Vortex-dynamics study of the frequency dependence of the superfluid onset temperature

Hideo Yano,* Tsuyoshi Jocha, and Nobuo Wada
Graduate School of Arts and Sciences, University of Tokyo, Tokyo 153-8902, Japan

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The superfluid transition in a thin ^{4}He film on an oscillating Mylar substrate at two frequencies was studied by using a two-torsional-oscillators technique. An onset in superfluid and a peak in dissipation in the same coverage film were observed at higher temperatures at high frequency than at low frequency. The frequency dependence agrees quantitatively with the prediction in the dynamic Kosterlitz-Thouless theory. In interpretation of the frequency dependence we found a peak in the inverse of the diffusion length of a vortex at a coverage of 1.0 atomic layer. A comparison with the isothermal compressibility of ^{4}He film adsorbed on graphite suggests that the diffusion length increases with compressibility in the vicinity of 1.0 atomic layer. We also report the ratio of the diffusion constant to the square of the core radius of the vortex in the submonolayer region as well as at coverages above 1 atomic layer. [S0163-1829(99)02425-X]

I. INTRODUCTION

The superfluid transition in ^{4}He film has been described successfully in the Kosterlitz-Thouless (KT) picture of a vortex-antivortex pair unbinding.[1,2] The vortex-antivortex pair is a relevant thermal excitation in a superfluid film. At a temperature where many pairs are created, screening effect results in unbinding of the pair and induces free vortices to destroy superfluidity. In a static flow limit the transition occurs at a temperature where free vortices are induced. In the experimental study, however, it is necessary to include the effects of a time-dependent flow velocity. Ambegaokar, Halperin, Nelson, and Siggia (AHNS) extended the KT transition by including the effects of the vortex dynamics in externally applied velocity fields.[3]

Based on the theoretical description the vortex dynamics has been investigated experimentally by various methods of a torsional oscillator,[2,4–8] a quartz microbalance,[9–11] a thermal transport,[4,12,13] and a thermal counter flow.[14] These experiments have manifested the property of the vortex such as a diffusion constant D and a vortex core radius r_0 in superfluid films adsorbed on various substrates. Particularly a Mylar sheet has been often used as a substrate ever since the excellent study by Bishop and Reppy.[2] For superfluid films on the Mylar substrate a value of D/r_0^2 has been obtained at coverages above 1 atomic layer by a fit to experimental data.[2,4,5,12,13] Below this coverage, however, the value was not obvious because of an unsuccessful fit. The height of a peak in dissipation measured with a torsional oscillator is lower than that expected in the AHNS theory below 1 atomic layer except some coverages.[5,15]

The low peak in dissipation is attributable to macroscopic inhomogeneities in substrate potential.[5,16] A Mylar sheet contains small bubbles and flakes on the surface. These dimensions are in the order of 10 μm, while the diffusion length of a vortex is in the order of 1000 Å.[16] It is reasonable to expect macroscopic inhomogeneities to occur on a scale larger than the diffusion length. The macroscopic inhomogeneities cause inhomogeneous broadening in the film density. The temperature T_p at the peak in dissipation increases linearly with coverage, while the half width of the peak remains less than 10^{-2} of T_p. As a result the macroscopic inhomogeneities make the peak lower and broader. For substrate models a simple macroscopic hypernetted-chain calculation[16] yields reasonable order-of-magnitude agreement with the height of the peak in the experiment.[15]

Another perspective is necessary for the purpose of the vortex dynamics study in thin films. Motion of the vortex causes a variation in T_p with frequency. The temperature T_p is insensitive to the macroscopic inhomogeneities, because these dimensions are much larger than the diffusion length. Frequency dependence measurements are suitable for the vortex dynamics study in the submonolayer region.

In this paper we present results of the frequency dependence of the superfluid onset temperature in thin ^{4}He films on Mylar at coverages below 2 atomic layers by a two-torsional-oscillators technique. We found a peak in the inverse of the diffusion length of the vortex at the coverage of 1.0 atomic layer. We also report the coverage dependence of D/r_0^2 of the vortex in the submonolayer region as well as at coverages above 1 atomic layer. In Sec. II we derive the frequency dependence of superfluid onset temperature from the theoretical description of the vortex dynamics. In Sec. III we describe a two-torsional-oscillators technique and experimental procedure. In Sec. IV we present experimental results and discuss the vortex dynamics. First we present the temperature dependence of the superfluid density and the dissipation. Second we present the variation of the superfluid onset temperature with frequency. From both temperature and frequency dependence we discuss the vortex dynamics.

II. SUPERFLUID ONSET IN DYNAMIC THEORY

A superfluid density and a dissipation in an alternated field are described essentially by expressions in the AHNS theory. The vortex dynamics has been investigated experimentally by a fit of the temperature dependence of the density and the dissipation. Although an excellent fit to experimental data has been often obtained, a successful fit is limited for films at coverages above 1 atomic layer. To investigate the vortex dynamics in thin films we address another relation between the superfluid onset temperature and frequency. The superfluid onset temperature and a dissipation peak temperature increase with frequency. Since the temperature dependence in the AHNS theory is from complicated expressions with five free parameters, it is difficult to derive the frequency dependence from the expressions. In this section we present a simple equation of the frequency dependence based on a result in more phenomenological theory given by Minnhagen.[17] Comparisons with experiments support the Minnhagen theory for two-dimensional superfluid.[18]

The dissipation ΔQ^{-1} shows a peak at the temperature T_p where a screening length λ is equal to a diffusion length $\sqrt{14D/\omega}$.[17] Here D is the diffusion constant of a superfluid vortex and ω is an oscillator angular frequency. The screening length λ is written as

$$\lambda = r_0 \exp\left[\frac{2\pi}{b\sqrt{(T-T_{KT})/T_{KT}}}\right], \tag{2.1}$$

where r_0 is the core radius of the vortex, b is a nonuniversal parameter, and T_{KT} is a Kosterlitz-Thouless transition temperature. From the relation of $\lambda = \sqrt{14D/\omega}$ at T_p, T_p is given as

$$\frac{T_p - T_{KT}}{T_{KT}} = 4\pi^2 b^{-2}\left[\frac{1}{2}\ln\frac{14D}{r_0^2\omega}\right]^{-2}. \tag{2.2}$$

Thus the temperature T_p increases with oscillator frequency. In the present experiment values of T_p were measured at two frequencies. From Eq. (2.2) a difference ΔT_p in T_p between two frequencies ω_1 and ω_2 can be written as

$$\frac{\Delta T_p}{T_{KT}} = 4\pi^2 b^{-2} l_D^{-3} \ln\frac{\omega_1}{\omega_2} + O\left[\left(\ln\frac{\omega_1}{\omega_2}\right)^2\right]. \tag{2.3}$$

Here l_D is the logarithm of the diffusion length scaled by r_0 and is given by

$$l_D = \frac{1}{2}\ln\left(\frac{14D}{r_0^2\sqrt{\omega_1\omega_2}}\right). \tag{2.4}$$

The higher order terms in Eq. (2.3) are estimated to be lower than a tenth of the first term using two frequencies in the present experiment and values of D/r_0^2 and b obtained by Bishop and Reppy.[2] The temperature T_p can be more easily detected than T_{KT} in the torsional experiment. Because the temperature T_p at 2.6 kHz is only about 10 mK higher than T_{KT} for a film with $T_{KT} = 1.2$ K,[2] Eq. (2.3) can be rewritten as

$$\frac{\Delta T_p}{T_p} \approx 4\pi^2 b^{-2} l_D^{-3} (\ln\omega_1 - \ln\omega_2). \tag{2.5}$$

Equation (2.5) says that a parameter $b^2 l_D^3$ relates directly to the difference ΔT_p. The vortex parameters of D/r_0^2 and b, however, cannot be obtained separately from this equation. Accordingly, we use a fit of the temperature dependence of the superfluid density and the dissipation in the AHNS theory along with Eq. (2.5). In Sec. IV A values of D/r_0^2 and b estimated by the fit are presented separately, although the parameters are adjustable for only films above 1 atomic layer. In Sec. IV B we discuss the vortex dynamics based on the difference ΔT_p data for all coverages. In Sec. IV C we present the parameter D/r_0^2 estimated from Eq. (2.5) by assuming a value of b.

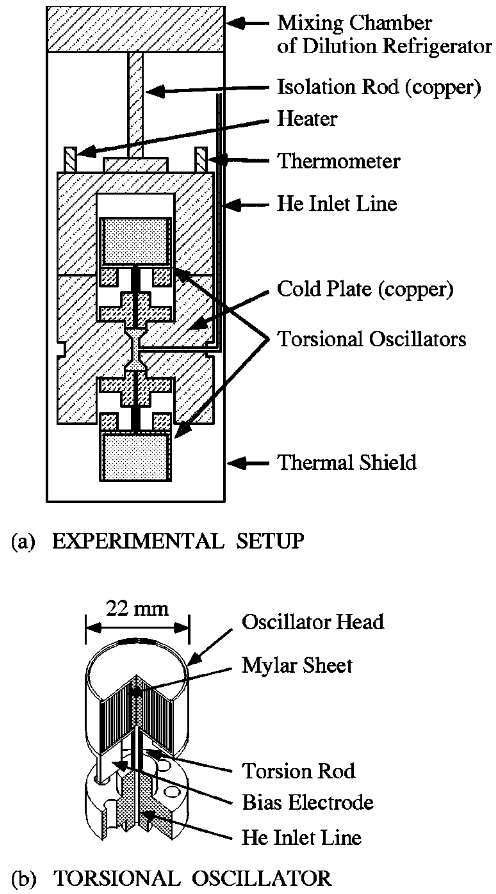

FIG. 1. (a) Cross section of the experimental setup in the vicinity of torsional oscillators. Two torsional oscillators are mounted on the same cold plate and are connected to the same inlet line. (b) Torsional oscillator. A Mylar sheet is rolled in an oscillator head.

III. EXPERIMENTAL DETAILS

A. Two-torsional-oscillators technique

A torsional oscillator is useful for the experiment of the superfluid density. Since its frequency is stabilized in 10^{-9} and its quality factor is 10^5 to 10^6, a self-balanced circuit accurately detects a frequency variation proportional to the superfluid density. The method, however, prevents modification of the frequency of the oscillator. Accordingly, we prepare two torsional oscillators whose frequency is different from each other. We must detect the superfluid onset temperature within the accuracy below 1 mK as mentioned in the next section, while the onset temperature increases with film coverage.[2] Therefore the same coverage film is necessary in the oscillators.

For unifying films we used a two-torsional-oscillators technique.[7,19] The cross section of our experimental setup is shown in Fig. 1(a). The torsional oscillators were mounted on the same cold plate made of copper. A helium-inlet line was connected to the oscillator cells through the cold plate. Measurements were simultaneously performed for two oscillators. These settings and measurements allow us to unify film in the oscillators and to obtain data at the same temperature for two frequencies.

The massive cold plate was mounted on a cryostat through an isolation rod. Since a characteristic frequency of the cold plate is much lower than the oscillator frequency, oscillations are prevented from the outside of the cryostat. The rod was made of copper and thermally connected the cold plate to a mixing chamber of a dilution refrigerator. The cold plate and the oscillators were surrounded with a thermal shield attached to the mixing chamber for avoiding a temperature gradient.

The schematic drawing of the torsional oscillator is shown in Fig. 1(b). The oscillation frequencies are 1016 and 1778 Hz and the quality factors (Q) are 1.0×10^6 and 1.3×10^5, respectively. The torsional oscillators are the same type as in a previous study.[7] A torsion rod and a head of the oscillator were made of Al alloy 5056. Since the density of the alloy is low and a torsion made of the alloy has high Q,[20] this material is well suited for superfluid film studies.

A Mylar strip of 4.5 μm thick, 12.5 mm wide, and 48 m long was wrapped around a spool attached to a cap. The cap with the Mylar strip was held in an oscillator head and was sealed with Stycast 1266 epoxy. To avoid different conditions between the oscillators we cut each strip from the same patch and wrapped it with the same tension. The surface area in each cell is computed to be 1.2 m^2 from the amount of the Mylar strip.

Oscillation of the head was detected through capacitive electrodes. Two electrodes attached to the head were made of brass and were biased to a static voltage of 160 V. External electrodes were attached to the cold plate. The gap between the external and bias electrodes was set at about 50 μm to achieve a capacitance of 2 pF. The detection of the oscillation was amplified electrically and was applied to the head through another part of the capacitive electrodes. A self-balanced circuit stabilized the frequency in 10^{-9} to 10^{-10}. The dissipation of the torsional oscillator was detected as the amplitude of the oscillation proportional to Q. The accuracy of Q is within 10^{-3} to 10^{-4} using a lock-in amplifier. To perform simultaneous measurements for two oscillators we used two self-balanced circuits.

B. Experimental procedure

Before filling gas into the cells we measured the frequency and the dissipation as a function of temperature. The temperature dependence is similar to those obtained in previous experiments.[5,20] The frequency of the 1016 Hz oscillator decreases by 1.9 mHz with an increase in temperature from 0.05 to 1.6 K. We obtained background variations by the fit of polynomial expressions.

The sample gas was filled into the cells at 1.2 K for each coverage. After heating up to 4.2 K we kept the temperature of the sample in the superfluid region more than 10 h for annealing. An oscillator-frequency change ΔF and a dissipation change ΔQ due to superfluid were measured as a function of temperature T. The temperature control was made within 0.2 mK by a proportion-integration-deviation action using a resistance thermometer and a heater attached to the cold plate. A temperature step was 0.5 mK in the vicinity of the superfluid onset temperature. At each temperature ΔF and ΔQ were measured after a 6 to 10 min wait because of the high Q value of the 1016 Hz oscillator.

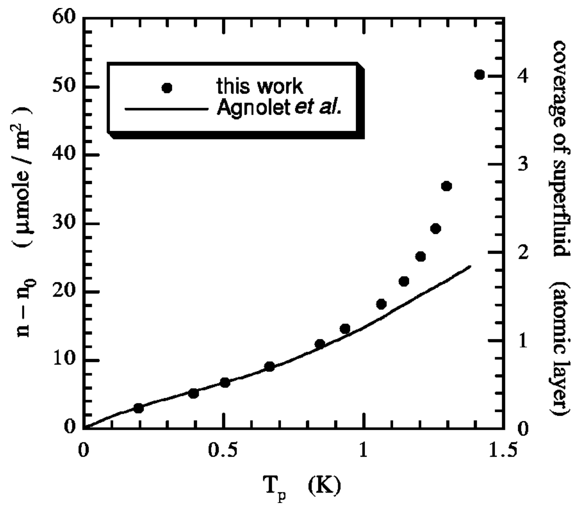

FIG. 2. Variation of superfluid coverage $n-n_0$ with temperature T_p at the peak in dissipation. Here n is a total amount of helium divided by surface area and n_0 is a nonsuperfluid coverage. Coverage data at high temperatures are incorrect because of desorbed gas in the cell. A correct superfluid coverage obtained by Agnolet *et al.* is also shown as a solid line (Ref. 5). We use T_p as the superfluid coverage rather than $n-n_0$ (see text). At the right side we show the coverage of superfluid in atomic layer.

Since the helium is desorbed from the substrate above 0.9 K, corrections are necessary on film coverages. In Fig. 2 we plot the coverage dependence of T_p in the present experiment as well as that in the experiment by Agnolet, McQueeney, and Reppy.[5] In the latter experiment film coverages were corrected from the gas amount in the cell. The discrepancy in T_p at high coverages shows the deflection in the film coverage. In the present paper we use T_p as the film coverage rather than the total amount of gas divided by surface area.

To verify thickness of a film between two oscillators we measured the frequency dependence under some conditions. As shown in Fig. 1 the oscillators were mounted on upper and lower side of a cold plate. Measurements for two combinations between the oscillator and the side yielded the same result. We also performed measurements for a different frequencies set.[7] The result of the relation between the superfluid onset and the frequency is consistent with that in Sec. IV B. These results indicate the same film thickness in two oscillators.

IV. RESULTS AND DISCUSSIONS

A. Temperature dependence of $2\Delta F/F_0$ and ΔQ^{-1}

We present results of the temperature dependence of $2\Delta F/F_0$ and ΔQ^{-1} at 1016 Hz in Fig. 3, where F_0 is the oscillator frequency. An abrupt change in $2\Delta F/F_0$ is proportional to the superfluid transition temperature T_c as shown by the solid line in Fig. 3(a). The proportionality is well known as the universal jump in the Kosterlitz-Thouless transition.[1] The height of a peak in ΔQ^{-1}, however, is not proportional to the temperature T_p at the peak for all coverages. Above 1

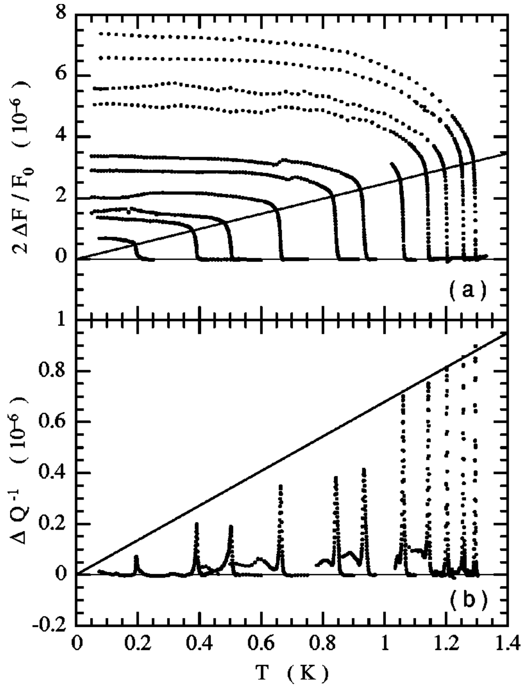

FIG. 3. Superfluid onset in ^{4}He films on an oscillating substrate at 1016 Hz for various coverages. (a) A superfluid density $2\Delta F/F_0$ has a steplike change just below superfluid onset temperature T_c. The superfluid density is proportional to T_c shown as a solid line. (b) The dissipation ΔQ^{-1} has a peak around T_c and the temperature T_p where ΔQ^{-1} retains a maximum is a few mK lower than T_c (see in Fig. 6). The peak height is proportional to T_p for films with T_p above 1 K shown as a solid line, but it is lower than the solid line below 1 K.

K the peak height increases linearly with T_p as a straight line in Fig. 3(b), while it is lower than the line below 1 K. Since a coverage of a film with $T_p = 0.94$ K corresponds to 1 atomic layer of a superfluid film as shown in Fig. 2, the low peaks were observed in the submonolayer region. To confirm the AHNS description we tried to perform a fit to data presented in Fig. 3. The successful fit, however, was limited to some coverages with T_p above 1 K. For other coverages the peak was lower and broader than that expected in the AHNS theory.

The unsuccessful fit of low and broad peaks in ΔQ^{-1} was also pointed out by McQueeney *et al.*[15] The peak height in their torsional oscillator experiment changes as a function of coverage with a periodicity of a half atomic layer. In Fig. 3(b) a periodicity in the peak height seems a half to a third atomic layer in the submonolayer region. Although the coverage dependence is not the same as each other in the whole coverage range, the periodic change in the peak height is reliable below 1 atomic layer. In a microscopic hypernetted-chain model Collins *et al.* found that the low and broad peak is caused by the macroscopic inhomogeneity of substrate potential.[16] The model, however, shows not a half-layer periodicity but a full-layer one. They suggested that the more sophisticated model derives short periodicity such a half layer. In the situation of the inhomogeneous potential an extra expression including an inhomogeneity effect is necessary for the fit.

The fit was performed in Fig. 4 using the same equations

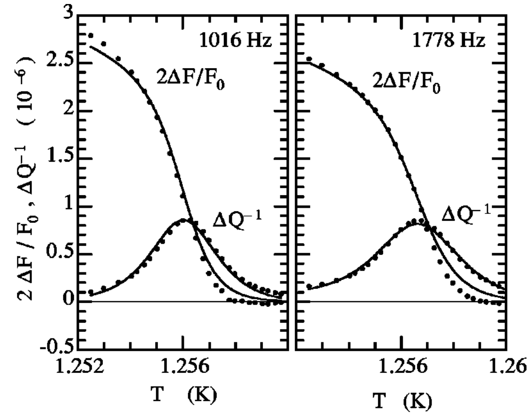

FIG. 4. Typical fits to data of $2\Delta F/F_0$ and ΔQ^{-1}. The fits were performed separately for each frequency.

in the study of Agnolet, McQueeney, and Reppy.[5] We obtained values of the vortex parameters separately for each frequency: $D/r_0^2 = 2.5\times10^{10}$ sec^{-1} and $T_{\mathrm{KT}} = 1.2466$ K for the 1016 Hz oscillator and $D/r_0^2 = 1.3\times10^{10}$ sec^{-1} and $T_{\mathrm{KT}} = 1.2456$ K for the 1778 Hz oscillator. Discrepancy between the values is attributable to uncertainties in the data. We plot the coverage dependence of the vortex parameters of D/r_0^2 and b from the successful fit in Fig. 5. As the coverage of the film increases with the peak temperature from 1 K to 1.4 K, D/r_0^2 increases by two orders of magnitude. The value b is practically constant in this coverage range.

Parameters of D/r_0^2 and b have been estimated from torsional oscillator experiments[2,5] and thermal conductivity experiments.[4,12,13] In Table I we list the typical values of D/r_0^2 and b obtained in the previous studies along with the present study. Our values of D/r_0^2 and b are in good agreement with those in the conductivity experiments rather than in the oscillator experiment. In the present study we used more reasonable equations[5] excluding a parameter b than those including b in the previous torsional oscillator experiments.[2,4] A difference in the fitting methods might reflect estimated values of D/r_0^2. For comparison we calculate a value of $\Delta T_p/T_p$ in Eq. (2.5): $\Delta T_p/T_p = 3.9\times10^{-4}$ with

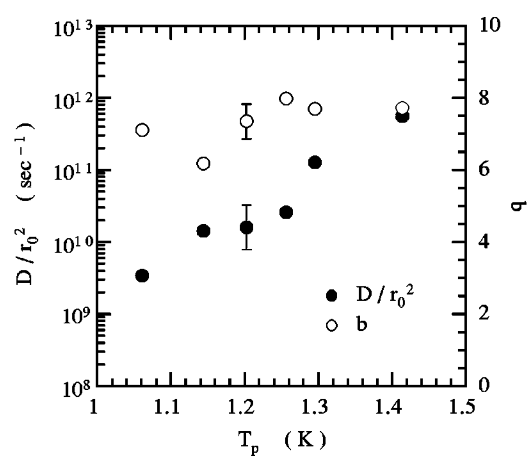

FIG. 5. Diffusion parameter D/r_0^2 and intrinsic parameter b of the vortex. Here D is the diffusion constant, r_0 the core radius, and b the parameter in Eq. (2.1). Error bars are shown only for data at $T_p = 1.2$ K for clarity.

TABLE I. Typical values of the parameters b and D/r_0^2 for a Mylar substrate found in various experiments conducted in the vicinity of the Kosterlitz-Thouless transition. The values are listed for films with T_{KT} below 1.4 K. The standard errors for b are 7% and for $\ln(D/r_0^2)$ they are 3% in the present study.

T_{KT} (K)	b	D/r_0^2 (sec^{-1})	Experiment
1.047	7.1	3.4×10^{9}	this work
1.128	6.2	1.4×10^{10}	this work
1.191	7.3	1.6×10^{10}	this work
1.247	8.0	2.5×10^{10}	this work
1.286	7.7	1.3×10^{11}	this work
1.405	7.7	5.5×10^{11}	this work
1.2043	5.5	3.1×10^{13}	oscillator (Ref. 2)
1.2794	4.9	6.1×10^{13}	oscillator (Ref. 4)
1.2794	6.9	6.2×10^{11}	conductivity (Ref. 4)
1.30	3.2	6.3×10^{11}	conductivity (Ref. 12)
1.2811	8.13	1.5×10^{10}	conductivity (Ref. 13)
1.3683	8.0	6×10^{9}	conductivity (Ref. 13)
1.3789	7.75	9×10^{9}	conductivity (Ref. 13)

$D/r_0^2=3.1\times10^{13}$ sec^{-1} and $b=5.5$, and $\Delta T_p/T_p=4.5\times10^{-4}$ with $D/r_0^2=6\times10^{13}$ sec^{-1} and $b=4.9$. These values are in good agreement with our value of $\Delta T_p/T_p=4.2\times10^{-4}$ at $T_{KT}=1.286$ K. This implies that the temperature dependences are the same.

In submonolayer films at coverages with T_p below 1 K D/r_0^2 was estimated for only two coverages. The estimated values are two to three orders lower than those above 1 K. If inhomogeneities lower the peak in dissipation, the estimated D/r_0^2 is incorrect. In this case the correct value is larger than the estimated one. From the temperature dependence we cannot decide values of D/r_0^2 in submonolayer films.

B. Frequency dependence of superfluid onset

The temperature dependence of $2\Delta F/F_0$ and ΔQ^{-1} was obtained at two frequencies of 1016 and 1778 Hz for various coverages by the two-torsional-oscillators method. Figure 6 provides typical data in the vicinity of the superfluid onset temperature. An onset temperature T_{onset} and a peak temperature T_p are higher at 1778 Hz than at 1016 Hz. A difference ΔT_p between two frequencies was obtained to be 0.5 ± 0.2 mK.

In the AHNS theory the difference ΔT_p is caused by the vortex motion in alternated fields. In the previous section we estimated the vortex parameters from the temperature dependence of $2\Delta F/F_0$ and ΔQ^{-1}. From Eq. (2.3) ΔT_p is computed to be 0.64 mK using the values of D/r_0^2 and b with $T_{KT}=1.247$ K in Table I. The computed value is in quantitative agreement with the value obtained in the present experiment. The agreement indicates that the superfluid onset temperature in alternated fields is dominated by the vortex dynamics in terms of a diffusion length and a screening length of a vortex. It is also evidence of the same condition of ^{4}He films in two oscillator-cells except frequency.

At all coverages T_{onset} and T_p are higher at 1778 Hz than at 1016 Hz. Differences ΔT_p divided by T_p are presented in Fig. 7, where T_p is the temperature of a dissipation peak at 1016 Hz. The coverage dependence of $\Delta T_p/T_p$ shows a peak at $T_p=0.94$ K, and $\Delta T_p/T_p$ increases with decreasing coverage below 0.5 K. The coverages of films with $T_p=0.94$ and 0.5 K correspond to 1.0 and 0.6 atomic layers, respectively, as shown in Fig. 2.

In the interpretation of Eq. (2.5) the coverage dependence indicates that the inverse of the diffusion length of the vortex varies with coverage accompanied with a peak at 1.0 atomic layer. It is probable that some property in film affects the diffusion length. In the case of graphite substrate ^{4}He film grows layer by layer through at least seven atomic layers and its isothermal compressibility exhibits minima at layer compression.[21] Since the inert layer is three atomic layers, the inverse of the compressibility exhibits a peak at one atomic layer of superfluid film. The coverage dependence of the compressibility is similar to the peak in $\Delta T_p/T_p$ at 1.0 atomic layer of superfluid film. The comparison suggests that the diffusion length of the vortex increases with compressibility in ^{4}He film.

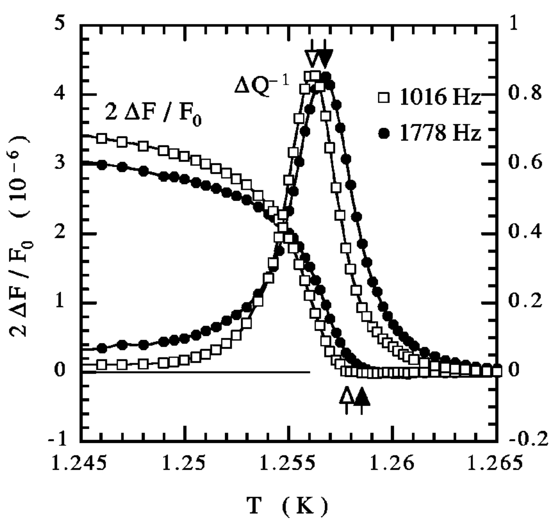

FIG. 6. Superfluid density $2\Delta F/F_0$ and dissipation ΔQ^{-1} for two frequencies of 1016 and 1778 Hz measured by a two-torsional-oscillators technique. Both a superfluid onset temperature and a dissipation peak temperature are higher at 1778 than at 1016 Hz. The arrows on the lower side indicate the superfluid onsets and those on the upper side indicate the dissipation peaks.

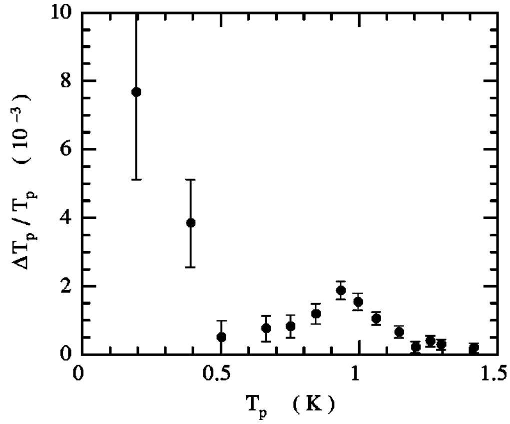

FIG. 7. Difference ΔT_p of dissipation peak temperatures between 1016 and 1778 Hz as a function of coverage. We plot a dissipation peak temperature T_p at 1016 Hz as the coverage.

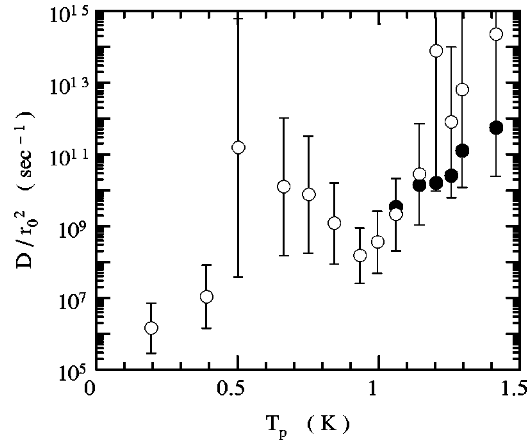

FIG. 8. Coverage dependence of the diffusion parameter D/r_0^2 (white circles) calculated from $\Delta T_p/T_p$ in Fig. 7 and Eq. (2.5). The diffusion parameter calculated from the temperature dependence in Fig. 5 is also plotted as solid circles.

The same peak in $\Delta T_p/T_p$ was found in ^{4}He films adsorbed on hydrogen by the two-torsional-oscillators method.[8] The coverage at the maximum of the peak is $T_p=1.15$ K, which corresponds to one atomic layer of superfluid film.[22] The compressibility in ^{4}He film on graphite preplated with hydrogen exhibits minimum at the same coverage.[21] This implies the existence of the universal relation between the diffusion length of the vortex and the isothermal compressibility in ^{4}He film.

C. Coverage dependence of vortex parameters

For the estimate of D/r_0^2 from the data of $\Delta T_p/T_p$ we must know the value of the nonuniversal parameter b. In the region of the thicker films with $T_p>1.6$ K the value of b is proportional to $d^{0.69}$, where d is the film thickness.[13] In the region between 1 and 1.4 K, however, b is nearly constant as shown in Table I. Therefore it is probable that b does not vary so much in the submonolayer region below 1 K. By assuming $b=7\pm1$ we estimate D/r_0^2 from $\Delta T_p/T_p$ in Fig. 8. At coverages with $T_p>1$ K the estimated values are consistent with the fitting values within errors from the temperature dependence of $2\Delta F/F_0$ and ΔQ^{-1}. In the region above 0.94 K D/r_0^2 increases monotonously with coverage, whereas the coverage dependence is not monotonous in the submonolayer region. The peak in the coverage dependence seems to be at 0.5 K which corresponds to 0.6 atomic layer. Below 0.5 K D/r_0^2 decreases two to three orders of magnitude than that at 1.0 atomic layer. In superfluid films adsorbed on 500 Å alumina powder, the core size of the vortex increases by two order of magnitude as the film is thinned at coverages from 0.6 to 0.05 K.[23] It is probable that the decrease in D/r_0^2 at low coverages is attributed to the increase in the vortex core size of r_0.

It is also likely that b increases with coverage. For submonolayer films vortex parameters have been estimated at the only coverage with $T_p=0.11$ K.[5] At the coverage a value of b is computed to be 4.4 from Eq. (2.2). Assuming $b=4.4$ D/r_0^2 at 0.2 K increases one order of magnitude from the values in Fig. 8. In this case our value of D/r_0^2 agrees well with that at 0.11 K in the previous experiment.

V. CONCLUSIONS

In summary, we investigated the motion of the superfluid vortex in alternated fields based on the experimental results of the superfluid density and the dissipation by a two-torsional-oscillators technique. The frequency dependence of the superfluid onset temperature is caused by the balance between a diffusion length and a screening length of the vortex. In the two-torsional-oscillators experiment we indicate that the difference of the onset temperature between two frequencies is related to the diffusion length scaled by a core radius of the vortex. The coverage dependence of the temperature difference shows a peak in the diffusion length at the coverage of 1 atomic layer. On comparison with the isothermal compressibility of ^{4}He film adsorbed on graphite the peak suggests that the diffusion length increases with compressibility in the vicinity of 1.0 atomic layer. We present the coverage dependence of the ratio of the diffusion constant to the square of the core radius of the vortex below 2 atomic layers. The ratio decreases three orders of magnitude above 1 atomic layer as decreasing coverage. In the submonolayer region the coverage dependence of the ratio is not monotonous: the peak seems to be at the 0.6 atomic layer.

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*Present address: Faculty of Science, Osaka City University, Sumiyoshi-ku, Osaka 558-8585, Japan.

[1] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[2] D. J. Bishop and J. D. Reppy, Phys. Rev. B **22**, 5171 (1980).

[3] V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. B **21**, 1806 (1980).

[4] G. Agnolet, S. L. Teitel, and J. D. Reppy, Phys. Rev. Lett. **47**, 1537 (1981).

[5] G. Agnolet, D. F. McQueeney, and J. D. Reppy, Phys. Rev. B **39**, 8934 (1989).
[6] P. W. Adams and W. I. Glaberson, Phys. Rev. B **35**, 4633 (1987).
[7] H. Yano, T. Jocha, and N. Wada, J. Low Temp. Phys. **101**, 513 (1995).
[8] H. Yano, T. Jocha, and N. Wada, Czech. J. Phys. **46**, 415 (1996).
[9] M. Chester and L. C. Yang, Phys. Rev. Lett. **31**, 1377 (1973).
[10] R. Brada, H. Chayet, and W. I. Glaberson, Phys. Rev. B **48**, 12 874 (1993).
[11] S. Ben Ezra and W. I. Glaberson, Czech. J. Phys. **46**, 433 (1996).
[12] J. Maps and R. B. Hallock, Phys. Rev. B **27**, 5491 (1983).
[13] D. Finotello and F. M. Gasparini, Phys. Rev. Lett. **55**, 2156 (1985); D. Finotello, Y. Y. Yu, and F. M. Gasparini, Phys. Rev. B **41**, 10 994 (1990).
[14] R. W. A. van de Laar, A. van der Hoek, and H. van Beelen, Physica B **216**, 24 (1995); A. van der Hoek and H. van Beelen, Czech. J. Phys. **46**, 415 (1996).
[15] D. F. McQueeney, G. Agnolet, G. K. S. Wong, and J. D. Reppy, Jpn. J. Appl. Phys., Suppl. **26**, 79 (1987).
[16] M. Collins, G. Agnolet, W. M. Saslow, and E. Krotscheck, Phys. Rev. B **47**, 8905 (1993).
[17] P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
[18] M. Wallin, Phys. Rev. B **41**, 6575 (1990).
[19] K. Shirahama, M. Kubota, S. Ogawa, N. Wada, and T. Watanabe, Phys. Rev. Lett. **64**, 1541 (1990).
[20] P. W. Adams and Jing-chun Xu, Rev. Sci. Instrum. **62**, 2461 (1991).
[21] G. Zimmerli, G. Mistura, and M. H. W. Chan, Phys. Rev. Lett. **68**, 60 (1992).
[22] P. J. Shirron and J. M. Mochel, Phys. Rev. Lett. **67**, 1118 (1991).
[23] H. Cho and G. A. Williams, Phys. Rev. Lett. **75**, 1562 (1995); J. Low Temp. Phys. **110**, 533 (1998).