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Theory of the Josephson effect in a superconductor/one-dimensional electron gas/superconductor junction

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We present a theory for the Josephson effect in an unconventional superconductor/one-dimensional electron gas/unconventional superconductor ($s/o/s$) junction, where the Josephson current is carried by components injected perpendicular to the interface. When superconductors on both sides have triplet symmetries, the Josephson current is enhanced at low temperature due to the zero-energy states formed near the interface. Measuring Josephson current in this $s/o/s$ junction, we can identify parity of the superconductor.
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Nowadays, novel interference effects of the quasiparticle tunneling in unconventional superconductor junctions, where pair potentials change sign on the Fermi surface, have been paid much attention.[1,2] One of the remarkable features is the formation of the zero-energy states (ZES's) localized near surfaces of unconventional superconductors.[1,3,4] The ZES's are detectable by tunneling spectroscopy as conductance peaks. Experimental observations of the ZES's on surfaces of high-T_c superconductors have been reported in several papers.[5–8] Motivated by these works, general formulas for the Josephson current in (even-parity) unconventional superconductors were presented by taking account of the ZES's.[9–12] Calculated results show several anomalous properties including the strong enhancement of the Josephson current at low temperature under the influence of the ZES formation.

Recently, Maeno *et al.* discovered superconductivity in Sr_2RuO_4, where symmetry of the pair potential is believed to be triplet.[13–15] In (odd parity) triplet superconductor junctions, it is also expected that the Josephson current is enhanced by the formation of the ZES similarly to the even-parity cases.[4,16] Since the ZES formation is a universal phenomenon for any pair potential with the sign change on the Fermi surface irrespective of parity of the pair potential, it is not so easy to determine the parity of the unconventional superconductor using usual Josephson junctions.

In order to distinguish odd-parity superconductors from even-parity ones, we propose a method using a superconductor/one-dimensional electron gas (1DEG)/superconductor ($s/o/s$) junction. Anomalous behaviors in the Josephson effect are expected only for odd parity superconductor in this junction configuration. This is because direction of quasiparticle injection, which is a decisive factor for the formation of the ZES's, is restricted to be normal to the interface. In this configuration, the appearance of the ZES is governed by the parity of the superconductor, as precisely discussed below. Thus the $s/o/s$ junction provides a simple strategy to determine the parity of the superconductor.

Recent rapid progress in the technology of superconductor/semiconductor hybrid structure makes it possible to fabricate and to study $s/o/s$ junctions. Hence the way is promising enough. Several theories have already been presented about the effect of interaction in 1DEG on the Josephson effect using superconductor/Luttinger liquid (LL)/superconductor (s/LL/s) junctions.[17,18] In these works, however, the superconductor is assumed to be BCS-type s wave and cases for unconventional superconductors are not clarified yet.

In this paper, a formula of the Josephson current is presented for $s/o/s$ junctions assuming that the 1DEG is noninteracting. The Josephson current is shown to be sensitive to the parity of the superconductor. We further study the effect of interaction for the 1DEG using the Tomonaga-Luttinger (TL) model. A Josephson-current formula for general $s/o/s$ junctions with normal boundary reflections is obtained by generalizing the method by Maslov *et al.*, which again shows sensitivity of the current to the parity of the superconductor.

Let us consider a semi-infinite superconductor with a flat interface at $x=0$ as shown in Fig. 1. The effective potentials for injected and reflected quasiparticles with spin index σ are given by $\Delta_{L\sigma}(\theta)$ and $\Delta_{L\sigma}(\pi-\theta)$, respectively. In usual Josephson junctions, ZES's at a surface are formed if a condition $\Delta_{L\sigma}(\theta)\Delta_{L\sigma}(\pi-\theta)<0$ is satisfied.[1,3,4] On the other hand, as we stated above, the most remarkable difference in $s/o/s$ junctions from usual Josephson junctions is that only the components of the current which flow perpendicular to the interface ($\theta=0$) contribute to the Josephson current. For singlet superconductors, since $\Delta_{L\sigma}(0)=\Delta_{L\sigma}(\pi)$, the condition for the ZES is never satisfied. On the other hand, for triplet superconductors, since $\Delta_{L\sigma}(0)=-\Delta_{L\sigma}(\pi)$ is satisfied, ZES's are always expected.[3] This is the reason why we propose a $s/o/s$ junction to distinguish the parity of the superconductor.

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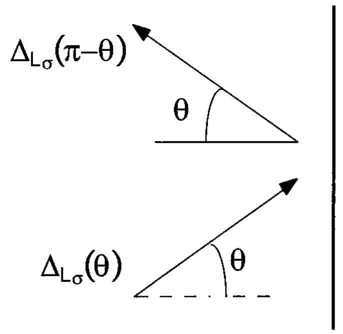

FIG. 1. A schematic illustration for the formation of the ZES at the surface of an unconventional superconductor.

To perform the simplest model calculation, we consider $s/o/s$ junctions with perfectly flat interfaces in the clean limit. In this model, the interface is perpendicular to the x axis and is located at $x=0$ and $x=d$ where d is the length of the 1DEG region. In real junctions, insulator is inevitably located between the superconductors and the 1DEG. We model the insulator by a delta functions, namely $H\delta(x)$ and $H\delta(x-d)$, where H denotes strength of the barrier. We assume that the superconductors are two-dimensional. The Fermi wave number k_F and the effective mass m are assumed to be equal in the left and right superconductors. In the 1DEG, the magnitude of the Fermi wave number and the effective mass are also chosen as k_F and m, respectively. In the following, we will calculate the Josephson current in the $s/o/s$ junction shown in Fig. 2. For simplicity, the Cooper pair is assumed to be formed by two electrons with antiparallel spins both for the singlet pairing and for the triplet pairing ($S=1,\ S_z=0$).

We first consider the case with noninteracting 1DEG. In the framework of the quasiclassical approximation, the effective pair potentials for the quasiparticles depend on their directions of their motions. We assume an electronlike quasiparticle (ELQ) is injected from the left. The effective pair potentials for the injected ELQ [a reflected holelike quasiparticle (HLQ)], the reflected ELQ, the transmitted ELQ and the transmitted HLQ are given by $\Delta_{L\sigma}(0)\exp(i\varphi_L)$, $\Delta_{L\sigma}(\pi)\exp(i\varphi_L)$, $\Delta_{R\sigma}(0)\exp(i\varphi_R)$, and $\Delta_{R\sigma}(\pi)\exp(i\varphi_R)$, respectively (see Fig. 2). The quantities φ_L and φ_R denote the macroscopic phases, which are measured along the x axis, of the left and right superconductors, respectively. The Josephson current through the junction is expressed in terms of the coefficients of the Andreev reflection[19] $[a_\sigma(\varphi)]$ as

$$R_N I(\varphi)=\frac{\pi k_B T}{e\sigma_T}\sum_{\omega_n,\sigma}\frac{\Delta_{L\sigma}(0)}{2\Omega_n}[a_\sigma(\varphi)-a_\sigma(-\varphi)] \quad (1)$$

with $\Omega_n=\sqrt{\omega_n^2+|\Delta_{L\sigma}(0)|^2}$, $\varphi=\varphi_L-\varphi_R$, and $\omega_n=2\pi k_B T(n+1/2)$ with an integer n.[20] Conductance of the junction in the normal state σ_T is given by

$$\sigma_T=\frac{\sigma_N^2}{\{1+(1-\sigma_N)^2+F(1-\sigma_N)\}}, \quad (2)$$

$$F=[2(2\sigma_N-1)\cos(2k_F d)+4\sqrt{\sigma_N(1-\sigma_N)}\sin(2k_F d)]$$

with $\sigma_N=4/(4+Z^2)$ and $Z=2mH/\hbar^2$. Coefficients of the Andreev reflection are obtained by solving the following equations:

$$\Psi(x=0_-)=\Psi(x=0_+),\quad \Psi(x=d_-)=\Psi(x=d_+),$$

$$\left.\frac{d}{dx}\Psi(x)\right|_{x=0_+}-\left.\frac{d}{dx}\Psi(x)\right|_{x=0_-}=\left.\frac{2mH}{\hbar^2}\Psi(x)\right|_{x=0_+},$$

$$\left.\frac{d}{dx}\Psi(x)\right|_{x=d_+}-\left.\frac{d}{dx}\Psi(x)\right|_{x=d_-}=\left.\frac{2mH}{\hbar^2}\Psi(x)\right|_{x=d_+}, \quad (3)$$

where $\Psi(x)$ denotes the two-component wave functions. In the following, we will consider two cases; (i) singlet superconductor/1DEG/singlet superconductor ($ss/o/ss$) junction $[\Delta_{L(R)\sigma}(0)=\Delta_{L(R)\sigma}(\pi)=s\Delta_0]$, (ii) triplet superconductor/1DEG/triplet superconductor ($ts/o/ts$) junction $[\Delta_{L(R)\sigma}(0)=\Delta_0,\ \Delta_{L(R)\sigma}(\pi)=-\Delta_0]$, with $s=1$ ($s=-1$) for up- (down-) spin electron injection. The Josephson current is expressed as

(i)$ss/o/ss$ junction case:

$$R_N I(\varphi)=\frac{\pi k_B T}{e\sigma_T}\sum_{\omega_n}\frac{4\gamma\eta^2\sigma_N^2\sin\varphi}{\sigma_N\Lambda+(1-\sigma_N)(1+\eta^2)^2 t}, \quad (4)$$

(ii)$ts/o/ts$ junction case:

$$R_N I(\varphi)=\frac{\pi k_B T}{e\sigma_T}\sum_{\omega_n}\frac{4\gamma\eta^2\sigma_N^2\sin\varphi}{\sigma_N\Lambda+(1-\sigma_N)(1-\eta^2)^2 t}, \quad (5)$$

where

$$\Lambda=(1+\gamma^2\eta^4+2\gamma\eta^2\cos\varphi)-(1-\sigma_N)(\gamma^2+\eta^4+2\gamma\eta^2\cos\varphi),$$

$$\eta=\frac{\Delta_0}{\Omega_n+\omega_n},\quad \gamma=\exp[-2|\omega_n|d/\hbar v_F],$$

$$t=1+\gamma^2-\gamma\left(t_s\delta+\frac{1}{t_s\delta}\right),\quad t_s=-\frac{2-iZ}{2+iZ},\quad \delta=\exp(2ik_F d). \quad (6)$$

Temperature dependence of the maximum Josephson current $I_C(T)$ of $ss/o/ss$ and $ts/o/ts$ junctions is plotted in Fig. 3. With increasing Z, magnitude of $R_N I_C(T)$ for $ss/I/ss$ junc-

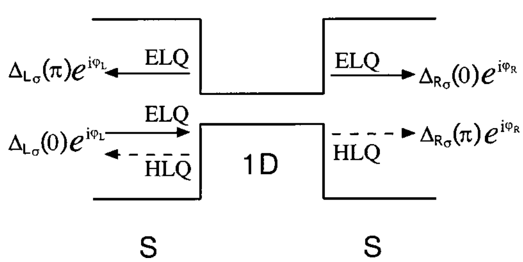

FIG. 2. A schematic illustration of the superconductor/1DEG/superconductor junction. The effective pair potentials for an injected ELQ (the reflected HLQ), the reflected ELQ, the transmitted ELQ and the transmitted HLQ are $\Delta_{L\sigma}(0)\exp(i\varphi_L)$, $\Delta_{L\sigma}(\pi)\exp(i\varphi_L)$, $\Delta_{R\sigma}(0)\exp(i\varphi_R)$, and $\Delta_{R\sigma}(0)\exp(i\varphi_R)$, respectively.

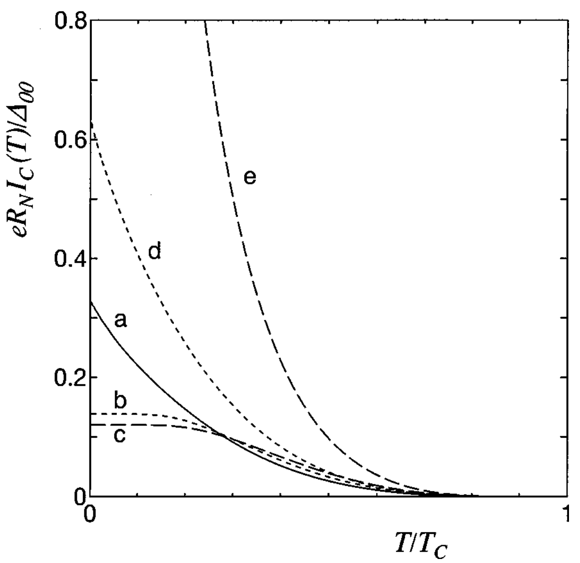

FIG. 3. The maximum Josephson current $I_C(T)$ in $ss/o/ss$ junctions for (a) $Z=0$, (b) $Z=1$, and (c) $Z=5$ and that in $ts/o/ts$ junctions for (d) $Z=1$ and (e) $Z=5$ with $dk_F=50$, $d/\xi=5$, and $\xi=\hbar v_F/\Delta_{00}$. For $Z=0$, both junctions show the same magnitude of $I_C(T)$. The Δ_{00} is the value of Δ_0 at zero temperature, where the temperature dependence of Δ_0 is assumed to obey the BCS relation.

tion is reduced. On the other hand, for $ts/o/ts$ junction, it is enhanced oppositely with increasing Z. The enhancement of the Josephson current for larger Z is due to the resonating current through the ZES formed near the interface. In real junctions, an insulating barrier inevitably exists near the interface. Such a situation corresponds to the larger magnitude of Z in our calculations. The present result suggests that we can distinguish the parity of the superconductor, whether $I_C(T)$ shows an upturn curvature (triplet case) or not (singlet case).

Now, we consider an effect of the interaction in the 1DEG. We derive the Josephson current formula for unconventional superconductors with arbitrary barrier heights by taking account of both the Andreev reflection and normal reflection at the interfaces. The effect of interaction in 1DEG is introduced following Maslov *et al.* using the TL model.[18,21] Basis for 1DEG is spanned by bound states formed in the superconducting gap. For simplicity, we consider here only the low-temperature limit and assume that relevant excitations determining the Josephson current have energy $|\varepsilon|\ll\Delta_0$. Within these conditions, a difference between $ss/LL/ss$ and $ts/LL/ts$ junctions appears only in the following generalized boundary conditions for the fermion field operators:

$$\psi_{\pm,s}(x+2d)=\lambda\psi_{\pm,s}(x), \tag{7}$$

$$\psi_{+,s}(x)=s\psi^{\dagger}_{-,-s}(-x). \tag{8}$$

Here $\psi_{\pm,s}$ represents right-going (left-going) fermion field with spin s. Extra phase factor λ, which is a function of k_F, d, σ_N, and φ, coincides with the factor in Eqs. (16a) and (16b) of Ref. 18, when a $ss/LL/ss$ junction with only the Andreev refrection at the boundary is considered. Following the bosonization technique for the open boundary conditions $\psi_{\pm,s}$ can be represented by chiral boson fields.[18,22] We see that only zero modes are affected by the parity of the superconductor through the boundary condition (7) and χ in Eq. (28) of Ref. 18 is replaced by a complicated function of φ for general situations considered here. Explicit formulas for ψ as well as λ will be presented elsewhere.[23] The current is obtained by $I(\varphi)=-(2ek_BT/\hbar)(\partial/\partial\varphi)\ln Z(\varphi)$ where $Z(\varphi)$ is the partition function. As Maslov *et al.* have claimed, the Josephson current in the present limit is determined by the zero mode (the topological excitations) and nonzero modes do not contribute.[18] Our general formula of the Josephson current for interacting 1DEG systems shows essentially the same feature as noninteracting cases in that $I(\varphi)$ is enhanced for $ts/LL/ts$ compared with $ss/LL/ss$.

In this paper, we propose a method to identify the parity of a superconductor using a $s/o/s$ junction. We derive a formula for the Josephson current assuming that the 1DEG is noninteracting. Anomalous behavior in the Josephson effect is expected only in triplet superconductor with odd parity. This is because the direction of quasiparticle injection, which is a decisive factor for the formation of ZES's, is selected to be normal to the interface.[3] For the singlet superconductor with even parity, the ZES's never appear in the present geometry as precisely discussed. In the present calculation, the suppression of the pair potential near the interface[24] is neglected. Even if we take into account this effect, qualitative features in the upturn curvature due to the ZES's at low temperatures will not be changed, then the present results are still valid.[10,25] We have further studied the effect of interaction for the 1DEG using the TL model. It is shown that the essential feature is determined by the parity of the superconductor and the influence of the interaction effect is not so important within the TL model at the low-temperature limit. We will report detailed properties of general $s/LL/s$ junctions in a forthcoming paper using a bosonization technique with further consideration of the interelectron interaction.[23]

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[1] C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994); J. Yang and C. R. Hu, Phys. Rev. B **50**, 16 766 (1994).
[2] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995); S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B **51**, 1350 (1995); S. Kashiwaya, Y. Tanaka, M. Koyanagi, and K. Kajimura, *ibid.* **53**, 2667 (1996).
[3] L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981).
[4] J. Hara and K. Nagai, Prog. Theor. Phys. **74**, 1237 (1986).
[5] J. Geerk, X. X. Xi, and G. Linker, Z. Phys. B **73**, 329 (1988).
[6] J. Lesueur, L. H. Greene, W. L. Feldmann, and A. Inam, Physica C **191**, 325 (1991).
[7] L. Alff, H. Takashima, S. Kashiwaya, N. Terada, H. Ihara, Y. Tanaka, M. Koyanagi, and K. Kajimura, Phys. Rev. B **55**, 14 757 (1997).

[8] J. Y. T. Wei, N.-C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. **81**, 2542 (1998).
[9] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **53**, 11 957 (1996); **56**, 892 (1997).
[10] Yu. S. Barash, H. Burkhardt, and D. Rainer, Phys. Rev. Lett. **77**, 4070 (1996).
[11] M. P. Samanta and S. Datta, Phys. Rev. B **55**, 8689 (1997).
[12] R. A. Riedel and P. F. Bagwell, Phys. Rev. B **57**, 6084 (1998).
[13] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz, and F. Lichtenberg, Nature (London) **372**, 532 (1994).
[14] T. M. Rice and M. Sigrist, J. Phys.: Condens. Matter **7**, 643 (1995).
[15] K. Machida, M. Ozaki, and T. Ohmi, J. Phys. Soc. Jpn. **65**, 3720 (1996).
[16] M. Yamashiro, Y. Tanaka, Y. Tanuma, and S. Kashiwaya, J. Phys. Soc. Jpn. **67**, 3224 (1998).
[17] R. Fazio, F. W. J. Hekking, and A. A. Odintsov, Phys. Rev. Lett. **74**, 1843 (1995).
[18] D. L. Maslov, M. Stone, P. M. Goldbart, and D. Loss, Phys. Rev. B **53**, 1548 (1996).
[19] A. F. Andreev, Zh. Éksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].
[20] A. Furusaki and M. Tsukada, Solid State Commun. **78**, 299 (1991).
[21] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[22] M. Fabrizio and A. O. Gogolin, Phys. Rev. B **51**, 17 827 (1995).
[23] T. Hirai, K. Kusakabe, Y. Tanaka, and S. Kashiwaya (unpublished).
[24] D. Rainer and P. A. Lee, Phys. Rev. B **35**, 3181 (1987).
[25] Y. Tanaka and S. Kashiwaya, Phys. Rev. B **58**, 2948 (1998).