Inhomogeneity of the superconducting state and consequent diamagnetism above T_c: Application to the cuprates

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The inhomogeneity of a superconducting system gives rise to an interesting situation when at $T > T_c^{\mathrm{res}}$ (T_c^{res} is characterized by a relatively sharp resistive transition) one can observe an anomalously large diamagnetic moment. Here we focus on the case when the sample can be described as containing a nonuniform distribution of magnetic impurities and, as a consequence, by an inhomogeneous distribution of critical temperatures. The analysis is in good agreement with experimental data on overdoped cuprates. [S0163-1829(99)01030-9]

I. INTRODUCTION

The present paper is concerned with magnetic properties of inhomogeneous superconducting systems. High-T_c oxides represent an important example of such systems, and this is particularly true for samples with nonstoichometric compositions such as overdoped cuprates.

Inhomogeneity leads to the situation when T_c becomes spatially dependent. This case was studied, for example, by A. Larkin and one of the authors.[1] In this paper we have explored the case when the sample contains small regions ($\rho \ll \delta$, δ is the penetration depth) with a local T_c higher than the average value $\overline{T_c}$. In other words, at $T > \overline{T_c}$, the superconducting grains are embedded in a normal matrix. A typical example of such a situation occurs when the sample contains magnetic impurities which are distributed in a nonuniform way. As is well known,[2,3] magnetic impurities act as pair breakers and their presence depresses T_c. As a result, the inhomogeneous scenario we just described can be perfectly realistic.

The present paper is directly related to a recent experimental study of overdoped Tl-based cuprates.[4] According to Ref. 4, above $T_c \cong 15\,\mathrm{K}$ one measures a noticeable diamagnetic response. Such response was also observed in the presence of a strong magnetic field; then $T_c \equiv T_c(H)$. The experimental data[4] will be discussed below (Sec. III) in detail; one can show that the theory developed here allows us to fully describe the phenomenon observed in Ref. 4.

The structure of the paper is as follows. Section II contains a theoretical analysis of the inhomogeneous system and an evaluation of the magnetic moment. The application to the overdoped cuprates and a comparison with experimental data are discussed in Sec. III.

II. THEORY

A. Main equations

Consider an inhomogeneous superconductor which contains magnetic impurities. Our approach is based on the method of integrated Green's functions.[5] The system is described in detail by the equations introduced by Larkin and one of the authors:[6]

$$\alpha\Delta - \beta\omega + \frac{D}{2}(\alpha\partial_-^2\beta - \beta\partial_{\mathbf{r}}^2\alpha) = \alpha\beta\Gamma, \tag{1a}$$

$$\alpha^2 + |\beta|^2 = 1, \tag{1b}$$

$$\Delta = 2\pi T|\lambda|\sum_{\omega>0}\beta. \tag{1c}$$

Here α and β are the usual and pairing Green's functions averaged over energy, Δ is the order parameter, $\Gamma \equiv \tau_s^{-1}$ is the spin-flip relaxation time. Because of the inhomogeneity, all of these quantities are spatially dependent. In addition, $\partial_\pm = \partial_{\mathbf{r}} \pm 2ie\mathbf{A}$, $\mathbf{A}$ is the vector potential, $\partial_{\mathbf{r}} = (\partial/\partial\mathbf{r})$. We consider the ''dirty'' case, so that D is the diffusion coefficient.

Assume that the sample contains a sufficient amount of magnetic impurities so that $\tau_s T_c^\circ < 1$; as a result $T_c \ll T_c^\circ$, where T_c is the average value of the critical temperature, and T_c° corresponds to the transition temperature with no magnetic impurities. In this case, with the use of Eqs. (1), we obtain

$$\beta = \beta_0 - \beta_1; \quad \alpha = 1 - |\beta_0|^2/2, \tag{2a}$$

$$\beta_0 = \left(\Gamma + \omega - \frac{D}{2}\partial_-^2\right)^{-1}\Delta, \tag{2b}$$

$$\beta_1 = \left(\Gamma + \omega - \frac{D}{2}\partial_-^2\right)^{-1}\left\{\frac{|\beta_0|^2}{2}(\Delta - \Gamma\beta_0) + \frac{D}{4}(|\beta_0|^2\partial_-^2\beta_0 - \beta_0\partial_{\mathbf{r}}^2|\beta_0|^2)\right\}. \tag{2c}$$

Based on Eqs. (1c) and (2), we obtain the following equation for the order parameter:

$$\Delta = 2\pi T|\lambda| \sum_{\omega>0} \left(\Gamma + \omega - \frac{D}{2}\partial_-^2\right)^{-1} \times \left\{ \Delta - \frac{\omega}{2}\beta_0|\beta_0|^2 + \frac{D}{4}\beta_0 \partial_{\mathbf{r}}^2 |\beta_0|^2 \right\}. \quad (3)$$

Let us focus on the temperature region $T > T_c(H)$. Then the order parameter is localized near some clusters. In such a region ($\mathbf{r} \cong \mathbf{r}$) the operator $[\Gamma + \omega - (D/2)\partial_-^2]$ has discrete eigenvalues.

Our goal is to evaluate the order parameter [at $T > Tc(H)$] for the inhomogeneous structure and then the corresponding magnetic moment. The order parameter can be found in the form

$$\Delta = C\Delta_0, \quad (4)$$

where C is a constant (its value will be calculated below) and Δ_0 is the solution of the equation

$$[\Gamma - (D/2)\partial_-^2]\Delta_0 = (\Gamma_\infty + \lambda_1)\Delta_0. \quad (5)$$

Here Γ_∞ is the value of Γ outside of the grain, and λ_1 is a minimum eigenvalue. One can see from Eq. (2b) that

$$\beta_0 = C\Delta_0(\omega + \Gamma_\infty + \lambda_1)^{-1}. \quad (6)$$

Inserting this expression into Eq. (3), we arrive, after the summation, at the following equation:

$$\ln(T_c^0/T) = \psi\left[\tfrac{1}{2} + \frac{\Gamma_\infty + \lambda_1}{2\pi T}\right] - \psi(\tfrac{1}{2}) + \frac{C^2}{12\Gamma_\infty^2} \frac{(\Delta_0^{*2}, \Delta_0^2)}{(\Delta_0^*, \Delta_0)}. \quad (7)$$

Here ψ is the Euler function, and the notation (f,g) corresponds to scalar product of the functions. The transition temperature $T_C \equiv T_C^{\text{aver}}$ is determined by Eq. (8) which can be obtained from Eq. (7) if we insert $C = \lambda_1 = 0$:

$$\ln(T_c^0/T_c) = \psi[\tfrac{1}{2} + (\Gamma_\infty/2\pi T_c)] - \psi(\tfrac{1}{2}) \quad (8)$$

which is a well-known equation obtained in Ref. 2. Equation (7) is the generalization of Eq. (8) for the inhomogeneous case studied here. For the region $\Gamma_\infty > T > T_c(H)$ we obtain the following expression for the constant C:

$$C^2 = 2\pi^2\alpha[1 - \tau^2 + B], \quad (9)$$

where

$$\alpha = T_c^2(\Delta_0^*, \Delta_0)(\Delta_0^{*2}, \Delta_0^2)^{-1}; \quad \tau = T/T_C;$$

$$B = -6\lambda_1\Gamma_\infty/(\pi T_C)^2. \quad (10)$$

Equations (4), (6), and (9) determine the temperature dependence of the order parameter and the pairing Green's function β_0. Let us turn now to the evaluation of the magnetic moment.

B. Magnetic moment

Based on Eqs. (4)–(6) and (9) one can evaluate the magnetic moment of the inhomogeneous system described above. Indeed, the current density is described by the expression[6]

$$j = -ie\upsilon D\pi T \sum_\omega (\beta^* \partial_- \beta - \beta \partial_+ \beta^*). \quad (11)$$

Here υ is the density of states. With the use of Eqs. (6) and (10), we obtain

$$j = -\frac{ie\upsilon DC^2}{2\pi T}\psi'\left(\frac{1}{2} + \frac{\Gamma_\infty + \lambda_1}{2\pi T}\right)(\Delta_0^* \partial_- \Delta_0 - \Delta_0 \partial_+ \Delta_0^*). \quad (11')$$

As was mentioned above, the inhomogeneous structure contains small regions with a local value of the critical temperature $T_{c;L}$ higher than the average value $\tilde{T}_c$. In this case, the order parameter Δ_0, which corresponds to the lowest eigenvalue can be taken as real. Then, we can obtain the following expression for the magnetic moment of an isolated cluster $M_z = L\int d\boldsymbol{\rho}[\boldsymbol{\rho}\mathbf{j}]_z$:

$$M_z = -(e^2\upsilon DC^2 HL/\pi T)\psi'\left(\frac{1}{2} + \frac{\Gamma_\infty + \lambda_1}{2\pi T}\right)K. \quad (12)$$

Here $K = \int d\boldsymbol{\rho}\rho^2\Delta_0^2$, and the vector potential has been chosen as $\mathbf{A} = \frac{1}{2}[\mathbf{Hr}]$; L is the effective thickness of the superconducting layer. Note also that because the cluster size is smaller than the penetration depth, one can neglect the spacial variation of the magnetic field.

Based on Eqs. (5) and (12), one can perform a final evaluation of the magnetic moment [see Eq. (12)]; as a first step, one should calculate the order parameter Δ_0 and the eigenvalue λ_1 [see Eq. (4)]. One can study several different cases. The simplest one corresponds to a small variation of Γ, so that one can use perturbation theory (see Appendix). Consider the most interesting case when the variation of the amplitude $\delta\Gamma(\mathbf{r}) = \Gamma_\infty - \Gamma$ has the form

$$\delta\Gamma(\mathbf{r}) = \begin{cases} \delta\Gamma(\rho); & \rho < \rho_0 \\ 0; & \rho > \rho_0. \end{cases} \quad (13)$$

Then Eq. (5) can be written in the form

$$\left\{\delta\Gamma(\rho) - \frac{D}{2}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) - e^2H^2\rho^2\right]\right\}\Delta_0(\rho) = \lambda_1\Delta_0(\rho). \quad (14)$$

A similar equation has been studied by Bezryadin, Pennetier, and one of the authors in Ref. 7. The solution is

$$\Delta_0(\rho) = \frac{1}{\sqrt{eH\rho^2}} \begin{cases} M_{(\lambda_1 + \delta\Gamma)/2eHD;0}(eH\rho^2); & \rho < \rho_0 \\ C_1 W_{\lambda_1/2eHD;0}(eH\rho^2); & \rho > \rho_0. \end{cases} \quad (15)$$

Here $W_{\lambda,\mu}(z)$ and $M_{\lambda,\mu}(z)$ are the Whittaker functions. It is convenient to use the following presentations of the Whittaker functions:

$$\Delta_0(\rho)=\exp(-eH\rho^2/2)\times\begin{cases}\phi(1/2-\tilde{\lambda}-\tilde{\delta}\Gamma;0;\alpha); & \rho<\rho_0\\ C_1[\Gamma(1/2-\tilde{\lambda})]^{-1}\int_0^\infty dtF(t,\rho); & \rho>\rho_0.\end{cases} \quad (16)$$

Here $\tilde{\lambda}=\lambda_1/2eHD$, $\delta\tilde{\Gamma}=\delta\Gamma/2eHD$, $\alpha=eH\rho^2$, $\phi\equiv\phi(\beta,\gamma,x)$ is the confluent hypergeometric function, and $F(t,\rho)=e^{-\alpha t}t^{-0.5-\tilde{\lambda}}(1+t)^{-0.5+\tilde{\lambda}}$. The eigenvalue λ_1 can be determined from the condition for the logarithmic derivative to be continuous,

$$-(\phi'/\phi)_{|\rho_0}=\left(\int dt\cdot t\cdot F(t,\rho)\right)\left(\int dtF(t,\rho)\right)^{-1}_{|\rho_0}. \quad (17)$$

It is important to emphasize that Eq. (17) reflects the strong impact of the proximity effect. Indeed, the superconducting cluster with its $T_{C,L}>\tilde{T}_c(H)$ is embedded in a normal matrix, and therefore it is necessary to take into account the proximity effect. Thus Eq. (17) is a boundary condition for the order parameter.

Equation (17) allows us to evaluate the eigenvalue λ_1. One can show (see the Appendix), that this quantity is equal to

$$\lambda_1=-\delta\Gamma+0.5D(z_0/\rho_0)^2. \quad (18)$$

Here $z_0\approx2.4$ is the lowest zero of the Bessel function, that is $J_0(z_0)=0$. In addition,

$$\Delta_0(\rho)=J_0(\rho z_0/\rho_0)\quad \text{for } \rho<\rho_0. \quad (19)$$

With the use of Eqs. (9), (12), (18), and (19), we can obtain the following expression for the magnetic moment:

$$M_z=-A(\tilde{B}-\tau^2)H. \quad (20)$$

Here

$$A=(8\pi^2e^2\upsilon DT_C^2/\Gamma_\infty)\rho_0^2z_0^{-4}n(\tilde{x}_{3;2}\tilde{x}_{1;2}/\tilde{x}_{1;4});\quad \tilde{B}=B+1. \quad (21)$$

B and λ_1 are determined by Eqs. (9) and (18), n is the cluster concentration, and $\tilde{x}_{n;i}=\int_0^{z_0}dx\cdot x^nJ_0^i(x)$. If $\delta\Gamma\ll\Gamma_\infty\cong\pi T_C^0$, the value of the local critical temperature $T_{C;L}$ greatly exceeds its average value.

One can see directly from Eq. (20), that it is possible to observe a noticeable diamagnetic moment. Indeed, if we assume realistic values: $p=10^{-20}\,\mathrm{cm\,sec^{-2}}$, $1=40$ Å (1 is a mean free path: $D=v_Fl/3$), $T_C=10$ K, $\Gamma_\infty=10^2$ K, $\delta\Gamma=50$ K, $\rho_0=80$ Å, and $n\cong0.1$, we obtain with the use of Eq. (10), (18), and (21) the following values of the parameters: $A\cong10^{-5}$, $B=3$, $|\lambda_1|=5$ K. Then, for example, at $T=11$ K, one can observe $\chi_D=-M_Z/H=3\times10^{-5}$, which is a value that greatly exceeds the usual value of the paramagnetic response of a normal metal $\chi_P\cong10^{-6}$.

The diamagnetic response can be observed in the region $\tau<\tilde{B}$ [see Eq. (20)]. It is important to note that the limitation on the value of $\tilde{B}$ is caused by the proximity effect. Indeed, the value of $|\lambda_1|$ which determines the upper limit of τ [see Eq. (9)], is defined by the equation $|\lambda_1|=\delta\Gamma-0.5D(z_0/\rho_0)^2$ [see Eq. (18)]. The value of $|\lambda_1|$ depends on an interplay of two terms. The first term reflects the impact of the magnetic scattering, and the second negative term describes the proximity effect. For example, a decrease in the size of the inhomogeneity ρ_0 leads to an increase of the second term and, accordingly, to decrease in value of $\tilde{B}$ [see Eq. (9)]; thus decreasing the temperature region ($\tau<\tilde{B}$) in which one can observe a diamagnetic response. This is natural, since the influence of the proximity effect to depress the superconductivity grows with a decrease in the size ρ_0 of the superconducting grain.

In order to observe the linear dependence (21), it is better to be in the region $T>T_C(H)+\delta$, where δ is the broadening of the transition determined, e.g., by resistive measurement.

Equation (19) can be generalized for the case when the magnetic scattering amplitude Γ depends on temperature. Such a scenario was considered by the authors in Refs. 8 and 9. As we know, the magnetic impurities act as pair breakers, and this leads to a depression in T_c as well as other parameters (H_{c2}, Josephson critical current, etc.). In Refs. 2 and 3 impurities were considered as independent and the amplitude Γ was proportional to n_s, where n_s is the concentration of the impurities. However, it is important to realize that a decrease in temperature leads to a correlation between the localized magnetic moment. Spin-flip scattering is frustrated, and, as result, Γ decreases as $T\rightarrow0$, that is $\Gamma\equiv\Gamma(T)$. In this case a generalized Eq. (9) has the form

$$C^2=2\pi^2\alpha f(T,T_c), \quad (9')$$

where

$$f(T,T_c)=\left[\left(\frac{\Gamma_\infty(T)}{\Gamma_\infty(T_c)}\right)^2-\tau^2\right]-\frac{6\Gamma_\infty^2(T)}{(\pi T_c)^2}\ln\left(\frac{\Gamma_\infty(T)}{\Gamma_\infty(T_c)}\right)+B_0\frac{\Gamma_\infty(T)}{\Gamma_0}, \quad (9'')$$

where $B_0=-6\lambda_1\Gamma_0/(\pi T_C)^2$ [cf. Eq. (9)], $\Gamma\equiv\Gamma_\infty(0)$. Correspondingly, the magnetic moment is equal to

$$M_z=-Af(T,T_C)H/\Gamma_\infty(T). \quad (22)$$

III. OVERDOPED CUPRATES: COMPARISON WITH EXPERIMENT

The theory described above can be applied to any inhomogeneous superconducting system, conventional, as well as the high-T_c oxides. In this section we focus on the overdoped cuprates because of a recent observation by Bergemann *et al.*[4] The authors[4] describe torque measurements performed on a $Tl_2Ba_2CuO_6$ overdoped sample ($T_c\cong15$ K). A diamagnetic moment, proportional to the external magnetic field, has been observed at $T>T_c$. Note that the value of $T_c\cong15$ K has been determined by resistive measurements. One can show that the theory described in Sec. II is directly related to the data obtained in Ref. 4.

As we know, the overdoping is provided by an additional oxidization and is accompanied by a drastic decrease in T_c

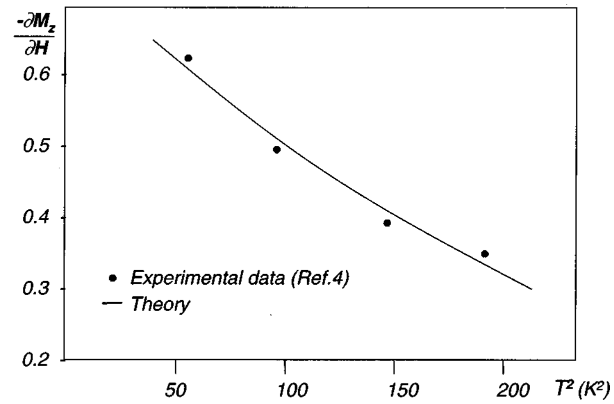

FIG. 1. Diamagnetic susceptibility for the overdoped $Tl_2Ba_2CuO_{6+\delta}$ ($T_C=15$ K). ●: experimental data (Ref. 4); solid line: theory.

($T_c^0\cong 90$ K$\rightarrow T_c\cong 15$ K). We suggest (see, e.g., Refs. 8 and 9) that this decrease is caused by the pair breaking effect of magnetic scatterers. Note that the presence of pair breakers has been observed experimentally (see, e.g., Refs. 10 and 11). Since the oxygen distribution is nonuniform, the sample becomes inhomogeneous with a nonuniform distribution of magnetic impurities. As a result, one can expect (see above) the appearance of a diamagnetic moment at $T>T_c^{\rm res}\cong 15$ K.

Let us use the approach developed in Sec. II, and, more specifically, Eq. (22) to analyze the data in Ref. 4. The authors[4] use a Tl-based cuprate sample which was studied before in Ref. 12 and displays an unconventional behavior of $H_{c2}(T)$ (positive curvature, sharp upturn near $T\cong 1$ K, a large value of $H_{c2}(0)$ relative to $H_{c2}^{\rm conv}$).[13,14] This behavior has been explained by us[8] by magnetic impurity scattering and an ordering trend of those impurities as $T\rightarrow 0$. The temperature dependence of the amplitude $\Gamma(T)$ for the sample studied in Ref. 12 and recently in Ref. 14 has been evaluated by us in Ref. 8 and has the form

$$\Gamma(T)=\Gamma_0(1.26T+\theta)(T+\theta)^{-1};\quad \Gamma_0=95.2\text{ K};\quad \theta=1\text{ K}. \tag{23}$$

Based on Eqs. (9″), (22), and (23) one can evaluate the magnetic moment. One can see directly from Fig. 1 that the data obtained in Ref. 4 follows a quadratic dependence [see Eqs. (21) and (9″)]. In addition, Eq. (22) contains two adjustable parameters and they are $A=2.5\times 10^{-6}$, $B_0=0.7$. One can see that the total set of data obtained in Ref. 4 is in very good agreement with the theory using Eq. (22) and these values of A and B.

Let us note also that Ref. 4 was aimed at the investigation of the problem of whether the $H_{c2}(T)$ curve obtained in Ref. 12 (for Bi-based overdoped cuprate a similar effect was observed in Ref. 15) represents the critical field (then this curve separates superconducting and normal regions) or the irreversibility line (in this case the line separates vortex lattice and vortex liquid regions). Torque magnetometry is an ideal tool for such a study. The measurements[4] demonstrated the absence of vortices above the curve, but, nevertheless, a puzzling diamagnetic response was observed in the region $H>H_{c2}$. Moreover, a similar response has been observed above $T_c\cong 15$ K, and this observation makes the problem go beyond the nature of the $H_{c2}(T)$ dependence.

We think that the phenomenon observed in Ref. 4 is due to the inhomogeneous nature of their sample. The curve $H_{c2}(T)$ as well as the value $T_c^{\rm res}\cong 15$ K separates superconducting and ''normal'' regions, and the ''normal'' region displays normal transport properties. As for the magnetic response, it is determined by the presence of small superconducting regions with $T_{C;L}>T_c^{\rm trans}$, and this leads to the phenomenon observed in Ref. 4 (see Fig. 1).

IV. CONCLUSION

Inhomogeneous superconducting systems, such as overdoped cuprates, are characterized by the coexistence of an anomalous diamagnetic moment along with normal resistive dissipation. Such an unusual state occurs at $T>T_c^{\rm res}$ and is caused by the presence of small superconducting regions $\rho_0<\xi_0$ (ξ_0 is the coherence length) with a value of local $T_{c;L}>T_c^{\rm res}$.

The magnetic moment for such a system is described by Eqs. (21) and (22). The proximity effect is playing an important role and was explicitly taken into account.

Note that typical normal metals are characterized by a small value of magnetic susceptibility ($\cong 10^{-6}$) whereas the magnetic response of a superconductor is larger by almost 5–6 orders of magnitude. As a result, even the presence of a small number of superconducting clusters leads to a drastic increase in the magnetic moment.

The experimental data presented in Ref. 4 can be explained by our approach described herein based on an inhomogeneous structure of the system.

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APPENDIX

I. For the case of small deviation $\delta\Gamma=\Gamma_\infty-\Gamma$ one can use the perturbation theory. Based on Eq. (5), we obtain

$$\lambda_1=eHD-\frac{\int d\boldsymbol{\rho}\,\delta\Gamma|\Delta_0|^2}{\int d\boldsymbol{\rho}|\Delta_0|^2}. \tag{A1}$$

The term $\delta\Gamma\equiv\delta\Gamma(\boldsymbol{\rho})$ is localized near some point ρ_0. The eigenfunction Δ_0 can be written in the form $\Delta_0=\exp(-eH\rho^2/2)$. Inserting this expression in Eqs. (9) and (12), we obtain the value of the magnetic moment for the single cluster (spot):

$$\hat{M}_Z=-a[T_c^2-T^2-b]/\Gamma_\infty, \tag{A2}$$

where $a=(8\pi^3 vDL/H)$; $b=[eHD-(eH/\pi)\int d\boldsymbol{\rho}\delta\Gamma]6\Gamma_\infty/\pi^2$. This expression is valid if $(1/\pi D)\int d\boldsymbol{\rho}\delta\Gamma\ll 1$ and corresponds to the region

$$T > T_c(H) = [T_c^2 - (6/\pi^2) eHD\Gamma_\infty]^{1/2}.$$

II. Boundary condition (17) allows us to evaluate the eigenvalue λ_1. If $|\lambda_1|/eHD \gg 1$ (this is valid in the realistic case $\delta\Gamma\rho_0^2/D \gg 1$; $eH\rho_0^2 < 1$), one can simplify Eq. (17). Since $\int dt F(t,\rho)_{|\rho_0} \cong \int_0^\infty dx\, x^{-1} \exp[-(\alpha/x) - \overline{\lambda x}]$, the ratio on the right-hand side of Eq. (17) greatly exceeds 1, then the lowest eigenvalue λ_1 is the solution of the equation $\phi[0.5 - \tilde{\lambda} - \delta\tilde{\Gamma}; 1; \alpha(\rho_0)] = 0$. In reality, $\alpha(\rho_0) \ll 1$. As a result, we arrive at the equation

$$J_0[(2\rho_0^2(\lambda_1 + \delta\Gamma)/D)^{1/2}] = 0, \tag{A3}$$

where J_0 is the Bessel function. The expression (18) determining the eigenvalue λ_1 directly follows from Eq. (A3).

[1] A. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **61**, 1221 (1971) [Sov. Phys. JETP **34**, 651 (1972)]; **61**, 2147 (1971) [**34**, 1144 (1972)].

[2] A. Abrikosov and L. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1961) [Sov. Phys. JETP **12**, 1243 (1961)].

[3] P. de Gennes, J. Phys.: Condens. Matter **3**, 79 (1964); *Superconductivity in Metals and Alloys* (Benjamin, New York, 1966).

[4] C. Bergemann, A. Tyler, A. Mackenzie, J. Cooper, S. Julian, and D. Farrell, Phys. Rev. B **57**, 14 387 (1998).

[5] See the review: A. Larkin and Yu. Ovchinnikov in *Nonequilibrium Superconductivity*, edited by D. Landenberg and A. Larkin (Elsevier, Amsterdam, 1986), p. 530.

[6] A. Larkin and Yu. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys. JETP **28**, 1200 (1969)]; **64**, 1096 (1973) [**37**, 557 (1973)].

[7] Yu. Ovchinnikov, Zh. Eksp. Teor. Fiz. **79**, 1496 (1980) [Sov. Phys. JETP **52**, 755 (1980)]; A. Bezryadin, Yu. Ovchinnikov, and B. Pannetier, Phys. Rev. B **53**, 8553 (1996).

[8] Yu. Ovchinnikov and V. Kresin, Phys. Rev. B **54**, 1251 (1996).

[9] V. Kresin, S. Wolf, and Yu. Ovchinnikov, Phys. Rep. **288**, 347 (1997).

[10] N. Phillips, R. Fisher, and J. Gordon, in *Progress in Low-Temperature Physics*, edited by D. Brewer (North-Holland, Amsterdam, 1992), Vol. 13, p. 267; J. Wade, J. Loram, K. Mirza, J. Cooper, and J. Tallon, J. Supercond. **7**, 261 (1994).

[11] C. Niedermayer, C. Bernhard, V. Bunninger, H. Gluckler, J. Tallon, E. Ansaldo, and J. Budnick, Phys. Rev. Lett. **71**, 1764 (1993).

[12] A. Mackenzie, S. Julian, G. Lonzarich, A. Carrington, S. Hughes, R. Liu, and D. Sinclair, Phys. Rev. Lett. **71**, 1238 (1993).

[13] L. Gor'kov, Zh. Eksp. Teor. Fiz. **37**, 833 (1960) [Sov. Phys. JETP **10**, 593 (1960)].

[14] E. Helfand and N. Werthamer, Phys. Rev. Lett. **13**, 686 (1964); Phys. Rev. **147**, 288 (1966).

[15] M. Osofsky, R. Soulen, S. Wolf, J. Broto, H. Rakoto, J. Ousset, G. Coffe, S. Askenazy, P. Pari, I. Bozovik, J. Eckstein, and G. Virshup, Phys. Rev. Lett. **71**, 2315 (1993).