

Superconductor-insulator transition in two dimensions

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The superconductor-insulator transition of ultrathin films of bismuth, grown on liquid-helium-cooled substrates, has been studied. The transition was tuned by changing both film thickness and perpendicular magnetic field. Assuming that the transition is controlled by a $T=0$ critical point, a finite-size scaling analysis was carried out to determine the correlation length exponent ν and the dynamical critical exponent z . The phase diagram and the critical resistance have been studied as a function of film thickness and magnetic field. The results are discussed in terms of bosonic models of the superconductor-insulator transition, as well as the percolation models which predict finite dissipation at $T=0$.

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I. INTRODUCTION

After about two decades of research, the superconductor-insulator (SI) transition in disordered films of metals remains a controversial subject, mainly due to contradictory results in both theoretical and experimental studies. This work aims to improve the understanding of this phenomenon, which might also be relevant for high- T_c superconductors and possibly connected to interesting metal-insulator transitions in two-dimensional (2D) electron systems.

The superconductor-insulator transition in ultrathin films of metals is believed to occur at the absolute zero of temperature when the quantum ground state of the system is changed by tuning disorder, film thickness, carrier concentration, or magnetic field. Unlike finite temperature phase transitions in which thermal fluctuations are crucial, $T=0$ phase transitions are driven purely by quantum fluctuations. At finite temperatures, an underlying quantum phase transition manifests itself in the scaling behavior of the resistance with the appropriate tuning parameter and the temperature, along with the coherence length and dynamical critical exponents ν and z , respectively.¹ Various models of the superconductor-insulator transition in disordered films can be roughly divided in two groups: those in which the superconductivity is destroyed by fluctuations of the amplitude of the order parameter, and those which focus only on the phase fluctuations.

If superconductivity is destroyed only by phase fluctuations, then Cooper pairs persist on the insulating side of the transition and the transition may be described by a model of interacting bosons in the presence of disorder. Based on this assumption, Fisher *et al.*² suggested a scaling theory and a phase diagram for a two-dimensional system as a function of temperature, disorder, and magnetic field.^{3,4} The superconducting phase is considered to be a condensate of Cooper pairs with localized vortices, and the insulating phase is a condensate of vortices with localized Cooper pairs. At the transition, both vortices and Cooper pairs are mobile as they exchange their roles, which leads to a finite resistance. Some important predictions of the model are the universal value of this critical resistance and specific values of the critical exponents ν and z .

This so-called “dirty boson” problem has been extensively studied using quantum Monte Carlo simulations,^{5–12} real-space renormalization-group techniques,^{13,14} strong-coupling expansions,¹⁵ and in other ways.^{16–20} Finite temperature behavior in the vicinity of a quantum critical point was also studied analytically.^{21,22} A transition from a superfluid to a Mott insulator was found in the pure case, and to a Bose glass insulator in the presence of disorder, but there is still considerable disagreement as to the universality class of the transition, as well as the value of the critical resistance.

An alternative picture of interacting electrons^{23–25} proposes a different mechanism: the density of states and the Cooper pairing are suppressed on the insulating side of the superconductor-insulator transition due to an enhanced Coulomb interaction. The SI transition occurs as a consequence of fluctuations in the amplitude, rather than the phase of the order parameter. In other words, Cooper pairs break up into single electrons at the transition. Therefore the superconducting gap would also vanish at the transition.

The model of interacting electrons has also been studied numerically. Quantum Monte Carlo simulations of an attractive fermion Hubbard model with on-site interactions²⁶ yielded a direct superconductor-to-insulator transition in two dimensions without an intervening metallic phase. The critical resistance was found to depend on the strength of the attractive interaction, as a function of which a crossover from a fermionic to a bosonic regime occurs. The results of this theory qualitatively resemble the experimental data. A recent calculation of the effect of disorder on the gap in the density of states, using a similar model,²⁷ showed that the existence of a gap on the insulating side of the transition depends on the coupling strength, allowing for a Fermi insulator at weak and a Bose insulator at strong coupling. However the mean-field analysis of Ref. 20 for the same model at $T=0$, which allows for an inhomogeneous pairing amplitude, leads to a finite gap at all values of the coupling in the insulating state. This is explained by the formation of superconducting islands separated by insulating regions.

Experimentally, the destruction of superconductivity by disorder has been studied in films of MoGe,^{28,29} InO_x,^{30–34} and Bi, Pb, Ga, and Al,^{35–37} among others. Evidence was found of T_c going to zero with increasing disorder²⁸ imply-

ing the destruction of Cooper pairs at the transition. Tunneling experiments also seem to support the fermionic picture. Valles, Dynes, and Garno found that the superconducting gap and the mean-field transition temperature are both suppressed as disorder is increased, and that the gap vanishes on the insulating side of the superconductor-insulator transition.³⁸ Hsu, Chervenak, and Valles carried out tunneling studies of the magnetic-field-driven superconductor-insulator transition in PbBi/Ge films, and found a large number of quasiparticle states near the Fermi energy.³⁹ They estimated the average number of Cooper pairs in a coherence volume to be on the order of one at the superconductor-insulator transition. This result, in combination with the disappearance of the energy gap, was interpreted as evidence of the superconductor-insulator transition being driven by fluctuations in the amplitude of the order parameter. Alternatively, it is possible for the superconducting energy gap to be reduced or the tunneling density of states to be broadened as a consequence of phase fluctuations.⁴⁰ Thus the absence of the gap in these tunneling studies does not necessarily mean that Cooper pairs are absent on the insulating side of the superconductor-insulator transition, but it may imply that a full picture might have to include fermionic degrees of freedom. Indeed Tešanović and co-workers have argued that superconductors near the upper critical field have a gapless spectrum for fermionic excitations.⁶⁵

Evidence of the importance of the bosonic picture can be found in the work of Paalanen, Hebard, and Ruel.³¹ These workers studied the magnetoresistance and the Hall effect in amorphous InO_x films and observed two distinct transitions: one at a critical field B_{xx}^c where the longitudinal resistance diverges and the system presumably undergoes a transition from a superconducting phase to a Bose glass insulator with localized Cooper pairs, and the other at a higher field B_{xy}^c , where the transverse resistance diverges and the Cooper pairs of the Bose glass insulator presumably unbind. The transition in the transverse resistance occurred at the same magnetic field where the longitudinal resistance showed a maximum. Since a Bose insulator might be expected to have a higher resistance than an insulator with localized single electrons, and from the disorder dependence of B_{xx}^c/B_{xy}^c , this was interpreted as evidence of the bosonic nature of the insulating state close to the superconductor-insulator transition. Similar behavior was observed by other groups.³⁴ Magnetoresistance studies of amorphous InO_x films by Gantmakher *et al.*³² also seem to support the bosonic picture. Furthermore, a linear component of the magnetoresistance observed in the insulating regime in amorphous Bi films can be interpreted as a signature of vortex motion.⁴¹

In the context of the scaling behavior, the thickness tuned transition of ultrathin films of amorphous Bi has been studied in zero magnetic field.³⁶ A scaling analysis of the magnetic field tuned SI transition has been carried out for thin films of InO_x (Ref. 30) and MoGe.²⁹ All of these investigations found $\nu \approx 1.3$ and $z \approx 1$, consistent with the theoretical predictions of the boson Hubbard model. On the other hand, the magnetic-field-driven transition in films of the high-temperature superconductor La_{2-*x*}Sr_{*x*}CuO₄ appears to be qualitatively different from the transitions found for ImO_x and MoGe.⁶⁶

Yet another interpretation of the experimental data has

recently been proposed by Shimshoni, Auerbach, and Kapitulnik⁴² and expanded upon by Mason and Kapitulnik.⁴³ In this picture, a film contains both insulating and superconducting puddles, and transport is dominated by tunneling or activated hopping between them. The SI transition then occurs as a consequence of the percolation of one phase or the other. Since the correlation length exponent in 2D classical percolation is 4/3, this is consistent with $\nu \approx 1.3$ observed in most experiments. This model also predicts a saturation of the resistance at very low temperatures, which seems to be supported by the experimental data of Ephron *et al.*⁴⁴ and Yazdani and Kapitulnik.²⁹ Similar effects have been observed in the much earlier work of Wang *et al.*⁴⁵ on underdoped high- T_c (cuprate) films. These ideas may be relevant to similar features of the results of Kravchenko *et al.* on two-dimensional electron gas systems.⁴⁶ In all studies in which there is flattening in $R(T)$ at low temperatures, one must be concerned with the possibility of electrical noise being the source of the effect. Also in multicomponent materials such as MoGe and underdoped cuprates there is always a possibility of second phases affecting the outcome. Furthermore, it has recently been proposed that the flattening in $R(T)$ at low temperatures may be a signature of Bose metal, a phase in which the Cooper pairs are mobile but do not condense.⁴⁷

The quantitative results of the study of the magnetic-field-tuned superconductor-insulator transition presented here for disordered metal systems are in serious disagreement with previous measurements of this transition, adding yet another puzzle to this problem, and calling for a re-examination of existing models. The thickness-tuned transition has also been studied in a nonzero magnetic field. This allows for the construction of a phase diagram and a direct comparison of the two different ways of tuning the SI transition, by varying thickness or magnetic field.

This paper is organized as follows: the finite-size scaling procedures used to determine the critical exponents are described in Sec. II. Experimental details are given in Sec. III. Section IV focuses on the magnetic-field-tuned transition, while the analysis of the thickness-tuned transition in finite magnetic field is presented in Sec. V. In Sec. VI, the phase diagram as a function of thickness and magnetic field is presented. The critical resistance and its apparent nonuniversality are discussed in Sec. VII. The results and their implications are summarized and further discussed in Sec. VIII. A brief account of a portion of this work has been previously reported.⁴⁸

II. SCALING PROCEDURES

The scale of fluctuations on either side of a quantum phase transition is set by a diverging correlation length $\xi \propto \delta^{-\nu}$ and a vanishing characteristic frequency $\Omega \propto \xi^{-z}$. Here δ is the deviation from the critical point $\delta = |K - K_c|$, where K is the control or tuning parameter, which drives the system through the transition (i.e., disorder, thickness, magnetic field, etc.), K_c is the critical value of K at the transition, ν is the correlation length exponent and z is the dynamical critical exponent. The exponents ν and z determine the universality class of the transition. They may not depend on the microscopic details of the physics of the system under study,

but on its dimensionality, the symmetry group of its Hamiltonian, and the range of interactions.

The resistance of a two-dimensional system in the quantum critical regime follows the scaling relation^{1,3}

$$R(\delta, T) = R_c f(\delta T^{-1/\nu z}). \quad (1)$$

Here $\delta = |d - d_c|$ in the case of the thickness-tuned transition and $\delta = |B - B_c|$ in the case of the magnetic-field-tuned transition. R_c is the critical resistance at $\delta = 0$ and $f(x)$ is a universal scaling function such that $f(0) = 1$.

The first step in the analysis of the experimental data is to determine the critical value of the tuning parameter and plot the resistance as a function of δ . The δ axis is then rescaled by a factor t :

$$R(\delta, t) = R_c f(\delta t), \quad (2)$$

where the parameter $t(T)$ is determined at each temperature by performing a numerical minimization which yields the best collapse of the data. If the resistance really follows the scaling law [Eq. (1)], it is obvious that $t(T)$ has to be a power law in temperature, $t(T) \equiv T^{-1/\nu z}$. The exponent product νz is then found by plotting $t(T)$ as a function of T on a log-log scale, and determining the slope which is then equal to $-1/\nu z$.

Similarly, at a constant temperature,²⁹

$$R(\delta, E) = R_c f(\delta E^{-1/\nu(z+1)}), \quad (3)$$

where E is the electric field across the sample. This time, the δ axis is rescaled by a field-dependent factor $t(E)$, which should be a power law in electric field $t(E) \equiv E^{-1/\nu(z+1)}$, and the exponent $\nu(z+1)$ can then be determined from the field dependence of the parameter $t(E)$.

The main advantage of this scaling procedure is that it requires neither prior knowledge of the critical exponents, nor the temperature and thickness dependence of the resistance. The critical exponents are determined empirically from the data, with the critical exponent product as the only adjustable parameter, while the critical value of the tuning parameter is determined independently. The temperature scaling determines the product νz , while the electric field scaling determines $\nu(z+1)$. Combining the two results, the correlation length exponent ν and the dynamical exponent z can be determined separately.

An alternative way to determine these critical exponent products is to evaluate a derivative of the resistance with respect to K at its critical value K_c :³⁰

$$(\partial R / \partial K)_{K_c} \propto R_c T^{-1/\nu z} f'(0), \quad (4)$$

where $K \equiv d$ at the thickness-tuned transition and $K \equiv B$ at the magnetic-field-tuned transition, and $f'(0)$ is a constant. Plotting $(\partial R / \partial K)_{K_c}$ as a function of T^{-1} on a log-log scale should yield a straight line, with a slope equal to $1/\nu z$. The same method can be applied to the electric-field scaling to determine $1/\nu(z+1)$, and then ν and z can be calculated from the results.

In the work described below, both scaling procedures were used to obtain the critical exponents, in order to check

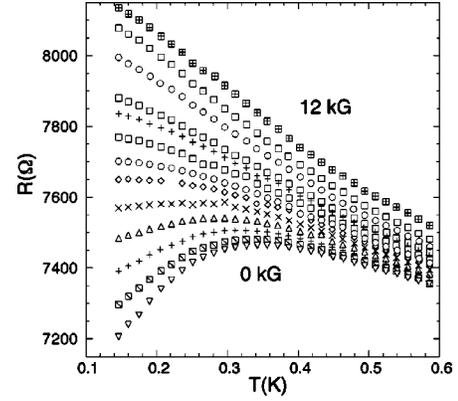


FIG. 1. Resistance per square as a function of temperature in different magnetic fields, ranging from 0 kG (bottom) to 12 kG (top), in 1 kG increments.

their consistency. The exponents obtained using two different methods were found to be the same, within the experimental uncertainty.

III. EXPERIMENTAL METHODS

Ultrathin Bi films were evaporated on top of a 10 Å thick layer of amorphous Ge, which was predeposited onto either SrTiO₃ or glazed alumina substrates. The substrate temperature was kept well below 20 K during all depositions and all the films were grown *in situ* under UHV conditions ($\sim 10^{-10}$ Torr). The film thickness was gradually increased through successive depositions in increments of 0.1–0.2 Å. Resistance measurements were carried out between the depositions using a standard dc four-probe technique, with currents up to 50 nA. A detailed temperature dependence of the resistance in zero field and in magnetic field was recorded at each film thickness in the temperature range between 0.14 and 15 K, where the lowest temperatures were achieved using a dilution refrigerator. As the film thickness increased from 7–15 Å, the temperature dependence of the resistance of the system changed from insulatorlike to superconductorlike at low temperatures, with no sign of re-entrant behavior typically observed in granular films.³⁵ The films that were superconducting in zero field were driven insulating by applying a magnetic field of up to 12 kG perpendicular to the plane of the sample using a superconducting split-coil magnet. The scaling procedures described above were applied to the magnetic-field-tuned transition, as well as to the thickness-tuned transition in both zero field and in a fixed magnetic field.

IV. MAGNETIC-FIELD-TUNED SI TRANSITION

The resistance as a function of temperature for seven films with varying degrees of disorder was studied in magnetic fields up to 12 kG applied perpendicular to the plane of the sample. A typical temperature dependence of the resistance as the magnetic field changes is shown in Fig. 1. In zero field, the resistance decreases with decreasing temperature suggesting the existence of superconducting fluctuations. A magnetic field destroys this downward curvature, and at some critical magnetic field B_c the resistance is independent of temperature. In magnetic fields higher than B_c the

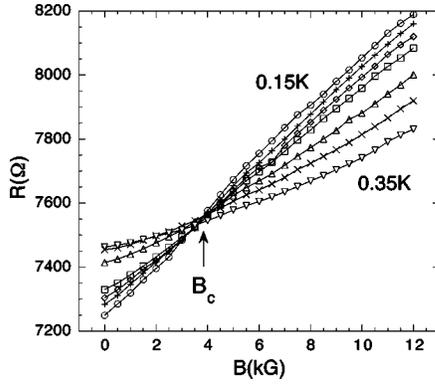


FIG. 2. Resistance per square as a function of magnetic field for a bismuth film close to the transition. Different curves represent different temperatures: 0.15, 0.17, 0.19, 0.2, 0.25, 0.3, and 0.35 K.

film is insulating, with $\partial R/\partial T < 0$. Figure 2 shows the resistance as a function of magnetic field for different temperatures.

If the sheet resistance is normalized by the value of the critical resistance at each thickness $R/R_c(d)$, then all the data can be collapsed onto a single curve. The collapse of the normalized resistance data as a function of δt for five samples is shown in Fig. 3. The critical exponent product νz , determined from the temperature dependence of the parameter t (inset of Fig. 3), is found to be $\nu z = 0.7 \pm 0.2$, apparently independent of the film thickness. The same exponent products were obtained using the alternative method of plotting $(\partial R/\partial B)_{B_c}$ vs T^{-1} on a log-log plot and determining the slope which is equal to $1/\nu z$, as shown on Fig. 4.

Electric-field scaling were also carried out for one of the samples. Unfortunately, there was not enough data available for the insulating side of the transition to carry out a complete analysis, but the data on the superconducting side were sufficient to obtain the value of the critical exponent product $\nu(z+1)$. The magnetic-field dependence of the sheet resistance for different values of electric field applied across the sample is shown on Fig. 5. The resistance data were then plotted as a function of $(B - B_c)$, and rescaled by a parameter $t(E)$ to obtain the best collapse of the data, shown in

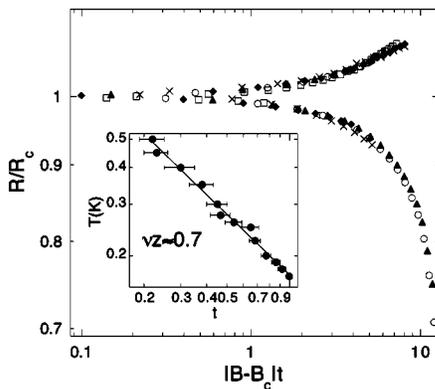


FIG. 3. Normalized resistance per square as a function of the scaling variable $T^{-1/\nu z} |B - B_c|$. Each symbol represents one film at different temperatures (only a small portion of the data is shown for clarity). Inset: The fitting a power law to the temperature dependence of the parameter t determines the value of νz .

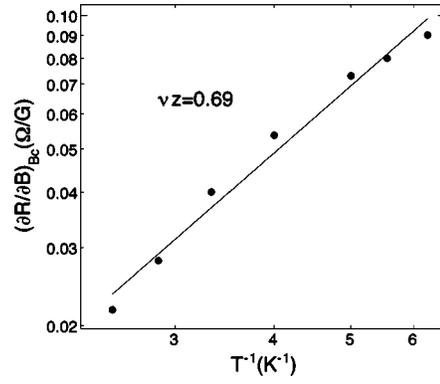


FIG. 4. The critical exponent product νz for the magnetic-field-tuned transition as determined by the inverse slope of $\partial R/\partial B$ at the critical value of B_c plotted vs $1/T$.

Fig. 6. For the electric-field dependence of the parameter $t(E)$, shown in the inset of Fig. 6, the best power-law fit was obtained for $\nu(z+1) \approx 1.4$. Combining this result with the result of the temperature scaling, it follows that $z \approx 1$ and $\nu \approx 0.7$ for the magnetic-field-tuned superconductor-insulator transition.

In contrast with our findings, previous studies of thin films of amorphous InO_x (Ref. 30) and MoGe (Ref. 29) both showed $\nu \approx 1.3$ and $z = 1$ for the magnetic-field-tuned superconductor-insulator transition. Our surprising result is also in obvious disagreement with the prediction of the scaling theory (from which $\nu \geq 1$ (Refs. 49 and 3) for a disordered system), as well as with the percolation-based models⁴² (from which $\nu \approx 1.3$ would be expected).

V. THICKNESS-TUNED SI TRANSITION

For very thin films, the resistance increases exponentially with decreasing temperature, while for the thicker films the resistance goes to zero as the films become superconducting. At the critical thickness d_c , the resistance is temperature independent, and the system is expected to stay metallic down to $T = 0$.

Using the same methods described above, the critical exponent product νz was determined to be 1.2 ± 0.2 when the

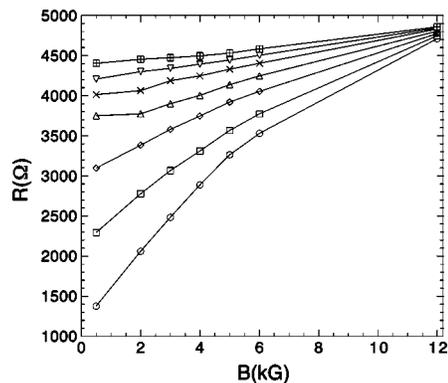


FIG. 5. Resistance per square as a function of magnetic field at different electric fields across the film: 0.5 (bottom), 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5 V/m (top). Only $B < B_c$ is shown where the resistance increases with increasing electric field. The temperature is 0.7 K.

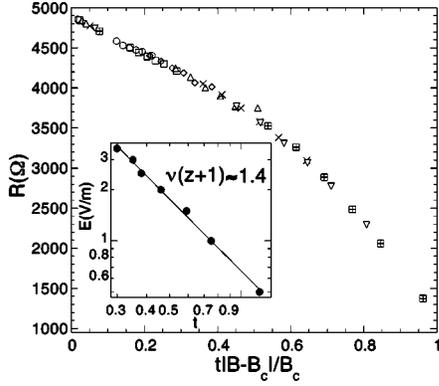


FIG. 6. Resistance per square as a function of the scaling variable $t|B - B_c|$ for different electric fields: 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5 V/m. Here $t = E^{-1/\nu(z+1)}$ is treated as an adjustable parameter to obtain the best collapse of the data. Inset: The fitting of a power law to the temperature dependence of the parameter t determines the value of νz .

superconductor-insulator transition was tuned by changing the film thickness in zero magnetic field.⁴⁸ A similar scaling behavior has been found in ultrathin films of Bi by Liu *et al.*,³⁶ with the critical exponent product $\nu z = 2.8$ on the insulating side and $\nu z = 1.4$ on the superconducting side of the transition. The fact that νz was found to be different on the two sides of the transition raises the question of whether the measurements really probed the quantum critical regime. It is likely that the scaling was carried out too deep into the insulating phase, forcing the scaling form [Eq. (1)] on films which were in a fundamentally different insulating regime. Such films should not be expected to scale together with the superconducting films, hence the discrepancy on the insulating side of the transition. In the present work, the measurements were carried out at lower temperatures than previously studied and with more detail in the range of thicknesses close to the transition. Both sides of the transition scaled with $\nu z \approx 1.2$, which is close to the value obtained by Liu *et al.* on the superconducting side of the transition. This result is also in very good agreement with the predictions of the scaling theory,⁴ renormalization-group calculations,^{13,14,19} and Monte Carlo simulations.^{7,9,6}

All previous experiments which studied the thickness or disorder tuned superconductor-insulator transition were carried out in zero magnetic field. An applied magnetic field is generally expected to change the universality class of the transition since it breaks time reversal symmetry. One would therefore expect the critical exponent product νz to be different in the presence of a finite magnetic field. Furthermore, the thickness-tuned transition in a finite magnetic field might be expected to be in the same universality class as the magnetic-field-tuned transition at fixed thickness.

The thickness-tuned superconductor-insulator transition in a finite magnetic field was probed by sorting the magnetoresistance data which were carefully taken as a function of temperature and magnetic field for each film. A detailed scaling analysis was carried out at fixed magnetic fields of 0.5, 1, 2, 3, 4.5, and 7 kG for one set of films, and 12 kG for a different set of films. For each value of the magnetic field, the resistance was plotted as a function of the film thickness at different temperatures, ranging from 0.14–0.5 K, in order

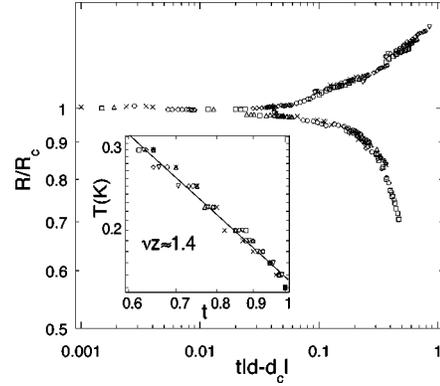


FIG. 7. Normalized resistance per square as a function of the scaling variable $t|d - d_c|$ in different magnetic fields: 0.5 (squares), 1.0 (circles), 3.0 (crosses), 4.5 (triangles), and 7.0 kG (diamonds). Inset: The fitting of a power law to the temperature dependence of the parameter t determines the value of νz .

to determine the critical thickness at that field. If the sheet resistance is normalized by the critical value at each field $R/R_c(B)$, then the normalized resistance data as a function of the scaling variable for all temperatures and all values of the magnetic field collapsed onto a single curve, as shown on Fig. 7. The critical exponent product determined from the parameter $t(T)$ (inset of Fig. 7) was found to be $\nu z = 1.4 \pm 0.2$, apparently independent of the magnetic field. Once again, the alternative scaling procedure yielded very similar results, as shown in Fig. 8.

This value of the product νz is a factor of two larger than that obtained for the magnetic field-tuned transition. It is, however, very close to that obtained from the analysis of the zero-field transition carried out using data from the same set of films, which was $\nu z = 1.2 \pm 0.2$. Given the experimental uncertainties, it is hard to say whether this difference in value of the exponent products of the thickness driven transitions in zero and finite magnetic field reflects a difference between the universality classes. These exponent products are close to those found in Monte Carlo simulations of the $(2+1)$ -dimensional classical XY model with disorder by Cha and Girvin.⁷

VI. THE PHASE DIAGRAM

Combining the data obtained from the thickness-tuned transitions in a fixed magnetic field and the field-tuned tran-

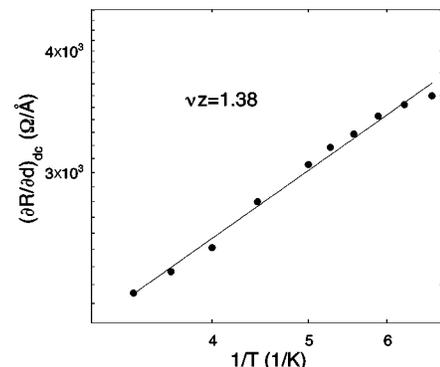


FIG. 8. The critical exponent product νz for the thickness-tuned transition as determined by the inverse slope of $\partial R/\partial d$ at the critical value of d_c plotted vs $1/T$.

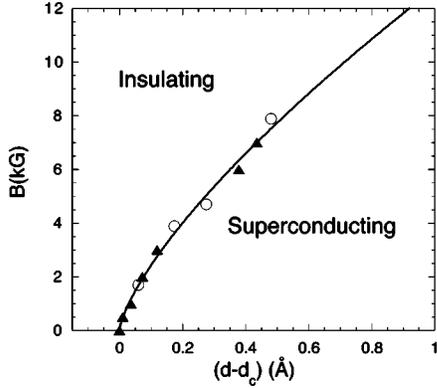


FIG. 9. The phase diagram in the d - B plane in the $T=0$ limit. The points on the phase boundary were obtained from thickness tuned transitions (triangles) and magnetic-field-tuned transitions (circles). The solid line is a power-law fit. Here d_c is taken to be the critical thickness in zero field.

sitions at the fixed thickness, one can construct a phase diagram with thickness and magnetic field as independent variables. This is shown in Fig. 9. The films characterized by parameters which lie above the phase boundary are “insulating” ($\partial R/\partial T < 0$ at finite temperatures), and the ones below it are “superconducting” ($\partial R/\partial T > 0$ at finite temperatures). The phase boundary is a power law:

$$B_c \propto |d - d_c|^x. \quad (5)$$

The best fit to the data yields $x=0.7$. Near the critical thickness for the zero-field transition, a simple dimensionality argument³ suggests that the critical magnetic field should scale as

$$B_c \propto \frac{\Phi_0}{\xi^2}, \quad (6)$$

where Φ_0 is the flux quantum. Since the correlation length is $\xi \propto |d - d_c|^{-\nu}$, one might expect the critical field to be

$$B_c \propto |d - d_c|^{2\nu}. \quad (7)$$

According to the phase boundary obtained in this experiment [see Eq. (5)], this would mean that $\nu=0.35$, a value not consistent with the results of the scaling analysis carried out on the same films. It also does not agree with $\nu=1.3$ obtained by Refs. 30 and 29. There is no obvious physical reason for such a small value of ν and implied large values of z , so this discrepancy is a mystery at this time. It is possible that the simple argument expressed in Eqs. (6) and (7) is too naive.

Another surprising feature of the experimental results is that the critical exponent product νz evidently depends on whether the phase boundary is crossed vertically (changing the thickness at a constant magnetic field), in which case $\nu z \approx 1.4$, or horizontally (changing the magnetic field at a fixed thickness), in which case $\nu_B z_B \approx 0.7$. One might expect the critical exponents to not depend on the direction in which the boundary is crossed. If, however, the actual tuning parameter were not film thickness, but some other physical parameter which was a function of thickness, a factor of 2 in the critical exponent product determined from an analysis

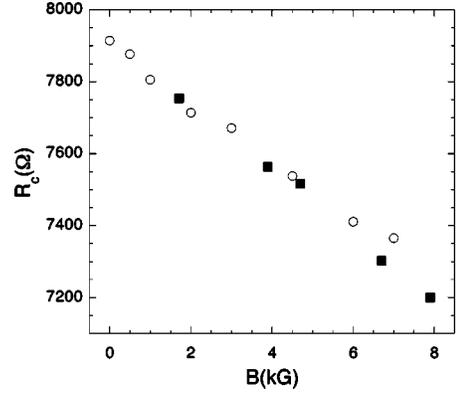


FIG. 10. The critical resistance as a function of the critical field for a series of bismuth films. Here R_c decreases with increasing thickness, as thicker films have lower normal-state resistances and higher critical fields.

using thickness rather than the “correct” control parameter might result. The correct control parameter might be some measure of disorder, electron screening, damping, or Cooper pair density. The detailed functional form of the thickness dependence of these parameters for quench-condensed films is not known.

Another possibility is that there are actually two phase boundaries, separating three different regimes, so that each exponent belongs to a different phase boundary. There has been some indication of a vortex liquid phase in between the superconducting (vortex glass) phase and the insulating (Bose glass) phase.^{50,51} Since there only appears to be one phase boundary, that is probably not the case. It is possible, however, that the two boundaries could be indistinguishable over the range of parameters explored in these studies, but would become apparent at higher fields, greater film thicknesses, or lower temperatures. These matters need to be investigated further.

VII. THE CRITICAL RESISTANCE

The critical resistance for the field-driven transition, contrary to the predictions of the dirty boson models, does not seem to be universal. Figure 10 shows that R_c decreases as the critical field increases, roughly in a linear fashion. Since thicker films have lower normal-state resistances and higher critical fields, this also means that R_c decreases with increasing thickness and decreasing normal-state resistance. Very similar behavior was observed by Yazdani and Kapitulnik.²⁹ In order to explain the nonuniversal behavior of the critical resistance, these authors proposed a two-channel conduction model, in which the conductance due to the electron (fermion) channel adds to the conductance due to the boson channel. When the unpaired electrons are strongly localized, the conduction is mostly due to bosons, and the resistance is close to $R_Q = h/4e^2$, as predicted by the boson Hubbard model. In the opposite limit, unpaired electrons contribute significantly to the conduction at the transition. Films with lower normal-state resistances would then have lower critical resistances due to the larger fraction of normal electrons. The critical resistances in our experiment, however, are all *greater* than R_Q and their values could only be explained this

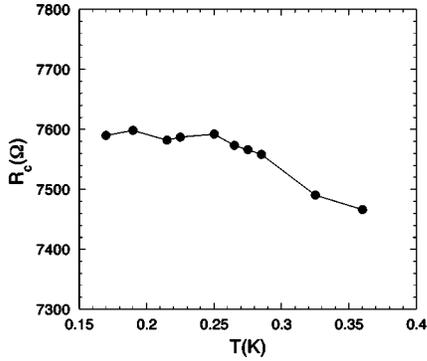


FIG. 11. The critical resistance as a function of temperature for a 12.353-Å-thick film.

way if the quantum resistance due to pairs was itself greater than R_Q .

The conductance due to the electronic channel in a magnetic field might also depend on the strength of the spin-orbit interactions, which is another difference between our samples and those of Refs. 30 and 29. The strength of the spin-orbit interactions is typically proportional to Z^4 , where Z is the atomic number. Since Bi is a heavy metal, spin-orbit interactions are stronger than in the lighter InO_x and MoGe. It is known that in the weakly localized systems with strong spin-orbit interactions the magnetoresistance is positive, while it is negative otherwise.^{52,53} If weakly localized unpaired electrons really contributed significantly to the conduction at the magnetic-field-tuned superconductor-insulator transition at the experimentally accessible finite temperatures, the contribution to the magnetoresistance due to localization effects could have a positive or a negative sign, depending on the strength of the spin-orbit interactions. This would make R_c bigger in the case of Bi films, and smaller in the case of InO_x and MoGe, consistent with experimental observations. There is, however, a striking similarity in the magnetic field and normal-state resistance dependence of the critical resistance of the Bi films and MoGe films of Ref. 29: even though their critical resistances fall on the opposite sides of R_Q , they both decrease with magnetic field roughly linearly, with almost the same slope.

Strictly speaking, the critical resistance is predicted to be universal only at $T=0$, while the finite temperature corrections are expected to be scaled with the Kosterlitz-Thouless transition temperature T_c :³

$$R_c(B_c, T) = R_c^* + O\left(\frac{T}{T_c}\right)^2, \quad (8)$$

where R_c^* is the universal resistance at $T=0$, and R_c is the critical resistance at some finite temperature as measured in the experiments. A closer look at the crossing plots such as that of Fig. 2, reveals that the critical resistance is indeed slightly temperature dependent. A considerable amount of noise over the accessible temperature range made it hard to compare this temperature dependence with Eq. (8), but qualitative behavior is shown in Fig. 11. Normal-state resistances of the MoGe films²⁹ are a factor of 3–10 lower than the Bi films considered here, which means that our samples are probing a different part of the phase diagram (normal-state resistances are inversely proportional to the film thickness in

our experiment), and the finite temperature corrections might be more important in one case than the other. Indeed, somewhat higher critical resistances were found in InO_x films if the temperature dependence of R_c is taken into account.³³

A recent analytical calculation of the critical resistance of a two-dimensional system at finite temperatures in the dirty boson model including Coulomb interactions²² yielded a critical resistance of $\approx 1.4R_Q$. The author suggested that the next order correction would bring the result closer to R_Q . This result is in excellent agreement with the critical resistance found in the present measurements, which was $1.1R_Q - 1.2R_Q$. Monte Carlo simulations of the (2+1)-dimensional XY model without disorder⁸ also find the critical resistance to be $R_c = 7.7 \text{ k}\Omega$, again very close to the value found in this work.

VIII. DISCUSSION

A lot of attention has been focused recently on the effects of dissipation on SI transitions.^{42,43,11,54–56} Within the picture proposed by Shimshoni, Auerbach, and Kapitulnik,⁴² the transition between the superconducting and the insulating state is of a percolative nature. On the insulating side of the transition, electrical transport occurs through activation or tunneling of Cooper pairs between superconducting domains. Likewise, on the superconducting side, vortices tunnel from one insulating domain to another. Using incoherent Boltzmann transport theory, Shimshoni, Auerbach, and Kapitulnik derive resistivity laws in different temperature regimes and predict finite dissipation at $T=0$ for all values of the magnetic field. Their results seem to be supported by measurements on several different systems: thin films,^{44,29} 2D Josephson junction arrays,⁵⁷ Si metal-oxide-semiconductor field-effect transistor (MOSFET's),⁴⁶ and QH systems,⁵⁹ where a saturation of the resistance at low temperatures is observed and attributed to dissipation effects. The percolative nature of the transition can explain the value of $\nu \approx 1.3$ found in most of the field-tuned experiments on thin films,^{30,29} as well as the apparent symmetry between insulating and the conducting phase observed in other experiments.^{57,46,58,59}

In contrast with the above-mentioned results, we do not observe any saturation in the temperature dependence of the resistance as the temperature decreases, or in other words, $\delta R / \delta T$ is nonzero down to the lowest temperatures, which were above 0.1 K. Of course, investigation down to even lower temperatures might lead to a different conclusion. However, a satisfactory fit to the resistivity laws predicted by Shimshoni, Auerbach, and Kapitulnik⁴² could not be obtained.

Mason and Kapitulnik⁴³ recently proposed an interesting phase diagram for the SI transition which takes into account the possibility of a coupling of the system to a dissipative bath. They argued that this coupling, which becomes important when the critical point is approached, can result in a metallic phase. In this picture, a direct SI transition is still possible for very weak coupling, while for a stronger coupling the system goes through a metallic phase and is truly superconducting only at the lowest magnetic fields.

The fact that the typical sheet resistances of our samples are about a factor of 5 higher than those in which resistance

leveling was observed⁴⁴ might just mean that our samples are in the weak-coupling regime. However, the correlation length exponent determined in our experiment for the magnetic-field-tuned transition, using two different methods, on different physical samples and at several levels of disorder was found to be $\nu \approx 0.7$, which is not consistent with the exponent expected from the classical 2D percolation theory, $\nu = 4/3$, even with much more generous error bars.

A coherence length exponent of 0.7 is also inconsistent with what was believed to be an exact theorem,⁴⁹ which predicts $\nu \geq 1$ in two dimensions in a presence of disorder. It is interesting to note that our exponent agrees with the result of the classical 3D XY model which is suggested to be relevant in the absence of disorder.² Numerical simulations of a $(2+1)$ -dimensional XY model⁸ and the Boson-Hubbard model at $T=0$ (Ref. 10) without disorder also find $z=1$ and $\nu=0.7$. However, recently it was suggested that the nature of disorder averaging may introduce a new correlation length, different from the intrinsic one, which might lead to $\nu < 1$ even for a disordered system.⁶⁰

There is also a possibility that the local dissipation coupled to the phase of the superconducting order parameter due to gapless electronic excitations might change the universality class of the system and lead to a nonuniversal critical resistance.¹¹ The critical resistance would then be expected to increase with increasing damping due to dissipation. The latter would be expected to increase with decreasing normal-state resistance. However, we observe that the critical resistance decreases as the normal-state resistance decreases, which is exactly the *opposite* of the behavior predicted by Wagenblast *et al.*¹¹

We should note that $\nu \approx 0.7$ was also found for the magnetic-field-tuned insulator-conductor transition in Si MOSFET samples,⁶¹ suggesting a possible connection between the two phenomena.

Our results for the magnetic-field-tuned SI transition seem to be consistent with the predictions of bosonic models, rather than percolation models. This is further supported by the transport studies in the insulating regime, where the mag-

netoresistance cannot be explained by the weak-localization theory only,⁴¹ and the temperature dependence of the resistance fits the predictions of Das and Doniach⁶² for the bosonic conduction. These observations still need to be reconciled with the results of the tunneling experiments which find no superconducting gap in the insulating regime. The tunneling experiments might, however, be emphasizing regions of the samples containing quasilocalized single electron states below the gap, or those in which the amplitude fluctuations break the system into superconducting ‘‘islands’’ with finite spectral gaps in the density of states, as recently predicted.²⁰ A highly nonuniform gap has also been predicted by Herbut⁶³ for the case of large disorder. This problem might be clarified using spatially resolved scanning tunneling spectroscopy at low temperatures, which may be able to detect *local* variations in the density of states.

Such studies might also help answer the question as to why ν different for the thickness- and magnetic-field-tuned transitions on the same samples. In the case of the thickness-tuned transition, the correlation length exponent is close to what might be expected from the percolation theory. There is a major difference between the magnetic-field-tuned and the thickness-tuned transitions: when the transition is tuned by the magnetic field, the microstructure of the sample stays fixed, while in the case of the thickness-tuned transitions it changes slightly with each film in the sequence. It may be that in this case the percolation effects become relevant, complicating the determination of the critical exponents.⁶⁴

Finally, the shape of the phase boundary poses a further challenge to theorists. We are currently investigating the role of the dissipation in this system in more detail, using a 2D electron gas as a substrate, similar to the experiment of Rimberg *et al.*⁵⁶

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¹S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, *Rev. Mod. Phys.* **69**, 315 (1997).

²M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

³M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990).

⁴M. P. A. Fisher, G. Grinstein, and S. M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990).

⁵R. T. Scalettar, G. G. Batrouni, and G. T. Zimányi, *Phys. Rev. Lett.* **66**, 3144 (1991); W. Krauth, N. Trivedi, and D. Ceperley, *ibid.* **67**, 2307 (1991).

⁶M. Makivić, N. Trivedi, and S. Ullah, *Phys. Rev. Lett.* **71**, 2307 (1993).

⁷M.-C. Cha, M. P. A. Fisher, S. M. Girvin, M. Wallin, and A. P. Young, *Phys. Rev. B* **44**, 6883 (1991).

⁸M.-C. Cha and S. M. Girvin, *Phys. Rev. B* **49**, 9794 (1994).

⁹E. S. Sorensen, M. Wallin, S. M. Girvin, and A. P. Young, *Phys. Rev. Lett.* **69**, 828 (1992); M. Wallin, E. S. Sorensen, S. M. Girvin, and A. P. Young, *Phys. Rev. B* **49**, 12 115 (1994).

¹⁰J. Kisker and H. Rieger, *Phys. Rev. B* **55**, R11 981 (1997).

¹¹Karl-Heinz Wagenblast, Anne van Otterlo, Gerd Schön, and Gergely T. Zimányi, *Phys. Rev. Lett.* **78**, 1779 (1997); **79**, 2730 (1997).

¹²Jack Lidmar, Mats Wallin, Carsten Wengel, S. M. Girvin, and A. P. Young, *Phys. Rev. B* **58**, 2827 (1998).

¹³K. G. Singh and D. S. Rokhsar, *Phys. Rev. B* **46**, 3002 (1992).

¹⁴L. Zhang and M. Ma, *Phys. Rev. B* **45**, 4855 (1992).

¹⁵J. K. Freericks and H. Monien, *Phys. Rev. B* **53**, 2691 (1996).

¹⁶A. Gold, *Z. Phys. B* **52**, 1 (1983).

¹⁷M. Ma, B. I. Halperin, and P. A. Lee, *Phys. Rev. B* **34**, 3136 (1986).

¹⁸M. Ma and E. Fradkin, *Phys. Rev. Lett.* **56**, 1416 (1986).

¹⁹Igor F. Herbut, *Phys. Rev. Lett.* **79**, 3502 (1997).

²⁰Amit Ghosal, Mohit Randeria, and Nandini Trivedi, *Phys. Rev. Lett.* **81**, 3940 (1998).

²¹Subir Sachdev, *Phys. Rev. B* **55**, 142 (1997).

²²Igor F. Herbut, *Phys. Rev. Lett.* **81**, 3916 (1998).

²³Sadamichi Maekawa and Hidetoshi Fukuyama, *J. Phys. Soc. Jpn.* **51**, 1380 (1981).

- ²⁴D. Belitz, *Phys. Rev. B* **40**, 111 (1989).
- ²⁵A. Finkel'shtein, *Pis'ma Zh. Eksp. Teor. Fiz.* **45**, 37 (1987) [*JETP Lett.* **45**, 46 (1987)]; *Physica B* **197**, 636 (1994).
- ²⁶Nandini Trivedi, Richard T. Scalettar, and Mohit Randeria, *Phys. Rev. B* **54**, R3756 (1990).
- ²⁷Carey Huscroft and Richard T. Scalettar, *Phys. Rev. Lett.* **81**, 2775 (1998).
- ²⁸J. M. Graybeal and M. R. Beasley, *Phys. Rev. B* **29**, 4167 (1984); *Phys. Rev. Lett.* **56**, 173 (1986).
- ²⁹Ali Yazdani and Aharon Kapitulnik, *Phys. Rev. Lett.* **74**, 3037 (1995).
- ³⁰A. F. Hebard and M. A. Paalanen, *Phys. Rev. Lett.* **65**, 927 (1990).
- ³¹M. A. Paalanen, A. F. Hebard, and R. R. Ruel, *Phys. Rev. Lett.* **69**, 1604 (1992).
- ³²V. F. Gantmakher, M. V. Golubkov, J. G. S. Lok, and A. K. Geim, *Zh. Eksp. Teor. Fiz.* **109**, 1765 (1996) [*JETP* **82**, 951 (1996)].
- ³³V. F. Gantmakher, M. V. Golubkov, V. T. Dolgopopov, G. E. Tsydynzhapov, and A. A. Shashkin, *Pis'ma Zh. Eksp. Teor. Fiz.* **68**, 337 (1998) [*JETP Lett.* **68**, 363 (1998)].
- ³⁴S. Okuma and N. Kokubo, *Phys. Rev. B* **56**, 410 (1997); **51**, 15 415 (1995).
- ³⁵H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman, *Phys. Rev. B* **40**, 182 (1989).
- ³⁶Y. Liu, K. A. McGreer, B. Nease, D. B. Haviland, G. Martinez, J. W. Halley, and A. M. Goldman, *Phys. Rev. Lett.* **67**, 2068 (1991); Y. Liu, D. B. Haviland, B. Nease, and A. M. Goldman, *Phys. Rev. B* **47**, 5931 (1993); D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989).
- ³⁷Wenhao Wu and P. W. Adams, *Phys. Rev. Lett.* **73**, 1412 (1994).
- ³⁸J. M. Valles, Jr., R. C. Dynes, and J. P. Garno, *Phys. Rev. Lett.* **69**, 3567 (1992).
- ³⁹S.-Y. Hsu, J. A. Chervenak, and J. M. Valles, Jr., *Phys. Rev. Lett.* **75**, 132 (1995).
- ⁴⁰R. A. Ferrell and C. J. Lobb (private communication); Soon-Gul Lee and Thomas R. Lemberger, *Phys. Rev. B* **37**, 7911 (1988).
- ⁴¹N. Marković, A. M. Mack, G. Martinez-Arizala, C. Christiansen, and A. M. Goldman, *Phys. Rev. Lett.* **81**, 701 (1998).
- ⁴²Efrat Shimshoni, Assa Auerbach, and Aharon Kapitulnik, *Phys. Rev. Lett.* **80**, 3352 (1998).
- ⁴³N. Mason and A. Kapitulnik, *Phys. Rev. Lett.* **82**, 5341 (1999).
- ⁴⁴D. Ephron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, *Phys. Rev. Lett.* **76**, 1529 (1996).
- ⁴⁵T. Wang, K. M. Beauchamp, D. D. Berkley, B. R. Johnson, J.-X. Liu, J. Zhang, and A. M. Goldman, *Phys. Rev. B* **43**, 8623 (1991).
- ⁴⁶S. V. Kravchenko, W. E. Mason, G. E. Bowker, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio, *Phys. Rev. B* **51**, 7038 (1995); S. V. Kravchenko, D. Simonian, M. P. Sarachik, W. E. Mason, and J. E. Furneaux, *Phys. Rev. Lett.* **77**, 4938 (1996).
- ⁴⁷D. Das and S. Doniach, *Phys. Rev. B* **60**, 1261 (1999).
- ⁴⁸N. Marković, C. Christiansen, and A. M. Goldman, *Phys. Rev. Lett.* **81**, 5217 (1998).
- ⁴⁹J. T. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, *Phys. Rev. Lett.* **57**, 2999 (1986).
- ⁵⁰J. A. Chervenak and J. M. Valles, Jr., *Phys. Rev. B* **54**, R15 649 (1996).
- ⁵¹Anne van Otterlo, Richard T. Scalettar, and Gergely T. Zimányi, *Phys. Rev. Lett.* **81**, 1497 (1998).
- ⁵²G. Bergmann, *Phys. Rep.* **107**, 1 (1984).
- ⁵³P. A. Lee and T. V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985).
- ⁵⁴A. Krämer and S. Doniach, *Phys. Rev. Lett.* **81**, 3523 (1998).
- ⁵⁵S. Chakravarty, G.-L. Ingold, S. Kivelson, and A. Luther, *Phys. Rev. Lett.* **56**, 2303 (1986); S. Chakravarty, G.-L. Ingold, S. Kivelson, and G. Zimányi, *Phys. Rev. B* **37**, 3283 (1988).
- ⁵⁶A. J. Rimberg, T. R. Ho, Ç. Kurdak, John Clarke, K. L. Campman, and A. C. Gossard, *Phys. Rev. Lett.* **78**, 2632 (1997).
- ⁵⁷H. S. J. van der Zant, W. J. Elion, L. J. Geerligs, and J. E. Mooij, *Phys. Rev. B* **54**, 10 081 (1996).
- ⁵⁸D. Simonian, S. V. Kravchenko, and M. P. Sarachik, *Phys. Rev. B* **55**, 13 421 (1997).
- ⁵⁹D. Shahar, M. Hilke, C. C. Li, D. C. Tsui, S. L. Sondhi, J. E. Cunningham, and M. Razeghi, *Solid State Commun.* **107**, 19 (1998).
- ⁶⁰F. Pázmándi, R. T. Scalettar, and G. T. Zimányi, *Phys. Rev. Lett.* **79**, 5130 (1997).
- ⁶¹P. Phillips, Yi Wan, Ivar Martin, Sergey Knysh, and Denis Dalidovich, *Nature (London)* **395**, 253 (1998).
- ⁶²D. Das and S. Doniach, *Phys. Rev. B* **57**, 14 440 (1998).
- ⁶³Igor F. Herbut, cond-mat/9902305 (unpublished).
- ⁶⁴K. Sheshadri, H. R. Krishnamurthy, Rahul Pandit, and T. V. Ramakrishnan, *Phys. Rev. Lett.* **75**, 4075 (1995).
- ⁶⁵K. Karpuiskaya *et al.*, *Phys. Rev. Lett.* **77**, 3033 (1996).
- ⁶⁶Saša Duken and Zlatko Tešanović, *Phys. Rev. B* **49**, 13 017 (1994); Zlatko Tešanović and Pedro D. Sacramento, *Phys. Rev. Lett.* **80**, 1521 (1998).