Exchange-induced enhancement of spin-orbit coupling in two-dimensional electronic systems

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(Received 20 January 1999)

We study theoretically the renormalization of the spin-orbit-coupling constant of two-dimensional electrons by electron-electron interactions. We demonstrate that, similarly to the g factor, the renormalization corresponds to the enhancement, although the magnitude of the enhancement is weaker than that for the g factor. For high-electron concentrations (small interaction parameter r_s) the enhancement factor is evaluated analytically within the static random phase approximation. For large $r_s \sim 10$, we use an approximate expression for effective electron-electron interaction, which takes into account the local field factor, and calculate the enhancement numerically. We also study the interplay between the interaction-enhanced Zeeman splitting and interaction-enhanced spin-orbit coupling. [S0163-1829(99)08731-7]

I. INTRODUCTION

Early experimental studies of magnetotransport in two-dimensional (2D) electron systems[1] indicated that the g factor of electrons in these systems may differ significantly from its bulk value. It was established[1] that the magnitude of the g factor for electrons confined to (100) Si surfaces exceeds $g=2$ and increases from $g=2.47$ to $g=3.25$ with decreasing the concentration of electrons from 6×10^{12} cm^{-2} to 10^{12} cm^{-2}.

Shortly after the publication of experimental results,[1] it was suggested by Janak[2] that the enhancement of the g factor can be accounted for by the electron-electron interactions. The argument of Janak represents a 2D version of the Fermi liquid theory[3] and goes as follows. In applied weak magnetic field B the quasiparticle energies for the two spin projections can be written as

$$E_{\uparrow}(k)=E^{(0)}(k)+\frac{\Delta_Z}{2}+\Sigma_{\uparrow}[k,E_{\uparrow}(k)],$$

$$E_{\downarrow}(k)=E^{(0)}(k)-\frac{\Delta_Z}{2}+\Sigma_{\downarrow}[k,E_{\downarrow}(k)], \tag{1}$$

where $\mathbf{k}$ is the momentum, $E^{(0)}(k)=\hbar^2k^2/2m$ is the spectrum of a free electron, $\Delta_Z=g\mu_B B$ is the bare Zeeman splitting, and $\Sigma(k,E_k)$ is the self-energy

$$\Sigma_{\uparrow,\downarrow}(k)=-\int\frac{d^2k'}{(2\pi)^2}V_{eff}(|\mathbf{k}-\mathbf{k}'|)f_0[E_F-E_{\uparrow,\downarrow}(k')], \tag{2}$$

where $V_{eff}(q)$ is the Fourier component of the effective interaction between the electrons, and f_0 is the Fermi function. Solving the system Eq. (1), Eq. (2) in the zero-temperature limit, the effective g-factor can be presented as

$$g^*=\frac{\Delta_Z^*}{\mu_B B}, \tag{3}$$

where

$$\Delta_Z^*=E_{\uparrow}(k_F)-E_{\downarrow}(k_F)=\frac{\Delta_Z}{1-\frac{m^*}{m}\lambda_Z}. \tag{4}$$

In Eq. (4), m^* is the effective mass

$$m^*=\hbar^2k_F\left(\frac{\partial E_k}{\partial k}\right)^{-1}_{k_F}, \tag{5}$$

and k_F is the Fermi momentum. The enhancement factor λ_Z is given by

$$\lambda_Z=\frac{m}{(2\pi\hbar)^2}\int_0^{2\pi}d\phi\,V_{eff}\left(2k_F\sin\frac{\phi}{2}\right). \tag{6}$$

In the random phase approximation (RPA) one has[5]

$$V_{eff}(q)=\frac{2\pi e^2}{\varepsilon_0(q+\sqrt{2}r_sk_F)}, \tag{7}$$

for $q<2k_F$, where ε_0 is the dielectric constant of the material, and $r_s=\sqrt{2}me^2/\varepsilon_0\hbar^2k_F$ is the interaction parameter of the 2D gas. With $V_{eff}(q)$ in the form Eq. (7), m^* and λ_Z can be evaluated analytically yielding[2,4]

$$\lambda_Z=\mathcal{F}(r_s), \tag{8}$$

$$\frac{m}{m^*}=1-\frac{\sqrt{2}}{\pi}r_s+\frac{r_s^2}{2}+(1-r_s^2)\mathcal{F}(r_s), \tag{9}$$

where the function $\mathcal{F}(r_s)$ is defined as

$$\mathcal{F}(r_s)=\frac{r_s}{\pi\sqrt{2-r_s^2}}\cosh^{-1}\left(\frac{\sqrt{2}}{r_s}\right),\quad r_s\leqslant\sqrt{2},$$

$$\mathcal{F}(r_s)=\frac{r_s}{\pi\sqrt{r_s^2-2}}\cos^{-1}\left(\frac{\sqrt{2}}{r_s}\right),\quad r_s\geqslant\sqrt{2}. \tag{10}$$

In the high-density limit ($r_s\ll1$) the enhancement factor (8) takes the form (see also Ref. 6)

$$\lambda_Z = \frac{r_s}{\sqrt{2}\pi} \ln\left(\frac{2^{3/2}}{r_s}\right). \tag{11}$$

Note that the theory[2] neglects the frequency dependence of V_{eff}. As a result, Eq. (9) predicts that interactions reduce the effective mass. In fact, taking the frequency dependence into account[7] leads to $m^*/m > 1$ already within the RPA (see, however, the recent numerical simulations[8]).

Later magnetotransport experiments[9,10] on quantum well structures in narrow band semiconductors provided an evidence for a splitting of the conduction band in a zero-magnetic field. The analysis of the beating patterns in electron Shubnikov-de Haas oscillations led the authors[9,10] to the conclusion that such a splitting can be accounted for by adding the spin-orbit (SO) term

$$\hat{H}_{SO} = \alpha \mathbf{k} \cdot (\boldsymbol{\sigma} \times \mathbf{n}), \tag{12}$$

to the Hamiltonian of a free electron. Here, α is the SO coupling constant, $\mathbf{k}$ is the wave vector, $\mathbf{n}$ is the unit vector normal to the plane of the quantum well, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. The term Eq. (12) was first introduced by Bychkov and Rashba[11,12] to explain the experimental results on electron spin resonance[13] and a cyclotron resonance of holes[14] in GaAs/Al_xGa_{1-x}As heterostructures.

In order to obtain more detailed information about the SO-induced splitting of the conduction band, the evolution of the Shubnikov-de Haas oscillations with a tilting of magnetic field was traced.[15] Subsequently, the energy spectrum of a 2D electron in a tilted magnetic field in the presence of the SO coupling was studied theoretically.[16,17]

Recently, a zero-field splitting in different 2D systems was inferred experimentally either from the Shubnikov-de Haas[18–21] or from the commensurability oscillations[22] (in a spatially modulated sample) patterns.

In the domain of weak-magnetic fields, the Shubnikov-de Haas oscillations are smeared out. However, it was demonstrated both experimentally[23] and theoretically[24] that the SO coupling still manifests itself in this domain through the weak-localization corrections to the conductance. Early works[23,24] in this direction were succeeded by detailed studies.[25]

In the present paper, we investigate theoretically the interplay between the SO coupling and the electron-electron interactions. Namely, we address the question whether the interactions cause the renormalization of the coupling constant α in Eq. (12) as it is the case for the g factor.

II. RENORMALIZATION OF THE SO SPLITTING

We assume that the bare SO splitting is weak enough $\alpha k_F \ll E_F$. With the SO term Eq. (12) the Hamiltonian of noninteracting electrons can be presented in the form

$$\hat{H} = E_+^{(0)}(k)\hat{P}^+(\mathbf{k}) + E_-^{(0)}(k)\hat{P}^-(\mathbf{k}), \tag{13}$$

where the projection operators $\hat{P}^+(\mathbf{k})$ and $\hat{P}^-(\mathbf{k})$ are defined as

$$\hat{P}^+(\mathbf{k}) = \frac{1}{2}\begin{pmatrix} 1 & ie^{-i\phi_{\mathbf{k}}} \\ -ie^{i\phi_{\mathbf{k}}} & 1 \end{pmatrix}, \quad \hat{P}^-(\mathbf{k}) = 1 - \hat{P}^+(\mathbf{k}), \tag{14}$$

so that $\hat{P}^+(\mathbf{k})\hat{P}^-(\mathbf{k}) = 0$. In Eqs. (13) and (14), $k = |\mathbf{k}|$ is the absolute value and $\phi_{\mathbf{k}} = \arctan(k_y/k_x)$ is the azimuthal angle of the wave vector $\mathbf{k}$. The energy spectrum consists of two branches

$$E_\pm^{(0)}(k) = \frac{\hbar^2 k^2}{2m} \pm \alpha k. \tag{15}$$

Following the Fermi liquid theory, the selfenergy in the presence of the SO coupling becomes an operator

$$\hat{\Sigma}(\mathbf{k}) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} V_{eff}(|\mathbf{k}-\mathbf{k}'|)\{\hat{P}^+(\mathbf{k}')f_0[E_F - E_+(k')] + \hat{P}^-(\mathbf{k}')f_0[E_F - E_-(k')]\}. \tag{16}$$

Our main observation is that in the presence of the interaction, the operator $\hat{\Sigma}(\mathbf{k})$ still retains the structure of Eq. (13)

$$\hat{\Sigma}(\mathbf{k}) = \Sigma^+(k)\hat{P}^+(\mathbf{k}) + \Sigma^-(k)\hat{P}^-(\mathbf{k}), \tag{17}$$

where $\Sigma^\pm(k)$ are the *scalar* functions of k

$$\Sigma^\pm(k) = -\frac{1}{2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} V_{eff}(|\mathbf{k}-\mathbf{k}'|)\{f_0[E_F - E_+(k')] + f_0[E_F - E_-(k')]\} \pm \frac{1}{2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2}\cos(\phi_{\mathbf{k}} - \phi_{\mathbf{k}'})V_{eff}(|\mathbf{k}-\mathbf{k}'|) \times \{f_0[E_F - E_+(k')] - f_0[E_F - E_-(k')]\}. \tag{18}$$

For renormalized energy spectrum, we have

$$E_+(k) = E_+^{(0)}(k) + \Sigma^+(k), \quad E_-(k) = E_-^{(0)}(k) + \Sigma^-(k). \tag{19}$$

By solving the system Eqs. (18) and (19), we get the following result for the renormalized SO splitting

$$\Delta_{SO}^* = E_+(k_F) - E_-(k_F) = \frac{\Delta_{SO}}{1 - \dfrac{m^*}{m}\lambda_{SO}}, \tag{20}$$

where $\Delta_{SO} = 2\alpha k_F$ is the bare SO splitting and the renormalization factor is determined as

$$\lambda_{SO} = \frac{m}{(2\pi\hbar)^2}\int_0^{2\pi} d\phi \cos\phi V_{eff}\left(2k_F \sin\frac{\phi}{2}\right). \tag{21}$$

If V_{eff} does not depend on ϕ (when interactions are short-ranged due, e.g., to the presence of a gate electrode close to the 2D plane), then we have $\lambda_{SO} = 0$. However, in general, the integral (21) is positive. Thus, we conclude that the exchange interaction leads to the *enhancement* of the SO coupling. Within the random phase approximation when V_{eff} has the form Eq. (7), the integral (21) can be calculated analytically and expressed through the function $\mathcal{F}(r_s)$ as follows

$$\lambda_{SO}=\frac{r_s^2}{2}-\frac{\sqrt{2}r_s}{\pi}+(1-r_s^2)\mathcal{F}(r_s). \tag{22}$$

Comparison of the last expression with Eq. (9) indicates that $1+\lambda_{SO}=m/m^*$. In fact, this relation holds not only for $V_{eff}(q)$ in the form (7), but for arbitrary static effective interaction and represents a 2D version of the corresponding relation in the Fermi-liquid theory.[26] To verify this relation, it is convenient to perform transformation to the real space where the interaction has the form $\tilde{V}_{eff}(\rho)$. Then from Eqs. (5) and (21) it is easy to check that

$$\lambda_{SO}=\frac{m}{m^*}-1=\frac{m}{\hbar^2}\int_0^\infty d\rho\,\rho\mathcal{J}_1^2(k_F\rho)\tilde{V}_{eff}(\rho), \tag{23}$$

where $\mathcal{J}_1(x)$ is the Bessel function. Combining Eq. (20) with Eq. (23), we get

$$\frac{\Delta_{SO}^*}{\Delta_{SO}}=1+\lambda_{SO}. \tag{24}$$

In Fig. 1, we plot both λ_Z and λ_{SO} as a function of interaction parameter r_s. It is seen that λ_Z is much bigger than λ_{SO}, which has a maximum at $r_s=0.52$ and does not exceed 6%. The decay of $\lambda_{SO}(r_s)\geqslant 1$ indicates that RPA is not suitable for the calculation of λ_{SO} in this domain. The physical origin of the failure of RPA is that it overestimates the screening effect at large r_s. This, in turn, leads to the drastic suppression of λ_{SO} as can be seen from Eq. (21). To extend the Fermi-liquid description to higher r_s, it is customary[27] to modify the random phase dielectric function as follows

$$\varepsilon(q)=\varepsilon_0\left[1-\frac{v(q)\chi_0}{1+v(q)G(q)\chi_0}\right], \tag{25}$$

where $v(q)=2\pi e^2/\varepsilon_0 q$ is the Fourier component of the Coulomb interaction and $\chi_0=-m/\pi\hbar^2$ is the Lindhard susceptibility of the free-electron gas. The factor $G(q)$ (local-field correction) describes the reduction of the screening at large q (small distances). For $G(q)=0$ we recover Eq. (7) for the effective interaction[5] $V_{eff}(q)=v(q)/\varepsilon(q)$.

In later works,[28,4,29,30] a different approximation for $V_{eff}(q)$ was put forward

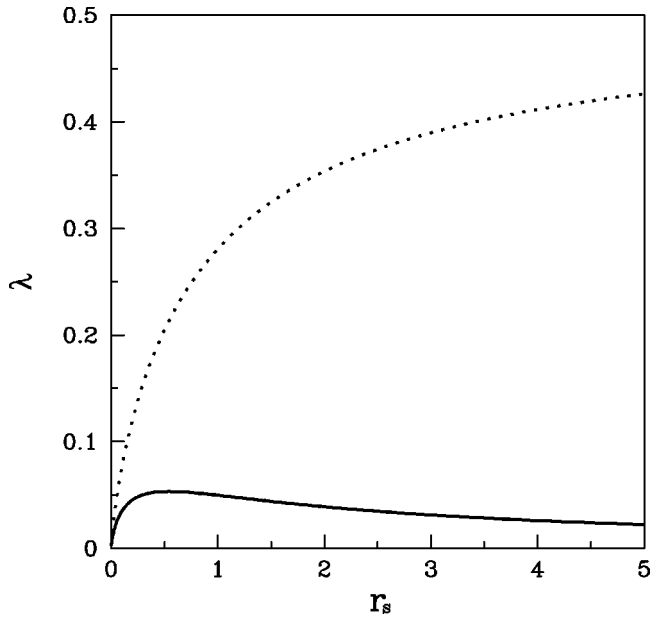

FIG. 1. The enhancement factors of Zeeman splitting (dotted line) and spin-orbit splitting (full line), calculated within the static random phase approximation, are plotted vs the interaction parameter r_s.

$$V_{eff}(q)=v(q)+v^2(q)[1-G(q)]^2\chi(q), \tag{26}$$

where $\chi(q)$ is defined as

$$\chi(q)=\frac{\chi_0}{1-v(q)[1-G(q)]\chi_0}. \tag{27}$$

For the local-field correction $G(q)$, the authors[28,4,29] adopted the following form

$$G(q)=\frac{G_\infty q}{\sqrt{q^2+q_1^2(r_s)}}, \tag{28}$$

where $q_1(r_s)=2a(r_s)k_F$, and $G_\infty(r_s)$, $a(r_s)$ are the numerical factors. Equation (26) is written neglecting the spin-fluctuation-induced vertex corrections. Combining Eqs. (26), (28), and (21), we get the following expression for the enhancement factor of SO coupling

$$\lambda_{SO}=\frac{r_s}{4\pi\sqrt{2}}\int_0^{2\pi}d\phi\cos\phi\,\frac{\sqrt{a^2+\sin^2\phi/2}[G_\infty r_s+\sqrt{2(a^2+\sin^2\phi/2)}]-G_\infty^2 r_s\sin\phi/2}{(r_s+\sqrt{2}\sin\phi/2)(a^2+\sin^2\phi/2)-G_\infty r_s\sin\phi/2\sqrt{a^2+\sin^2\phi/2}}. \tag{29}$$

In Fig. 2, we present the dependence $\lambda_{SO}(r_s)$ calculated numerically within the region up to $r_s\sim 8$. Following,[28,4,29] we took $G_\infty(r_s)$ from numerical calculations (at discrete values of r_s) of the pair correlation function[31] and following Ref. 28 assumed $a(r_s)\approx 1.5G_\infty$ within the entire domain. It is seen that instead of falling down (as in Fig. 1) $\lambda_{SO}(r_s)$ increases with r_s, when the local factor is included. Note, however, that approximately constant value for $a(r_s)$ was established only within a limited interval $r_s\leqslant 3$ in Refs. 4 and 28. We used the same value for calculation at higher r_s in order to illustrate that the enhancement factor can take appreciable values in this domain.

The alternative approach to the effective interaction in 2D electron gas with $r_s\gg 1$ is described in Ref. 32. In this paper the local-field factor in the conventional form (25) of $\varepsilon(q)$ was fitted in such a way that the static characteristics of the

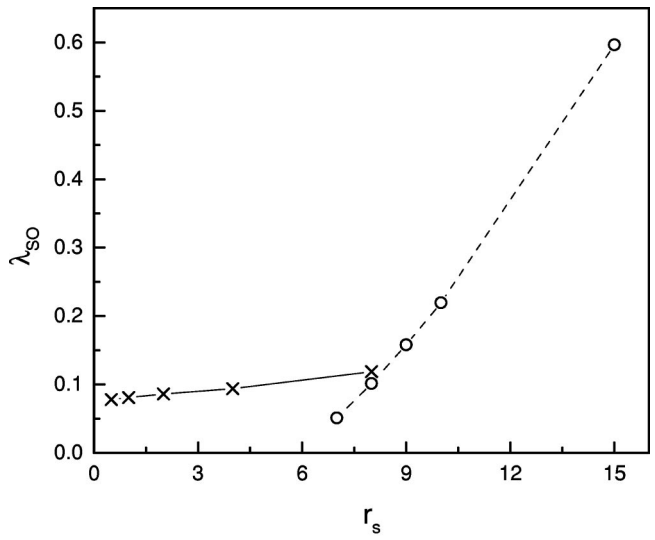

FIG. 2. The enhancement factor λ_{SO}, calculated numerically with local-field correction taken into account, is plotted vs the interaction parameter r_s. Full curve corresponds to the approach of Refs. 4 and 29 with G_∞ taken from Ref. 31 at points marked with crosses. Dashed curve is calculated using the local field factor taken from Ref. 32 at points marked with empty circles.

system, calculated with this $\varepsilon(q)$, are consistent with the Monte Carlo results of Tanatar and Ceperley.[33] According to Ref. 32 the local-field correction has the form

$$G(q)=\frac{1}{2}\left[\frac{q}{(q^2+4b_1^2k_F^2)^{1/2}}+\frac{q}{(q^2+4b_2^2k_F^2)^{1/2}}\right], \quad (30)$$

where the parameters $b_1(r_s)$, $b_2(r_s)$ are listed in Ref. 32 at discrete values of r_s up to $r_s=40$. The numerical results for λ_{SO} calculated for these values by substituting $G(q)$ in the form (30) into Eq. (25) are shown in Fig. 2. They indicate that for $r_s\sim 10$ the enhancement is quite pronounced.

Let us summarize the results of this section. The analytical expression (22) for the enhancement factor λ_{SO} has been derived within the RPA, and shown in Fig. 1. This result applies for $r_s\leqslant 1$ although the plot is extended up to $r_s=5$. For larger r_s, in order to get the correct behavior of $\lambda_{SO}(r_s)$ one should go beyond RPA. The numerical results are presented in Fig. 2 with the local-field correction taken into account. We would like to emphasize that the local-field factor starts to play the crucial role for the enhancement of spin-orbit coupling already at moderate $r_s\approx 1$. This can be seen from *qualitatively* different behavior of $\lambda_{SO}(r_s)$ in Figs. 1 and 2.

III. NONZERO EXTERNAL FIELD

Now let us address the situation when the Zeeman splitting and SO coupling are present simultaneously. We will study the interplay between the interaction-induced enhancement of the g factor and of the SO coupling. First assume that Zeeman splitting is caused by a perpendicular magnetic field. The bare Hamiltonian in this case takes the form

$$\hat{H}=E_+^{(0)}(k)\hat{\mathcal{P}}_{0\perp}^{+}(\mathbf{k})+E_-^{(0)}(k)\hat{\mathcal{P}}_{0\perp}^{-}(\mathbf{k}), \quad (31)$$

where the modified projection operators

$$\hat{\mathcal{P}}_{0\perp}^{+}(\mathbf{k})=\frac{\gamma_0(k)}{1+\gamma_0^2(k)}\begin{pmatrix}\gamma_0^{-1}(k) & ie^{-i\phi_{\mathbf{k}}}\\ -ie^{i\phi_{\mathbf{k}}} & \gamma_0(k)\end{pmatrix},$$

$$\hat{\mathcal{P}}_{0\perp}^{-}(\mathbf{k})=1-\hat{\mathcal{P}}_{0\perp}^{+}(\mathbf{k}), \quad (32)$$

are introduced. In Eq. (32) $\gamma_0(k)$ is defined as

$$\gamma_0(k)=\sqrt{1+\left(\frac{\Delta_Z k_F}{\Delta_{SO}k}\right)^2}-\frac{\Delta_Z k_F}{\Delta_{SO}k}. \quad (33)$$

The bare energy spectrum is given by

$$E_\pm^{(0)}(k)=\frac{\hbar^2k^2}{2m}\pm\frac{1}{2}\sqrt{\Delta_{SO}^2\left(\frac{k}{k_F}\right)^2+\Delta_Z^2}, \quad (34)$$

so that the splitting of the spectrum at $k=k_F$ equals

$$\Delta=\sqrt{\Delta_{SO}^2+\Delta_Z^2}. \quad (35)$$

The general expression Eq. (16) for the selfenergy retains its form in the present case after changing $\hat{P}^{\pm}(\mathbf{k})$ by $\hat{\mathcal{P}}_{\perp}^{\pm}(\mathbf{k})$, where the renormalized projection operators have the form of Eq. (32)

$$\hat{\mathcal{P}}_{\perp}^{+}(\mathbf{k})=\frac{\gamma(k)}{1+\gamma^2(k)}\begin{pmatrix}\gamma^{-1}(k) & ie^{-i\phi_{\mathbf{k}}}\\ -ie^{i\phi_{\mathbf{k}}} & \gamma(k)\end{pmatrix},$$

$$\hat{\mathcal{P}}_{\perp}^{-}(\mathbf{k})=1-\hat{\mathcal{P}}_{\perp}^{+}(\mathbf{k}), \quad (36)$$

with renormalized parameter $\gamma(k)$, which should be determined selfconsistently together with renormalized spectrum $E_\pm(k)$. Since in the present case the operators $\hat{\mathcal{P}}_{\perp}^{\pm}(\mathbf{k})$ differ from $\hat{\mathcal{P}}_{0\perp}^{\pm}(\mathbf{k})$, the consequence (17) of Eq. (16) is not valid anymore. Instead, we get the following system of equations

$$\frac{E_+(k)+\gamma^2(k)E_-(k)}{1+\gamma^2(k)}=\frac{E_+^{(0)}(k)+\gamma_0^2(k)E_-^{(0)}(k)}{1+\gamma_0^2(k)}+\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\frac{V_{eff}(|\mathbf{k}-\mathbf{k}'|)}{1+\gamma^2(k')}\times\{f_0[E_F-E_+(k')]+\gamma^2(k')f_0[E_F-E_-(k')]\}, \quad (37)$$

$$\frac{E_-(k)+\gamma^2(k)E_+(k)}{1+\gamma^2(k)}=\frac{E_-^{(0)}(k)+\gamma_0^2(k)E_+^{(0)}(k)}{1+\gamma_0^2(k)}+\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\frac{V_{eff}(|\mathbf{k}-\mathbf{k}'|)}{1+\gamma^2(k')}\times\{f_0[E_F-E_-(k')]+\gamma^2(k')f_0[E_F-E_+(k')]\}, \quad (38)$$

$$\frac{\gamma(k)}{1+\gamma^2(k)}[E_+(k)-E_-(k)]$$

$$=\frac{\gamma_0(k)}{1+\gamma_0^2(k)}[E_+^{(0)}(k)-E_-^{(0)}(k)]$$

$$+\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\cos(\phi_{\mathbf{k}}-\phi_{\mathbf{k}'})V_{eff}(|\mathbf{k}-\mathbf{k}'|)$$

$$\times\{f_0[E_F-E_+(k')]-f_0[E_F-E_-(k')]\}. \quad (39)$$

Subtracting Eq. (38) from Eq. (37), we get

$$\frac{1-\gamma^2(k)}{1+\gamma^2(k)}[E_+(k)-E_-(k)]$$

$$=\frac{1-\gamma_0^2(k)}{1+\gamma_0^2(k)}[E_+^{(0)}(k)-E_-^{(0)}(k)]$$

$$+\int\frac{d^2\mathbf{k}'}{(2\pi)^2}V_{eff}(|\mathbf{k}-\mathbf{k}'|)\frac{1-\gamma^2(k')}{1+\gamma^2(k')}$$

$$\times\{f_0[E_F-E_+(k')]-f_0[E_F-E_-(k')]\}. \quad (40)$$

Now we can apply to Eq. (39) and Eq. (40) the same argument that led to renormalization of Δ_{SO} and Δ_Z, respectively. In the zero-temperature limit this results in the following system of equations

$$\frac{\gamma(k)}{1+\gamma^2(k)}[E_+(k)-E_-(k)]\left[1-\frac{m^*}{m}\lambda_{SO}\right]$$

$$=\frac{\gamma_0(k)}{1+\gamma_0^2(k)}[E_+^{(0)}(k)-E_-^{(0)}(k)], \quad (41)$$

$$\frac{1-\gamma^2(k)}{1+\gamma^2(k)}[E_+(k)-E_-(k)]\left[1-\frac{m^*}{m}\lambda_Z\right]$$

$$=\frac{1-\gamma_0^2(k)}{1+\gamma_0^2(k)}[E_+^{(0)}(k)-E_-^{(0)}(k)]. \quad (42)$$

Dividing Eq. (42) by Eq. (41) we get a closed quadratic equation for $\gamma(k)$

$$\gamma^2(k)+\gamma(k)\frac{1-\gamma_0^2(k)}{\gamma_0(k)}\frac{1-\frac{m^*}{m}\lambda_{SO}}{1-\frac{m^*}{m}\lambda_Z}-1=0. \quad (43)$$

Substituting the solution of this equation back into Eq. (37), we get the renormalized splitting of the spectrum $\Delta^*=E_+(k_F)-E_-(k_F)$

$$\frac{\Delta^*}{\Delta}=\frac{\sqrt{(1-\gamma_0^2)^2\left(1-\frac{m^*}{m}\lambda_{SO}\right)^2+4\gamma_0^2\left(1-\frac{m^*}{m}\lambda_Z\right)^2}}{(1+\gamma_0^2)\left(1-\frac{m^*}{m}\lambda_{SO}\right)^2\left(1-\frac{m^*}{m}\lambda_Z\right)^2}. \quad (44)$$

Using the definition (33) of γ_0, we can rewrite the last result in the following concise form

$$\frac{\Delta^*}{\Delta}=\sqrt{\frac{\Delta_Z^2}{\Delta_Z^2+\Delta_{SO}^2}\left(\frac{1}{1-\frac{m^*}{m}\lambda_Z}\right)^2+\frac{\Delta_{SO}^2}{\Delta_Z^2+\Delta_{SO}^2}\left(\frac{1}{1-\frac{m^*}{m}\lambda_{SO}}\right)^2}. \quad (45)$$

Finally, with the use of Eqs. (4) and (20), we arrive at the conclusion that renormalized splitting Δ^* is related to renormalized values Δ_Z^* and Δ_{SO}^* in the same way as the bare values [Eq. (32)]

$$\Delta^*=\sqrt{\Delta_{SO}^{*2}+\Delta_Z^{*2}}. \quad (46)$$

Consider now the case when the Zeeman splitting is caused by a parallel magnetic field applied along the x direction. Then the Hamiltonian can be written as

$$\hat{H}=\begin{pmatrix}\frac{\hbar^2k^2}{2m} & \frac{\Delta_Z}{2}+i\alpha k e^{-i\phi_{\mathbf{k}}}\\ \frac{\Delta_Z}{2}-i\alpha k e^{i\phi_{\mathbf{k}}} & \frac{\hbar^2k^2}{2m}\end{pmatrix}$$

$$=E_+^{(0)}(k)\hat{\mathcal{P}}_{0\|}^+(\mathbf{k})+E_-^{(0)}(k)\hat{\mathcal{P}}_{0\|}^-(\mathbf{k}), \quad (47)$$

where the energy spectrum

$$E_{\pm}^{(0)}(k)=\frac{\hbar^2k^2}{2m} \pm\frac{1}{2}\sqrt{\Delta_Z^2+\Delta_{SO}^2\left(\frac{k}{k_F}\right)^2+2\Delta_Z\Delta_{SO}\left(\frac{k}{k_F}\right)\sin\phi_{\mathbf{k}}}, \tag{48}$$

depends both on the amplitude and orientation of $\mathbf{k}$ with respect to the magnetic field. In Eq. (47), the projection operators $\hat{\mathcal{P}}_{0\|}^{\pm}(\mathbf{k})$ are defined as

$$\hat{\mathcal{P}}_{0\|}^{+}(\mathbf{k})=\frac{1}{2}\begin{pmatrix}1 & ie^{-i\varphi_{\mathbf{k}}}\\ -ie^{i\varphi_{\mathbf{k}}} & 1\end{pmatrix},\ \widehat{\mathcal{P}_{0\|}^{-}}(\mathbf{k})=1-\hat{\mathcal{P}}_{0\|}^{+}(\mathbf{k}), \tag{49}$$

with the angle $\varphi_{\mathbf{k}}$ related to the azimuthal angle of vector $\mathbf{k}$ as follows

$$\varphi_{\mathbf{k}}=\arctan\left(\frac{\alpha k\cos\phi_{\mathbf{k}}}{\alpha k\sin\phi_{\mathbf{k}}+\dfrac{\Delta_Z}{2}}\right). \tag{50}$$

The bare splitting of the energy spectrum at $|\mathbf{k}|=k_F$ is equal to

$$\Delta(\phi)=\sqrt{\Delta_Z^2+\Delta_{SO}^2+2\Delta_Z\Delta_{SO}\sin\phi}. \tag{51}$$

Performing calculations similar to those for perpendicular field, it is easy to check that in the present case the relation between the renormalized splitting Δ^* and Δ_Z, Δ_{SO} preserves the form (51)

$$\Delta^*(\phi)=\sqrt{\Delta_Z^{*2}+\Delta_{SO}^{*2}+2\Delta_Z^*\Delta_{SO}^*\sin\phi}. \tag{52}$$

IV. RELATION TO THE LANDAU PARAMETERS

The above calculations were based on the concept of effective interaction between electrons, $V_{eff}(q)$. Generally speaking, Fermi-liquid theory relates the observable values to the bare parameters of electron gas by means of interaction function[3,26] having the form

$$f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}')=f^s(\mathbf{k},\mathbf{k}')+(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')f^a(\mathbf{k},\mathbf{k}') =\frac{\pi\hbar^2}{m^*}[F^s(\mathbf{k},\mathbf{k}')+(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')F^a(\mathbf{k},\mathbf{k}')], \tag{53}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$ are spin matrices, $f^s(\mathbf{k},\mathbf{k}')$ and $f^a(\mathbf{k},\mathbf{k}')$ are the symmetric and antisymmetric parts of the interaction function, respectively. In Eq. (53), $F^{s(a)}(\mathbf{k},\mathbf{k}')=\sum_{l=0}^{\infty}F_l^{s(a)}\cos(l\phi_{\mathbf{k}\mathbf{k}'})$ are dimensionless quantities. The concept of effective interaction used above is equivalent to the assumption $f^a\equiv f^s$. The way to extend our theory in order to take into account the difference between f^s and f^a is to modify the self-energy Eq. (16) as follows

$$\hat{\Sigma}(\mathbf{k})=\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\hat{P}^+(\mathbf{k}')\{V_{eff}(|\mathbf{k}-\mathbf{k}'|)f_0[E_F-E_+(k')] +W_{eff}(|\mathbf{k}-\mathbf{k}'|)f_0[E_F-E_-(k')]\} +\int\frac{d^2\mathbf{k}'}{(2\pi)^2}\hat{P}^-(\mathbf{k}')\{V_{eff}(|\mathbf{k}-\mathbf{k}'|)f_0[E_F-E_-(k')] +W_{eff}(|\mathbf{k}-\mathbf{k}'|)f_0[E_F-E_+(k')]\}. \tag{54}$$

Here $V_{eff}(q)$ corresponds to the effective interaction in Eq. (16), whereas $W_{eff}(q)$ accounts for the difference between f^s and f^a. It is straightforward to check that with selfenergy operator Eq. (54), the projection operators $\hat{P}^+(\mathbf{k})$ and $\hat{P}^-(\mathbf{k})$, which, in principle, should be determined self-consistently, still retain the form Eq. (14). Thus, we can repeat the derivation for the enhancement of Δ_{SO} in a similar way as in Sec. II. The difference is, however, that the relation Eq. (23) does not hold anymore. Indeed, the renormalization of the effective mass[8] is now determined by the effective interaction $V_{eff}(q)+W_{eff}(q)$ through $m^*/m=1+\frac{1}{2}F_1^s$, while λ_{SO} is determined by the first Fourier harmonics of $V_{eff}(q)-W_{eff}(q)$; accordingly, λ_Z is determined by the zero's Fourier component of $V_{eff}(q)-W_{eff}(q)$. Consequently, in terms of the dimensionless Landau parameters,[3] we get the following generalization of Eq. (24)

$$\frac{\Delta_{SO}^*}{\Delta_{SO}}=\frac{1}{1+\dfrac{1}{2}F_1^a}. \tag{55}$$

The dependence of Landau parameters on r_s in two dimensions has been the subject of extensive Monte Carlo studies in Ref. 8. For F_1^a the results listed in Ref. 8 at $r_s=1,2,3$, and $r_s=5$ are $F_1^a=-0.19,-0.24,-0.26$, and $F_1^a=-0.27$. Substitution of these values in Eq. (55) leads to $\Delta_{SO}^*/\Delta_{SO}=1.11$, 1.14, 1.15, and 1.16, respectively. These values agree within 30% with the results shown in Fig. 2.

V. CONCLUSION

The main goal of this paper is to demonstrate that alongside with fundamental characteristics, $\lambda_Z(r_s)$, of interacting electron gas, which describes the enhancement of the g factor, and was studied in many works, there exists another fundamental characteristics $\lambda_{SO}(r_s)$, which describes the interaction-induced enhancement of the SO coupling. We calculated this function analytically in the limit of high concentrations and estimated numerically at low concentrations. Note that throughout the paper we assumed the bare SO coupling to be small: $\Delta_{SO}\ll E_F$. However, in the limit of high concentrations ($r_s\ll 1$) the corresponding condition is more strict: $\Delta_{SO}\ll r_sE_F$, which is equivalent to $\alpha\ll e^2/\varepsilon_0$. In the intermediate region $r_sE_F\ll\Delta_{SO}\ll E_F$, instead of Eq. (22), the enhancement factor is given by

$$\lambda_{SO}=\frac{r_s}{\sqrt{2}\pi}\ln\left(\frac{E_F}{\Delta_{SO}}\right). \tag{56}$$

Since experimentally the concentration of carriers is varied by changing the gate voltage,[18,20,22] there exists another simple reason for the dependence of the SO coupling on the concentration. Indeed, the change of the gate voltage causes the redistribution of the confining potential, which, in turn,[34] affects the parameter α. This mechanism should be dominant at high concentrations when λ_{SO} is small.

Note in conclusion, that if the bare SO splitting is caused by the Dresselhaus mechanism,[35] which originates from the absence of the inversion symmetry in the bulk, the renormalization of the corresponding splitting of the spectrum, Δ_D, has the same form as Eq. (24): $\Delta_D^*=(1+\lambda_{SO})\Delta_D$. As a result, when both Δ_D and Δ_{SO} are present, the splitting of the energy spectrum is given by the same formula as for noninteracting electrons[36,37]

$$\Delta^*(\phi)=\sqrt{\Delta_D^{*2}+\Delta_{SO}^{*2}+2\Delta_D^*\Delta_{SO}^*\sin 2\phi}. \qquad (57)$$

Finally, let us point out that in conventional magnetotransport oscillations experiments performed up to now[15,17–21] the typical concentrations of electrons were quite high $\sim 10^{12}$ cm^{-2}. As a result, the typical values of the interaction parameter r_s were rather low ($r_s<1$). Only in 2D hole gas[22] the condition ($r_s\geqslant 1$) was fulfilled. For low values of r_s our theory predicts that the renormalization of the SO coupling is weak. However, in recent experiments on the electron gases in silicon metal-oxide-semiconductor field-effect transistors[38–40] and AlAs quantum wells,[41] as well as in hole gases in SiGe quantum wells,[42] GaAs inverted semiconductor-insulator-semiconductor structures,[43] and GaAs-$Al_xGa_{1-x}As$ heterostructures[44] the values of r_s ranged from[38–40] $r_s\approx 6$ to[43,44] $r_s\approx 24$. For such large r_s we predict a strong renormalization of the SO coupling, which might be of relevance for metal-insulator transition observed in these systems. For example, the strong renormalization of SO coupling at large r_s might cause an instability of electronic spectrum in a clean system, so that the system in zero-magnetic field would undergo a transition into an exotic ''chiral phase'' at some critical density. As it is seen from Eq. (55), the condition for such an instability is $1+\frac{1}{2}F_1^a=0$.

Another possibility for strong effect of λ_{SO} on the properties of low-density 2D electron gas is that it can cause a significant redistribution of electrons between the branches of the spectrum. Note that in the latest publication[45] the critical density for metal-insulator transition in *n*-type GaAs was reported to be $n_c\approx 1.3\times 10^{10}$ cm^{-2}. To accommodate all these electrons within the lower branch of the spectrum, corresponding to chirality ''$-$'' [see Eq. (15)], the effective coupling constant α should exceed $\alpha_c=(\hbar^2 n_c/2\pi m^*)^{1/2}\approx 3.65\times 10^{-12}$ eV·m. On the other hand, the constant α for relatively high-density GaAs/$Al_xGa_{1-x}As$ structure with $n\approx 4.0\times 10^{11}$ cm^{-2} can be extracted from Ref. 19 to be $\alpha=\Delta_{SO}/\sqrt{8\pi n}\approx 1.7\times 10^{-12}$ eV·m. Thus, a two times interaction-induced enhancement of SO coupling would be enough to drive all electrons into the state with the ''$-$'' chirality.

ACKNOWLEDGMENTS

We are grateful to R. R. Du, D. Mattis, and Y. S. Wu for useful discussions. We also thank L. E. Zhukov for assistance in preparation of the manuscript. M. E. R. acknowledges the discussions with H. U. Baranger and Y. Imry and the hospitality of the Aspen Center for Physics.

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