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Spin diffusion in a two-dimensional electron gas

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Spin transport properties of spin-polarized two-dimensional electron systems are studied, with application to quantum wells or heterostructures of III-V semiconductors. We present numerical solutions of a quantum transport equation, which is a 2×2 matrix in spinor space. It is shown that the ''spin-rotation term,'' which leads to anomalous spin diffusion and spin waves in three-dimensional rarefied spin polarized ^{3}He and Hydrogen, is also present in two-dimensional degenerate electron systems at low temperatures. Calculated longitudinal and transverse spin diffusion coefficients show different temperature dependence due to the fact that the phase space available in electron-electron scatterings associated with each diffusion process is different. We also discuss the possible experimental setups to achieve spin-polarized electrons in semiconductor quantum wells. [S0163-1829(99)02031-7]

I. INTRODUCTION

There has been growing interest in the spin degrees of freedom of carrier electrons in semiconductor quantum wells (QW's) and heterostructures. Many intensive studies are in progress motivated by the prospect of developing electronic[1] and optical devices based on the carrier spins in semiconductors. Compound semiconductors are mainly investigated because the symmetry of the band structure enables one to generate highly spin-polarized carriers by circularly polarized optical excitations,[2] and the recent technological progress enables the injection of spin-polarized carriers from ferromagnetic metallic electrodes.[3–6]

Quantitative understanding of the carrier spin dynamics within the semiconductor nanostructure is essential for the development of device design. While spin relaxation mechanisms are extensively studied both in theoretically and experimentally, to the best of our knowledge, there have been few reports on electron spin transport properties in semiconductor nanostructures. In this paper, we present a theory and numerical calculations of spin diffusion coefficients in two-dimensional electron system (2DES).

It has been known that the spin diffusion[7] behavior of strongly interacting Fermi system in the degenerate regime, or highly polarized Fermi/ Bose system in Boltzmann regime, is quite different from that of free particles. As for highly degenerate Fermi liquid, Leggett and Rice[8–10] pointed out that the effective transverse spin diffusion coefficients measured by spin echo experiments should depend on the spin-tipping angle by the initial r.f. pulse (Leggett-Rice effect). This effect is caused by an additional term in the equation of motion for spin currents, representing the precession of the spin current about the molecular field made by other spins. Their results also imply the existence of spin waves first predicted by Silin[11] and Platzman and Wolff.[12] Bashkin[13] and Lhuillier and Laloé[14] independently predicted a similar effect in dilute Fermi and Bose gases in the Boltzmann regime, provided that the spin polarization is very high. In Lhuillier and Laloë's microscopic transport theory, the exchange effect in interparticle collisions, which is a direct consequence of particle indistinguishability, plays an important role at low temperatures where de Broglie wavelength is comparable to the interaction range. When the direction of spin polarizations varies spatially, this exchange effect rotates spin directions during scattering, leading to a similar spin precession effect as in the Fermi liquid model, which they called the ''identical spin-rotation effect.''

Later, the connection between the Fermi liquid model for degenerate system and the microscopic collision theory for Boltzmann gas were investigated and unified in the quantum kinetic theory using Green's-function technique.[15–20] We should also point out the Meyerovich's work,[21] which showed that the transverse spin diffusion coefficient $D_\perp$ would be different from the longitudinal one $D_\parallel$ in highly degenerate system. $D_\parallel$ diverges at low temperatures as T^{-2}, as expected in the transport of degenerate Fermi system. But $D_\perp$ was predicted to show saturation at low temperatures and converge to a finite value as $T\rightarrow 0$.

On the experimental side, there have been many investigations to observe the predicted phenomena. Spin waves in degenerate Fermi system were first observed in bulk metallic Na and K.[22] Spin waves in weakly polarized, degenerate Fermi system were also found in normal liquid ^{3}He (Ref. 23) and ^{3}He-^{4}He mixture.[24] Direct evidence of Leggett-Rice effects were observed in bulk liquid ^{3}He and ^{3}He-^{4}He mixture at very low temperatures.[25] Spin-rotation effects in highly polarized gases in the Boltzmann regime were observed in spin-polarized atomic Hydrogen,[26] in optically polarized ^{3}He gas,[27] and in dilute liquid ^{3}He-^{4}He mixture in high-magnetic fields.[28] Crossover between the two regimes have been intensively investigated in liquid helium.[29–31]

The foregoing experiments are done for bulk system of helium and hydrogen in gaseous or liquid phase, and electrons in metal. We are interested in the possibility of observing the same effects in 2DES in semiconductors. Recently, high-quality samples are available with very low rate of impurity-related scatterings so that electrons confined in heterostructures can be treated as nearly ideal 2DES. Experimentally, there are two advantages in 2DES in semiconductors over the previously investigated systems: (1) electron spin polarization can be controlled from weak to full by applying linearly or circularly polarized optical excitations without any magnetic fields. (2) sheet densities of electrons can be easily varied ranging from very dilute to high density such that degeneracy temperature can be varied easily unlike electrons in metals. One drawback of optically excited 2DES in semiconductors is that spin relaxation times are short compared to other experimentally studied systems.

The purpose of this report is to calculate the longitudinal and transverse spin diffusion coefficients in 2DES pertaining to GaAs QW's and examine whether or not ‘‘identical spin-rotation effects’’ cause important effects in realistic experimental conditions. To evaluate the size of the effect, a rigorous quantum transport equation must be solved, since no experimental values of phenomenological parameters are available.

This paper is organized as follows: In Sec. II, the quantum transport equation for spin-polarized electrons is briefly described, followed by our approximate approach to obtain the solutions for 2DES. In Sec. III, the results of numerical calculations for spin diffusion coefficients and spin-rotation parameters are shown with detailed discussions. We also discuss the plausible experimental setups to realize spin-polarized electrons in semiconductors. Conclusions are given in Sec. IV.

II. THEORY

We consider two-dimensional electrons interacting through Coulomb interactions. Other interactions such as electron-phonon, electron-ionized impurity or the exchange interaction of electrons and magnetic impurities are not included in the present study. The Hamiltonian of the system is

$$
\begin{aligned}
H=&\int d^2r\,\psi_i^\dagger(\mathbf{r},t)\left(-\frac{\hbar^2\nabla^2}{2m}\right)\psi_i(\mathbf{r},t)\\
&+\frac{1}{2}\int d^2r_1\int d^2r_2\,\psi_i^\dagger(\mathbf{r}_1,t_1)\psi_j^\dagger(\mathbf{r}_2,t_2)V(|\mathbf{r}_1-\mathbf{r}_2|)\\
&\times\psi_j(\mathbf{r}_2,t_2)\psi_i(\mathbf{r}_1,t_1)+\int d^2r\,\psi_i^\dagger(\mathbf{r},t)U_{ij}\psi_j(\mathbf{r},t),
\end{aligned}
\tag{2.1}
$$

where $\psi_i(\mathbf{r},t)$ is an electron field operator with spin z-component i, m is the *band effective mass* ($0.067m_0$ in GaAs), the Coulomb interaction $V(|\mathbf{r}_1-\mathbf{r}_2|)=e^2/(\varepsilon_0|\mathbf{r}_1-\mathbf{r}_2|)$ (dielectric constant $\varepsilon_0=12.91$ in GaAs), and the repeated indices (i,j) are assumed to be summed over. The external field U_{ij} is $-\frac{1}{2}g\mu_B\mathbf{B}(\mathbf{r},t)\cdot\boldsymbol{\tau}_{ij}$, where $\mathbf{B}(\mathbf{r},t)$ is a magnetic field and $\boldsymbol{\tau}_{ij}$ is a Pauli spin matrix. In the following we will drop the term of the external magnetic field assuming that the spins are aligned by optical excitations. Even if the field is present, provided that the field strength is sufficiently low so that the orbital motion is not affected (we discuss this point at the end of Sec. III), its effect can trivially be included in the following derivations by using a rotating frame.

We consider 2DES long after optical excitations, long enough so that the electrons are close to the local, quasithermal equilibrium in momentum space, but not too long to lose spin polarizations. The distribution function is given by the local equilibrium function plus a small correction term, $n_{k\pm}(\mathbf{R},T)=n^0_{k\pm}(\mathbf{R},T)+\delta n_{k\pm}(\mathbf{R},T)$ ($\pm$ for the spin-up and -down components). The total density of spin-up and spin-down electrons, $n(\mathbf{R},T)=n_+(\mathbf{R},T)+n_-(\mathbf{R},T)$, is assumed to be spatially and temporally constant. Their difference, i.e., the spin density $M(\mathbf{R},T)=n_+(\mathbf{R},T)-n_-(\mathbf{R},T)$, varies slowly on a hydrodynamic scale.[32] The spin polarization is given by $P=(n_+-n_-)/(n_++n_-)$. In the present study, we can calculate the spin transport coefficients only when the spin polarizations is small due to the approximations used in the derivations.

In transport problems in semiconductors, the *semiclassical* Boltzmann transport model is widely used, in which the quantum mechanics appears only in the calculation of scattering rates and Pauli blocking in collision terms. In order to solve transport problems associated with spin degrees of freedom, which are quantum mechanical in origin, we have to treat the problem quantum mechanically from the outset. Here we use quantum kinetic equation derived from the equation of motion for the nonequilibrium real-time Green's functions.[33–37] For the spin-polarized Fermi liquid (^{3}He-^{4}He mixture in 3D), the derivation of spin-transport coefficients from the quantum kinetic equation is described in great detail in Refs.18 and 20. We apply their formulation to 2DES. The present calculations differ from these previous studies in two points; electrons are confined in two dimensions instead of three, and they are interacting via a Coulomb force. We adopt the Green's-function method to calculate many-body effects of electrons such as self-energies in the drift terms and the screening of the Coulomb interactions in the collision terms.

A. Drift terms

We start from the quantum kinetic equation for nonequilibrium Green's functions $\underline{G}^<$. After using the gradient expansion and applying the Kadanoff-Baym Ansatz,[33] we arrive at the quantum transport equation for the electron distribution functions $\underline{n}_p$,

$$
\begin{aligned}
&\frac{\partial\underline{n}_p(\mathbf{r},t)}{\partial t}+\frac{1}{2}\left\{\frac{\partial\underline{\varepsilon}_p(\mathbf{r},t)}{\partial\mathbf{p}},\frac{\partial\underline{n}_p(\mathbf{r},t)}{\partial\mathbf{r}}\right\}\\
&\qquad-\frac{1}{2}\left\{\frac{\partial\underline{\varepsilon}_p(\mathbf{r},t)}{\partial\mathbf{r}},\frac{\partial\underline{n}_p(\mathbf{r},t)}{\partial\mathbf{p}}\right\}+\frac{i}{\hbar}[\underline{\varepsilon}_p(\mathbf{r},t),\underline{n}_p(\mathbf{r},t)]\\
&\quad=\left(\frac{\partial\underline{n}_p(\mathbf{r},t)}{\partial t}\right)_{col.},
\end{aligned}
\tag{2.2}
$$

where underlining indicates a 2×2 matrix in spinor space. The distribution function is decomposed into the spin scalar f_p and vector $\boldsymbol{\sigma}_p$ parts,

$$\underline{n}_p(\mathbf{r},t)=\tfrac{1}{2}[f_p(\mathbf{r},t)\underline{I}+\boldsymbol{\sigma}_p(\mathbf{r},t)\cdot\underline{\boldsymbol{\tau}}].$$

The quasiparticle energy, $\underline{\varepsilon}_p(\mathbf{r},t)$, which will be given in the Hartree-Fock approximation (HFA), is also decomposed into spin-scalar and spin-vector parts:

$$\underline{\varepsilon}_p(\mathbf{r},t)=\varepsilon_p(\mathbf{r},t)\underline{I}+\mathbf{h}_p(\mathbf{r},t)\cdot\underline{\boldsymbol{\tau}}.$$

The first three terms of Eq. (2.2) correspond to the drift terms in the classical Boltzmann equations. The last term on the left-hand side represents the effect of the molecular field, which is called the ''spin-rotation term'' and is essential to the present analysis.

Now, we apply the Chapman-Enskog expansion;[38] the true solution is divided into a local equilibrium distribution and a correction term, $\underline{n}_p^0+\delta\underline{n}_p$. The drift term is evaluated to the lowest order with the local equilibrium distribution function $\underline{n}_p^0$ while the spin-rotation and collision terms are evaluated with the variational function $\underline{n}_p^0+\delta\underline{n}_p$. We set the Cartesian coordinate $(\hat{\boldsymbol{e}}_x,\hat{\boldsymbol{e}}_y,\hat{\boldsymbol{e}}_z)$ where z axis is along the growth axis of the quantum well and x-y in the 2D plane. The 2D transport equation is expressed in the x-y coordinates. We assume that the local equilibrium spin-polarization direction is close to the direction $\hat{\boldsymbol{e}}_z$, so that all physical quantities in 2×2 spinor space can be written in diagonal form when the quantization axis is taken along $\hat{\boldsymbol{e}}_z$, neglecting small off-diagonal components. Then

$$\begin{aligned}\underline{n}_p^0(\mathbf{r},t)&=\tfrac{1}{2}[f_p^0(\mathbf{r},t)\underline{I}+\boldsymbol{\sigma}_p^0(\mathbf{r},t)\cdot\underline{\boldsymbol{\tau}}]\\&=\tfrac{1}{2}[f_p^0(\mathbf{r},t)\underline{I}+\sigma_p^0(\mathbf{r},t)\hat{\boldsymbol{e}}_\sigma(\mathbf{r},t)\cdot\underline{\boldsymbol{\tau}}]\\&\cong\begin{pmatrix}n_{p+}^0(r,t)&0\\0&n_{p-}^0(r,t)\end{pmatrix},\end{aligned}\tag{2.3}$$

where $\hat{\boldsymbol{e}}_\sigma$ is a directional vector of the spin polarization, which is close to $\hat{\boldsymbol{e}}_z$, $f_p^0(\mathbf{r},t)=n_{p+}^0(\mathbf{r},t)+n_{p-}^0(\mathbf{r},t)$ is a spin-scalar part, and $\sigma_p^0(\mathbf{r},t)=n_{p+}^0(\mathbf{r},t)-n_{p-}^0(\mathbf{r},t)$ is a spin-vector part of the distribution function. The local equilibrium distribution functions for spin-up and -down electrons are given by the Fermi distribution function,

$$n_{p\pm}^0(\mathbf{r},t)=\{\exp[\beta(\varepsilon_{p\pm}^0(\mathbf{r},t)-\mu_\pm(\mathbf{r},t))]+1\}^{-1},$$

where $\mu_\pm$ is a chemical potential, and $\varepsilon_{p\pm}^0$ is a diagonal part of quasiparticle energy [see Eq. (2.4)]. The local spin density vector is defined as

$$\mathbf{M}(\mathbf{r},t)=M(\mathbf{r},t)\hat{\boldsymbol{e}}_\sigma(\mathbf{r},t)=[n_+(\mathbf{r},t)-n_-(\mathbf{r},t)]\hat{\boldsymbol{e}}_\sigma(\mathbf{r},t).$$

The quasiparticle energy is also written in spin-diagonal form:

$$\begin{aligned}\underline{\varepsilon}_p^0(\mathbf{r},t)&=\varepsilon_p^0(\mathbf{r},t)\underline{I}+h_p^0(\mathbf{r},t)\hat{\boldsymbol{e}}_\sigma\cdot\underline{\boldsymbol{\tau}}\\&\cong\begin{pmatrix}\dfrac{p^2}{2m}+\hbar\Sigma_{HF+}(\mathbf{p};\mathbf{r},t)&0\\0&\dfrac{p^2}{2m}+\hbar\Sigma_{HF-}(\mathbf{p};\mathbf{r},t)\end{pmatrix}\end{aligned}\tag{2.4}$$

where $\hbar\Sigma_{HF\pm}$ is a self-energy in HFA. Here, we define

$$\hbar\Sigma_{HFD}^{HFS}(\mathbf{p};\mathbf{r},t)=\hbar\Sigma_{HF+}(\mathbf{p};\mathbf{r},t)\pm\hbar\Sigma_{HF-}(\mathbf{p};\mathbf{r},t).$$

Putting these expressions into Eq. (2.2) and we arrive at the drift terms of the transport equations for the distribution functions f_p^0 and $\boldsymbol{\sigma}_p^0$:

$$\begin{aligned}&\frac{\partial f_p^0}{\partial t}-\frac{\partial M}{\partial r_i}\left(\frac{v_{p+,i}}{2G_+}\frac{\partial n_{p+}^0}{\partial\varepsilon_{Kin}}-\frac{v_{p-,i}}{2G_-}\frac{\partial n_{p-}^0}{\partial\varepsilon_{Kin}}\right)\\&\quad-v_{p+,i}\frac{\partial\hbar\Sigma_{HF+}}{\partial r_i}\frac{\partial n_{p+}^0}{\partial\varepsilon_{Kin}}-v_{p-,i}\frac{\partial\hbar\Sigma_{HF-}}{\partial r_i}\frac{\partial n_{p-}^0}{\partial\varepsilon_{Kin}},\end{aligned}\tag{2.5}$$

$$\begin{aligned}&\frac{\partial\boldsymbol{\sigma}_p^0}{\partial t}-\hat{\boldsymbol{e}}_\sigma\left[\frac{\partial M}{\partial r_i}\left(\frac{v_{p+,i}}{2G_+}\frac{\partial n_{p+}^0}{\partial\varepsilon_{Kin}}+\frac{v_{p-,i}}{2G_-}\frac{\partial n_{p-}^0}{\partial\varepsilon_{Kin}}\right)\right.\\&\quad\left.+v_{p+,i}\frac{\partial\hbar\Sigma_{HF+}}{\partial r_i}\frac{\partial n_{p+}^0}{\partial\varepsilon_{Kin}}-v_{p-,i}\frac{\partial\hbar\Sigma_{HF-}}{\partial r_i}\frac{\partial n_{p-}^0}{\partial\varepsilon_{Kin}}\right]\\&\quad+\frac{\partial\hat{\boldsymbol{e}}_\sigma}{\partial r_i}\left[v_{p,i}^{av}(n_{p+}^0-n_{p-}^0)-\frac{1}{2}\hbar\Sigma_{HFD}\left(v_{p+,i}\frac{\partial n_{p+}^0}{\partial\varepsilon_{Kin}}\right.\right.\\&\quad\left.\left.+v_{p-,i}\frac{\partial n_{p-}^0}{\partial\varepsilon_{Kin}}\right)\right],\end{aligned}\tag{2.6}$$

where i denotes spatial direction x or y, $\varepsilon_{Kin}=p^2/2m$, the quasiparticle velocity is defined as $v_{p\pm,i}=\partial(\varepsilon_{Kin}+\hbar\Sigma_{HF+})/\partial p_i$, the mean velocity $v_{p,i}^{av}=(v_{p+,i}+v_{p-,i})/2$, and $G_\pm$ is given by

$$G_\pm=-\int\frac{d^2p}{h^2}\frac{\partial n_{p\pm}^0}{\partial\varepsilon_{Kin}}.\tag{2.7}$$

We should notice that the present forms of transport equations in Eqs. (2.5) and (2.6) are valid only when the local equilibrium spin-polarization direction is parallel to the growth axis (the normal of 2D plane), which is also a spin-quantization axis. When the polarization direction is tipped away from the growth axis by a finite angle, the finite off-diagonal elements appear in 2×2 spinor matrices such as in $\underline{n}_p^0(\mathbf{r},t)$ and $\underline{\varepsilon}_p^0(\mathbf{r},t)$, resulting in the extra terms in the transport equations. This is a distinctive feature of 2D system. In 3D case, due to the isotropy of all directions, it is always possible to set the direction of local equilibrium spin polarization as z axis at each point of space and write the transport equation in this coordinate system. Since we are interested in the case when the spin polarization is parallel to growth axis assuming the optical spin orientation in semiconductor QW structures, we will calculate transport coefficients for this case below.

The particle current in the spatial direction i is given by[10]

$$\begin{aligned}J_i(\mathbf{r},t)&=\int\frac{d^2p}{h^2}\left(\frac{\partial\varepsilon_p(\mathbf{r},t)}{\partial p_i}f_p(r,t)+\frac{\partial\mathbf{h}_p(\mathbf{r},t)}{\partial p_i}\cdot\boldsymbol{\sigma}_p(\mathbf{r},t)\right)\\&=\int\frac{d^2p}{h^2}v_{p,i}^0f_p(\mathbf{r},t),\end{aligned}\tag{2.8}$$

where $v_{p,i}^0=p_i/m=\partial\varepsilon_{Kin}/\partial p_i$ is a velocity of the noninteracting particle. The spin current of the spin component i flowing in the spatial direction j is given by

$$J_{\sigma_i,j}(\mathbf{r},t)=\int\frac{d^2p}{h^2}\left(\frac{\partial\varepsilon_p(\mathbf{r},t)}{\partial p_j}\sigma_{p,i}(\mathbf{r},t)+\frac{\partial h_{p,i}(\mathbf{r},t)}{\partial p_j}f_p(\mathbf{r},t)\right)$$

$$=\int\frac{d^2p}{h^2}v^0_{p,j}\boldsymbol{\sigma}_{p,i}(\mathbf{r},t). \tag{2.9}$$

The goal of the derivation is to obtain the equation for the spin current $J_{\boldsymbol{\sigma},j}(\mathbf{r},t)$. We multiply Eq. (2.6) by $v^0_{p,j}$ and integrate over $\mathbf{p}$. Then, we find

$$\frac{\partial J_{\sigma,i}(\mathbf{r},t)}{\partial t}+\hat{\boldsymbol{e}}_\sigma\frac{\partial M(\mathbf{r},t)}{\partial r_i}A(k_{F\pm},\mathbf{r},t)$$

$$+M(\mathbf{r},t)\frac{\partial\hat{\boldsymbol{e}}_\sigma}{\partial r_i}B(k_{F\pm},\mathbf{r},t), \tag{2.10}$$

where

$$A(k_{F\pm},\mathbf{r},t)=\frac{1}{m}\left(\frac{n_+(\mathbf{r},t)}{2\overline{G}_+}+\frac{n_-(\mathbf{r},t)}{2\overline{G}_-}\right), \tag{2.11}$$

$$B(k_{F\pm},\mathbf{r},t)=\frac{\pi\Xi}{h^2M(\mathbf{r},t)}$$

$$+\frac{1}{2mM(\mathbf{r},t)}[n_+(\mathbf{r},t)\hbar\Sigma_{HFD}(k_{F+},\mathbf{r},t)$$

$$+n_-(\mathbf{r},t)\hbar\Sigma_{HFD}(k_{F-},\mathbf{r},t)], \tag{2.12}$$

where $\overline{G}_\pm^{-1}=G_\pm^{-1}-\alpha_\pm$ with $\alpha_\pm=2e^2/(\varepsilon_0\sqrt{\pi n_\pm})$, and Ξ is defined as

$$\Xi=\int_0^\infty dp\,p v^0_p v^{av}_p(n^0_{p+}-n^0_{p-}).$$

In further evaluating the quantities in Eqs. (2.11) and (2.12), we run into a difficulty due to the inadequacy of using the HFA: The derivative $\partial\Sigma_{HF}(k)/\partial k$ diverges at the Fermi surface (in 2D as well as in 3D), and $G_\pm$, Ξ and $\hbar\Sigma_{HFD}/M$ are not defined at $k=k_{F\pm}$. This also leads to the vanishing quasiparticle effective mass, $m^*=\hbar^2k_F/[\partial\varepsilon(k)/\partial k]|_{k=k_F}$.[34,39] This is an artifact due to the infinite range of the bare Coulomb interaction used in HFA, and the finite effective mass is recovered when the higher order terms are included. [The effective mass calculated in random phase approximation (RPA) (Ref. 40) is 1.0–1.15$\times m$, depending on the density.] To circumvent this problem, although the selfconsistency of the theory is lost, we will assume through the following derivations that the energy dispersion of quasiparticles near the Fermi surface is given by the quadratic form as

$$\varepsilon_{k\pm}=\frac{\hbar^2k^2}{2m^*_\pm}+E_{0\pm}, \tag{2.13}$$

using a finite effective mass m^*. We can then calculate as,

$$G_\pm=\frac{m^*_\pm}{2\pi\hbar^2},\qquad \Xi=\frac{2\pi^2\hbar^4nM}{m}\frac{m^*_++m^*_-}{m^*_+m^*_-},$$

$$\frac{\hbar\Sigma_{HFD}(k_{F_\pm})}{M}=-\frac{4\pi e^2}{\varepsilon_0(k_{F_+}+k_{F_-})}, \tag{2.14}$$

where we used the fact that the electron distribution is degenerate and the spin polarization is small, $k_{F_+}\approx k_{F_-}$. We keep the limitation of the present scheme in mind when we apply it.

B. Variational functions

The variational functions are carefully chosen in the similar form to the drift terms. From Eq. (2.5), the spin-scalar part of the variational functions is

$$\delta f_p=-C_\parallel\frac{\partial M}{\partial r_i}\left(\frac{v_{p+,i}}{2\overline{G}_+}\frac{\partial n^0_{p+}}{\partial\varepsilon_{Kin}}-\frac{v_{p-,i}}{2\overline{G}_-}\frac{\partial n^0_{p-}}{\partial\varepsilon_{Kin}}\right), \tag{2.15}$$

where $C_\parallel$ is a variational constant. This form guarantees a vanishing particle current in Eq. (2.8). The spin-vector variational function consists of a longitudinal (parallel to the local spin polarization) and a transverse part,

$$\delta\boldsymbol{\sigma}_p=\delta\boldsymbol{\sigma}_{p\parallel}+\delta\boldsymbol{\sigma}_{p\perp},$$

where the longitudinal part is given by

$$\delta\boldsymbol{\sigma}_{p\parallel}=-\hat{\boldsymbol{e}}_\sigma C_\parallel\frac{\partial M}{\partial r_i}\left(\frac{v_{p+,i}}{2\overline{G}_+}\frac{\partial n^0_{p+}}{\partial\varepsilon_{Kin}}+\frac{v_{p-,i}}{2\overline{G}_-}\frac{\partial n^0_{p-}}{\partial\varepsilon_{Kin}}\right), \tag{2.16}$$

and the transverse part is

$$\delta\boldsymbol{\sigma}_{p\perp}=\frac{1}{2}v^{av}_{p,i}\sigma^0_p(C_{x,i}\hat{\boldsymbol{e}}_x-iC_{y,i}\hat{\boldsymbol{e}}_y). \tag{2.17}$$

In the transverse part, $C_{x,i}$ and $C_{y,i}$ are variational constants and the directional vectors are defined as

$$\frac{\partial\hat{\boldsymbol{e}}_\sigma}{\partial r_i}=\phi_{x,i}\hat{\boldsymbol{e}}_x+\phi_{y,i}\hat{\boldsymbol{e}}_y,\qquad \hat{\boldsymbol{e}}_\sigma\times\frac{\partial\hat{\boldsymbol{e}}_\sigma}{\partial r_i}=\phi_{x,i}\hat{\boldsymbol{e}}_y-\phi_{y,i}\hat{\boldsymbol{e}}_x, \tag{2.18}$$

where $\phi_{x,i}$ and $\phi_{y,i}$ are constants. The mean field contributions associated with $\hbar\Sigma_{HFD}$ is small and will be neglected.

C. Spin-rotation term

From the last term in l.h.s. of Eq. (2.2), the spin-vector part of the spin-rotation term is

$$-\frac{2}{\hbar}[\mathbf{h}_p(\mathbf{r},t)\times\boldsymbol{\sigma}_p(\mathbf{r},t)].$$

The spin-scalar part vanishes. This term is evaluated with $\delta\boldsymbol{\sigma}_{p\perp}$, multiplied by $v^0_{k,j}$, and integrated over $\mathbf{k}$ to find the spin-rotation term for the transport equation for the spin current $J_{\boldsymbol{\sigma},j}(r,t)$. It can easily be shown that this term is proportional to $\mathbf{M}\times J_{\boldsymbol{\sigma},i}$ in spinor space. Then, we can write

$$\mu_{SR}\mathbf{M}\times J_{\boldsymbol{\sigma},i}, \tag{2.19}$$

where

$$\mu_{SR}=\frac{e^2}{2\hbar\varepsilon_0 nM^2}\int\frac{d^2k}{(2\pi)^2}\frac{d^2k'}{(2\pi)^2}|\mathbf{k}-\mathbf{k}'|(n^0_{k+}-n^0_{k-})\times(n^0_{k'+}-n^0_{k'-}). \tag{2.20}$$

The spin-rotation parameter is obtained by numerically integrating this expression.

D. Collision terms

We find the collision term of the following form:

$$\left(\frac{\partial\underline{n}_k}{\partial t}\right)_{Coll.}=\frac{1}{\hbar}\int\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_1'}{(2\pi)^2}\frac{d^2k_2'}{(2\pi)^2}(2\pi)^3\times\delta^2(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2')\times\delta(\varepsilon_{k1}+\varepsilon_{k2}-\varepsilon_{k1'}-\varepsilon_{k2'})\times\frac{1}{2}|V_{RPA}(|\mathbf{k}_1-\mathbf{k}_1'|,\varepsilon_{k1}/\hbar-\varepsilon_{k1'}/\hbar)|^2\times(\mathrm{Tr}[\underline{n}_{k2'}\tilde{\underline{n}}_{k2}]\{\underline{n}_{k1'},\tilde{\underline{n}}_{k1}\}-\mathrm{Tr}[\tilde{\underline{n}}_{k2'}\underline{n}_{k2}]\{\tilde{\underline{n}}_{k1'},\underline{n}_{k1}\}), \tag{2.21}$$

where $\tilde{\underline{n}}_k=\underline{I}-\underline{n}_k$. We put the distribution function $\underline{n}_k(\mathbf{r},t)=\underline{n}^0_k(\mathbf{r},t)+\delta\underline{n}_k(\mathbf{r},t)$ into Eq. (2.21) and evaluate it to the lowest order of the variation $\delta\underline{n}_k$. V_{RPA} is a dynamically screened Coulomb interaction given in RPA.

Here we encounter the problem of the Kadanoff-Baym Ansatz specific to the spin-polarized system. We use the Kadanoff-Baym Ansatz, $G^<=in_kA$, to obtain the quantum transport equation [Eq. (2.2)] for $\underline{n}_k$ from the quantum-kinetic equation for $\underline{G}^<$. (Here A is a spectral function.) Since both $\underline{n}_k$ and $\underline{A}$ are 2×2 matrices, $i\underline{n}_k\underline{A}(\mathbf{k},\omega)$ is generally different from $i\underline{A}(\mathbf{k},\omega)\underline{n}_k$, and we do not have any principle on which order to take. The drift term is not subject to this problem since the quasiequilibrium functions that are diagonal in spinor space is used in evaluations. However, in the collision term, the different order of matrix multiplication leads to different results as shown explicitly below. The problem arises from the fact that the relation that is valid only in equilibrium system without spin polarization is applied to the polarized system.

The off-diagonal components of the collision term, which are associated with the transverse spin diffusion, give different results when the matrix multiplication order is changed. For example, we have terms like

$$\delta(\underline{\underline{\varepsilon_{k_1-}}}+\varepsilon_{k_2-}-\varepsilon_{k_1'+}-\varepsilon_{k_2'-})|V_{RPA}(|\mathbf{k}_1-\mathbf{k}_1'|,\underline{\underline{\varepsilon_{k_1-}}}/\hbar-\varepsilon_{k_1+}/\hbar)|^2\tilde{n}^0_{k_1'+}\tilde{n}^0_{k_2'-}\delta\sigma_{k_1-}n^0_{k_2-} \tag{2.22}$$

$$\delta(\underline{\underline{\varepsilon_{k_1+}}}+\varepsilon_{k_2-}-\varepsilon_{k_1'+}-\varepsilon_{k_2'-})|V_{RPA}(|\mathbf{k}_1-\mathbf{k}_1'|,\underline{\underline{\varepsilon_{k_1+}}}/\hbar-\varepsilon_{k_1+}/\hbar)|^2\tilde{n}^0_{k_1'+}\tilde{n}^0_{k_2'-}\delta\sigma_{k_1-}n^0_{k_2-}. \tag{2.23}$$

We marked the differences with double underlines. The energy δ function in the collision integral ensures the energy conservation before and after the collision, and specifies the point in (k,ω) space at which the interaction $V_{RPA}(k,\omega)$ is sampled. When the spin-polarization is small, we can approximate $\varepsilon_{k+}\approx\varepsilon_{k-}$. If the interaction is a smooth function of energy transfer, and which is true in the RPA-screened Coulomb interaction except at the plasmon peak, the quasiparticle energy $\varepsilon_{k\pm}$ can be replaced with their average ε^{av}_k without introducing significant error. We adopt this approximation in calculating the collision terms below.

We next multiply $(\partial\underline{n}_k/\partial t)_{Coll.}$ with $v^0_{k,j}$ and integrate over $\mathbf{k}$ to find the collision term of the transport equation for the spin current $J_{\boldsymbol{\sigma},j}(r,t)$. After some calculations we find

$$-\frac{1}{\tau_\parallel}J_{\boldsymbol{\sigma}_\parallel,j}-\frac{1}{\tau_\perp}J_{\boldsymbol{\sigma}_\perp,j}, \tag{2.24}$$

where the longitudinal part $J_{\boldsymbol{\sigma}_\parallel,j}$ is the spin current in the spin direction $\delta\sigma_{p\parallel}$ (parallel to $\hat{\boldsymbol{e}}_z$) flowing in the spatial direction j, while the transverse part $J_{\boldsymbol{\sigma}_\perp,j}$ is the spin current in the spin direction $\delta\boldsymbol{\sigma}_{p\perp}$ (parallel to $C_{x,j}\hat{\boldsymbol{e}}_x-iC_{y,j}\hat{\boldsymbol{e}}_y$) flowing in the spatial direction j. The relaxation rate associated with the longitudinal spin diffusion is given by

$$\frac{1}{\tau_\parallel}=\frac{1}{2}\frac{\hbar\beta}{m^*_+m^*_-}\frac{m^*_+\bar{G}_++m^*_-\bar{G}_-}{n_+\bar{G}_-+n_-\bar{G}_+}\times\int\frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_1'}{(2\pi)^2}\frac{d^2k_2'}{(2\pi)^2}(2\pi)^3\times\delta^2(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2')\delta(\varepsilon^{av}_{k_1}+\varepsilon^{av}_{k_2}-\varepsilon^{av}_{k_1'}-\varepsilon^{av}_{k_2'})\times(\mathbf{k}_1-\mathbf{k}_1')^2\left|V_{RPA}\left(|\mathbf{k}_1-\mathbf{k}_1'|,\frac{\varepsilon^{av}_{k_1}}{\hbar}-\frac{\varepsilon^{av}_{k_1'}}{\hbar}\right)\right|^2\times\tilde{n}^0_{k_1'+}\tilde{n}^0_{k_2'-}n^0_{k_1+}n^0_{k_2-}, \tag{2.25}$$

where β is an inverse temperature $(kT)^{-1}$ and $\varepsilon^{av}_k=\frac{1}{2}(\varepsilon_{k+}+\varepsilon_{k-})$. This expression has the interpretion that the two incident electrons, one spin-up with $\mathbf{k}_1$ and one spin-down with $\mathbf{k}_2$ scatter through the screened Coulomb potential into the final states, one spin-up with $\mathbf{k}_1'$ and one spin-down with $\mathbf{k}_2'$. The relaxation rate associated with the transverse spin diffusion, $(\tau_\perp)^{-1}$, is given in more lengthy form as

$$\frac{1}{\tau_\perp}=\frac{\hbar^2}{m}\left(\frac{1}{m^*_+}+\frac{1}{m^*_-}\right)\frac{2\pi\hbar}{\Xi}\sinh\left(\frac{\beta\Delta}{2}\right)\times\int\frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_1'}{(2\pi)^2}\frac{d^2k_2'}{(2\pi)^2}(2\pi)^3\times\delta^2(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2')\delta(\varepsilon^{av}_{k_1}+\varepsilon^{av}_{k_2}-\varepsilon^{av}_{k_1'}-\varepsilon^{av}_{k_2'})\times(\mathbf{k}_1-\mathbf{k}_1')^2\left|V_{RPA}\left(|\mathbf{k}_1-\mathbf{k}_1'|,\frac{\varepsilon^{av}_{k_1}}{\hbar}-\frac{\varepsilon^{av}_{k_1'}}{\hbar}\right)\right|^2\times\tilde{n}^0_{k_1'+}\tilde{n}^0_{k_2'-}(e^{\beta\Delta/2}n^0_{k_1-}n^0_{k_2-}+e^{-\beta\Delta/2}n^0_{k_1+}n^0_{k_2+}), \tag{2.26}$$

where $\Delta=\mu_+-\mu_-$.

E. Diffusion coefficients

The Eqs. (2.10), (2.19), and (2.24) comprise the equation for the spin current $J_{\boldsymbol{\sigma},j}(r,t)$,

$$\frac{\partial J_{\sigma,i}(\mathbf{r},t)}{\partial t}+\hat{\boldsymbol{e}}_{\sigma}\frac{\partial M(\mathbf{r},t)}{\partial r_i}A(k_{F\pm},\mathbf{r},t)$$

$$+M(\mathbf{r},t)\frac{\partial \hat{\boldsymbol{e}}_{\sigma}}{\partial r_i}B(k_{F\pm},\mathbf{r},t)+\mu_{SR}\mathbf{M}\times J_{\boldsymbol{\sigma},i}$$

$$=-\frac{1}{\tau_{\parallel}}J_{\boldsymbol{\sigma}_{\parallel},j}-\frac{1}{\tau_{\perp}}J_{\boldsymbol{\sigma}_{\perp},j}. \tag{2.27}$$

This equation has the similar form to the one developed in Fermi liquid theory by Leggett[9] except that the coefficient in the drift terms associated with the longitudinal component $A(k_{F\pm},\mathbf{r},t)$ is different from the coefficient $B(k_{F\pm},\mathbf{r},t)$ associated with the transverse component. The steady-state solution of this equation is readily found to be

$$J_{\boldsymbol{\sigma},j}=-D_{\parallel}\frac{\partial M}{\partial r_j}\hat{\boldsymbol{e}}_{\sigma}-\frac{D_{\perp}}{1+\gamma^2M^2}\left(M\frac{\partial \hat{\boldsymbol{e}}_{\sigma}}{\partial r_j}+\gamma M^2\hat{\boldsymbol{e}}_{\sigma}\times\frac{\partial \hat{\boldsymbol{e}}_{\sigma}}{\partial r_j}\right), \tag{2.28}$$

where $D_{\parallel}=A\tau_{\parallel}$ is the longitudinal spin diffusion coefficient, expressing the diffusion of spin component parallel to the local spin polarization: the magnitude of polarization changes but the direction remains unaffected. Generally diffusion coefficients are expressed as (mean velocity)^2 $\times$(scattering time), and in the present case A has the dimension of (velocity)^2. $D_{\perp}=B\tau_{\perp}$ is called the transverse spin diffusion coefficient, which expresses, along with the denominator, $1+\gamma^2M^2$, the diffusion of spin component perpendicular to the local polarization and the spin precession about the effective field. $\gamma=-\mu_{SR}\tau_{\perp}$ is called a spin-rotation parameter, whose magnitude determines the transverse spin diffusion and the spin wave propagations (see below).

When the spin current is given as Eq. (2.27), it can be shown that the system supports damped spin waves.[14] The longitudinal component shows simple damping without oscillation; the damping rate is given by $D_{\parallel}k^2$, where k is a wave number of spatial modulations of the spin polarization. The transverse component exhibits the damped traveling wave with its angular frequency and the damping rate given by

$$\frac{D_{\perp}k^2\gamma M}{1+\gamma^2M^2}\quad\text{and}\quad\frac{D_{\perp}k^2}{1+\gamma^2M^2}, \tag{2.29}$$

respectively.

III. RESULTS AND DISCUSSION

A. Calculation conditions

We first summarize the conditions and the limitations of the present calculations. We consider the two-dimensional electrons with sheet density $n=n_{+}+n_{-}=2\times10^{11}$ cm^{-2} (corresponding Fermi temperature; 83 K), with the spin polarizations $P=(n_{+}-n_{-})/(n_{+}+n_{-})$ less than 0.1. We calculate in the degenerate region only; the temperatures are less than 20 K. The quasiparticle effective mass $m^*_{\pm}$ is necessary to calculate $G_{\pm}$, Ξ, and $\tau_{\parallel,\perp}$, which is not calculated in HF scheme we adopted in this study. (The effective mass could be given selfconsistently if the self-energies in the drift terms were calculated in RPA or higher order approximations: $1.0\times m<m^*<1.15\times m$ in RPA.[40]) We tentatively put $m^*_{\pm}=m$ in the present calculations. We should keep in mind that the present results are subject to this uncertainty.

The spin polarization is restricted to a small region because we approximated $\varepsilon_{k\pm}$ by ε^{av}_k in the transverse collision term due to the problem of Kadanoff-Baym Ansatz in the spin-polarized system. The self-energies in the HFA also limit the range of spin polarizations: Since the HFA does not include the correlation terms, the selfenergy for spin-up electrons does not contain the contribution from the spin-down electrons. This leads to a quite unrealistic situation when the spin polarization is high, i.e., provided we prepared 2DES with all spins pointing downward, and put a single spin-up electron into this 2DES, then in HFA this single electron would move as if it were a free particle without knowing the presence of spin-down electrons. This problem can be removed when the electron self-energies are evaluated including higher order terms, which is deferred to our future publications.

B. Spin diffusion coefficients and spin-rotation parameters

We will show the results of numerical calculations in this subsection. The coefficient of the spin-rotation term, μ_{SR}, in Eq. (2.20) can be directly computed assuming the $T=0$ form of the distribution functions. Since we consider the degenerate system, the finite temperature correction is confirmed to be less than 1%. The eightfold integral in $\tau_{\parallel,\perp}$, Eqs. (2.25) and (2.26), can be reduced using the method given in Refs.41 and 42. We find

$$\frac{1}{\tau_{\parallel}}=\frac{1}{2}\frac{\hbar\beta}{m^*_{+}m^*_{-}}\frac{m^*_{+}\overline{G}_{+}+m^*_{-}\overline{G}_{-}}{n_{+}\overline{G}_{-}+n_{-}\overline{G}_{+}}\frac{4\pi}{(2\pi)^5}$$

$$\times\int d^2k_1n^0_{k_1+}d^2q\left|V_{RPA}\left(q,\frac{\varepsilon^{av}_{k_1}}{\hbar}-\frac{\varepsilon^{av}_{\mathbf{k}_1+\mathbf{q}}}{\hbar}\right)\right|^2$$

$$\times q^2\tilde{n}^0_{\mathbf{k}_1+\mathbf{q},+}n_B(\varepsilon_{\mathbf{k}_1+\mathbf{q},+}-\varepsilon_{k_1+})$$

$$\times\operatorname{Im}P_{-}\left(q,\frac{\varepsilon_{k_1+}}{\hbar}-\frac{\varepsilon_{\mathbf{k}_1+\mathbf{q},+}}{\hbar}\right), \tag{3.1}$$

where $\mathbf{q}=\mathbf{k}_1-\mathbf{k}'_1$ and $n_B(a)=1/(e^{\beta a}-1)$. $P_{\pm}(q,\omega)$ has the same functional form as the RPA polarization function, except that the electron mass is replaced with the quasiparticle effective mass. And for $\tau_{\perp}$, we find

$$\frac{1}{\tau_{\perp}}=\frac{\hbar^2}{m}\left(\frac{1}{m^*_{+}}+\frac{1}{m^*_{-}}\right)\frac{2\pi\hbar}{\Xi}\sinh\left(\frac{\beta\Delta}{2}\right)\left(\frac{1}{T^a_{\perp}}+\frac{1}{T^b_{\perp}}\right), \tag{3.2}$$

where

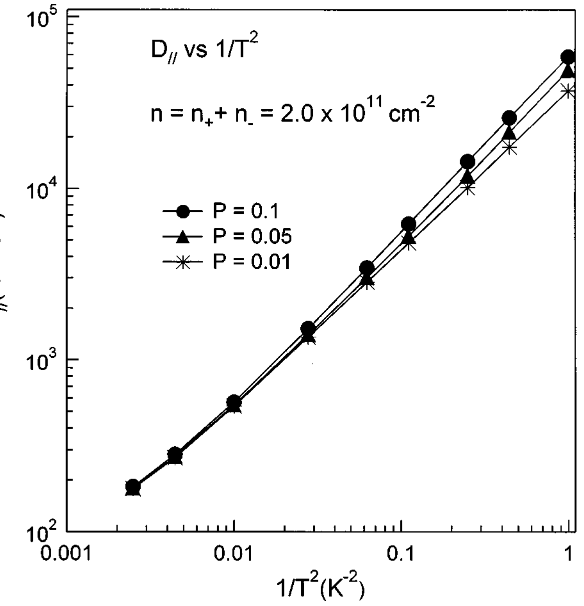

FIG. 1. The temperature dependence of the longitudinal spin diffusion coefficients with $P=0.01$, 0.05 and 0.1 and at the electron sheet density 2.0×10^{11} cm^{-2} (Fermi temperature 83 K).

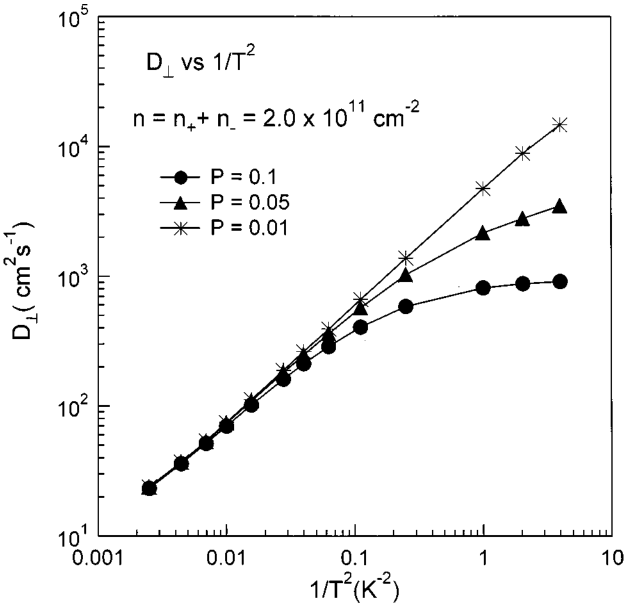

FIG. 2. The transverse spin diffusion coefficients vs $1/T^2$. The spin polarizations and the density are the same as Fig. 1

$$\frac{1}{T_\perp^a}=\frac{4\pi}{(2\pi)^5}e^{+\frac{\beta\Delta}{2}}\int d^2k_1 n^0_{k_1-}d^2q \times\left|V_{RPA}\left(q,\frac{\varepsilon^{av}_{k_1}}{\hbar}-\frac{\varepsilon^{av}_{\mathbf{k}_1+\mathbf{q}}}{\hbar}\right)\right|^2 q^2\tilde{n}^0_{\mathbf{k}_1+\mathbf{q},+} \times n_B(\varepsilon_{\mathbf{k}_1+\mathbf{q},+}-\varepsilon_{k_1+})\,\mathrm{Im}\,P_-\left(q,\frac{\varepsilon_{k_1+}}{\hbar}-\frac{\varepsilon_{\mathbf{k}_1+\mathbf{q},+}}{\hbar}\right) \tag{3.3}$$

$$\frac{1}{T_\perp^b}=\frac{4\pi}{(2\pi)^5}e^{-\frac{\beta\Delta}{2}}\int d^2k_1 n^0_{k_1+}d^2q \times\left|V_{RPA}\left(q,\frac{\varepsilon^{av}_{k_1}}{\hbar}-\frac{\varepsilon^{av}_{\mathbf{k}_1+\mathbf{q}}}{\hbar}\right)\right|^2 q^2\tilde{n}^0_{\mathbf{k}_1+\mathbf{q},-} \times n_B(\varepsilon_{\mathbf{k}_1+\mathbf{q},-}-\varepsilon_{k_1-})\,\mathrm{Im}\,P_+\left(q,\frac{\varepsilon_{k_1+}}{\hbar}-\frac{\varepsilon_{\mathbf{k}_1-\mathbf{q},-}}{\hbar}\right). \tag{3.4}$$

These expressions are valid at finite temperatures. We obtain $\tau_{\|,\perp}$ by numerically integrating Eqs. (3.1)–(3.4).

Figure 1 shows the plots of longitudinal spin diffusion coefficients $D_\|$ vs T^{-2} at three spin polarizations, 0.01, 0.05, and 0.1. $D_\|$ is nearly proportional to T^{-2} but it shows a slight deviation (bulging downward) from T^{-2} dependence. The structure of $\tau_\|$ in Eq. (2.25) shows that the electrons close to the Fermi surface within the width kT only are allowed to participate in the scatterings due to the kinematics and the Pauli exclusion. This structure of the collision integral generally appears in the scattering rates of degenerate Fermion system, leading to T^2 dependence in 3D. However in a 2D system, it has been shown that the same structure of the collision integral leads to the logarithmic correction factor of the scattering rate as $-(k_BT/E_F)^2\ln(k_BT/E_F)$.[43,44] Thus, we expect that $D_\|\propto(E_F/k_BT)^2/\ln(E_F/k_BT)$, and $D_\|$ will diverge as $T\rightarrow0$. (The coefficient A shows only a small temperature dependence.) The small deviation of $D_\|$ from T^{-2} dependence in Fig. 1 corresponds to the logarithmic correction characteristic of 2D system.

The increase of $D_\|$ with the spin polarization can also be understood in terms of the phase space available to the scattering. When the angle between two incident electrons with $\mathbf{k}_1$ and $\mathbf{k}_2$ is changed from 0 to π, the scattering angle, θ, between $\mathbf{k}_1$ and $\mathbf{k}_1'$ can also vary from 0 to π when $P=0$ ($k_{F+}=k_{F-}$). However, with increasing P, θ is restricted to angles smaller than π. This causes a reduction of phase space in the scattering, leading to a smaller scattering rate (or larger spin diffusion). The calculation for the helium system in Ref. 17 shows the similar increase of $D_\|$ with P in small P region but it has a peak at some value of P and then drops at large P region.

Figure 2 shows the transverse spin diffusion coefficients $D_\perp$ plotted against T^{-2}. It shows saturation as $T\rightarrow0$. This trend, which is not expected for the ordinary transport coefficients in the degenerate Fermions, was pointed out by Meyerrovich[21] and discussed in detail by Jeon and Mullin.[18,20] The saturation has its origin in the form of the transverse collision term. Careful examination of Eq. (2.26) shows that the electrons between the two Fermi surfaces of spin-up and spin-down electrons can participate in the scattering. (When $P>0$, electrons with $k_{F+}>k>k_{F-}$.) Thus, even when $T\rightarrow0$, the finite number of electrons are involved in the scattering, and the increase of $D_\perp$ saturates. The larger the spin polarization, the more electrons in $k_{F+}>k>k_{F-}$, and the smaller the saturated $D_\perp$. The number of electrons between the two Fermi surfaces is proportional to $\pi(k_{F+}^2-k_{F-}^2)$, thus $D_\perp\propto(k_{F+}^2-k_{F-}^2)^{-2}\propto(n_+-n_-)^{-2}=P^{-2}$. (The same relation is expected in 3D.) This anomalous temperature dependence is also observed experimentally in the He system.[30,31] At higher temperatures at which kT is larger than the Fermi energy difference, $D_\perp$ increases in proportion

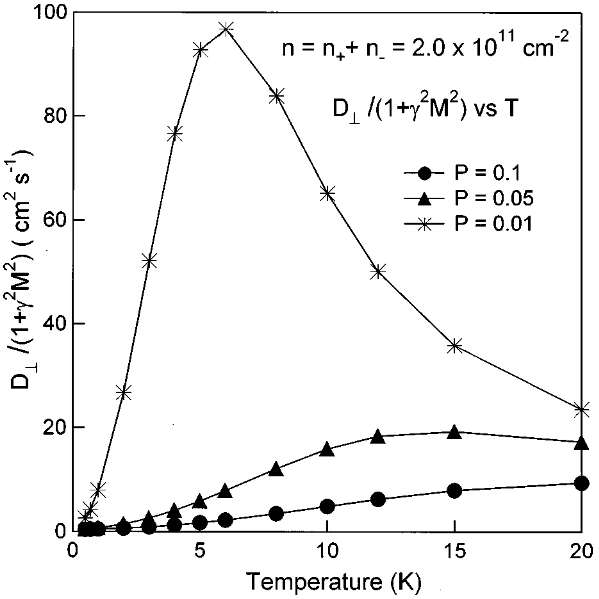

FIG. 3. The temperature dependence of the effective transverse spin diffusion coefficients. The spin polarizations and the density are the same as Fig. 1.

to T^{-2} as in the longitudinal case. (The logarithmic correction is not evident in the temperature range we calculated.)

Figure 3 shows the plots of $D_\perp/(1+\gamma^2M^2)$ vs temperatures. As shown in Eq. (2.28) it is interpreted as an effective spin diffusion coefficient in the transverse mode. The magnitudes of this value is crucial to the experimental detection. The temperature dependence is determined by $\tau_\perp$ in the denominator as $\gamma=-\mu_{SR}\tau_\perp$ and in the numerator as $D_\perp=B\tau_\perp$.

We show the temperature dependence of the spin-rotation parameters γ multiplied by M in Fig. 4. The same parameter is calculated in Ref. 20 for ^{3}He-^{4}He mixtures. Comparing their values with ours scaled by the normalized temperature T/T_F, the γM's show similar a trend as the temperature is decreased. The magnitude of this parameter is crucial to the observation of spin waves in 2DES. To detect the spin waves the damping time must be much longer than the oscillation period. As shown in Eq. (2.29), γM corresponds to the ratio, (damping time)/(period). The present calculation shows it is much larger than unity at low temperatures. This implies the possibility of the experimental detection of spin waves in 2DES.

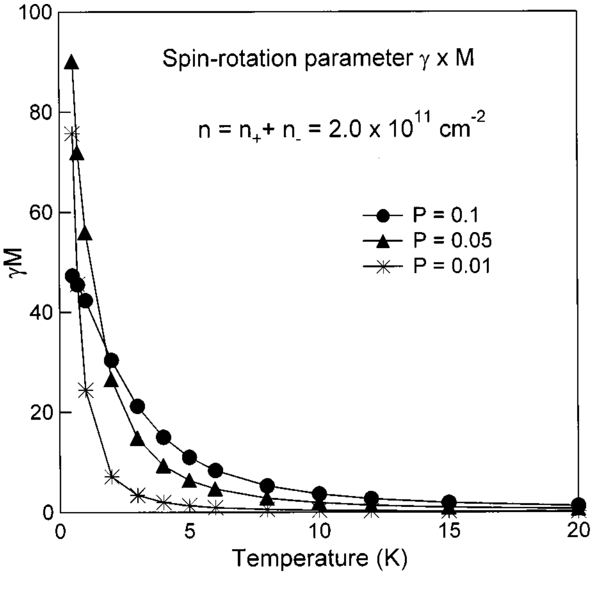

FIG. 4. The temperature dependence of the spin-rotation parameters multiplied by the spin densities, γM. The spin polarizations and the density are the same as Fig. 1.

C. Experimental prospects of spin-polarized 2DES

We would like to discuss whether and how the spin-polarized 2DES could be realized to observe the physical quantities calculated in the preceding sections. We have calculated spin transport coefficients of 2DES using the physical parameters of GaAs heterostructures or QW's. In these structures, owing to the development of crystal growth techniques, high-quality samples are available with very low scattering rates other than electron-electron scattering. In III-V (or in II-VI) semiconductors, due to the symmetry of electronic states in the conduction and valence bands, the circularly polarized photons can generate spin-polarized electrons and holes. Especially in undoped QW's, the spin polarization of electrons reaches almost unity just after photogeneration since the degeneracy in valence bands is lifted. In the modulation doped QW's the partially spin polarized system composed of the unpolarized carriers due to the doping and the polarized photocarriers can be prepared. In this way we can set up a spin-polarized electrons without applying the external field.

The experimental observation depends on how long the electron spin polarization can be kept. The following three mechanisms are responsible for the spin relaxations in III–V compound semiconductors:[2] Elliott-Yafet mechanism; the electronic states in the conduction bands depart from the spin eigenstates since the spin-orbit coupling mixes the conduction bands with valence bands. This mechanism is considered to be less effective in GaAs due to its large band gap, and also due to the low scattering rate in high-quality samples. Dyakonov-Perel (DP) mechanism; due to the lack of inversion symmetry of the crystal, the degeneracy of the spin-up and -down states is lifted in the conduction bands, leading to the energy splitting in the two spin states. This is equivalent to the presence of an effective internal magnetic field with magnitude and orientation dependent on **k**. The electron spins precess around this field and lose spin polarization. Bir-Aronov-Pikus (BAP) mechanism; the valence bands are not spin eigenstates (except at the zone center) due to the strong mixing. Thus, the hole spins relax in very short periods comparable to momentum relaxation times. The electrons lose their spin polarization in the exchange scattering with holes with random spin orientations.

Experimentally observed spin relaxation times scatter widely between a few picoseconds to hundreds.[45] Different experimental conditions and sample preparations make the systematic interpretation of results difficult. But there are qualitative features in relaxation mechanisms. The spin relaxation by BAP becomes slower at low temperatures since the phase space available to e-hole scattering decreases with temperature. Furthermore, the spin relaxation times can be extended by spatially separating electrons from holes in the coupled double QW structures with an electric field perpendicular to 2D plane, or type-II QW's. The separation also contributes to the extended radiative life times of photogenerated carriers. Maialle and Degani[46] have recently predicted a very long (~ 100 ns) electron spin relaxation times in double QW's if only BAP were considered. The spin relaxation rate of DP mechanism can also be reduced. Lommer *et al.*[47] predicted that the spin splitting of conduction bands at Fermi surface can be eliminated by a density-dependent external field in GaAs/Al$_x$Ga$_{1-x}$As heterostructures. The magnitude of field is approximately 0.2 T when the electron sheet density is 2×10^{11} cm^{-2}. In the real system, the structural asymmetry (Rashba term) should also be included, and the confirmation of this cancellation mechanism awaits further experimental investigations. Recently extremely long spin life times (~ 10 ns or longer) have been experimentally observed both in QW of ZnSe/Zn$_x$Cd$_{1-x}$Se and in bulk GaAs when the n-type dopant density is carefully selected.[48] Though it is not certain whether the similar long spin life time can be achieved in the experimental configurations we consider here, they are quite encouraging results.

With the progress in the crystal growth technique, high-quality quantum well structures of III-V and II-VI semiconductors with diluted magnetic impurities are being available. In these systems, in addition to the molecular field of conduction electrons that we considered here, the exchange coupling between conduction electrons and localized spins plays an important role. In the presence of exchange coupling, the spin polarization of conduction electrons M does not precess about the external field H but about the effective field $H+\alpha M_d$, where M_d is a spin polarization of localized spins and α is some coefficient. Langreth and Wilkins[37] have given a detailed analysis on this case in the Kadanoff-Baym's scheme similar to our method in a 3D metallic system. They have shown that the spin dynamics is strongly modified due to the exchange coupling of electrons with localized spins.

We finally consider the magnitude of the external magnetic field which can be applied to the system. We derived the equations without the external magnetic field and, as mentioned before, the effect of weak field can be included by using rotating frame. However, with increasing magnetic field, the electron orbital motion is quantized in Landau levels, and the system turns into a different state representing a quantum Hall effect, to which our present theory does not apply. The typical energy scale of the electron motion is considered to be the kinetic energy at the Fermi surface, which is 7.1 meV at the density 2×10^{11} cm^{-2}. While the effect of the external field to the orbital motion can be estimated by the interval of Landau levels,

$$\frac{1.16\times 10^{-4}}{m_{eff}}\times B,$$

where m_{eff} is an electron effective mass measured in the free electron mass, B is a magnetic field in Tesla. This yields 1.7 meV at the field strength 1 T. This estimation ensures the validity of our present formulation and calculations when the external field is smaller than 1 T.

IV. CONCLUSIONS

We have derived a quantum transport equation for the spin-polarized two-dimensional electron system from the microscopic Hamiltonian. The transport equation reduces to a similar form as the one based on the Fermi liquid theory, containing the spin-rotation term from which spin-collective phenomena originates. We then numerically calculated longitudinal and transverse spin diffusion coefficients and spin-rotation parameters pertaining to GaAs QW's and heterostructures. The longitudinal coefficients diverge as the temperature approaches zero, while the transverse coefficients remain finite at all temperatures, as was predicted and observed in liquid He. The temperature and the spin-polarization dependence of the transport coefficients can be interpreted in terms of the structures of collision integrals. Though there are some ambiguities in the magnitudes of calculated values due to the approximations we used, we expect that the diffusion coefficients and spin-rotation parameters are within the reach of experimental observations if the electron spin polarization is kept long enough. The spin relaxation times reported for simple GaAs QW's is no more than 1 ns so far, which is too short for observations. But it should be noticed that the structures used in those experiments are not optimized for long spin life times. There is still a prospect of fabricating semiconductor nanostructures with elongated spin life times. Further theoretical and experimental investigations are required to determine whether or not the present results can be actually observed in semiconductor samples.

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sion, consider the system where the magnitude of the spin polarization is the same everywhere but its direction is tilted gradually from the direction z to x by a small angle if we go from $x=x_0$ to $x=x_0+\Delta x$. In this case the *transverse* spin current flows in the spatial direction x whose spin direction is in x, perpendicular to the spin polarization.

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