $\theta = 0$  and  $\theta = \pi/2$ . These two phases of  $\theta$  can cause important differences (cf. Figs. 3 and 4). It is known that for any  $\theta \neq 0$ , time-reversal symmetry is locally broken<sup>19</sup> which corresponds to a phase transition to an *almost* fully gapped phase (except at the points  $\pm\pi/2, \pm\pi/2$  due to common nodal points from both channels) from a partially ungapped phase of  $d_{x^2-y^2}$  symmetry. On the other hand, the  $\theta = 0$  phase still remains nodeful, although the nodal lines shifts a lot from the usual  $k_x = k_y$  lines of the  $d_{x^2-y^2}$ .

The solid lines represent the amplitude of the  $d_{x^2-y^2}$  channel whereas the dashed lines indicate that of  $s_{x^2+y^2}$ . These Figs. (1 and 2) clearly demonstrate that the *usual*  $d_{x^2-y^2}$  and  $s_{x^2+y^2}$  symmetries do not mix with each other

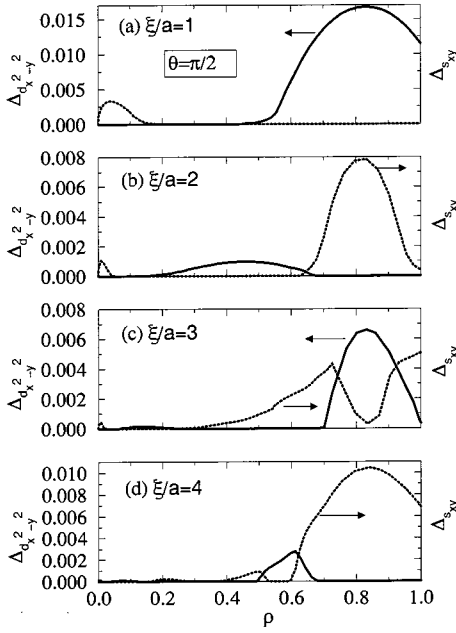


FIG. 3. Amplitudes of the  $\Delta_{d_{x^2-y^2}}$  (solid lines) and  $\Delta_{s_{xy}}$  (dashed lines) as a function of band filling  $\rho$  for  $\theta = \pi/2$  (i.e.,  $d_{x^2-y^2} + is_{x^2+y^2}$ ) phase in various values of  $\xi/a$ . While the usual  $d_{x^2-y^2}$  does not mix with usual  $s_{xy}$  (a), higher anisotropic  $d_{x^2-y^2}$  and  $s_{xy}$  (c) and (d) can mix with each other freely even in the absence of orthorhombicity. It is worth noticing that the change in the dominant pairing symmetry with the typical length  $\xi/a$  (e.g., for  $\xi/a = 2$  the only dominant symmetry is  $s$  wavelike). Panel (a) should particularly be contrasted with that of Fig. 4

[cf. Figs. 1(a) and 2(a)] but the *higher-order*  $d_{x^2-y^2}, s_{x^2+y^2}$  symmetries do mix with each other [cf. Figs. 1(c) and 2(c) and (d)]. In fact, as the interaction becomes longer ranged (i.e.,  $\xi/a = 1, 2, 3, 4$  as is demonstrated in Figs. 1, 2(a), (b), (c), and (d), respectively) the dominant symmetry changes drastically; as the typical length  $\xi$  is odd multiple of the lattice constant, the dominant symmetry at lower doping is  $d_{x^2-y^2}$ -like whereas when the  $\xi$  is an even multiple of the lattice constant, the dominant symmetry at lower doping is something in the  $s$ -wave family (see also Figs. 3 and 4).

As the typical length  $\xi$  is increased, the predominant symmetry at the optimal doping<sup>20</sup> changes from  $d$ -wave at  $\xi = a$  to an extended  $s$ -wave  $s_{x^2+y^2}, s_{xy}$  for  $\xi = 2a$ , to again a predominant  $d$ -wave symmetry at  $\xi = 3a$  and finally for  $\xi = 4a$  to extended  $s$ -wave symmetry for  $\theta = \pi/2$ . These phase diagrams (Figs. 1 – 4) drawn at  $T = 1$  mK do not change the scenario even for  $\theta = 0$ , in the mixed phase of  $d$  wave with  $s_{x^2+y^2}$  symmetry but causes significant change for that with  $s_{xy}$  symmetry (cf. Fig. 4). More significantly, the case of  $\xi = 2a$  is universal (i.e., independent of  $\theta$  and  $s_{x^2+y^2}$  or  $s_{xy}$  mixing with  $d$  wave), the dominant symmetry at zero temperature is  $s$ -wave-type. This work therefore has revealed in a significant way the change in predominant pairing symmetry as the interaction range is changed at  $T = 0$ . It is to be noted that in contrast to hole doped material, the electron doped materials (like  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ ) have no signature of dominant  $d$ -wave symmetry. Furthermore, the antiferromagnetic phase in the electron doped systems is more extended or exists until larger doping in comparison to the hole doped

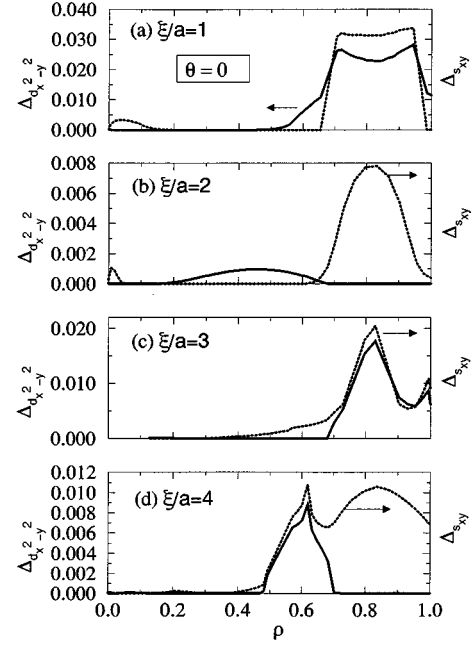


FIG. 4. Same as that in Fig. 3 except for  $\theta = 0$ , i.e.,  $d_{x^2-y^2} + s_{xy}$  phase that preserves the time reversal symmetry. The notable difference is that the usual  $d_{x^2-y^2}$  and  $s_{xy}$  components can mix with each other freely in absence of orthorhombicity, signifying the importance of the phase  $\theta$  of the non- $d$ -wave symmetry, in contrast to Fig. 3(a).

material. Therefore, considering models related to spin-fluctuation mediated superconductivity, the longer range attraction should be more important. In the present picture, we showed that such longer range interaction causes change in the pairing symmetry which might make this study to have important bearings for the high- $T_c$  compounds.

Some interesting features of the data presented is that optimal doping remains unchanged irrespective of  $\xi$  that causes a significant *crossover* in the dominant symmetry of the order parameter. The position of the  $d$  wave does not change appreciably except in the case of  $\xi/a = 4$  while the extended  $s$ -wave region moves drastically with  $\xi$ . In particular, for  $\xi/a = 1$ , the extended  $s$ -wave family has finite amplitude only at densities close to zero ( $\rho \sim 0$ ) (cf. Figs. 1–3) leading to no mixed phases except the outstanding case of  $\theta = 0$  for  $s_{xy}$  (cf. Fig. 4). In  $\xi/a = 2$  case, the extended  $s$ -wave family completely takes over the position of the  $d$  wave that it had in the case of  $\xi/a = 1$ . For  $\xi/a = 3$ , the  $d$  wave regains its position although both the amplitude and width decreases to about 50% to that of the  $\xi/a = 1$  case and the  $s$  wave shifts towards larger doping having its amplitude minimum at the maximum of the  $d$  wave. For  $\xi/a = 4$  the extended  $s$  wave dominates and the  $d$  wave either becomes a minor component or does not appear at all. Furthermore, in the optimal doping, whichever symmetry dominates causes the amplitude of the other minimum; i.e., the dominant symmetry always expels the other one at the optimum doping.

Following the above discussion, it is obvious that Fig. 4 represents an exceptional case. Figure 4 represents phase diagram of superconductors having mixed phase symmetry like  $\Delta_{d_{x^2-y^2}}(0)f^d(\xi k) + e^{i\theta}\Delta_{s_{xy}}(0)f^{s_{xy}}(\xi k)$  with  $\theta = 0$  (the case



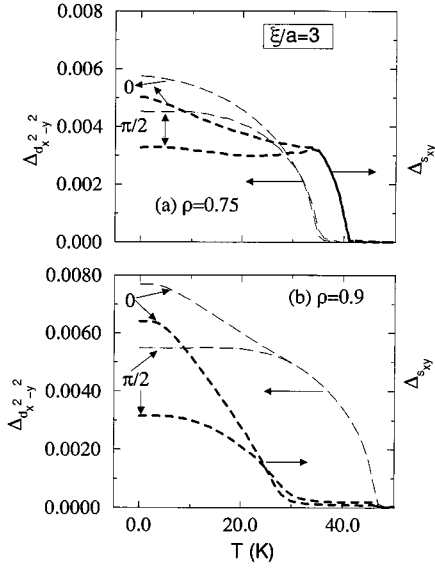


FIG. 5. Temperature dependencies of the superconducting gap in the  $d_{x^2-y^2}$  and  $d_{s_{xy}}$  channel for their real and complex mixing for different band fillings (a)  $\rho=0.75$  and (b)  $\rho=0.9$ . When the  $s_{xy}$  component has larger  $T_c$ , its thermal growth is suppressed at the onset of the  $d_{x^2-y^2}$  component [cf. (a)] but that of the  $d_{x^2-y^2}$  amplitude is not influenced by the corresponding onset of the  $s_{xy}$  [cf. (b)]. In general, for  $\theta=0$  the gaps open up at a faster rate with decreasing temperature than that for  $\theta=\pi/2$ .

of  $\theta=\pi/2$  is discussed in Fig. 3 and should be contrasted with Fig. 4). The phase diagram comprises the amplitudes of the respective symmetry channels as a function band filling  $\rho$ . In striking contrast to Figs. 1–3, there is strong mixing of  $d_{x^2-y^2}$  with  $s_{xy}$  for  $\xi/a=1, 3$ , and 4. In fact, mixing between the two symmetries is so strong that it is difficult to find out the predominant symmetry for the cases  $\xi/a=1$  and 3. In this mixed symmetry, for  $\theta=\pi/2$  and  $\xi/a=4$  [cf. Fig. 3(d)], the  $d$ -wave amplitude is practically zero whereas for  $\theta=0$  [cf. Fig. 4(d)] it has strong mixing regime. This is the only mixed phase where both of the symmetries at optimal doping have large values [see Figs. 4(a) and (c)] unlike those in Figs. 1–3. The results of this figure thus convincingly point out the role of the phase between the two mixing symmetries. All the experimentally observed properties of cuprates will be consistent with the scenario of Fig. 4, including the sign change of the order parameter as well as gap nodes. The strong interplay between the two order parameters of mixed  $d-s_{xy}$  symmetry has also been reflected in their thermal behaviors (cf. Fig. 5). In Figs. 5 and 6 we display the temperature dependencies of the amplitudes (in eV) of different symmetry order parameters for  $\xi/a=3$  as maximum mixing is found in this case. When the  $s_{xy}$  component determines the bulk  $T_c$ , [e.g., at  $\rho=0.75$  in Fig. 5(a)] the amplitude of the  $s_{xy}$  component is suppressed with the onset of the  $d$ -wave component. However, when the bulk  $T_c$  is determined by the  $d$  wave, the amplitude of the  $d$  wave is not affected by the onset of the  $s_{xy}$  component. In a study of mixed phase with usual  $d+is$  phase with  $s$  as isotropic  $s$  wave, it was shown earlier<sup>19,21</sup> that the  $d$ -wave component gets suppressed with the onset of  $s$  wave but not the reverse. In contrast to Fig. 5, the temperature dependencies of the amplitudes of the  $d$  and

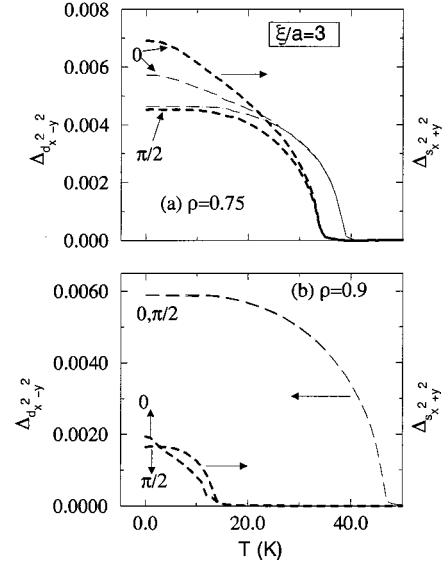


FIG. 6. Temperature dependencies of the superconducting gap in the  $d_{x^2-y^2}$  and  $s_{x^2-y^2}$  channel for their real and complex mixing for different band fillings (a)  $\rho=0.75$  and (b)  $\rho=0.9$ . Unlike the  $s_{xy}$  case (cf. Fig. 5), the temperature dependencies of the gap amplitudes in the respective channels do not influence each other. Similar to that in Fig. 5, for  $\theta=0$  the gaps open up at a faster rate with decreasing temperature than that for  $\theta=\pi/2$ .

$s_{x^2+y^2}$  symmetries remain unaffected by each other as displayed in Fig. 6. In general, however, the growth of the amplitudes of different symmetries with lowering in temperature is faster in the case of  $\theta=0$  than that for  $\theta=\pi/2$ . This once again emphasizes the role of the phase  $\theta$ . Temperature dependencies for other values of  $\xi/a$  are qualitatively the same as those shown in Figs. 5 and 6.

So far we have discussed the interplay of order parameters in mixed phases like  $\Delta_{d_{x^2-y^2}} + e^{i\theta} \Delta_{s_{x^2+y^2}}$ ,  $\alpha = x^2 + y^2$  or  $xy$ . This excluded discussion of some other exotic  $d$  and  $s_{x^2+y^2}$  symmetries that can arise from the fourth-neighbor attraction as discussed earlier in the context of Eqs. (1) and (3). More specifically, a combination of (8th+9th) and (10th+11th) terms of Eq. (1) can give rise to mixed pairing symmetries such as  $\Delta(k) = \Delta_{d_{x^2-y^2}}(0)F^d(k) + e^{i\theta}\Delta_{s_{x^2+y^2}}(0)G^s(k)$  where  $F^d(k) = f^d(k)[1 + f^{d_{xy}}(k) + f^{s_{xy}}(k)]$ ,  $G^s(k) = g(k)[f^{d_{xy}}(k) + f^{s_{xy}}(k) - 1]$ . These exotic symmetries are not discussed in the literature. Following the same procedure as deriving Eq. (5), one can find the gap equation for the components  $\Delta_{d_{x^2-y^2}}(0)$  and  $\Delta_{s_{x^2+y^2}}(0)$ , although a bit complicated, it arrives at the same gap equation as Eq. (5) with the pair vertex  $V_{j \rightarrow} \rightarrow V_j/2$  and  $f_k^1 = F^d(k), f_k^2 = G^s(k)$ . Solving the gap equations together with the number Eq. (6) simultaneously, no mixing between these unconventional  $d$ - and  $s$ -wave symmetries was found. Within the same parameter as in earlier figures (i.e.,  $V_1/V_2=0.71$ ),  $d$  wave remains very strong at lower dopings (within the range  $1 \geq \rho > 0.70$ ) whereas the  $s$ -wave amplitude appears very close to zero band filling. Therefore in Fig. 7 we present the momentum anisotropy of the unconventional  $d$ -wave gap originated from fourth-neighbor attraction. It is clear that gap anisotropy is undoubtedly very different from the *usual* nearest-neighbor  $d$ -wave symmetry, although basic features of change in sign, nodes,

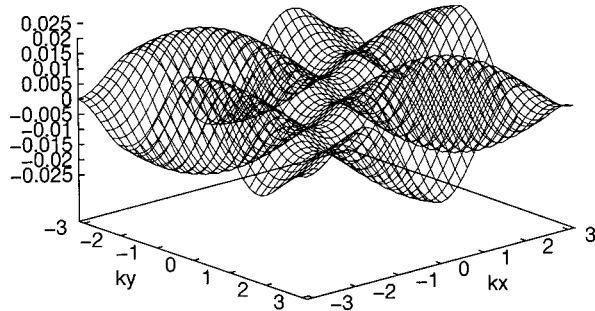


FIG. 7. Momentum anisotropy of the *higher anisotropic d-wave* symmetry. This higher anisotropic  $d_{x^2-y^2}$  symmetry originates from the fourth-neighbor attraction in an anisotropic lattice [cf.  $V_4$  terms in Eq. (1)]. The remarkable difference in the  $k$  anisotropy of this  $d$ -wave symmetry compared to the *usual d-wave* symmetry is worth noticing. This  $d$  wave has  $2\Delta(k)_{\max}/k_B T_c = 5$  at  $\rho = 0.8$ .

etc., remain the same as that of the ordinary  $d$  wave. This gap symmetry at  $\rho = 0.8$  gives rise to a BCS gap ratio  $2\Delta(k)_{\max}/k_B T_c = 5.0$  against 4.29 in the case of *usual d* wave. Such higher anisotropic  $d$ -wave symmetries will have the advantage of avoiding electronic repulsion in strongly correlated systems like the cuprates.

#### IV. CONCLUSIONS

We have studied the superconducting phase with the two-component order-parameter scenario, such as  $d_{x^2-y^2} + e^{i\theta} s_\alpha$ , where  $\alpha = xy, x^2 + y^2$ . We showed, that in absence

of orthorhombicity, the usual  $d_{x^2-y^2}$  does not mix with the usual  $s_{x^2+y^2}$  symmetry gap in an anisotropic band structure. But the  $s_{xy}$  symmetry *does* mix with the usual  $d$ -wave symmetry for  $\theta = 0$ . Even in absence of orthorhombicity, the *higher anisotropic d-wave* symmetry mixes with *higher anisotropic* extended  $s$ -wave symmetry. This is obtained by considering longer ranged two-body attractive potential in the spirit of tight-binding lattice than the usual nearest neighbor. This study revealed that the dominant pairing symmetry changes drastically from  $d$ - to  $s$ -like as the attractive pair potential is obtained from longer ranged attraction — if the interaction is sufficiently short ranged that can be mapped into a nearest-neighbor potential, at low doping, the system is described by *pure*  $d_{x^2-y^2}$  order parameter. Such consideration of longer range attraction has also been revealed by recent ARPES data.<sup>18</sup> The role of longer range pair potential on pairing symmetry within weak-coupling theory of superconductivity has thus been established. We showed that the momentum distribution of the *higher anisotropic d-wave* symmetries is quite different from the usual  $d$ -wave symmetries. We found that the typical interplay in the temperature dependencies of these higher-order  $d$ - and  $s$ -wave pairing symmetries can be different from what is known. In brief, we believe such study of higher anisotropic symmetries is potentially important and will stimulate further studies in contrast to the usual  $d$ - and  $s$ -wave symmetries.

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