Possible existence of topological excitations in quantum spin models in low dimensions

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(Received 11 December 1998)

The possibility of the existence of topological excitations in the anisotropic quantum Heisenberg model in one and two spatial dimensions is studied using the coherent state method. It is found that a part of the Wess-Zumino term contributes to the partition function, as a topological term for ferromagnets in the long-wavelength limit in both one and two dimensions. In particular, the *XY* limit of the two-dimensional anisotropic ferromagnet is shown to retain the topological excitations, as expected from the quantum Kosterlitz-Thouless scenario. [S0163-1829(99)13429-5]

I. INTRODUCTION

Quantum spin systems in low dimensions have acquired considerable significance in condensed matter physics in recent times. In particular, two-dimensional (2D) spin-$\frac{1}{2}$ quantum Heisenberg antiferromagnet (QHAF) evoked a lot of interest in light of the discovery of high-temperature superconductors.[1] Many interesting theoretical and experimental works probing the magnetic property of various 2D systems and also that of many quasi-one-dimensional systems brought into notice important features of anisotropic quantum spin models.[2–4] Parallel to this, a possible extension of Kosterlitz-Thouless (KT) scenerio to quantum ferromagnetic spin models has also been attempted.[5]

However, in spite of this endeavor, many crucial questions have remained unanswered and in particular the origin of the existence of topological excitations in quantum ferromagnetic and antiferromagnetic models seems to be mysterious. The existence of topological excitations in isotropic 1D AF is well known.[3,6] The case of 1D ferromagnetism (both isotropic and anisotropic), on the other hand, has drawn lesser attention.[3,6] One possible reason for this could be the lack of proper theoretical analyses of the quantum nature of the problem, which we describe in this paper. In the 2D case even for AF, the issue of the existence of topological excitations is still not settled fully, although most of the theoretical calculations rule out such a possibility.[6,7] Moreover, the high-T_c oxides in the insulating antiferromagnetic phase seem to be governed by anisotropic (2D) Heisenberg models, whereas the theoretical efforts have mostly been confined to the isotropic case only.[1,2,6,7] The 2D ferromagnetic situation has remained even less understood so far.[5,6]

This motivated us to study the anisotropic quantum Heisenberg ferromagnetic and antiferromagnetic models in 1D and 2D, in a unified manner.

II. MATHEMATICAL FORMULATION

We analyze the quantum actions for *XXZ* ferromagnets and antiferromagnets in 1D and 2D by the spin coherent state method.[6] The philosophy behind this procedure is that the existence of a topological term in the full quantum partition function of a quantum spin system implies topological excitations in the system.[8] Keeping in mind the physically relevant situations, we choose the anisotropy of the above spin models to the *XY* like.

In the following we perform all the calculations on the lattice with a finite lattice parameter a in the long-wavelength limit. We write down the expressions of the quantum Euclidean action in the quasicontinuum limit, so that we have a clear understanding of the topological terms, while the physical system retains its lattice structure.

III. CALCULATIONS

The quantum Euclidean action $S_E[\mathbf{n}]$ for the spin coherent fields $\mathbf{n}(t)$ can be written as[6,9]

$$S_E[\mathbf{n}] = -i\mathcal{S} S_{\mathrm{WZ}}[\mathbf{n}] + \frac{\mathcal{S}\delta t}{4}\int_0^\beta dt\, \partial_t[\mathbf{n}(t)]^2 + \int_0^\beta dt\, H(\mathbf{n}), \tag{1}$$

where $\mathcal{S}$ is the magnitude of the spin and

$$H(\mathbf{n}) = \langle \mathbf{n}|H(\mathbf{S})|\mathbf{n}\rangle, \tag{2}$$

$H(\mathbf{S})$ being the spin Hamiltonian in the representation $\mathcal{S}$. The Wess-Zumino term S_{WZ} is given by[6]

$$S_{\mathrm{WZ}}[\mathbf{n}] = \int_0^\beta dt \int_0^1 d\tau\, \mathbf{n}(t,\tau)\cdot \partial_t \mathbf{n}(t,\tau) \wedge \partial_\tau \mathbf{n}(t,\tau) = \mathcal{A}, \tag{3}$$

with $\mathbf{n}(t,0) \equiv \mathbf{n}(t)$, $\mathbf{n}(t,1) \equiv \mathbf{n}_0$, and $\mathbf{n}(0,\tau) \equiv \mathbf{n}(\beta,\tau)$, $t \in [0,\beta]$, $\tau \in [0,1]$.

In Eq. (3), $\mathcal{A}$ is the area of the cap bounded by the trajectory Γ parametrized by $\mathbf{n}(t)$ on the sphere:

$$\mathbf{n}\cdot\mathbf{n} = 1. \tag{4}$$

Here $|\mathbf{n}\rangle$ is the spin coherent state as defined in Ref. 6. The spin Hamiltonian for *XXZ* Heisenberg ferromagnets is given by

$$H(\mathbf{S}) = -g \sum_{\langle \mathbf{r},\mathbf{r}'\rangle} \tilde{\mathbf{S}}(\mathbf{r})\cdot\tilde{\mathbf{S}}(\mathbf{r}') - g\lambda_Z \sum_{\langle \mathbf{r},\mathbf{r}'\rangle} S_z(\mathbf{r}) S_Z(\mathbf{r}'), \tag{5}$$

with $g > 0$ and $0 < \lambda_Z < 1$. $\mathbf{r},\mathbf{r}'$ run over the lattice, and $\langle \mathbf{r},\mathbf{r}'\rangle$ signifies nearest-neighbor interaction and $\mathbf{S} = (\tilde{\mathbf{S}}, S_Z)$.

A. Linear chain

The quantum Euclidean action in the quasicontinuum limit can be written as

$$S_E[\mathbf{n}]=-i\mathcal{S}S_{\mathrm{WZ}}^{\mathrm{ntop}}-\frac{i\mathcal{S}}{2}\int_{-L}^{L}dx\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}+\frac{\mathcal{S}\delta t}{2a}\int_{-L}^{L}dx\int_0^{\beta}dt(\partial_t\mathbf{n})^2+\frac{\mathcal{S}^2\cdot ga}{2}\int_{-L}^{L}dx$$
$$\times\int_0^{\beta}dt[(\partial_x\tilde{\mathbf{n}})^2+\lambda_Z(\partial_x n_Z)^2],\tag{6}$$

where $S_{\mathrm{WZ}}^{\mathrm{ntop}}=2\Sigma_{r=-m}^{m-1}S_{\mathrm{WZ}}[\mathbf{n}(2ar)]$ is the nontopological part of the WZ term S_{WZ} on the chain, $L=2ma$, and $\mathbf{n}=(\tilde{\mathbf{n}},n_Z)$. We have written down Eq. (6) in the long-wavelength limit. We analyze the S_{WZ} term in the following manner:

$$\sum_r S_{\mathrm{WZ}}[\mathbf{n}(ar)]=\sum_{r=-m}^{m}S_{\mathrm{WZ}}[\mathbf{n}(2ar)]+\sum_{r=-m}^{m-1}S_{\mathrm{WZ}}[\mathbf{n}\{(2r+1)a\}]$$
$$=2S_{\mathrm{WZ}}[\mathbf{n}(-2am)]+S_{\mathrm{WZ}}[\mathbf{n}(2am)]+2\sum_{r=-m+1}^{m-1}S_{\mathrm{WZ}}[\mathbf{n}(2ar)]+\frac{1}{2}\int_{-L}^{L}dx\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}$$
$$=2S_{\mathrm{WZ}}[\mathbf{n}(-2am)]+S_{\mathrm{WZ}}[\mathbf{n}(2am)]+2\sum_{r=-m}^{m-p}S_{\mathrm{WZ}}[\mathbf{n}(2ar)]+2\left[\int_{-L}^{L-(p-1)2a}dx+\int_{-L}^{L-(p-2)2a}dx+\cdots\right.$$
$$\left.+\int_{-L}^{L-2a}dx\right]\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}+\frac{1}{2}\int_{-L}^{L}dx\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n},\tag{7}$$

where $1\leqslant p\leqslant 2m$. Since we keep the lattice parameter a finite and we take $\delta t\rightarrow 0$, Eq. (6) takes the form

$$S_E[\mathbf{n}]=-i\mathcal{S}S_{\mathrm{WZ}}^{\mathrm{ntop}}-\frac{i\mathcal{S}}{2}\int_{-L}^{L}dx\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}$$
$$+\frac{\mathcal{S}^2ga}{2}\int_{-L}^{L}dx\int_0^{\beta}dt[(\partial_x\tilde{\mathbf{n}})^2+\lambda_Z(\partial_x n_Z)^2].\tag{8}$$

In order that the Euclidean action Eq. (8) is finite for very large L, we have

$$\lim_{|\mathbf{x}|\rightarrow\infty}\partial_x\tilde{\mathbf{n}}=0,\qquad \lim_{|\mathbf{x}|\rightarrow\infty}\partial_x n_Z=0,\tag{9}$$

i.e., $\mathbf{n}\rightarrow\mathbf{n}_0(t)$ on the circle $x^2+t^2=R^2$; $R\rightarrow\infty$ with $\mathbf{n}\cdot\mathbf{n}=1$.

This defines a mapping from (x,t) space to the internal space $\mathbf{n}\cdot\mathbf{n}=1$. However, there is a smaller class of $\mathbf{n}$ fields on the (x,t) space which satisfies Eq. (9) with $\mathbf{n}_0(t)$ independent of t, denoted by $\mathbf{n}_0$. In that case the boundary points in the (x,t) space are identified with a single point, and we have a topological mapping $S^2_{\mathrm{phys}}\rightarrow S^2_{\mathrm{int}}$ with $\pi_2(S^2)=Z$.[10] The winding number in this case is given by[8]

$$Q=\frac{1}{4\pi}\int dx\,dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n},\tag{10}$$

where $Q\in Z$. Thus, for field configurations represented by

$$\{\mathbf{n}(x,t):\lim_{|\mathbf{x}|\rightarrow\infty}\mathbf{n}(x,t)=\mathbf{n}_0\},\tag{11}$$

Eq. (7) can be written as

$$\sum_r S_{\mathrm{WZ}}[\mathbf{n}(ar)]$$
$$=2\sum_{\gamma=-m}^{m-p}S_{\mathrm{WZ}}[\mathbf{n}(2ar)]$$
$$+2\left[\int_{-L}^{L-(p-1)2a}dx+\int_{-L}^{L-(p-2)2a}dx+\cdots+\int_{-L}^{L-2a}dx\right]$$
$$\times\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}+\frac{1}{2}\int_{-L}^{L}dx\int_0^{\beta}dt\,\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n},\tag{12}$$

where we have made use of Eq. (3). Notice that only the last term in Eq. (12) covers the entire chain where the boundary points are identified via the configuration (11). Therefore the topological content in S_{WZ} is the last integral in Eq. (12) which is same as Eq. (10). The rest of the terms in Eq. (12) are nontopological. From Eq. (7) the nontopological part can be written as

$$S_{\mathrm{WZ}}^{\mathrm{ntop}}=2\sum_{r=-m}^{m-1}S_{\mathrm{WZ}}[\mathbf{n}(2ar)].\tag{13}$$

Due to translational invariance of $\mathbf{n}$ on the chain, each term in Eq. (13) describes the same cap of area $\mathcal{A}$ given by Eq. (3). Thus Eq. (13) can be written as

$$S_{\mathrm{WZ}}^{\mathrm{ntop}}=2\sum_{r=-m}^{m-1}\mathcal{A}=2(2m-1)\mathcal{A}=\text{const}.\tag{14}$$

Therefore Eq. (8) becomes

$$S_E[\mathbf{n}] = -\frac{i\mathscr{s}}{2}\int_{-L}^{L} dx \int_0^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} + \frac{\mathscr{s}^2 ga}{2}\int_{-L}^{L} dx\int_0^{\beta} dt[(\partial_x\tilde{\mathbf{n}})^2 + \lambda_Z(\partial_x n_Z)^2]. \tag{15}$$

Equation (12) corresponding to an antiferromagnet ($g<0$) reads

$$S_{\mathrm{WZ}} = -\frac{1}{2}\int_{-L}^{L} dx\int_0^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}, \tag{16}$$

with $S_{\mathrm{WZ}}^{\mathrm{ntop}}$ vanishing due to a staggering operation. So we get back the same result as obtained for isotropic antiferromagnets.[6]

B. Two-dimensional square lattice

The spin Hamiltonian in this case is given by Eq. (5) where $\mathbf{r}$ runs over the 2D square lattice. The quantum action for the anisotropic ferromagnet in the long-wavelength limit is given by

$$S_E[\mathbf{n}] = -i\mathscr{s}\sum_{\mathbf{r}} S_{\mathrm{WZ}}[\mathbf{n}(\mathbf{r})] + \frac{g\mathscr{s}^2}{2}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt[(\partial_x\tilde{\mathbf{n}})^2 + \lambda_Z(\partial_x n_Z)^2 + (\partial_y\tilde{\mathbf{n}})^2 + \lambda_Z(\partial_y n_Z)^2](x,y,t). \tag{17}$$

Finiteness of the action (17) gives

$$\mathbf{n}(x,y,t)\rightarrow\mathbf{n}_0(t) \tag{18}$$

on the two-sphere $x^2+y^2+t^2=R^2$, $R\rightarrow\infty$, and $\mathbf{n}\rightarrow\mathbf{n}=1$. This boundary condition gives a mapping from the (x,y,t) space to the internal sphere $\mathbf{n}\cdot\mathbf{n}=1$. However, Eq. (18) admits a class of $\mathbf{n}$ fields where $\mathbf{n}_0(t)$ is independent of t denoted by $\mathbf{n}_0$, in which case we have a mapping $S^3_{\mathrm{phys}}\rightarrow S^2_{\mathrm{int}}$ and $\Pi_3(S^2)=Z$.[10] Thus, for the field configurations,

$$\{\mathbf{n}(x,y,t)\colon \lim_{|\mathbf{x}|\rightarrow\infty}\mathbf{n}(x,y,t)=\mathbf{n}_0\} \tag{19}$$

following the same line of arguments as in the case of linear chain, we can show that the S_{WZ} term corresponding to 2D square lattices can be written as

$$\sum_{\mathbf{r}} S_{\mathrm{WZ}}[\mathbf{n}(\mathbf{r})] = S_{\mathrm{WZ}}^{\mathrm{ntop}} + \frac{1}{2a}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt[\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} + \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_y\mathbf{n}](x,y,t), \tag{20}$$

$$S_{\mathrm{WZ}}^{\mathrm{ntop}} = \{2(2m-1)\}^2\mathcal{A}. \tag{21}$$

For the mapping $S^3_{\mathrm{phys}}\rightarrow S^2_{\mathrm{int}}$, we can parametrize the (x,y,t) space with boundary points identified by y planes in which case

$$\frac{1}{4\pi}\int_{-L}^{L} dx\int_0^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}$$

will be a winding number through Eq. (10) for each y. This happens because of the fact that each (x,t) plane (i.e., $y=\mathrm{const}$) has its boundary points identified for field configurations satisfying Eq. (19) and the (x,t) plane can be thought of as a sphere S^2 passing through the north pole of S^3. So for each y we have a mapping from the corresponding (x,t) plane ($=S^2$) to S^2_{int}. Therefore we can write

$$\frac{1}{4\pi}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}(x,y,t) = \int_{-L}^{L} Q(y)\,dy. \tag{22}$$

By similar arguments

$$\frac{1}{4\pi}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_y\mathbf{n}(x,y,t) = \int_{-L}^{L} Q(x)\,dx. \tag{23}$$

Note that we can apply the above principle for the mapping $S^3_{\mathrm{phys}}\rightarrow S^2_{\mathrm{int}}$ since $\pi_3(S^2)=Z$.[10] Thus, using Eqs. (20) and (21), the action (17) becomes

$$S_E[\mathbf{n}] = -\frac{i\mathscr{s}}{2a}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt[\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} + \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_y\mathbf{n}] \times(x,y,t) + \frac{g\mathscr{s}^2}{2}\int_{-L}^{L} dx\, dy\int_0^{\beta} dt[(\partial_x\tilde{\mathbf{n}})^2 + \lambda_Z(\partial_x n_Z)^2 + (\partial_y\tilde{\mathbf{n}})^2 + \lambda_Z(\partial_y n_Z)^2](x,y,t). \tag{24}$$

The first integral in Eq. (24) is a topological term as follows from Eqs. (22) and (23). Let us point out that the right-hand side of Eq. (20) vanishes identically on the lattice in the long-wavelength limit under a staggering operation in the case of antiferromagnets.

IV. CONCLUSIONS

We have presented a unified scheme for analyzing the topological terms in the effective action corresponding to the long-wavelength limit of *XY*-like anisotropic quantum Heisenberg ferromagnets and antiferromagnets in one and two spatial dimensions for any value of the spin. Our calculation brings out clearly the hidden topological contribution from the S_{WZ} term, which influences statistical mechanics of ferromagnets in one dimension. This is probably manifested in the ‘‘solitonlike excitations’’ occurring in many experimental systems corresponding to these models.[3,4] It may be also interesting to point out that in 1D the roles of kink and antikink are interchanged as we go from ferromagnets to antiferromagnets due to sign reversal in the respective topological terms [see Eqs. (15) and (16)]. In the 2D situation in the limit $\lambda_Z\rightarrow 0$, these excitations probably lead to the proposed ‘‘vortex-antivortex’’ scenario in the ‘‘quantum KT’’ picture.[5] On the contrary the 2D AF model does not exhibit any topological excitation in its long-wavelength behavior.

Let us conclude by pointing out that our whole calculational approach is meaningful only in the low-temperature regime where the spin-spin correlation length is appreciably large.[6,11]

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