Quantum-classical depinning-rate transition of a domain wall

Gwang-Hee Kim*
Department of Physics, Sejong University, Seoul 143-747, Republic of Korea
(Received 16 April 1999)

It is shown that a domain wall exhibits first- and second-order transitions between classical and quantum regimes for the depinning rate. The phase boundary between two transitions and its crossover temperature are controlled by the magnetic field applied at some angle with the anisotropy axis and the intrinsic quantities of the system via the effective mass of the domain wall. These features can be observed with the use of existing experimental techniques. [S0163-1829(99)07933-3]

Recently, spin systems have aroused considerable interest with the discovery that they provide examples which exhibit first- or second-order transition (FST) between the classical and quantum behavior of the escape rate.[1–7] In general transitions in a metastable system can occur via quantum tunneling through the barrier and the classical thermal activation. At high temperature there is a jumping of a system induced by thermal fluctuations whose escape rate $\Gamma(T)$ is proportional to $\exp(-U/k_BT)$ where U is the height of barrier. At a temperature low enough to neglect the thermal activation the transition occurs due to quantum tunneling with $\Gamma\sim\exp(-U/\hbar\omega)$ where ω is the oscillation frequency around the minimum of the inverted potential. In this situation it is expected that there exists the crossover temperature T_0 between the classical thermal activation and quantum tunneling, in which the two mechanisms of the escape coexist. Which order of the transition can occur in the process is determined by the behavior of the WKB exponent around T_0, i.e., around the barrier. If $d\Gamma(T)/dT$ is discontinuous (continuous) at the crossover temperature, the transition becomes first (second) order.

The question of the transition was studied by Affleck,[8] and Larkin and Ovchinikov[9] who demonstrated the second order transition at the crossover temperature by using the instanton technique. Chudnovsky[1] gave the criterion allowing one to establish which transition takes place, based on shape of the potential. Later, theoretical investigations for the transition in spin systems have been performed by several groups.[2–7] Up to now theoretical studies have been focused on the single domain ferromagnetic particle. However, whether the depinning of the domain wall is a first- or second-order transition is unknown. Thus it will be interesting to study which order of the transition occurs in the depinning process. In this paper we investigate the proper conditions for FST in a domain wall system, and present the phase diagram between the first- and the second-order transition and its crossover temperature.

As is well known, a domain wall is a soliton connecting two stable spin configurations separated by an energy barrier associated with magnetic anisotropy energy and pinned by an impurity, lowering the anisotroy energy locally.[10] In order for a domain wall to get depinned via quantum tunneling,[11,12] it is necessary to apply an exteranl magnetic field which controls the height and width of the barrier and the effective mass of the system. In this work we will consider a system with a magnetic field applied at some angle θ_H to the easy axis of magnetization and show that the corresponding field plays an important role in exhibiting FST via the effective mass of the domain wall.

Consider the domain wall of the slab geometry, as shown in Fig. 1. Assuming that the domain wall thickness λ is sufficiently larger than the lattice constant a between spins, we can use the continuum approximation for the magnetization. Introducing the magnetocrystalline anisotropy with the biaxial symmetry and the exchange interaction, the energy density is given by

$$E[\theta(\mathbf{r},t),\phi(\mathbf{r},t)]=K_{||}\sin^2\theta+K_{\perp}\sin^2\phi\sin^2\theta+\frac{1}{2}C[(\nabla\theta)^2+(\nabla\phi)^2\sin^2\theta], \quad (1)$$

where $K_{||}$ is the parallel anisotropy constant, $K_{\perp}\equiv K_{\perp,a}+2\pi M_0^2$. Here $K_{\perp,a}$ is the transverse anisotropy constant and $2\pi M_0^2$ comes from the demagnetization energy for the slab geometry.

Following the analysis discussed in Ref. 13, for the sample with width $w<\pi\sqrt{C/2K_{\perp}}$, the system can be treated as quasi-one-dimensional. The corresponding domain wall is perpendicular to the x axis, where the magnetization rotates in the easy plane (xz plane) and changes along the x axis.

Now, using the Landau-Lifshitz equation[14]

$$\frac{d\mathbf{M}}{dt}=-\gamma\mathbf{M}\times\frac{\delta E}{\delta\mathbf{M}}, \quad (2)$$

where $\gamma=g\mu_B/\hbar$ is the gyromagnetic factor, the soliton solution which describes the motion of the domain wall becomes

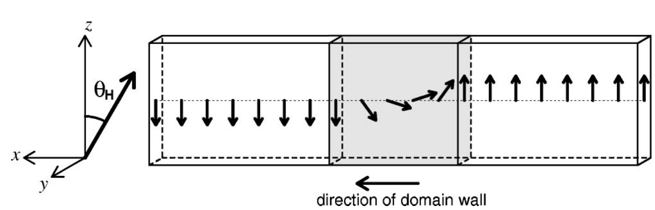

FIG. 1. A configuration of magnetization is shown in a thin long slab geometry where the wall plane is parallel to the easy axis (z) and the spin configuration spatially varies along in the x direction.

$$\theta_s(x-Q)=2\arctan\exp\left(\frac{x-Q}{\lambda}\right), \tag{3}$$

$$\frac{dQ}{dt}=kv_0\frac{\sin\phi_s\cos\phi_s}{\sqrt{1+k\sin^2\phi_s}}, \tag{4}$$

where $k=K_\perp/K_\parallel$, $v_0=\gamma\sqrt{2CK_\parallel}/M_0$, and the wall position is centered at Q along the x axis. Also, we represent $\lambda=\lambda_0/\sqrt{1+k\sin^2\phi_s}$, where $\lambda_0=\sqrt{C/2K_\parallel}$ is the width of the static wall which represents the compromise between exchange and anisotropy energy. Assuming that $\dot{Q}(=dQ/dt)$ is much smaller than the Walker critical velocity[10] $v_0(\sqrt{1+k}-1)$, from Eq. (1) the energy is expressed as

$$\int d^3\mathbf{r}E[\theta_s(\mathbf{r},t),\phi_s(\mathbf{r},t)]\simeq E_0+\frac{1}{2}M\left(\frac{dQ}{dt}\right)^2, \tag{5}$$

where $E_0=2A_w\sqrt{2CK_\parallel}$ and the wall mass[15] is $M=A_w(M_0^2/\gamma^2K_\perp)\sqrt{2K_\parallel/C}$ with A_w the cross sectional area of the sample. Even though the spins lie in the easy plane in a static domain wall, as is noted in Eqs. (3) and (4), its dynamical motion induces the spins to precess which leads to the magnetization out of the plane, i.e., $\hat{M}_y\propto\dot{Q}$. In this respect the inertial term in Eq. (5) comes from the precession of the spins. If the size of the defect is much smaller than the wall thickness λ, the wall is pinned by a potential form[11,13]

$$V_p=-V_0\mathrm{sech}^2(Q/\lambda), \tag{6}$$

where V_0 is proportional to the volume of the defect. Here it is noticed that λ depends on $\dot{Q}$ via ϕ_s. In case that a concentration of defects is small, the pinning energies become small, in which the radius of curvature of the wall is much larger than λ. Since it is shown[12] that weak curvature has very little effect on wall tunneling, the wall can be assumed to be flat and remained flat during the tunneling process. An external magnetic field with a negative x component in the xz plane leads to the additional energy[16]

$$-2A_wM_0H_zQ-\pi A_w\lambda M_0H_x\left(\frac{k+1}{2}\right)\left(\frac{\dot{Q}}{kv_0}\right)^2. \tag{7}$$

The reason why we need the field with a negative x component is associated with the adjustment of the range of the first-order transition via the effective mass in Eq. (9). In order words, as the second term in the effective mass (9) which comes from the directional dependence of the field contributes more, the magnitude of b in Eq. (9) for the phase boundary is expected to be smaller, which is more relevant to the experimental situations, as will be seen later. Denoting Q/λ_0 to be X, from Eqs. (5), (6), and (7) the total energy for the domain wall becomes

$$\frac{1}{2}m\left(\frac{dX}{dt}\right)^2+U(X), \tag{8}$$

where $U(X)=V_p^{(0)}(X)-h_zX$, $h_z=2A_wM_0H_z\lambda_0$, $V_p^{(0)}(X)=-V_0\mathrm{sech}^2X$, and the effective mass is represented as

$$m=M\lambda_0^2[1-m_H+bf(X)], \tag{9}$$

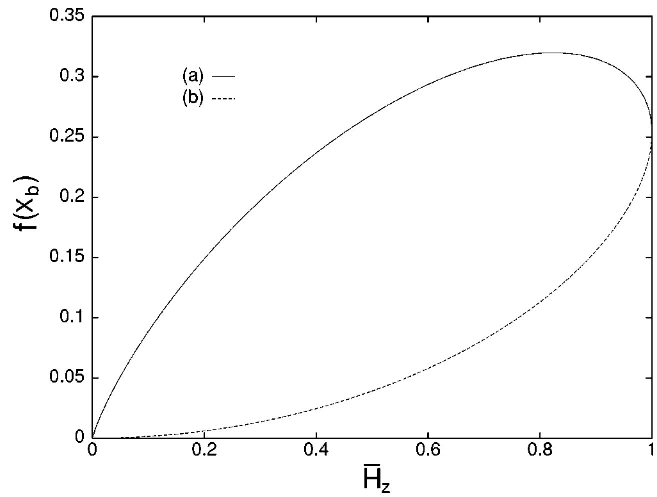

FIG. 2. The $\overline{H}_z$ dependence of the quantity $f(X_b)$, where (a) $\tanh X_i(=1/\sqrt{3})\leqslant\tanh X_b\leqslant 1$, and (b) $0\leqslant\tanh X_0\leqslant 1/\sqrt{3}$. Note that the maximum of $f(X_b)$ occurs $\overline{H}_z\simeq 0.82$.

with $m_H=a\tan\theta_H$, $a=(\pi/4)M_0H_z(1/K_\parallel+1/K_\perp)$, $b=2V_0/E_0$, and $f(X)=X\tanh X\mathrm{sech}^2X$. Here we note that the third term in Eq. (9) originated from the pinning potential (6) via λ for $\dot{Q}\ll\sqrt{k}v_0$.

Let us analyze the structure of the potential $U(X)$. Since the external magnetic field brings the system into a metastable state by tilting the potential, the domain wall has a chance to move out of the potential. As the field is increased, the metastable state disappears in the situation $U''(X_i)=0$, i.e., $X_i=\mathrm{arctanh}(1/\sqrt{3})$ and $U'(X_i)|_{h_z^c}=0$ where $'$ indicates the differentiation of the function with respect to X. This critical magnetic field $h_z^c[=(4\sqrt{3}/9)V_0]$ corresponds to the classical depinning field H_z^c multiplied by $2A_wM_0\lambda_0$. Denoting X_0 and X_b to be the X coordinate of the metastable position and the top of the potential, respectively, we have $0<X_0\leqslant X_i\leqslant X_b$. Since the second and the third term in Eq. (9) are expected to be important to determine the phase boundary, it is meaningful to estimate $f(X_b)$ depending on the magnitude of $\overline{H}_z(=H_z/H_z^c)\leqslant 1$. Noting that $\overline{H}_z[\equiv g(X_b)]$ is a function of X_b with $1/\sqrt{3}\leqslant\tanh X_b\leqslant 1$ via $U'(X_b)=0$, we have $0\leqslant f(X_b)\lesssim 0.32$, as is shown in Fig. 2.

Now, interested in the transition rate $\Gamma\sim\exp(-F_{\min}/T)$ where $F_{\min}$ is the minimum of the effective "free energy"[2,4] $F\equiv E+TS(E)-U_{\min}$ with respect to E, we consider the least action in the imaginary time

$$S_{\min}(E)=2\int_{X_1(E)}^{X_2(E)}dX\sqrt{2m(X)[U(X)-E]}, \tag{10}$$

where $X_1(E)$ and $X_2(E)$ are the turning points for the particle with energy $-E$ in the inverted potential $-U(x)$. As was previously mentioned, the effective mass depends on the coordinate and the direction of the field, which will be important in obtaining the first-order transition. In order to have the phase boundary between the first- and the second-order transition, we need to consider the behavior of $S_{\min}(E)$ around the top of the barrier. Expanding the integrand in Eq. (10) near X_b which corresponds to the top of the barrier, and introducing dimensionless energy variable[2,4] $p[\equiv(U_{\max}$

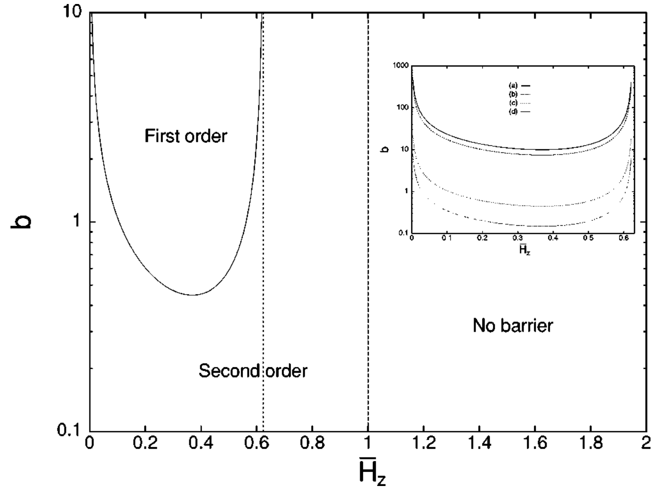

FIG. 3. The $\overline{H}_z$ dependence of the phase diagram for a given $m_H=0.97$. Inset: The phase boundary for $m_H=0$ (a), 0.5 (b), 0.97 (c), and 0.99 (d). The minimal values of b are approximately 14.9 (a), 7.5 (b), 0.45 (c), and 0.15 (d) at $\overline{H}_z \simeq 0.37$.

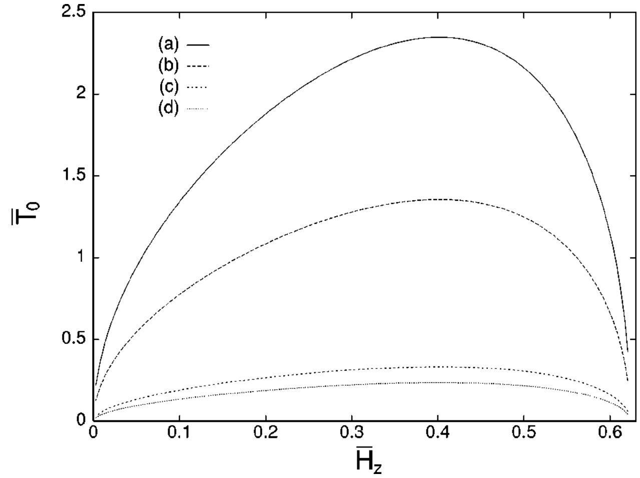

FIG. 4. The scaled quantum-classical crossover temperature $\overline{T}_0$ at the phase boundary between first- and second-order transitions where $m_H=0.99$ (a), 0.97 (b), 0.5 (c), and 0 (d). Note that the maximal values of $\overline{T}_0$ are approximately 2.48 (a), 1.36 (b), 0.33 (c), and 0.23 (d) at $\overline{H}_z \simeq 0.4$.

$-E)/(U_{max}-U_{min})]$ where U_{max} (U_{min}) corresponds to the top (bottom) of the potential, we have the minimal action near the top of the barrier

$$S_{min}(p)=\pi\sqrt{\frac{2m(X_b)}{U_2}}\Delta U[p+\beta p^2+O(p^3)], \quad (11)$$

where[17] $m(X_b)>0$ and

$$\beta=\frac{\Delta U}{16U_2}\left\{\frac{12U_4U_2+15U_3^2}{2U_2^2}+3\left(\frac{m'(X_b)}{m(X_b)}\right)\left(\frac{U_3}{U_2}\right)+\frac{m''(X_b)}{m(X_b)}-\frac{1}{2}\left(\frac{m'(X_b)}{m(X_b)}\right)^2\right\}, \quad (12)$$

with the derivatives of the potential denoted by $U_2=-U''(X_b)/2(>0)$, $U_3=U^{(3)}(X_b)/3!$ and $U_4=U^{(4)}(X_b)/4!$ Then, the effective free energy can with the help of Eq. (11) be written as

$$F_{min}(p)=\Delta U[1+\alpha p+\beta p^2+O(p^3)], \quad (13)$$

where $\alpha=T/T_0^{(c)}-1$ with $T_0^{(c)}=\sqrt{|U''(X_b)|/m(X_b)}/(2\pi)$. Owing to the general conditions for FST discussed in Ref. 1, FST can occur depending on the monotonic or nonmonotonic behavior of the period of oscillation $\tau(p)$ in the inverted potential. In other words, whether the slope of $\tau(p)$ is negative or positive near the top of the barrier determines FST. Since the minimum of the effective free energy determines the transition rate in the exponential approximation, $\Gamma\sim\exp(-F_{min})/T$, the phase transitions are governed by the behavior of the free energy, to be more specific, the sign of second derivative of the free energy around the top of the barrier, just as the Landau model of phase transition. Accordingly, the factor α changes signs at the phase transition temperature $T=T_0^{(c)}$. If the factor β is negative (positive), the system becomes the first- (second-) order transition, and thereby $\beta=0$ determines the phase boundary between them. Writing $b=(9/4\sqrt{3})H_z^c/(K_{||}/M_0)$, one can see from Fig. 3 that the phase boundary strongly depends on m_H, b, and $\overline{H}_z$. The less b is, the smaller the range of $\overline{H}_z$ for the first-order transition becomes. For a given m_H there exists a minimum of b in the phase boundary, e.g., $b_m\simeq 0.45$ for $m_H=0.97$ and the corresponding $\overline{H}_z$ is approximately 0.37 whose value is independent of m_H. As is noted in Fig. 3, the first-order transition can be observed in the field range, $0<H_z/H_z^c\lesssim 0.62$ for $m_H=1$, in which it occurs irrespective of the magnitude of b.[18]

The crossover temperature at the phase boundary can be obtained by using the relation between b and $\overline{H}_z$ at the boundary for a given m_H and $T_0^{(c)}(m_H,b,\overline{H}_z)$. As follows from Fig. 4, the corresponding crossover temperature increases as m_H increases, and has a maximum for a given m_H, e.g., $\overline{T}_0(m_H=0.97,\quad \overline{H}_z\simeq 0.4)\simeq 1.36$ where $\overline{T}_0$ is defined as

$$\overline{T}_0=T_0/\frac{1}{2\pi}\sqrt{\frac{3\sqrt{3}}{2}(\gamma H_z^c)\left(\frac{2\gamma K_\perp}{M_0}\right)},$$
$$=\sqrt{\frac{(1-\tanh^2X_b)(-1+3\tanh^2X_b)}{[1-m_H+bf(X_b)]}}. \quad (14)$$

Here we also note that $\overline{H}_z$ for the maximum of $\overline{T}_0$ is independent of m_H.

Our final note concerns the brief illustration of the results with concrete numbers. In order to do that, we have taken the physical quantities, e.g., from Ref. 13. For instance, b is calculated to be about 0.1, 0.08, and 1.03 and the scaled factor in the denominator of $\overline{T}_0$ in Eq. (14) 5.4, 27.4, and 155 in unit of mK for YIG, Ni, and $SrRuO_3$, respectively. Correspondingly, among these samples $SrRuO_3$ is best candidate for FST because of the large value of b and the large value of the scaled factor for T_0 mainly originating from larger coercivity H_z^c (Figs. 3 and 4). Thus, it would be interesting to study the phase transition for the materials as large coercivity as possible.

In conclusion, we have considered phase transition between quantum and classical regimes for the depinning rate of a domain wall. The effective free energy has been calculated near the top of the barrier. We have discussed the phase boundary between the first- and the second-order transition depending on the magnetic field and its crossover temperature. The investigations presented here opens new possibilities to observe such transitions experimentally in a domain wall system.

I am indebted to B. Barbara for many fruitful discussions. This work was supported by the Basic Science Research Institute Program, Ministry of Education, Project No. 015-D00118.

---

*Electronic address: gkim@kunja.sejong.ac.kr

[1] E. M. Chudnovsky, Phys. Rev. A **46**, 8011 (1992).

[2] E. M. Chudnovsky and D. A. Garanin, Phys. Rev. Lett. **79**, 4469 (1997).

[3] D. A. Garanin and E. M. Chudnovsky, Phys. Rev. B **56**, 11 102 (1997).

[4] D. A. Garanin, X. Martínes, and E. M. Chudnovsky, Phys. Rev. B **57**, 13 639 (1998).

[5] D. A. Garanin and E. M. Chudnovsky, Phys. Rev. B **59**, 3671 (1999).

[6] J.-Q. Liang, H. J. W. Müller-Kirsten, D. K. Park, and F. Zimmerschied, Phys. Rev. Lett. **81**, 216 (1998).

[7] G.-H. Kim, Phys. Rev. B **59**, 11 847 (1999); J. Appl. Phys. **86**, 1062 (1999).

[8] I. Affleck, Phys. Rev. Lett. **46**, 388 (1981).

[9] A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma Zh. Éksp. Teor. Fiz. **37**, 322 (1983) [JETP Lett. **37**, 382 (1983)].

[10] A. P. Malozemoff and J. C. Slonczewski, *Magnetic Domain Walls in Bubble Materials* (Academic Press, New York, 1979).

[11] P. C. E. Stamp, Phys. Rev. Lett. **66**, 2802 (1991).

[12] E. M. Chudnovsky, O. Iglesias, and P. C. E. Stamp, Phys. Rev. B **46**, 5392 (1992); P. C. E. Stamp, E. M. Chudnovsky, and B. Barbara, Int. J. Mod. Phys. B **6**, 1355 (1992).

[13] H.-B. Braun, J. Kyriakidis, and D. Loss, Phys. Rev. B **56**, 8129 (1997).

[14] V. G. Bar'yakhtar, M. V. Chetkin, B. A. Ivanov, and S. N. Gadetskii, *Dynamics of Topological Magnetic Solitons* (Springer-Verlag, Berlin, 1994); W. F. Brown, Jr., *Magnetostatic Principles in Ferromagnetism* (North Holland, Amsterdam, 1962).

[15] The mass presented here differs from the one $\widetilde{M}$ used in Ref. 12. Noting that $\widetilde{M}/M = k/(\sqrt{1+k}-1)^2$, the two masses agree in the limit of $k \gg 1$ while $\widetilde{M}/M \simeq 4/k$ for $k \ll 1$. The details have been discussed in Ref. 13.

[16] G.-H. Kim, Phys. Rev. B **58**, 12 138 (1998).

[17] In this paper our discussion will be devoted to the effective mass with the parameter range $0 \leqslant m_H \leqslant 1$. If m_H is greater than 1, the effective mass $m(X_b)$ becomes zero at some field and thereby β is expected to go to minus infinity. In this situation special care should be taken of the effective free energy close to the top of the barrier. Our preliminary calculation shows that there exists the first-order transition in the limit of the field around the classical depinning field. A more detailed study of the situation will be presented elsewhere.

[18] Here we have considered the situation when a single domain wall is coupled to defects within a very thin slab wire. However, in bulk samples the rate of magnetic relaxation is determined by the statistical average over a large number of individual tunneling events for the ensemble of domain walls. Although this problem is much more complicated than studied here, we believe that FST can occur even in this situation because each effective mass still depends on the coordinate and the direction of the field, which plays an important role in the phase transition.