Local magnon modes and resonances for dynamical skyrmions in Heisenberg two-dimensional ferromagnets

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We investigate bound magnon states localized on dynamical topological solitons (dynamical skyrmions) in two-dimensional (2D) Heisenberg weakly uniaxial ferromagnets. The exact analytical expressions for zero modes are derived. It is shown that in addition to zero modes with ‘‘angular momentum’’ numbers $m=0$, ± 1 truly local modes can exist in 2D ferromagnets on dynamical vortices provided that the magnon dispersion relation has a gap. We study real local modes for dynamical solitons with topological charges $\nu=1,2$. Eigenfrequencies of these modes are calculated numerically. It may be possible to observe resonance effects on eigenfrequencies of internal oscillations of dynamical skyrmions in 2D weakly easy-axis ferromagnets.
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I. INTRODUCTION

There has recently been a great interest in quasi-two-dimensional (2D) layered magnetic compounds.[1] It has been motivated by technical advances in the creation of real two-dimensional layered magnetic materials and theoretical understanding based on vortex (soliton) concept. The presence of mobile strongly nonlinear excitations (solitons) contributes to the central peak (CP), which for quasi-one-dimensional (1D) ferromagnets (FM) $CsNiF_3$ was observed by Kjems and Steiner in 1976.[2] It is now well understood, that kinks in 1D systems destruct long-range order at finite temperatures. In 2D easy-plane magnets vortices give rise to the Berezinskii-Kosterlitz-Thouless transition[3] when the dissociation of vortex-antivortex pairs takes place above the critical temperature T_{KT}. This transition occurs also in many other 2D systems such as crystals and superfluid films.[4] Two-dimensional behavior seems to be important for exploring the properties of ceramic superconductors.[5] For high-T_c superconductors there is a possible interplay between antiferromagnetism and superconductivity as noted in Ref. 6. In the superconductivity context moving self-localized magnons in easy-axis 2D Heisenberg model with a hole have been considered by Takeno, Kubota, and Kawasaki in Ref. 7. They have shown that the lesser value of the easy-axis exchange anisotropy constant the faster propagation of the localized mode leaving behind the one localized at the hole site. Other important examples of current issues, which should be mentioned are the Haldane gap problem,[8] ‘‘skyrmions’’ under the quantum Hall effect conditions,[9] and in the ultrathin magnetic films.

For topological solitons of boson fields an axially symmetric solution (skyrmion), minimizing the Hamiltonian has been found by Skyrme.[10] The space structure of a skyrmion can be derived by mapping of the plane in which the spins are placed onto sphere of the order parameter. Belavin and Polyakov have established the existence of a local static distortion of the homogeneous state of 2D isotropic Heisenberg ferromagnets, based on the classical nonlinear σ model.[11] Recently Waldner[12] estimated the skyrmion energy from the heat capacity measurements and interpreted the results as an indirect manifestation of skyrmions in quasi-2D magnets.

We pay a special attention to ferro- and antiferromagnets with easy-axis anisotropy. Namely for weak easy-axis antiferromagnets, Haldane has made his conjecture about the intrinsic difference between integer-spin and half-integer-spin magnetic systems.[8]

Two-dimensional magnon droplets with nonzero angular momentum or dynamical skyrmions (DS) can exist in the uniaxial 2D FMs.[13,14] These excitations are similar to precessing bubble domains[15,16] with a radius, which is less or comparable to the magnetic length. They are stable both in the topological and dynamical senses due to the spin precession unlike their instability in the static description, according to the Derick-Hobart theorem.

All types of solitons (domain walls, vortices, point defects and so on) have an internal degrees of freedom, which in principle can be observed in electron spin resonance (ESR) or inelastic neutron scattering (INS) experiments. Such magnetic soliton resonances have been detected in 1D Ising-type antiferromagnets (AFM) $CsCoCl_3$[17] and investigated theoretically for the case of Heisenberg AFM in the work.[18] Resonances on domain walls in 3D thulium ortoferrite have been observed in Ref. 19. Currently there are no obvious examples of experimental observation of soliton internal resonances in 2D magnets. Magnon modes in isotropic,[20] easy-plane Heisenberg,[21,22] XY-type[23] 2D FM have been predicted and investigated. Two magnon modes, localized on the in-plane vortices were studied analytically in 2D easy-plane AFM within continuous spectrum.[24] In all of these works only *quasi-local* modes have been predicted. A truly local mode within the continuous spectrum in easy-plane 2D AFM has been investigated in Ref. 25 Truly local modes (LM) can exist also on dynamical topological solitons, if magnons have finite activation dispersion law and eigenfrequencies of these modes lie in the frequency range, determined by the strength of anisotropy.[26,27] Near isotropic uniaxial FMs are of special interest because even very weak easy-axis anisotropy can change the situation crucially, leading to the appearence of local modes.

Continuum description of the DS is reasonable because in the near isotropic case the spin field has no singularity in the core region of the vortex unlike vortices in gapless easy-plane FM on a discrete lattice for which Wysin has made his ansatz.[21,22] The DS have an interesting internal structure which may be detected experimentally.

The aim and spirit of the present work is to show that in classical 2D FMs dynamical skyrmions have truly localized internal magnon modes which should be detected in resonance experiments with quasi-2D magnetic materials with weak easy-axis magnetic anisotropy.

The paper is organized as follows. In Sec. II the 2D Heisenberg easy-axis model is presented. In Sec. III equations of motion, describing magnons and DS are derived in the continuum limit and an approximate solution is cited.[13] In Sec. IV we derive equations of motion for eigenmodes. In Sec. V we describe the numerical method for finding truly local modes and eigenfrequencies. A discussion and concluding remarks are presented in Sec. VI. Appendix A is devoted to transformation of coordinates avoiding the singularity at $r=0$ which is useful when numerically integrating equations of motion.

II. THE MODEL

We start with the Hamiltonian for a classical two-dimensional ferromagnetic Heisenberg model with easy-axis anisotropy

$$\mathcal{H}=-J\sum_{\langle i,j\rangle}[\mathbf{S}_i\cdot\mathbf{S}_j+\lambda((S_i^z)^2-\mathbf{S}^2)], \tag{1}$$

where $\mathbf{S}_i$ denotes a spin vector at the ith site with lattice constant a and $\langle i,j\rangle$ designate nearest neighbor sites. The first term in Eq. (1) describes the ferromagnetic exchange interaction of strength $J>0$ between nearest neighbors, the second term $(\lambda>0)$ is the easy-axis anisotropy. We consider the most interesting near isotropic case $(\lambda\ll 1)$. The continuum limit of this Hamiltonian can be easily obtained by using angular variables for $\mathbf{S}$, $\mathbf{S}=S(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$

$$\mathcal{H}=\frac{JS^2}{2}\int\left[(\nabla\theta)^2+\sin^2\theta\left((\nabla\phi)^2+\frac{1}{l_0^2}\right)\right]d^2x, \tag{2}$$

where l_0 is the magnetic length. Dynamical equations for spin in the form of the Landau-Lifshitz equation are[14]

$$\nabla^2\theta-\sin\theta\cos\theta\left((\nabla\phi)^2+\frac{1}{l_0^2}\right)+\frac{1}{\omega_0 l_0^2}\frac{\partial\phi}{\partial t}\sin\theta=0,$$

$$\nabla\cdot(\sin^2\theta\nabla\phi)-\frac{1}{\omega_0 l_0^2}\frac{\partial\theta}{\partial t}\sin\theta=0, \tag{3}$$

where

$$l_0=\frac{a}{2\sqrt{\lambda}},\qquad \omega_0=4\lambda JS. \tag{4}$$

Here ω_0 is the homogeneous ferromagnetic resonance frequency and we set $\hbar=1$. It is more convenient to use l_0 and ω_0 instead of λ to study linear and nonlinear excitaions in this model.

III. MAGNONS AND DYNAMICAL SKYRMIONS

The easy-axis symmetry corresponds to a doubly degenerate classical ground state. In the long wavelength limit the easy-axis 2D model has a well known finite activation dispersion law for magnons

$$\omega(\vec{q})=\omega_0(1+(\mathbf{k}l_0)^2), \tag{5}$$

where $\mathbf{k}$ is the magnon wave vector.

Let us now consider nonlinear excitations. We can rewrite the first equation of Eq. (3) in polar coordinates. In the unperturbed DS with the topological charge ν spin has the following polar angle:

$$\phi_0(\chi,t)=\nu\chi+\omega t+\varphi_0, \tag{6}$$

where χ, ω and φ_0 are polar coordinate in the x-y plane, precession frequency and initial polar angle of the spin respectively. After substituting the expression for $\phi_0(\chi,t)$ into Eq. (3) for azimuthal angle $\theta_0(r)$ we get

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)-\sin\theta_0\cos\theta_0\left(\frac{\nu^2}{r^2}+\frac{1}{l_0^2}\right)+\frac{\omega}{\omega_0 l_0^2}\sin\theta_0=0. \tag{7}$$

Based on the matching method in Ref. 13 an approximate solution has been found to describe the space distribution of azimuthal angle of spin in the unperturbed DS with boundary conditions $\theta_0\to\pi$ for $r\to 0$ and $\theta_0\to 0$ for $r\to\infty$

$$\theta_0(r)=2\tan^{-1}\left[\frac{2}{(|\nu|-1)!}\left(\frac{k_0R}{2}\right)^{|\nu|}K_\nu(k_0r)\right], \tag{8}$$

where $k_0=\sqrt{(1-\omega/\omega_0)}/l_0$ and R is the radius of soliton. We call this nonlinear excitation a dynamical skyrmion (DS), because in the region $r<R$ it looks like a skyrmion, the spin of which precesses with a frequency close to $\omega=\omega_0/|\nu|$, provided that $R\ll l_0$. The energy of such DS is given by

$$E=4JS(\pi S|\nu|+\lambda N/|\nu|), \tag{9}$$

where N is the number of spin deviations from homogeneous state, which is proportional to R^2

$$N=\frac{S}{a^2}\int(1-\cos\theta_0)d^2x. \tag{10}$$

The dependence of the energy and precession frequency of the unperturbed DS on the number of spin deviations $E(N)$ and $\omega(N)$ was studied numerically in the above-mentioned Ref. 13. The side and front views of the DS with topological charge $\nu=1$ and $\nu=2$ are shown in Fig. 1 and Fig. 2, respectively. Both DS have the same radius, but the second one can be shown to be more localized (see Appendix A).

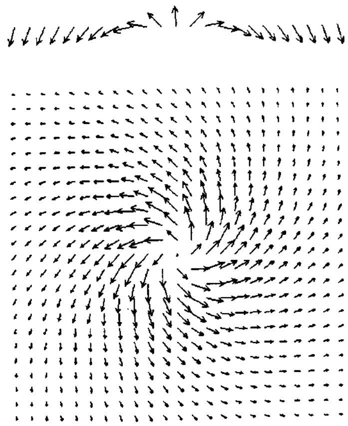

FIG. 1. Front and side views of the dynamical skyrmion with topological charge $\nu=1$.

IV. LOCAL MODES

In order to study small oscillations of the precessing spin vector of DS we introduce small deviations of the angle variables θ and ϕ from the ones corresponding to the unperturbed vortex, $\theta=\theta_0(r)+\vartheta(r,\chi,t)$, $\phi=\phi_0(\chi,t)+\mu(r,\chi,t)/\sin(\theta_0(r))$. The variables μ and ϑ are the projections of $\vec{M}$ on local axes $\vec{e}_1$ and $\vec{e}_2$ connected with the vortex: $\mu=\vec{M}\vec{e}_1$, $\vartheta=-\vec{M}\vec{e}_2$. Here

$$\vec{e}_1=\vec{e}_y\cos\phi_0-\vec{e}_x\sin\phi_0,$$

$$\vec{e}_2=\vec{e}_z\sin\phi_0-\cos\theta_0(\vec{e}_x\cos\phi_0+\vec{e}_y\sin\phi_0). \tag{11}$$

The axis $\vec{e}_3$ coincides with the direction of $\vec{M}$ in the unperturbed vortex. Linearizing Eqs. (3) we get the following coupled set of two partial differential equations:

$$[-\nabla^2+U_1(r)]\vartheta+\frac{2\nu\cos\theta_0}{r^2}\frac{\partial\mu}{\partial\chi}=\frac{1}{\omega_0 l_0^2}\frac{\partial\mu}{\partial t},$$

$$[-\nabla^2+U_2(r)]\mu-\frac{2\nu\cos\theta_0}{r^2}\frac{\partial\vartheta}{\partial\chi}=-\frac{1}{\omega_0 l_0^2}\frac{\partial\vartheta}{\partial t}, \tag{12}$$

where

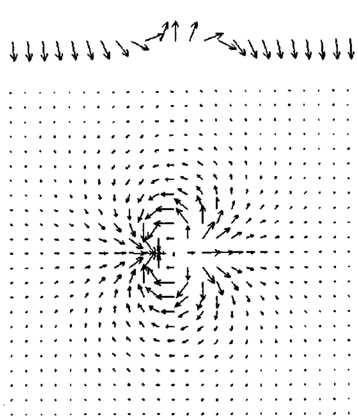

FIG. 2. The same as in Fig. 1 but for dynamical skyrmion with topological charge $\nu=2$.

$$U_1(r)=\left(\frac{\nu^2}{r^2}\right)\cos 2\theta_0+U_{\rm an}(r),$$

$$U_2(r)=\cot\theta_0\nabla^2\theta_0-(\nabla\theta_0)^2, \tag{13}$$

and introduce the ''anisotropy potential'' $U_{\rm an}(r)$ and $\gamma=\omega/\omega_0$

$$U_{\rm an}(r)=\frac{1}{l_0^2}(\cos 2\theta_0-\gamma\cos\theta_0). \tag{14}$$

Multiplying Eq. (7) by $r^2 d\theta_0/dr$ and integrating from 0 to r we get

$$(\nabla\theta_0)^2=\left(\frac{\nu^2}{r^2}+\frac{1}{l_0^2}\right)\sin^2\theta_0-\frac{1}{l_0^2}(2\gamma(1-\cos\theta_0)+I(r)), \tag{15}$$

where

$$I(r)=\frac{2}{r^2}\int_0^r(\sin^2\theta_0-2\gamma(1-\cos\theta_0))\rho d\rho.$$

Substituting Eq. (15) into Eq. (13) we derive the following identity for $\Delta U(r)=U_2(r)-U_1(r)>0$:

$$\Delta U(r)=\frac{1}{l_0^2}(2\gamma(1-\cos\theta_0)+I(r)). \tag{16}$$

Another equivalent identity for the $\Delta U(r)$ is

$$\Delta U(r)=\left(\frac{\nu^2}{r^2}+\frac{1}{l_0^2}\right)\sin^2\theta_0-(\nabla\theta_0)^2. \tag{17}$$

We look for the solution of Eq. (12) in the form of the ansatz

$$\vartheta=\sum_k\sum_{m=-\infty}^{+\infty}f_{k,m}(r)\cos(m\chi+\omega_{k,m}t+\delta_m),$$

$$\mu=\sum_k\sum_{m=-\infty}^{+\infty}g_{k,m}(r)\sin(m\chi+\omega_{k,m}t+\delta_m), \tag{18}$$

here k and m stand for numbers of eigenstates and ''angular momentum,'' respectively. We introduce dimensionless eigenfrequencies of the small oscillations $\gamma_{k,m}=\omega_{k,m}/\omega_0$ and δ_m are arbitrary phases. Further for convenience we will use f, g and γ_m instead of $f_{k,m}(r)$, $g_{k,m}(r)$ and $\gamma_{k,m}$ respectively. Substituting Eq. (18) into Eq. (12) we get the following equations for f and g:

$$\hat{H}_m f+V_m(r)g=0,$$

$$[\hat{H}_m+\Delta U(r)]g+V_m(r)f=0, \tag{19}$$

where

$$\hat{H}_m=-\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)+\frac{1}{r^2}(m^2+\nu^2\cos 2\theta_0)+U_{an}(r), \tag{20}$$

$$V_m(r)=\frac{2m\nu\cos\theta_0}{r^2}-\frac{\gamma_m}{l_0^2}. \tag{21}$$

We denote an eigenvector of solutions as

$$\overline{F}_m(r)=\begin{pmatrix} f \\ g \end{pmatrix}. \tag{22}$$

At first let us study symmetry relations for the Eq. (19). This equation is invariant with respect to the following transformation $m\to -m$, $\gamma_m\to -\gamma_m$ and $g\to -g$. Thus knowing solution for some m we can find another one by making use of this transformation.

A. Zero modes with $m=0,\pm 1$

As well known zero modes play an essential role when constructing the soliton thermodynamics. Their main feature is fully determined by the soliton profile. In this subsection we give the exact analytical expression for zero modes in easy-axis 2D FMs. In order to derive them we must put $\gamma_m=0$ for $m=0,\pm 1$. Zero modes have the following form:

$$f=m\frac{\partial\theta_0}{\partial r}r^{1-|m|},$$

$$g=-\frac{\nu\sin\theta}{r^{|m|}}. \tag{23}$$

Numerical simulations confirm this expression. Zero modes become apparent in inelastic neutron scattering experiments and essentially contributes in central peak due to interference between ones and DS. They may serve as a good test for the investigation of truly local modes.

B. Truly local modes

These modes do not exist in isotropic FM, because they have nonzero limit in infinity. They appear in the anisotropic case and have well pronounced exponential decay. Let us make analysis of the asymptotics of solutions of equations (19). Near the core the most important contribution comes from the isotropic exchange interaction so we can neglect by the ''anisotropy potential.'' The ''potentials difference'' $U_{\rm an}(r)\to 0$ in the region near the skyrmion center. Then we immediately recover equations for zero modes of static skyrmions in isotropic magnet[20] or for DS (23) in the case $\nu=1$. Far away from the core region ($r\gg R$) we have the following asymptotics for $U_m(r)$, $V_m(r)$ and $\Delta U(r)$:

$$U_m(r)\to\frac{m^2+\nu^2}{r^2}+k_0^2,\quad V_m(r)\to\frac{2m\nu}{r^2},\quad \Delta U(r)\to 0. \tag{24}$$

Asymptotics for $\overline{F}_m$ in the region $r\gg R$ are given by

$$\overline{F}_m(r)\to aK_{m+\nu}(k_+r)\begin{pmatrix}1\\1\end{pmatrix}+bK_{\nu-m}(k_-r)\begin{pmatrix}1\\-1\end{pmatrix}, \tag{25}$$

here $k_\pm(\gamma_m)=\sqrt{1-\gamma\mp\gamma_m}/l_0^2$ and a,b are arbitrary constants determined by boundary conditions at $r=0$. A significant difference between the near isotropic and isotropic cases is the existence of truly local modes with exponential decay for frequencies $|\omega_m|<\omega_0-\omega$.

V. NUMERICAL RESULTS

First we solve numerically an ordinary differential equation (7) for $\theta_0(r)$ and look for an appropriate value of ω for given R by employing the shooting method with boundary conditions $\theta_0\to\pi$ for $r\to 0$ and $\theta_0\to 0$ for $r\to\infty$. The problem arising with the singularity at $r=0$ can be effectively avoided by coordinate transformation (see Appendix A). Then the numerically obtained solution is used for finding magnon normal modes on the DS. The coupled set of linearized Landau-Lifshitz equations is solved by applying the shooting method. We considered ''angular momentum'' numbers $m=0,\pm 1$ for topological charges $\nu=1$ and $m=0$, $\pm 1\pm 2$ for $\nu=2$ when literally local modes exist. We have two shooting parameters Δ_m and γ_m. The first one is a ''mixing'' parameter between symmetric and antisymmetric modes and the second is a dimensionless eigenfrequency $\gamma_m=\omega_m/\omega_0$.

The eigenvalue problem is solved by matching of numerical solution with modified Bessel functions of the first kind near the region of soliton's center and with ones of the second kind (25) far away from DS. For this purpose the following boundary conditions for $r\to 0$ are applied:

$$\overline{F}_m(r)=\cos(\Delta_m)\begin{pmatrix}1\\1\end{pmatrix}I_{\nu-m}(\kappa_-r)+\sin(\Delta_m)\times\begin{pmatrix}1\\-1\end{pmatrix}I_{\nu+m}(\kappa_+r), \tag{26}$$

$$\frac{d\overline{F}_m}{dr}=\frac{d}{dr}\left(\cos(\Delta_m)\begin{pmatrix}1\\1\end{pmatrix}I_{\nu-m}(\kappa_-r)+\sin(\Delta_m)\times\begin{pmatrix}1\\-1\end{pmatrix}I_{\nu+m}(\kappa_+r)\right), \tag{27}$$

where

$$\kappa_\pm=\frac{\sqrt{1+\gamma\pm\gamma_m}}{l_0}.$$

The eigenfrequency spectrum for local modes for DS with $\nu=1,m=-1$ and $\nu=2,m=2$ for the different values of dimensionless radius of DS $\epsilon=R/l_0$ are presented in Fig. 3. It is seen that both eigenfrequencies tend to the $1-\gamma$, $\gamma=\omega/\omega_0$ when $\epsilon\to 0$. Eigenfrequencies γ_m together with $1-\gamma$ versus r are presented in the above mentioned Fig. 3.

Numerical solutions for the radial parts of the eigenfunctions f, g and analytical ones for $R/l_0=0.2$ and $\nu=1$ and $\nu=2$ are presented in Fig. 4.

VI. CONCLUSIONS AND DISCUSSIONS

We have investigated truly localized magnon modes of dynamical skyrmions and found exact solutions for zero modes in easy-axis 2D ferromagnets. Zero modes exist for dynamical skirmions with any topological charge ν but with ''angular momentum'' numbers $m=0,\pm 1$. These zero

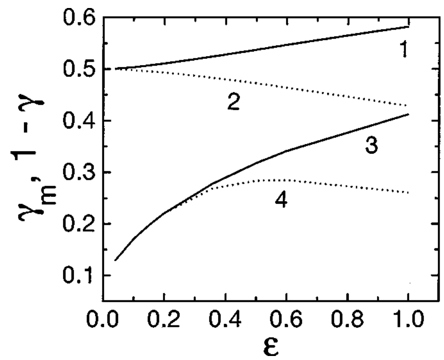

FIG. 3. The dependence of dimensionless eigenfrequency $\gamma_m = \omega_m/\omega_0$ versus ratio of the radius of dynamical skyrmion with $\nu = 1,2$ to the magnetic length $\epsilon = R/l_0$ at fixed value of l_0 (curves 1 and 3). The dependence of $1-\gamma$ versus ϵ for DS with $\nu = 1$ and $\nu = 2$, respectively (curves 2 and 4). The eigenfrequencies γ_{-1} and γ_2 for DS with $\nu = 1$ and $\nu = 2$, respectively.

modes are important because the interference between them and dynamical skyrmions gives rise to the contribution in the central peak. The truly local modes are in a certain frequency range, determined by the easy-axis anisotropy constant. The eigenfrequencies ω_m satisfy the relation $|\omega_m| < \omega_0 - \omega$, where ω_0 is the frequency of homogeneous ferromagnetic resonance and ω is the precession frequency of soliton. The analysis of these modes has been done numerically using the shooting method. The existence of all of these modes are connected with the high order hidden symmetry of the Landau-Lifshitz equation.

The phenomenon predicted in our paper is believed to be observed experimentally. For example one can generate precessing skyrmions in 2D ferromagnetic materials by making use of microwave pumping. Applying of an additional oscillating weak magnetic field causes peaks of absorption at the excitation frequencies of normal modes of skyrmions generated by microwave pumping. The condition of successful generation of precessing skyrmions demands the weakness of easy-axis anisotropy. Technical advance in the synthesis of ultrathin magnetic films gives a possibility to create samples with the very weak easy-axis anisotropy (see, for example Ref. 28).

The results concerning this problem together with detailed investigation of the dynamics of moving dynamical skyrmion will be presented in future papers.

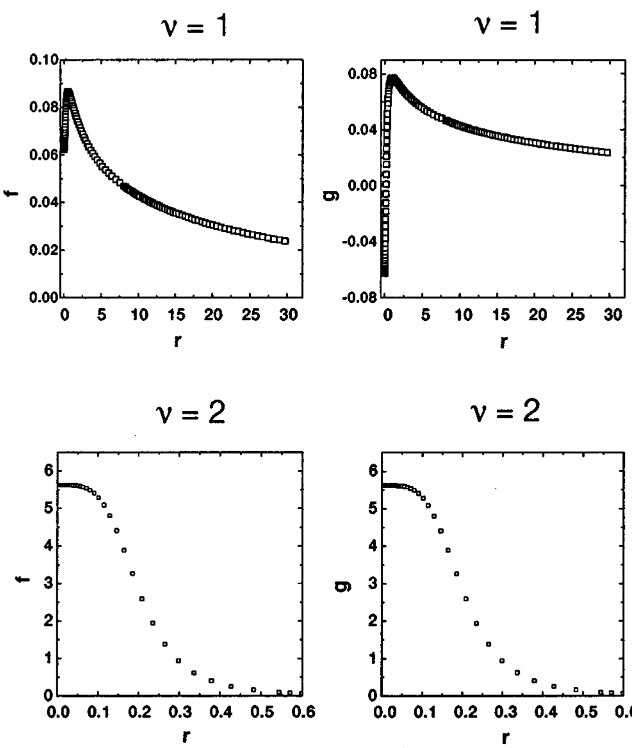

FIG. 4. The radial part of the eigenfunctions for real local modes for DS with topological charges $\nu = 1$ and $\nu = 2$ at $R/l_0 = 0.2$.

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APPENDIX A: TRANSFORMATION OF COORDINATES

This appendix shows that the $r = 0$ singularity problem can be solved by introducing a new variable $x = \ln(r/R)$, where R is a vortex radius. We performed numerical simulations with equations derived using this transformation. First consider the special case of the Belavin-Polyakov vortex.[11] This equation is important for us because for real magnets $\lambda \ll 1$ and in core region we have reconstruction of conformal invariance, therefore this vortex solution is a good ansatz for $r \leqslant R, x \leqslant 0$. The equation describing this vortex can be obtained from the model (1) if we put $\lambda = 0$, which corresponds to 2D isotropic magnet. Rewriting Eq. (7) for the azimuthal angle θ using the new variable x, we obtain

$$\frac{d^2\theta_0}{dx^2} - \nu^2 \sin\theta_0 \cos\theta_0 = 0, \tag{A1}$$

which is nothing else than a well known pendulum or static sine-Gordon equation. The solution of the Eq. (A1) with boundary conditions $\theta(r=0, x=-\infty) = \pi$ and $\theta(r=\infty, x=\infty) = 0$ is

$$\theta_0 = 2\tan^{-1}[\exp(-|\nu|(x-x_0))]. \tag{A2}$$

Here x_0 is the ''coordinate center'' and the absolute value of topological charge $|\nu|$ plays the role of ''inverse width'' of soliton. So the translational invariance of solution (A2) transforms into the conformal invariance of skyrmion in r coordinate. There are also solutions of linearized Eq. (A1) corresponding to magnon modes, localized on ''in-plane'' vortices in easy-plane AFM[24]

$$\theta = \frac{\pi}{2} + \vartheta_0 \cos(\nu x + \delta). \tag{A3}$$

Let us now rewrite the equations of motion for local modes (12) in x variable

$$\hat{H}_m f_m(x) + V(x) g_m(x) = 0,$$

$$(\hat{H}_m(x) + \Delta U(x)) g_m(x) + V(x) f_m(x) = 0, \tag{A4}$$

where

$$\hat{H}_m(x)=\left(-\frac{d^2}{dx^2}+m^2+\nu^2\cos(2\theta_0)+U_\epsilon(x)\right), \quad \text{(A5)}$$

where

$$\epsilon=\frac{R}{l_0}, \quad U_\epsilon(x)=\epsilon^2 e^{2x}(\cos(2\theta_0)-\gamma\cos\theta_0), \quad \text{(A6)}$$

$$V(x)=2m\nu\cos\theta_0-\gamma_m\epsilon^2 e^{2x}. \quad \text{(A7)}$$

For the anisotropic case to find the solutions for zero modes we must put $\gamma_m=0$ then for $m=0,\pm 1$ we find the following expressions:

$$f_m(x)=me^{-|m|x}\frac{\partial\theta_0(x)}{\partial x}, \quad g_m(x)=-\nu e^{-|m|x}\sin\theta_0(x). \quad \text{(A8)}$$

Thus it is more convenient to integrate numerically the equations of motion using this simple transformation of coordinates.

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