Co-operative Kondo effect in the two-channel Kondo lattice

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We examine how the properties of a single-channel Kondo lattice model are modified by additional screening channels. Contrary to current wisdom, additional screening channels appear to constitute a relevant perturbation which destabilizes the Fermi liquid. This instability involves two stages. When a heavy Fermi surface develops, it generates zero modes for Kondo singlets to fluctuate between screening channels of different symmetry, producing a divergent composite pair susceptibility. Additional screening channels couple to these divergent fluctuations, promoting an instability into a superconducting state with long-range composite order. We discuss possible implications for heavy fermion superconductivity. [S0163-1829(99)04629-9]

I. INTRODUCTION

One of the remarkable properties of localized magnetic moments is their ability to transform the electronic properties of their host. These effects are dramatic in heavy fermion compounds.[1,2] Since the mid 1970's, several hundred heavy fermion compounds have been discovered, characterized by a dense lattice of magnetic rare-earth or actinide ions immersed in a conducting host. These materials bypass the normal development of ordered antiferromagnetism to form a new kind of electron fluid.[16] The resulting metallic state contains quasiparticles with effective masses up to a thousand times greater than a bare electron. For example, in $CeCu_6$ (Ref. 3) the presence of only 14% Cerium in the copper host increases the effective mass of the electrons by a factor of 1600.

In a small handful of heavy fermion compounds, the heavy electron fluid becomes superconducting.[1] Local moments, usually harmful to superconductivity actually participate in this superconducting condensation process and a significant fraction of the local moment entropy is quenched as part of the condensation process. In UBe_{13}, for example, the spin-condensation entropy is about $0.2k_B \ln 2$ per spin.[4] One of the great challenges is to understand how microscopic order parameter in these systems involves the spin operators of the local moments.

The concept of ‘‘composite pairing,’’ where a Cooper pair and local moment form a bound-state combination that collectively condenses may provide a way to address this problem.[5–10] A composite ‘‘triplet’’ involves a bound-state between a spin and singlet Cooper pair,

$$\Lambda_t(x)=\langle\Psi_{N-2}|\psi_{\downarrow}(x)\psi_{\uparrow}(x)\mathbf{S}(x)|\Psi_N\rangle, \tag{1}$$

but a composite singlet involves a triplet and a spin-flip

$$\Lambda_s(x)=\langle\Psi_{N-2}|\psi_{1\downarrow}(x)\psi_{2\downarrow}(x)\mathbf{S}^{+}(x)|\Psi_N\rangle, \tag{2}$$

where 1 and 2 refer to two conduction electron channels. Such composite order parameters were originally considered in the context of odd-frequency pairing[5–7] but more recent work has emphasized that composite order may coexist with BCS pairing in cases where the spin plays a central role in the condensation process.[8,9] Unfortunately, we know very little about how such composite pairing might come to pass. A divergent composite singlet susceptibility is known to occur in the symmetric two-channel Kondo impurity model,[5] and more recent studies suggest that a large composite susceptibility may persist into the two-channel Kondo lattice.[11]

In this paper we introduce a model for heavy fermion behavior where the local moments couple to a *single* conduction band via two orthogonal scattering channels. We find that when two scattering channels of the same parity share a common Fermi surface, *constructive* interference develops between the channels. The scattering of electrons in the Kondo effect is described by an SU(2) matrix $\mathcal{V}_\Gamma$ ($\Gamma=1,2$) associated with each channel. A key result of our paper relates the composite order to the gauge invariant interference term between these two matrices

$$\mathcal{V}^{\dagger}{}_2\mathcal{V}_1=-\frac{J_1J_2}{2}\begin{bmatrix} F & \Lambda \\ -\Lambda^{\dagger} & F^{\dagger}\end{bmatrix}, \tag{3}$$

where

$$F=\psi^{\dagger}{}_1\boldsymbol{\sigma}\psi_2\cdot\mathbf{S},$$

$$\Lambda=\psi_1(i\sigma_y)\boldsymbol{\sigma}\psi_2\cdot\mathbf{S}, \tag{4}$$

represent the singlet composite order in the particle-hole, and particle-particle channels, respectively, and J_1 and J_2 de-

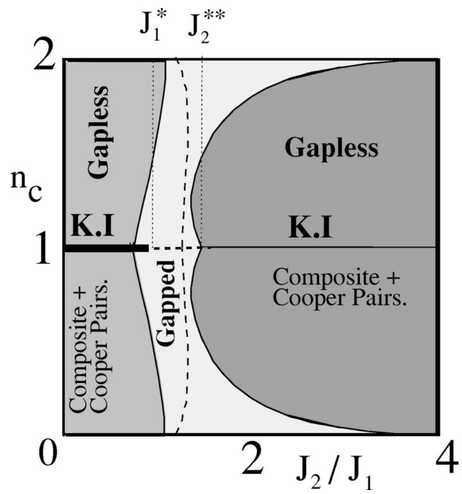

FIG. 1. Mean-field phase diagram for a two-dimensional two-channel Kondo lattice with an *s*-wave and *d*-wave interaction channel. n_c denotes the filling of the conduction band. J_2/J_1 is the ratio of coupling constants in the two channels. The phase diagram was computed for a tight-binding model, keeping $\max(J_1,J_2)=4t$, the bandwidth. KI denotes Kondo insulator phases, that exist at halffilling, but which undergo a superconductor-insulator transition at a critical value of J_2/J_1. The lightly shaded region is dominated by composite pairing, and there is a gap for quasiparticle excitations. Along the dotted lines, the conduction electrons are entirely unpaired, and a pure composite pair condensate is formed. In the darkly shaded regime, Cooper and composite pairs coexist, and a gapless anisotropic superconductor is formed.

scribe the Kondo coupling constants in the two channels. In a single impurity, the Kondo effect in the stronger channel, suppresses any Kondo effect in weaker channels.[12,13] A key feature of our lattice mechanism, is that channel interference co-operatively enhances the Kondo effect in the weaker channel, driving the development of composite pairing for arbitrarily weak second-channel coupling.

The development of phase coherence between the two channels is signaled by the condensation of composite singlet pairs at a new temperature scale

$$T_c \sim \sqrt{T_{K1}T_{K2}}, \tag{5}$$

where T_{K1} and T_{K2} are the Kondo temperatures of the primary and secondary channels, respectively. The underlying gap symmetry of the quasiparticles in this new superconducting phase reflects the interference phenomena and is is given by a product of the form factors $\Phi_{1\mathbf{k}}$ and $\Phi_{2\mathbf{k}}$ from each channel:

$$\Delta_{\mathbf{k}} = \Delta_0 \Phi_{1\mathbf{k}} \Phi_{2\mathbf{k}}. \tag{6}$$

In the typical composite paired state, composite pairs coexist with Cooper pairs, as envisaged in the works of Bonca and Balatsky and also Poilblanc.[8,9] One of the novel features of this mechanism, is that it permits both gapless *and* gapfull anisotropic superconductivity. In the region where the coupling constants J_1 and J_2, for the two channels, are of comparable strength, the nodes in the excitation spectrum gravitate to the center of the unit cell, where they mutually annihilate to produce a gapped phase (Fig. 1, Fig. 7). In this phase, the BCS order parameter is very small, and actually vanishes along a line in the phase diagram.

At half filling, a Kondo lattice generally forms a Kondo insulating phase.[14] With the addition of a second-channel coupling, we find that at a critical ratio of coupling constants, there is a second-order transition from the Kondo insulating phase into a pure condensate of superconducting pairs. This leads to a phase diagram, where a first order line representing the Kondo insulator terminates at a superconducting-insulating transition, as illustrated in Fig. 1.

A brief description of our work in this area has already been published.[15] This paper is intended to provide a detailed account and discussion of the co-operative two-channel Kondo effect.

II. MULTICHANNEL SCATTERING EFFECTS IN INTERACTING KONDO LATTICES

The classical approach to heavy fermion physics involves local moments which couple exclusively to conduction electron states with the same local *f* symmetry. This assumption derives from the observation that spin-exchange between the conduction electrons and the local moments occurs predominantly via hybridization in the *f* channel.

However, more careful considerations suggest[16–19] that electron-electron interactions can cause new spin-exchange channels to open up between a local moment and the conduction sea. There are several mechanisms by which these new spin exchange channels can open up, including the vicinity to a quantum critical point,[17] interactions in the conduction sea,[18,19] and intra-atomic Hunds interactions.[20]

The first mechanism, identified long ago in a little-known paper by Larkin and Melnikov[17] may be particularly important for heavy fermion systems which lie at the brink of magnetism. Larkin and Melnikov studied the single impurity Kondo effect in the vicinity of a magnetic quantum phase transition, where the local moment polarizes electrons at increasingly greater distances. The critical magnetic order thereby induces the spin to scatter electrons in a large number of angular momentum channels up to a maximum value $l_0 \sim k_F\xi$, where ξ is the spin correlation length. The large screening cloud causes the matrix element for spin exchange to become

$$J \rightarrow J_{\mathbf{k},\mathbf{k}'} = J\chi(\mathbf{k}-\mathbf{k}'), \tag{7}$$

where χ is the strongly momentum dependent susceptibility of the magnetic host. When decomposed into partial wave states, they found that this led to a Kondo coupling to electrons in all channels with angular momentum $l \leq l_0 = k_F\xi$.

More recent work has made it clear that new spin exchange channels open up whenever charge fluctuations are suppressed by interactions in the conduction sea.[18,19] Consider the situation shown in Fig 2, where a local moment hybridizes with nearby orbitals in a *d*-channel. The spin-exchange between the local moment is written

$$H_I = J(\mathbf{S} \cdot \Psi^{\dagger}{}_{d\alpha} \boldsymbol{\sigma}_{\alpha\beta} \Psi_{d\beta}), \tag{8}$$

where

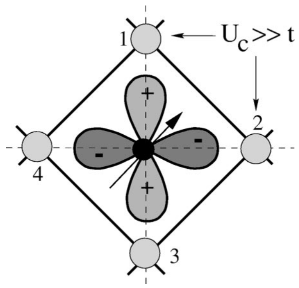

FIG. 2. Magnetic moment in an interacting environment. Localized electron at center of plaquet hybridizes in the d_{xy} channel with nearby atoms. The on-site interaction at each atomic site U_c is taken to be far larger than the electron bandwidth t.

$$\Psi^{\dagger}_{d\sigma}=\frac{1}{2}(c^{\dagger}_{1\sigma}-c^{\dagger}_{2\sigma}+c^{\dagger}_{3\sigma}-c^{\dagger}_{4\sigma}) \tag{9}$$

creates an electron in the d channel. Notice that the spin exchange involves processes where the electrons ''hop and flip'' between neighboring orbitals. If large repulsive interactions are present in the conduction sea, then an electron can no longer ''hop and flip'' onto a site that is already occupied. This restriction means that creation operators must be replaced by Hubbard operators

$$c_{j\sigma}\rightarrow c_{j\sigma}(1-n_{j-\sigma})=X_{j\sigma}. \tag{10}$$

To see how this modifies the spin-exchange processes one can use a Gutzwiller approximation

$$X^{\dagger}{}_j\boldsymbol{\sigma}X_l\rightarrow c^{\dagger}{}_j\boldsymbol{\sigma}c_l\times\begin{cases}1 & (j=l),\\ (1-x) & (j\neq l),\end{cases} \tag{11}$$

where x is the concentration of carriers per site. This approximation correctly describes the complete suppression of hop and flip processes in the limit where $x=1$. With this replacement the transformed Kondo interaction develops three new scattering channels

$$H_I=J_1\mathbf{S}(\Psi^{\dagger}{}_d\boldsymbol{\sigma}\Psi_d)+J_2\mathbf{S}(\Psi^{\dagger}{}_s\boldsymbol{\sigma}\Psi_s + \Psi^{\dagger}{}_{p_x}\boldsymbol{\sigma}\Psi_{p_x}+\Psi^{\dagger}{}_{p_y}\boldsymbol{\sigma}\Psi_{p_y}), \tag{12}$$

where $J_1=(1-3x/4)J,\ J_2=(x/4)J,$

$$\Psi^{\dagger}_{s\sigma}=\frac{1}{2}(c^{\dagger}_{1\sigma}+c^{\dagger}_{2\sigma}+c^{\dagger}_{3\sigma}+c^{\dagger}_{4\sigma})\quad s\ \text{channel},$$

$$\Psi^{\dagger}_{p_y\sigma}=\frac{1}{\sqrt{2}}(c^{\dagger}_{1\sigma}-c^{\dagger}_{3\sigma}),$$

$$\Psi^{\dagger}_{p_x\sigma}=\frac{1}{\sqrt{2}}(c^{\dagger}_{2\sigma}-c^{\dagger}_{4\sigma})\quad p\ \text{channel} \tag{13}$$

create electrons in the secondary channels. Electrons in the secondary channels are able to exchange spin with the local moment even though they do not hybridize with it.

In more complex Uranium heavy fermion systems, intraatomic interactions play a vital role in opening up second-channel couplings.[20] In uranium atoms, the Hunds interactions have the effect of suppressing fluctuations in the ''shape'' of the localized orbital, so that electrons scattering off a localized orbital tend to exchange spin, while preserving their orbital quantum numbers. In a tetragonal crystal for example where the low lying state of the f^2 ion is a magnetic non-Kramers doublet[20,21]

$$|\pm\rangle=\alpha|\pm1\rangle+\beta|\mp3\rangle, \tag{14}$$

spin fluctuations within this doublet involve the exchange of spin with conduction electrons in two different ''shape'' channels, with equal Kondo coupling constants.

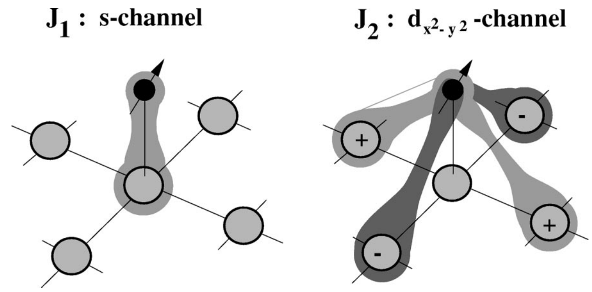

FIG. 3. Illustrating spin coupled to electrons via an s, and a $d_{x^2-y^2}$ channel.

III. TWO-CHANNEL KONDO LATTICE MODEL

This discussion motivates us to examine how additional spin exchange channels might modify the physics of a Kondo lattice. To this end, we shall consider a Kondo lattice model where two orthogonal scattering channels dominate the spin exchange process:

$$H=\sum \epsilon_{\mathbf{k}\sigma}c^{\dagger}{}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+\sum_{\Gamma j} J_{\Gamma}\psi^{\dagger}{}_{\Gamma j}\boldsymbol{\sigma}\psi_{\Gamma j}\cdot\mathbf{S}_j, \tag{15}$$

where $\psi^{\dagger}_{\Gamma j}=(\psi^{\dagger}_{\Gamma j\uparrow},\ \psi^{\dagger}_{\Gamma j\downarrow})$ $(\Gamma=1,\ 2)$ is a two component spinor

$$\psi^{\dagger}{}_{\Gamma j\sigma}=N_s^{-1/2}\sum_{\mathbf{k}}\Phi_{\Gamma\mathbf{k}}c^{\dagger}{}_{\mathbf{k}\sigma}e^{-i\mathbf{k}\cdot\mathbf{R}_j}, \tag{16}$$

that creates an electron at site j in one of two orthogonal Wannier states, with form-factor $\Phi_{\Gamma\mathbf{k}}$. Here N_s is the number of sites. We shall show that channel interference becomes strong when two channels have the same spatial parity.

A simple example of our model is a two-dimensional tight-binding lattice of conduction electrons, where

$$\epsilon_{\mathbf{k}}=-2t(\cos k_x+\cos k_y)-\mu, \tag{17}$$

and μ is the chemical potential, interacting with a local moment at each site in an s and a d channel, so that

$$\Phi_{1\mathbf{k}}=1\quad (s\ \text{channel}),$$
$$\Phi_{2\mathbf{k}}=(\cos k_x-\cos k_y)\quad (d_{x^2-y^2}\ \text{channel}), \tag{18}$$

as shown in Fig. 3. A slightly more appropriate example would be a three-dimensional lattice, where

$$\epsilon_{\mathbf{k}}=-2t(\cos k_x+\cos k_y+\cos k_z)-\mu, \tag{19}$$

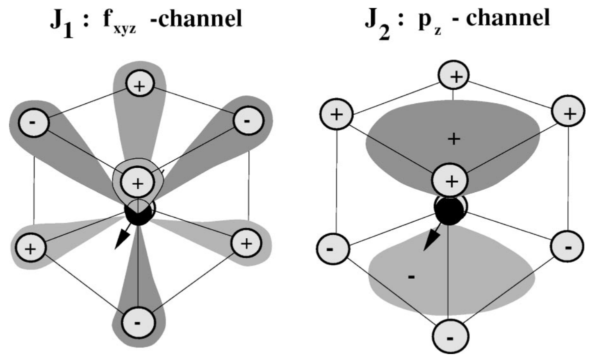

FIG. 4. Illustrating spin coupled to electrons via a primary f_{xyz} and a secondary p_z channel.

with a local moment at the center of each cube of atoms interacting in a primary f_{xyz} channel and a secondary p_z channel

$$\Phi_{1\mathbf{k}}=\sqrt{8}\sin k_x \sin k_y \sin k_z \quad (f_{xyz} \text{ channel}),$$

$$\Phi_{2\mathbf{k}}=\sqrt{2}\sin k_z \quad (p_z \text{ channel}), \tag{20}$$

as shown in Fig. 4.

Unlike earlier treatments of two-channel Kondo problems, our model involves a *single* conduction electron band, and there is no globally conserved ''channel quantum number.'' In a heavy fermion system, the orbital channels are locally well defined, but an electron scattering in one channel at one site, can then scatter in a different channel at a second site. This is important, for it can lead to interference effects between the Kondo effect in different channels which are completely absent in models with an artificial channel quantum number conservation. To illustrate this important point, we shall contrast the properties of our model with the channel symmetric ''control model''

$$H^C=\sum_{\mathbf{k}\Gamma\sigma}\epsilon_{\mathbf{k}}c^{\dagger}{}_{\Gamma\mathbf{k}\sigma}c_{\Gamma\mathbf{k}\sigma}+\sum_{\Gamma j}J_{\Gamma}c^{\dagger}{}_{\Gamma j}W\boldsymbol{\sigma}c_{\Gamma j}\cdot\mathbf{S}_{\mathbf{j}}, \tag{21}$$

where now

$$c^{\dagger}{}_{\Gamma j\sigma}=N_s{}^{-1/2}\sum_{\mathbf{k}}c^{\dagger}{}_{\Gamma\mathbf{k}\sigma}e^{-i\mathbf{k}\cdot\mathbf{R}_j} \tag{22}$$

($\Gamma=1,\ 2$). In the control, electrons in different channels do not interfere, and we shall show that this prevents the development of composite pairing.

IV. COMPOSITE PAIRING INSTABILITY OF THE ONE-CHANNEL KONDO LATTICE

To examine the effect of second-channel couplings, we introduce the composite operator

$$\Lambda^{\dagger}=\sum_j i\psi^{\dagger}{}_{1j}\boldsymbol{\sigma}\sigma_2\psi^{\dagger}{}_{2j}\cdot\mathbf{S}_{\mathbf{j}}. \tag{23}$$

This operator transfers singlets between channels by adding a triplet and flipping the local moment. To see how this works, consider a single site, where

$$|\Psi_{s1}\rangle=\frac{1}{\sqrt{2}}[\psi^{\dagger}{}_{1\uparrow}|\downarrow\rangle-\psi^{\dagger}{}_{1\downarrow}|\uparrow\rangle],$$

$$|\Psi_{s2}\rangle=\frac{1}{\sqrt{2}}[\psi^{\dagger}{}_{2\uparrow}|\downarrow\rangle-\psi^{\dagger}{}_{2\downarrow}|\uparrow\rangle], \tag{24}$$

represent Kondo singlets in channels one and two respectively. The action of the single site composite operator $\Lambda^{\dagger}=i\psi^{\dagger}{}_1\boldsymbol{\sigma}\sigma_2\psi^{\dagger}{}_2\cdot\mathbf{S}$ is as follows:

$$\Lambda^{\dagger}|\Psi_{s1}\rangle=2\psi^{\dagger}{}_{1\downarrow}\psi^{\dagger}{}_{1\uparrow}|\Psi_{s2}\rangle,$$

$$\Lambda^{\dagger}|\Psi_{s2}\rangle=2\psi^{\dagger}{}_{2\uparrow}\psi^{\dagger}{}_{2\downarrow}|\Psi_{s1}\rangle, \tag{25}$$

showing that the composite operator $\Lambda^{\dagger}$ transfers a Kondo singlet between channels, leaving an electron pair behind in the channel formerly occupied by a Kondo singlet.

We now show how channel interference in the one band model causes the susceptibility of this composite operator to develop a BCS-like divergence in the Fermi liquid ground state. Suppose that $J_2=0$ and J_1 is sufficiently large for a Kondo effect to develop in channel one. In the corresponding Fermi liquid ground state $|\Phi\rangle$, the composite pair susceptibility is given by

$$\chi_{\Lambda}=\sum_{\lambda}\left\{\left(\frac{\langle\Phi|\Lambda^{\dagger}|\lambda\rangle\langle\lambda|\Lambda|\Phi\rangle}{E_{\Phi}-E_{\lambda}}\right)+(\Lambda\rightleftharpoons\Lambda^{\dagger})\right\}. \tag{26}$$

To evaluate the matrix elements appearing in this expression, we need to decompose the composite operator in terms of quasiparticle operators. The essence of the Kondo effect is the development of Fermionic bound states between the local moments, and the conduction electrons. At low energies, the operator $(\mathbf{S}_{\mathbf{j}}\cdot\boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}$ then behaves as a single bound-state fermion, represented by the contraction

$$\overbrace{(\mathbf{S}_j\cdot\boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}}(j)=\overline{z}f_{j\alpha}. \tag{27}$$

where $\overline{z}$ is the amplitude for bound-state formation. By making this contraction, we imply that in all matrix elements between low-lying excitations $|a\rangle$ and $|b\rangle$ of the Fermi liquid, $(\mathbf{S}_{\mathbf{j}}\cdot\boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}$ can be replaced by a Fermi operator as follows:

$$\langle a|\overbrace{(\mathbf{S}_j\cdot\boldsymbol{\sigma}_{\alpha\beta})\psi_{1\beta}}(j)|b\rangle=\overline{z\langle a}|f_{j\alpha}|b. \tag{28}$$

It is the contraction of the exchange term which gives rise to a resonant hybridization between f and conduction electrons

$$J_1[\psi^{\dagger}{}_{1j}\overbrace{(\mathbf{S}_{\mathbf{j}}\cdot\boldsymbol{\sigma})\psi_{1j}}+\text{H. c.}]=J_1\overline{z}[\psi^{\dagger}{}_{1j}f_j+\text{H. c.}] \tag{29}$$

so that at low energies, the Kondo Hamiltonian can be replaced by an effective Anderson model.

The low energy eigenstates of the one channel Kondo lattice model are then an admixture of electron and composite fermion $a_{\mathbf{k}\sigma}=\cos\delta_{\mathbf{k}}c_{\mathbf{k}\sigma}+\sin\delta_{\mathbf{k}}f_{\mathbf{k}\sigma}$, with Hamiltonian $H^*=\Sigma_{\mathbf{k}\sigma}E_{\mathbf{k}}a^{\dagger}{}_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}$. The volume of the Fermi surface now counts both the conduction and composite f electrons.[22–24] In the one band model, the conduction and composite f electron share a single Fermi surface and they may be decomposed as follows:

$$c_{\mathbf{k}\sigma} = \cos\delta_{\mathbf{k_F}} a_{\mathbf{k}\sigma} + \cdots,$$

$$f_{\mathbf{k}\sigma} = \sin\delta_{\mathbf{k_F}} a_{\mathbf{k}\sigma} + \cdots, \tag{30}$$

where the high-energy components that do not affect the low-energy matrix elements have been omitted . Near the Fermi surface the scattering is resonant and $\delta_{\mathbf{k_F}} \sim \pi/2$. Moreover, the small conduction electron admixture at the Fermi surface must reflect the symmetry of the screening channel, so that $\cos\delta_{\mathbf{k_F}} \propto \Phi_{1\mathbf{k_F}}$.

We can now apply the contraction procedure to evaluate the matrix elements of the composite operator. Let us begin with the control model. Applying the contraction procedure we obtain

$$\begin{aligned}\langle\lambda|\Lambda^\dagger|\Phi\rangle_C &= -i\sum_j \langle\lambda|\mathbf{S}_j\cdot(c^\dagger{}_{1j}\boldsymbol{\sigma}\sigma_2 c^\dagger_{2j})|\Phi\\ &= z\sum_{\mathbf{k},\sigma}\langle\lambda|\sigma c^\dagger_{2\mathbf{k}\sigma} f^\dagger{}_{-\mathbf{k}-\sigma}|\Phi\rangle.\end{aligned} \tag{31}$$

In the control model, $c^\dagger_{2\mathbf{k}}$ and $f^\dagger_{-\mathbf{k}} \sim a^\dagger_{-\mathbf{k}}$, respectively, create light and heavy electrons on completely different Fermi surfaces. The mismatch between the volume and the dispersion of the Fermi surfaces for channel one and two assures that the excitation energy $E_\lambda - E_\Phi = \boldsymbol{\epsilon}_{\mathbf{k}} + E_{\mathbf{k}}$ is always finite:

$$\begin{aligned}\langle\lambda|\hat{\Lambda}^\dagger|\Phi\rangle_C &\propto \sum_{\mathbf{k},\sigma}\sigma\ \langle\lambda|c^\dagger{}_{\mathbf{2k}\sigma}a^\dagger{}_{-\mathbf{k}-\sigma}|\Phi\rangle,\\ E_\lambda - E_\Phi &= \boldsymbol{\epsilon}_{\mathbf{k}} + E_{\mathbf{k}} > 0.\end{aligned} \tag{32}$$

The channel susceptibility χ_Λ is consequently *finite*. We conclude that with perfect channel symmetry, a small second-channel coupling is *irrelevant*.

Now let us remove the channel symmetry and return to the physical model. Now we have

$$\begin{aligned}\langle\lambda|\Lambda^\dagger|\Phi\rangle &= i\sum_j \langle\lambda|\mathbf{S}_j\cdot(\psi^\dagger{}_{1j}\boldsymbol{\sigma}\sigma_2\psi^\dagger{}_{2j})|\Phi\rangle\\ &= z\sum_{\mathbf{k},\sigma}\Phi_{2\mathbf{k}}\langle\lambda|\sigma c^\dagger_{\mathbf{k}\sigma}f^\dagger{}_{-\mathbf{k}-\sigma}|\Phi\rangle.\end{aligned} \tag{33}$$

Unlike the previous case, this pair creation operator can be decomposed in terms of quasiparticles on a single heavy Fermi surface. Transforming to quasiparticle operators using Eq. (30) introduces a factor $\cos(\delta_{\mathbf{k}})\sin(\delta_{\mathbf{k}}) \sim \Phi_{1\mathbf{k}}$ into the sum, so that

$$\begin{aligned}\langle\lambda|\hat{\Lambda}^\dagger|\Phi\rangle &\propto \sum_{\mathbf{k},\sigma}\sigma\ \Phi_{1-\mathbf{k}}\Phi_{2\mathbf{k}}\langle\lambda|a^\dagger{}_{\mathbf{k}\sigma}a^\dagger{}_{-\mathbf{k}-\sigma}|\Phi\rangle,\\ E_\lambda - E_\Phi &= 2E_{\mathbf{k}}.\end{aligned} \tag{34}$$

This relation describes the decomposition of the composite pair operator in terms of the low-lying quasiparticles (see Fig. 5). Notice that the matrix element is proportional to $\Phi_{1-\mathbf{k}}\Phi_{2\mathbf{k}}$, showing that this amplitude involves an interference between the two channels. Furthermore, the two form factors must must have the same parity, or the composite operator vanishes on the Fermi surface. Since the excitation energy, $2E_{\mathbf{k}}$ vanishes on the heavy Fermi surface, it follows that there are now a large number of zero modes for the transfer of singlets between channels.

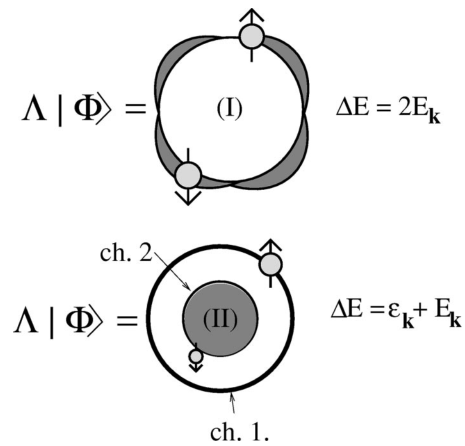

FIG. 5. The action of the composite operator on heavy Fermi liquid creates (i) a pair of heavy fermions (channel interference) and (ii) a heavy and light electron (channel conservation).

It follows that the composite pair susceptibility χ_Λ now contains a singular term. Substituting the above results into the general expression for the composite pair susceptibility, we find

$$\chi_\Lambda \propto \sum_{\mathbf{k}} \frac{(\Phi_{1\mathbf{k}}\Phi_{2\mathbf{k}})^2}{2E_{\mathbf{k}}} \to \infty, \tag{35}$$

which diverges logarithmically in the thermodynamic limit. We see that once channel symmetry is broken, the composite pair susceptibility χ_Λ is directly proportional to the BCS pair susceptibility of the heavy quasiparticles, where the symmetry of the channel is given by the *product* of the two screening channels.

This has immediate consequences for the effect of a finite J_2 on the Fermi liquid ground state. Once channel symmetry is broken, the susceptibility to transfer singlets by creating composite pairs diverges. Any finite J_2 will polarize the transfer of singlets into channel two, thereby coupling J_2 to this divergent susceptibility. Thus the loss of channel symmetry causes a coupling to a second channel to become a relevant perturbation. This will force J_2 to scale to strong coupling. A similar conclusion will hold when J_2 is large and J_1 is small. The simplest way to connect up the renormalization flows in the vicinity of the strong-coupling Fermi liquid fixed points with the flow away from the weak coupling fixed point is by hypothesizing the presence of a new attractive Kondo lattice fixed point that is common to both channels (Fig. 6).

V. SU(2) FORMALISM

The key to the development of a field theory for composite pairing, lies in the use of the Abrikosov pseudofermion representation for the local moments

$$\begin{aligned}\mathbf{S}_j &= f^\dagger{}_{j\alpha}\left(\frac{\boldsymbol{\sigma}}{2}\right)_{\alpha\beta} f_{j\beta},\\ n_f(j) &= 1.\end{aligned} \tag{36}$$

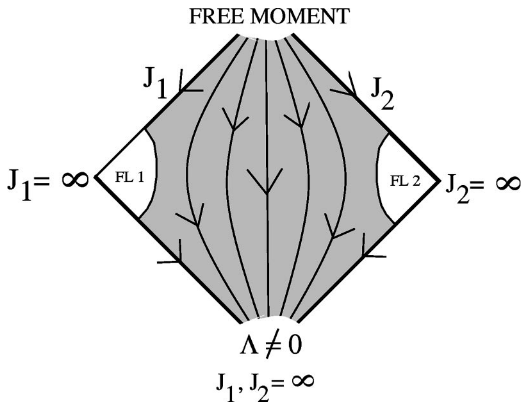

FIG. 6. Conjectured renormalization group flows for the co-operative two-channel Kondo effect. The Fermi liquid formed in channel one or two is unstable to common a two-channel state with composite order.

Since f-charge fluctuations have been removed, the Kondo lattice model (KLM) is defined within the subspace constrained by the (Gutzwiller) requirement $n_f=1$ at each site. The absence of f-charge fluctuations is manifested as a local SU(2) gauge invariance of the Heisenberg spin operator $\mathbf{S}_j$,[25]

$$f^\dagger{}_{j\sigma}\to\begin{cases} e^{i\phi}f^\dagger{}_{j\sigma}, \\ \cos\theta f^\dagger{}_{j\sigma}+\mathrm{sgn}\ \sigma\ \sin\theta f_{j-\sigma}. \end{cases} \quad (37)$$

To illustrate this feature, consider the spin raising operation S_+. This process can proceed by first annihilating a down electron, then creating an up electron, written $S_j^+=f^\dagger{}_{j\uparrow}f_{j\downarrow}$. Alternatively, it can proceed by first creating an up electron, forming the $n_f=2$ state, then annihilating a down electron, written $S_j^+=-f_{j\downarrow}f^\dagger{}_{j\uparrow}$. In fact, one can accomplish the spin raising operation by an arbitrary linear combination of the above:

$$S_j^+=(\cos\theta f^\dagger{}_{j\uparrow}+\sin\theta f_{j\downarrow})(\cos\theta f_{j\downarrow}-\sin\theta f^\dagger{}_{j\uparrow}). \quad (38)$$

In other words, there is no distinction between a particle or a hole when all charge fluctuations are removed.[25]

The SU(2) symmetry implies that the constraint $n_f=1$ is actually component of a triplet of local ''Gutzwiller constraints''

$$\left.\begin{array}{c} f^\dagger{}_{j\uparrow}f_{j\uparrow}-f_{j\downarrow}f^\dagger{}_{j\downarrow} \\ f^\dagger{}_{j\uparrow}f^\dagger{}_{j\downarrow} \\ f_{j\downarrow}f_{j\uparrow} \end{array}\right\}=0, \quad (39)$$

which can be written in the compact form

$$\tilde{f}^\dagger{}_j\boldsymbol{\tau}\tilde{f}_j=0, \quad (40)$$

where $\tilde{f}^\dagger{}_j=(f^\dagger{}_{j\uparrow},f_{j\downarrow})$ is a Nambu spinor, and $\boldsymbol{\tau}\equiv(\tau_1,\tau_2,\tau_3)$ represents the triplet of Pauli matrices. The first two constraints are particularly important in any consideration of a paired state, providing the main driving force for anisotropic pairing.

The partition function for our model is given by $Z=\mathrm{Tr}[P_G e^{-\beta H}]$, where $P_G=\Pi_j(n^f_{j\uparrow}-n^f_{j\downarrow})^2$ is the Gutzwiller projection for one f-spin per site. Following earlier work, we rewrite the Gutzwiller projection as an integral over the SU(2) group

$$(n^f_{j\uparrow}-n^f_{j\downarrow})^2=\int d[W_j]\ \hat{g}_j, \quad (41)$$

where $\hat{g}_j=e^{if^\dagger{}_jW_jf_j}$ is the SU(2) operator, $W_j=\theta_j\hat{\mathbf{n}}_j\cdot\boldsymbol{\tau}$, ($\theta_j\in[0,2\pi]$) and $d[W]=\sin^2\theta d\theta d\hat{n}/(4\pi^2)$ is the Haar measure[26] over the SU(2) group. Introducing this into the partition function permits us to write it as a path integral

$$Z=\int \mathcal{D}[f,c,W]e^{-\int_0^\beta(\mathcal{L}_1+H)d\tau}, \quad (42)$$

where

$$\mathcal{L}_1=\sum_{\vec{k}} c^\dagger{}_{\vec{k}}\partial_\tau c_{\vec{k}}+\sum_j f^\dagger{}_j(\partial_\tau-iW_j)f_j \quad (43)$$

is the Berry phase.

The antiferromagnetic interaction between the localized moments and the conduction electrons can be decoupled in the particle-hole channel as follows:

$$J_\Gamma\left[\boldsymbol{\sigma}_\Gamma\cdot\mathbf{S}-\frac{1}{2}\right]=-\frac{J_\Gamma}{2}\{a_\Gamma^\dagger,a_\Gamma\}, \quad (44)$$

where

$$a_\Gamma=\sum_\sigma f^\dagger{}_\sigma\psi_{\Gamma\sigma}. \quad (45)$$

The SU(2) gauge symmetry guarantees that there is in fact, a continuous family of ways to decouple the interaction. Thus, by making the transformation $f_\sigma\to\sigma f^\dagger_{-\sigma}$, we can decouple the interaction in the Cooper channel as follows:

$$J_\Gamma\left[\boldsymbol{\sigma}_\Gamma\cdot\mathbf{S}-\frac{1}{2}\right]=-\frac{J_\Gamma}{2}\{b_\Gamma^\dagger,b_\Gamma\}, \quad (46)$$

where

$$b_\Gamma=\sum_\sigma \sigma f_{-\sigma}\psi_{\Gamma\sigma}. \quad (47)$$

We now decouple the interaction simultaneously in both channels, by first writing

$$H_I=-\frac{J_\Gamma}{4}[\{a_\Gamma^\dagger,a_\Gamma\}+\{b_\Gamma^\dagger,b_\Gamma\}], \quad (48)$$

then decoupling each term as follows:

$$-\frac{J_\Gamma}{4}\{a_\Gamma^\dagger,a_\Gamma\}\to[a_\Gamma^\dagger V^\Gamma+\mathrm{H.c.}]+\frac{2}{J_\Gamma}V^{\Gamma*}V^\Gamma,$$

$$-\frac{J_\Gamma}{4}\{b_\Gamma^\dagger,b_\Gamma\}\to[b_\Gamma^\dagger\Delta^\Gamma+\mathrm{H.c.}]+\frac{2}{J_\Gamma}\Delta^{\Gamma*}\Delta^\Gamma. \quad (49)$$

It is convenient at this point, to introduce a Nambu spinor representation for the conduction electrons

$$c_{\mathbf{k}}=\begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^{\dagger}{}_{-\mathbf{k}\downarrow} \end{pmatrix}. \qquad (50)$$

The corresponding spinor for the localized electron Wannier states is

$$\tilde{\psi}_{\Gamma j}=\begin{pmatrix} \psi_{\Gamma\uparrow} \\ \psi^{\dagger}{}_{\Gamma\downarrow} \end{pmatrix}=\sum_{\mathbf{k}} \Phi_{\Gamma\mathbf{k}}\begin{pmatrix} c_{\mathbf{k}\uparrow} \\ \mathrm{p}c^{\dagger}{}_{-\mathbf{k}\downarrow} \end{pmatrix} e^{i\mathbf{k}\cdot\mathbf{R}_j}, \qquad (51)$$

where p is the parity of the form-factor $\Phi_{\Gamma\mathbf{k}}=\mathrm{p}\Phi_{\Gamma-\mathbf{k}}$. The decoupled interaction can now be written in the symmetric form (Appendix A).

$$J_{\Gamma}(\mathbf{S}\cdot\sigma_{\Gamma})\rightarrow[\tilde{f}^{\dagger}\mathcal{V}_{\Gamma}\tilde{\psi}_{\Gamma}+\mathrm{H.c.}]+\frac{1}{J_{\Gamma}}\mathrm{Tr}[\mathcal{V}^{\dagger}{}_{\Gamma}\mathcal{V}_{\Gamma}], \qquad (52)$$

where V_{Γ} is directly proportional to an SU(2) matrix g_{Γ}

$$\mathcal{V}_{\Gamma}=\begin{bmatrix} V & \Delta \\ \Delta^* & -V^* \end{bmatrix}^{\Gamma}=iV_0^{\Gamma}g_{\Gamma}. \qquad (53)$$

The integration measure for $\mathcal{V}_{\Gamma}$ is

$$d[\mathcal{V}_{\Gamma}]=dV_{\Gamma}dV_{\Gamma}^*d\Delta_{\Gamma}d\Delta_{\Gamma}^*. \qquad (54)$$

Repeating this decoupling procedure at each site in the path integral, enables us to write

$$Z=\int \mathcal{D}[f,c;W,\mathcal{V}]e^{-\int_0^{\beta}(\mathcal{L}_1+H)d\tau},$$

$$H=\sum_{\vec{k}} \epsilon_{\vec{k}}c^{\dagger}{}_{\vec{k}}\tau_3 c_{\vec{k}}+H_I,$$

$$H_I=\sum_{\Gamma i}\left\{[\tilde{f}^{\dagger}{}_i\mathcal{V}_{\Gamma i}\tilde{\Psi}_{\Gamma_i}+\mathrm{H.c.}]+\frac{1}{J_{\Gamma}}\mathrm{Tr}[\mathcal{V}^{\dagger}{}_{\Gamma i}\mathcal{V}_{\Gamma i}]\right\}. \qquad (55)$$

VI. GAUGE FIXING

Our model now has the following time dependent SU(2) gauge invariance:

$$\tilde{f}_j\rightarrow g_j\tilde{f}_j,$$

$$\mathcal{V}_{\Gamma j}\rightarrow g_j\mathcal{V}_{\Gamma j},$$

$$W_j\rightarrow g_j(W_j+i\partial_{\tau})g^{\dagger}{}_j, \qquad (56)$$

associated with the absence of *f*-charge fluctuations. When we develop a saddle-point expansion for the functional integral, we need to deal with the local zero modes associated with this gauge invariance. Following standard gauge theory practice, this means that we need to fix the gauge. We choose the ''radial gauge,'' where the Kondo matrix $\mathcal{V}$ is proportional to a unit matrix in the channel with the largest Kondo coupling constant. This is the SU(2) analog of the radial gauge used by Read and Newns in their U(1) treatment of the single channel[27,28] Kondo lattice. Suppose that

$$\mathcal{V}_1(j)=iv_1(j)h_j, \qquad (57)$$

where $v_1(j)$ is real and h_j is an SU(2) matrix. To fix the gauge, we absorb h_j into a redefinition of the fields by setting $g_j=h^{\dagger}{}_j(\tau)$ and making the gauge transformation (56). In the radial gauge,

$$\mathcal{V}_{1j}(\tau)=iv_{1j}(\tau)\underline{1},$$

$$W_j(\tau)=h^{\dagger}{}_j(\tau)(W_j+i\partial_{\tau})h_j(\tau), \qquad (58)$$

so the formerly static field W_j is elevated to the status of a dynamic field $W_j(\tau)$. The measure for the bosonic fields inside the path integral is now

$$d[\mathbf{W},\mathcal{V}]=(v_1)^3dv_1d[\mathcal{V}_2]d^3\mathbf{W}, \qquad (59)$$

at each site and time slice.

VII. LINK BETWEEN COMPOSITE ORDER AND CHANNEL INTERFERENCE

Under the local SU(2) gauge transformation $\tilde{f}_j\rightarrow g_j\tilde{f}_j$, Eq. (56), the two amplitudes $\mathcal{V}_{1j}$ and $\mathcal{V}_{2j}$ which describe the Kondo effect at site j transform in precisely the same way. The only gauge invariant term we can form from a single channel is trivially proportional to the unit matrix $\mathcal{V}^{\dagger}_{\Gamma j}\mathcal{V}_{\Gamma j}\propto=\underline{1}$. But in a two-channel Kondo problem, the interference term

$$\mathcal{V}^{\dagger}{}_{2j}\mathcal{V}_{1j} \qquad (60)$$

is also gauge invariant, since $\mathcal{V}^{\dagger}{}_{2j}\mathcal{V}_{1j}\rightarrow\mathcal{V}^{\dagger}{}_{1j}g^{\dagger}{}_jg_j\mathcal{V}_{2j}=\mathcal{V}^{\dagger}{}_{2j}\mathcal{V}_{1j}$ and for this reason, is expected to have a simple physical significance. To identify the meaning of the interference term we introduce a source term into the Hamiltonian that couples to it

$$H\rightarrow H+\sum_j \mathrm{Tr}[\mathcal{V}_{2j}^{\dagger}\mathcal{V}_{1j}\alpha_j+\mathrm{H.c.}], \qquad (61)$$

where $\alpha_j=\alpha_j^0+i\boldsymbol{\alpha}_j\cdot\boldsymbol{\tau}$ is a unitary matrix, with four real coefficients $(\alpha^0,\boldsymbol{\alpha})$ at each site . If we now reverse the Hubbard Stratonovich transformation, by integrating over the fields $\mathcal{V}_{\Gamma}$ (Appendix B) the Hamiltonian acquires the additional term

$$H\rightarrow H+\mathrm{Tr}[\mathcal{M}_j^{\dagger}\alpha_j+\mathrm{H.c.}], \qquad (62)$$

where now

$$\mathcal{M}^{\dagger}{}_j=-\frac{J_1J_2}{2}\begin{bmatrix} F & \Lambda \\ -\Lambda^{\dagger} & F^{\dagger} \end{bmatrix}_j \qquad (63)$$

and

$$F_j=\psi^{\dagger}{}_{1j}\boldsymbol{\sigma}\psi_{2j}\cdot\mathbf{S}_j,$$

$$\Lambda_j=\psi_{1j}i\sigma_y\boldsymbol{\sigma}\psi_{2j}\cdot\mathbf{S}_j \qquad (64)$$

represents the composite order in the particle-hole and particle-particle channels, respectively. By comparing Eqs. (61) and (62), we obtain a special relationship between the inter-channel interference and the composite order,

$$\mathcal{V}^{\dagger}{}_{2j}\mathcal{V}_{1j} = -\frac{J_1 J_2}{2}\begin{bmatrix} F & \Lambda \\ -\Lambda^{\dagger} & F^{\dagger}\end{bmatrix}_j . \tag{65}$$

Notice incidentally that the off-diagonal terms are odd under interchange of the channel index.

We thus learn that *if the Kondo effect develops coherently in two channels, composite order develops*. This enables us to understand why composite order develops critical correlations in the symmetric two-channel Kondo model.[5] In a lattice, true long-range order becomes possible.

Let us briefly consider the possible phases that might develop. If $\mathcal{V}_\Gamma$ develops a finite amplitude in both channels then the composite order in the ground state will have the form

$$\begin{bmatrix} \langle \psi|F|\psi\rangle & \langle \psi|\Lambda|\psi\rangle \\ -\langle \psi|\Lambda^{\dagger}|\psi\rangle & \langle \psi|F^{\dagger}|\psi\rangle \end{bmatrix}_j = -\left(\frac{2}{J_1 J_2}\right)\mathcal{V}^{\dagger}{}_{2j}\mathcal{V}_{1j} . \tag{66}$$

Suppose the amplitudes of $\mathcal{V}_\Gamma$ are constant, then in the ''radial gauge''

$$\mathcal{V}_{1j} = i v_1 \underline{1},$$

$$\mathcal{V}_{2j} = i v_2 e^{-i\phi_j \mathbf{n}_j\cdot\boldsymbol{\tau}}, \tag{67}$$

where the vector n_j develops a vacuum expectation value. The composite order matrix is then

$$\begin{bmatrix} \langle \psi|F|\psi\rangle & \langle \psi|\Lambda|\psi\rangle \\ -\langle \psi|\Lambda^{\dagger}|\psi\rangle & \langle \psi|F^{\dagger}|\psi\rangle \end{bmatrix}_j = M_0 e^{i\phi_j \mathbf{n}_j\cdot\boldsymbol{\tau}}, \tag{68}$$

where $M_0 = 2 v_1 v_2/(J_1 J_2)$. Two kinds of phase are possible.

Composite magnetism, where $\mathbf{n}_j = \hat{\mathbf{z}}$. In this phase, the order parameter matrix is diagonal and

$$\langle \psi|\psi^{\dagger}{}_{1j}\boldsymbol{\sigma}\psi_{2j}\cdot\mathbf{S}_j|\psi\rangle = M_0 e^{i\phi_j}. \tag{69}$$

This phase breaks time-reversal symmetry, forming an orbital magnet where the spin becomes correlated with electrons in two orbitals.

Composite singlet pairing, where $\phi_j = \pi/2$. If $\hat{\mathbf{n}}(x) = \cos\theta(x)\hat{\mathbf{x}} + \sin\theta(x)\hat{\mathbf{y}}$, whereupon

$$\Lambda_s(x_j) = i\langle \psi|\psi_{1j}\sigma_y\boldsymbol{\sigma}\psi_{2j}\cdot\mathbf{S}_j|\psi\rangle = iM_0 e^{-i\theta_j}. \tag{70}$$

The second possibility is particularly interesting, because the composite pair susceptibility diverges in the Fermi liquid phase. This is the main topic of the of the paper.

VIII. MEAN FIELD THEORY OF THE COMPOSITE PAIRED STATE

We now develop a mean-field theory for the uniform composite paired state. With this theory, we show that the two strong-coupling Fermi liquid phases of our two-channel Kondo model share a common instability into a phase with uniform composite order.

We seek a uniform solution, where all mean-field parameters have no dependence on position. In this case the mean-field Hamiltonian is most compactly represented in momentum space as

$$H_{MF} = \sum_{\mathbf{k}} (\tilde{c}^{\dagger}{}_{\mathbf{k}}, \tilde{f}^{\dagger}{}_{\mathbf{k}})\begin{bmatrix} \epsilon_{\mathbf{k}}\tau_3 & \mathcal{V}^{\dagger}{}_{\mathbf{k}} \\ \mathcal{V}_{\mathbf{k}} & \mathbf{W}\cdot\boldsymbol{\tau}\end{bmatrix}\begin{pmatrix}\tilde{c}_{\mathbf{k}} \\ \tilde{f}_{\mathbf{k}}\end{pmatrix}, \tag{71}$$

where now

$$\mathcal{V}_{\mathbf{k}} = \mathcal{V}^1\Phi_{1\mathbf{k}} + \mathcal{V}^2\Phi_{2\mathbf{k}}. \tag{72}$$

Strictly speaking, here we should have written the form factors as $\Phi_{\Gamma\mathbf{k}\tau_3}$, to take account of the possibility of an odd-parity scattering channel. However, provided both channels have the same parity, we can always cast $\mathcal{V}_{\mathbf{k}}$ in the above form. For even parity channels, $\Phi_{\Gamma\mathbf{k}\tau_3} = \Phi_{\Gamma\mathbf{k}}$ directly. For odd-parity channels, $\Phi_{\Gamma\mathbf{k}\tau_3} = \Phi_{\Gamma\mathbf{k}}\tau_3$, but in this case the τ_3 can be absorbed by a gauge transformation $\tilde{f}_j \to \tau_3\tilde{f}_j$.

To examine uniform pairing, we shall take

$$\mathcal{V}_1 = i v_1 \underline{1},$$

$$\mathcal{V}_2 = v_2 \mathbf{n}\cdot\boldsymbol{\tau},$$

$$\mathcal{W} = \lambda\tau_3, \tag{73}$$

where $\mathbf{n} = \cos\theta\hat{\mathbf{y}} - \sin\theta\hat{\mathbf{x}}$ describes the phase of the composite pairing. For convenience we shall take $\hat{\mathbf{n}} = \hat{\mathbf{y}}$, so that

$$\mathcal{V}_{\mathbf{k}} = i v_{1\mathbf{k}} + v_{2\mathbf{k}}\tau_2, \tag{74}$$

where we have introduced the notation $v_{\Gamma\mathbf{k}} = v_\Gamma\Phi_{\Gamma\mathbf{k}}$.

Ostensibly, our mean-field theory is that of a BCS superconductor, with the Hamiltonian described by

$$\mathcal{H}(\mathbf{k}) = \begin{bmatrix} \epsilon_{\mathbf{k}}\tau_3 & \mathcal{V}^{\dagger}{}_{\mathbf{k}} \\ \mathcal{V}_{\mathbf{k}} & \mathbf{W}\cdot\boldsymbol{\tau}\end{bmatrix}. \tag{75}$$

However, there is one important distinction: here the pairing takes place between charged conduction electrons and the neutral f spins, and is merely a manifestation of the formation of composite pairs. For this reason, it is actually not possible to say whether the pairing is channel one or in channel two. In the gauge we have chosen, the scattering in channel one is ''normal'' and pairing takes place in channel two. But suppose we make the gauge transformation (56) with $g_j = -i\tau_2$, then

$$\mathcal{V}_{\mathbf{k}} = i v_{1\mathbf{k}} + v_{2\mathbf{k}}\tau_2 \to i\tau_2\mathcal{V}_{\mathbf{k}} = v_{1\mathbf{k}}\tau_2 - i v_{2\mathbf{k}},$$

$$\mathbf{W} = \lambda\tau_3 \to i\tau_2\mathbf{W}(-i\tau_2) = -\lambda\tau_3, \tag{76}$$

which transforms the Hamiltonian to one which is now pairing in channel *one*, and ''normal'' in channel two. We are forced to recognize the superconductivity can not be identified with either channel, but instead derives from a coherence between the two channels.

Suppose we now integrate out the f electrons: now we find that the conduction electron Green function has the form

$$G(\kappa)^{-1} = \omega - \epsilon_{\mathbf{k}}\tau_3 - \Sigma(\kappa), \tag{77}$$

where $\kappa \equiv (\mathbf{k},\omega)$ and where the self-energy term

$$\Sigma(\kappa) = \mathcal{V}^{\dagger}{}_{\mathbf{k}}(\omega - \lambda\tau_3)^{-1}\mathcal{V}_{\mathbf{k}} \tag{78}$$

describes the resonant scattering off the quenched local moments. If we expand the self-energy, we see that it contains both normal and anomalous components

$$\Sigma(\kappa)=\Sigma_N(\kappa)+\Sigma_A(\kappa). \tag{79}$$

Notice that although this self-energy contains off-diagonal terms, it is invariant under the local SU(2) gauge transformations. The normal components are channel diagonal

$$\Sigma_N(\kappa)=\frac{v_{1\mathbf{k}}^2}{\omega-\lambda\tau_3}+\frac{v_{2\mathbf{k}}^2}{\omega+\lambda\tau_3} \tag{80}$$

but the anomalous terms depend on channel interference

$$\Sigma_A(\kappa)=-\frac{2\lambda v_{1\mathbf{k}}v_{2\mathbf{k}}}{\omega^2-\lambda^2}\tau_1, \tag{81}$$

and are directly proportional to the composite order parameter Λ. For $\lambda\neq 0$, composite order induces conventional pairing amongst the conduction electrons. We shall later see that the independent existence of the composite order means that a finite Meissner stiffness develops *even when* $\lambda=0$, *and conventional pairing is absent*.

The mean-field free energy per site can be written as follows:

$$F=-T/N_s\sum_{\mathbf{k}i\omega_n}\mathrm{Tr}\ \ln[i\omega_n-\mathcal{H}(\mathbf{k})]+2\sum_\Gamma\frac{v_\Gamma^2}{J_\Gamma}. \tag{82}$$

The eigenvalues of the mean-field Hamiltonian $\mathcal{H}(\mathbf{k})$ occur in two pairs $(-\omega_{\mathbf{k}\eta},\omega_{\mathbf{k}\eta})$, where $\eta=\pm$, corresponding to two bands of quasiparticle excitations. We may rewrite the characteristic determinant of $\mathcal{H}(\mathbf{k})$, $\mathrm{Det}[\omega-\mathcal{H}(\mathbf{k})]$, in terms of the $G(\kappa)$

$$\mathrm{Det}[\omega-\mathcal{H}(\mathbf{k})]=\mathrm{Det}[G(\kappa)^{-1}](\omega^2-\lambda^2). \tag{83}$$

To evaluate the determinant, we write $G(\kappa)^{-1}$ in the following form:

$$G(\kappa)^{-1}=(A-B\tau_3+C\tau_1)(\omega^2-\lambda^2)^{-1}, \tag{84}$$

where

$$A=\omega(\omega^2-\lambda^2-v_{\mathbf{k}}^2),$$

$$B=\epsilon_{\mathbf{k}}(\omega^2-\lambda^2)+\lambda v_{\mathbf{k}-}^2,$$

$$C=2\lambda v_{1\mathbf{k}}v_{2\mathbf{k}}, \tag{85}$$

$v_{\mathbf{k}}^2=v_{1\mathbf{k}}^2+v_{2\mathbf{k}}^2$, and $v_{\mathbf{k}-}^2=v_{1\mathbf{k}}^2\pm v_{2\mathbf{k}}^2$. The eigenvalue equation $\mathrm{Det}[\omega-\mathcal{H}(\mathbf{k})]=0$ then becomes

$$(A^2-B^2-C^2)(\omega^2-\lambda^2)^{-1}=0. \tag{86}$$

Inserting the resuls (85) into Eq. (86), we obtain

$$\mathrm{Det}[\omega-\mathcal{H}(\mathbf{k})]=\omega^4-2\omega^2\alpha_{\mathbf{k}}+\gamma_{\mathbf{k}}^2, \tag{87}$$

where

$$\alpha_{\mathbf{k}}=v_{\mathbf{k}}^2+\tfrac{1}{2}(\epsilon_{\mathbf{k}}^2+\lambda^2) \tag{88}$$

and

$$\gamma_{\mathbf{k}}=\sqrt{(\lambda\epsilon_{\mathbf{k}}-v_{\mathbf{k}-}^2)^2+(2v_{1\mathbf{k}}v_{2\mathbf{k}})^2}. \tag{89}$$

The eigenvalues of $\mathcal{H}(\mathbf{k})$ are thus given by

$$\omega_{\mathbf{k}\pm}=\sqrt{\alpha_{\mathbf{k}}\pm(\alpha_{\mathbf{k}}^2-\gamma_{\mathbf{k}}^2)^{1/2}}, \tag{90}$$

and the mean-field free energy is

$$F=-\frac{2T}{N_s}\sum_{\mathbf{k},\eta}\ln[2\cosh(\beta\omega_{\mathbf{k}\eta}/2)]+2\sum_{\Gamma=1,2}\frac{(v_\Gamma)^2}{J_\Gamma}. \tag{91}$$

By minimizing the Free energy with respect to λ, v_1, and v_2, we can now determine the mean-field phase diagram.

IX. PHASE DIAGRAM

We now discuss the mean-field phase diagram. There are three types of stable mean-field solutions.

Normal phase, v_1 *or* $v_2\neq 0$, $v_1v_2=0$. At high temperatures, either v_2 or v_1 is finite finite, signaling a Kondo effect in the stronger channel. There are thus *two* types of normal phase with different Fermi surface geometry, depending on which channel is the strongest. Suppose $v_2=0$, then the normal state spectrum (90) attains the simpler form

$$\omega_{\mathbf{k}\eta}\rightarrow E_{\mathbf{k}\pm}=\frac{1}{2}[(\epsilon_{\mathbf{k}}+\lambda)\pm\sqrt{(\epsilon_{\mathbf{k}}-\lambda)^2+4v_{1\mathbf{k}}^2}],$$

corresponding to a band formed by an admixture between the conduction electrons, and the composite *f* electrons in channel one. This phase describes a heavy fermion metal.

Gapless composite paired state, $v_1v_2\neq 0$. In the generic composite paired ground state, the quasiparticle excitation contains nodes. The condition for gapless excitations is

$$\gamma_{\mathbf{k}}^2=0, \tag{92}$$

which implies that

$$(\lambda\epsilon_{\mathbf{k}}-v_{\mathbf{k}-}^2)=0,$$

$$v_{1\mathbf{k}}v_{2\mathbf{k}}=0. \tag{93}$$

The first condition defines the locus of points on the underlying Fermi surface, while the second condition defines the nodes of the order parameter. Gapless quasiparticles form at the intersection of the order parameter nodes with the Fermi surface. This occurs when one or the other channel is dominant and in this case, the conduction propagator (77) can be written

$$G(\kappa)^{-1}=Z_{\mathbf{k}}^{-1}[\omega-E_{\mathbf{k}}^*\tau_3-\Delta_{\mathbf{k}}\tau_1], \tag{94}$$

where the gap symmetry is determined by the *product* of form factors

$$\Delta_{\mathbf{k}}=\Delta_0\Phi_{1\mathbf{k}}\Phi_{2\mathbf{k}} \tag{95}$$

and

$$Z_{\mathbf{k}}^{-1}=1+\frac{v_{\mathbf{k}}^2}{\lambda} \tag{96}$$

is a mass-renormalization constant, and $E_{\mathbf{k}}^*=Z_{\mathbf{k}}(\epsilon_{\mathbf{k}}-v_{\mathbf{k}-}^2/\lambda)$, $\Delta_0=2Z_{\mathbf{k}}v_1v_2\lambda^{-1}$ describe the kinetic and pair-

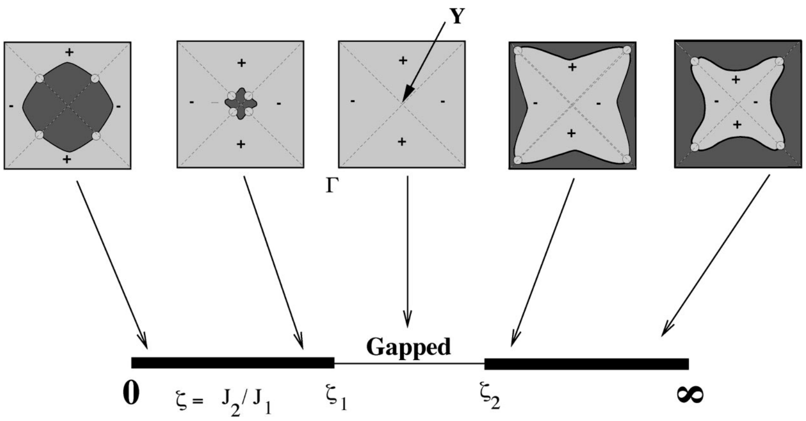

FIG. 7. Evolution of the gap nodes (open circles) and the underlying Fermi surface in the composite superconductor, as the ratio $\zeta = J_2/J_1$ evolves from zero to infinity. In the intermediate region, where $\zeta_1 < J_2/J_1 < \zeta_2$, the underlying Fermi surface collapses around the zone center, and the nodes annihilate one another to produce a gapped state.

ing contributions to the quasiparticle energy. The heavy electrons in this state are paired, and thus correspond to an anisotropic BCS superconductor a spectrum $\omega_{\mathbf{k}} = \sqrt{(E^*_{\mathbf{k}})^2 + (\Delta_{\mathbf{k}})^2}$.

Gapped composite paired state, $v_1 v_2 \neq 0$. The gapped composite paired phase occurs when $\zeta = J_2/J_1$ lies between two critical values

$$\zeta_1 < J_2/J_1 < \zeta_2 . \tag{97}$$

If the weaker coupling constants is increased to the point where it is comparable with the stronger channel, the underlying Fermi surface collapses around the zone center, causing the the nodes to mutually annihilate. At a a still larger coupling constant, the nodes reappear at the zone corners (Fig. 7). This phenomenon is perhaps easiest to visualize when the conduction sea is half filled. In this case, the normal state is a Kondo insulator with no Fermi surface.[14] The mean-field theory predicts that when J_2 exceeds a critical value, composite pairing can take place forming a pure composite paired state. Although the quasiparticle spectrum is gapped, as in a Kondo insulator

$$\omega_{\mathbf{k}\pm} = \frac{1}{2}\left[\sqrt{\epsilon_{\mathbf{k}}^2 + 4v_{+\mathbf{k}}^2} \pm \epsilon_{\mathbf{k}}\right], \tag{98}$$

the composite order parameter $\Lambda = 2v_1 v_2/(J_1 J_2)$ is finite and there is a superconducting response. When this gapped state is doped, it preserves its gap.

In the paired phase, the mean-field equations are given by the three conditions

$$\frac{\partial F}{\partial \lambda}, \quad \frac{\partial F}{\partial v_1}, \quad \frac{\partial F}{\partial v_2} = 0. \tag{99}$$

The first of these equations imposes the constraint $\tilde{f}\tau_3 f = 0$, whereas the second and third determine the magnitude of the Kondo effect in the two channels. Written out explicitly, the mean-field equations are

$$\frac{1}{N_s}\sum_{\mathbf{k}\eta} \frac{\tanh\left(\frac{\omega_{\mathbf{k}\eta}}{2T}\right)}{2\omega_{\mathbf{k}\eta}} \times \left\{ \begin{pmatrix} \lambda \\ \Phi_{1\mathbf{k}}^2 \\ \Phi_{2\mathbf{k}}^2 \end{pmatrix} + \frac{\eta}{(\alpha_{\mathbf{k}}^2 - \gamma_{\mathbf{k}}^2)^{1/2}} \mathrm{A} \right\} = \begin{pmatrix} 0 \\ \frac{2}{J_1} \\ \frac{2}{J_2} \end{pmatrix},$$

$$\mathrm{A} = \begin{pmatrix} \alpha_{\mathbf{k}}\lambda - \epsilon_{\mathbf{k}}(\epsilon_{\mathbf{k}}\lambda - v_{\mathbf{k}-}^2) \\ \frac{1}{2}(\epsilon_{\mathbf{k}} + \lambda)^2 \Phi_{1\mathbf{k}}^2 \\ \frac{1}{2}(\epsilon_{\mathbf{k}} - \lambda)^2 \Phi_{2\mathbf{k}}^2 \end{pmatrix}. \tag{100}$$

Suppose channel one is dominant, then in the normal state $v_2 = 0$, which yields two equations for the normal state

$$\sum_{\mathbf{k},\eta=\pm} \tanh\left(\frac{\beta E_{\mathbf{k}\eta}}{2}\right)\left[1 - \frac{\epsilon_{\mathbf{k}} - \lambda}{E_{\mathbf{k}\eta} - E_{\mathbf{k}-\eta}}\right] = 0,$$

$$\frac{2}{J_1} = \chi_K(T), \tag{101}$$

where

$$\chi_K(T) = \frac{1}{N_s}\sum_{\mathbf{k},\eta=\pm} \tanh\left(\frac{\beta E_{\mathbf{k}\eta}}{2}\right)\frac{\Phi_{1\mathbf{k}}^2}{E_{\mathbf{k}\eta} - E_{\mathbf{k}-\eta}}. \tag{102}$$

By setting $v_2 = 0^+$ in the full set of mean-field equations, we find that the transition temperature into the composite paired state is given by

$$\frac{2}{J_2} = \chi_C(T_c), \tag{103}$$

where

$$\chi_C(T) = \frac{1}{N_s}\sum_{\mathbf{k},\eta=\pm} \tanh\left(\frac{\beta E_{\mathbf{k}\eta}}{2}\right)\frac{\Lambda_{\mathbf{k}\eta}^2}{2E_{\mathbf{k}\eta}} \tag{104}$$

and

$$\Lambda_{\mathbf{k}\eta}^2=\Phi_{2\mathbf{k}}^2\left[1+\frac{(\epsilon_{\mathbf{k}_F}-\lambda)^2}{E_{\mathbf{k}-}^2-E_{\mathbf{k}+}^2}\right] \tag{105}$$

is identified as the matrix element $\Lambda_{\mathbf{k}\eta}^2\sim|\langle \mathbf{k}_F\uparrow,-\mathbf{k}_F\downarrow|\Lambda|\Phi\rangle|^2$ associated with the action of the composite pair operator on the Fermi liquid ground state $|\Phi\rangle$. In all the above expressions, we have simplified the algebra using the identities

$$E_{\mathbf{k}\eta}-E_{\mathbf{k}-\eta}=\eta\sqrt{(\epsilon_{\mathbf{k}}-\lambda)^2+(2v_{1\mathbf{k}})^2},$$
$$E_{\mathbf{k}\eta}+E_{\mathbf{k}-\eta}=\epsilon_{\mathbf{k}}+\lambda. \tag{106}$$

We now discuss the detailed phase diagram that results from the mean-field equations.

A. Instability of the heavy electron metal

We have argued that the presence of a heavy fermi surface leads to new zero modes for the transfer of singlets between different screening channels, so that when a second channel develops a finite coupling, a composite pair instability immediately results. We now describe in detail, how this result emerges naturally from our mean-field theory.

Let us begin by setting the scale of the Kondo temperature within this mean-field theory. In mean-field theory, the crossover into the Fermi-liquid regime is crudely delineated by a mean-field phase transition. We use this temperature as a definition of the single site Kondo temperature. At the ''transition temperature,'' $v_1=0^+$, so

$$\chi_K(T_K)=\frac{1}{N_s}\sum_{\mathbf{k}}\tanh\left(\frac{\beta\epsilon_{\mathbf{k}}}{2}\right)\Phi_{1\mathbf{k}}^2\frac{1}{\epsilon_{\mathbf{k}}}\approx 2N(0)\langle\Phi_{1\mathbf{k}}^2\rangle\ln\left(\frac{D}{T_K}\right), \tag{107}$$

where we have replaced the momentum sum by an integral over energy, so that $N(0)$ is the density of states at the Fermi surface, and $\langle\Phi_{1\mathbf{k}}^2\rangle$ denotes a Fermi surface average of the Form factor. With this definition the mean-field Kondo temperature takes the form

$$T_{K1}\sim De^{-1/J_1N(0)\langle\Phi_{1\mathbf{k}}^2\rangle}. \tag{108}$$

This quantity sets the characteristic size of the mean-field parameters in the normal state

$$\lambda\sim(V_1)^2N(0)\sim T_{K1}. \tag{109}$$

The composite pair instability of the normal state is directly related to a divergence in the fluctuations associated with the Kondo effect in channel two. To see this, we expand the mean-field expression for the Free energy to Gaussian order in $\mathcal{V}_2=v_{2x}\tau_x+v_{2y}\tau_y$, which gives

$$F=F_0+|v_2|^2\left[\frac{2}{J_2}-\chi_C\right], \tag{110}$$

where $v_2=v_{2x}+iv_{2y}$ and χ_C is given above. From this result, we can read off the fluctuations in the order parameter

$$\langle\delta v_2\delta v_2^*\rangle=\frac{\mathrm{T}}{2/J_2-\chi_C}. \tag{111}$$

But since the composite order parameter is given by $\delta\Lambda=2v_1\delta v_2/(J_1J_2)$, it follows that the composite pair susceptibility is given by

$$\chi_\Lambda=[\langle\delta\Lambda\,\delta\Lambda^*\rangle-\langle\delta\Lambda\,\delta\Lambda^*\rangle_{\chi_C=0}]/T=\left(\frac{v_1}{J_1}\right)^2\frac{\chi_C}{1-(J_2\chi_C/2)}. \tag{112}$$

The denominator in this expression vanishes at T_c, explicitly confirming that the composite pair susceptibility diverges at the mean-field transition between the normal, and the paired state.

To gain some insight into the composite pair instability described by Eqs. (103) and (104), we divide the bare composite susceptibility, χ_C into a ''high'' and a ''low'' energy component

$$\chi_C=\sum_{|E_{\mathbf{k}\eta}|>T_{K1}}+\sum_{|E_{\mathbf{k}\eta}|<T_{K1}}\{\cdots\}=\chi_h+\chi_l, \tag{113}$$

where the former describes the local Kondo effect in the weaker channel, the latter, the channel interference taking place on the heavy Fermi surface. At energies $|E_{\mathbf{k}\eta}|>>T_{K1}$,

$$\sum_\eta\tanh\left(\frac{\beta E_{\mathbf{k}\eta}}{2}\right)\frac{\Lambda_{\mathbf{k}\eta}^2}{2E_{\mathbf{k}\eta}}\rightarrow\frac{1}{|\epsilon_{\mathbf{k}}|}\Phi_{2\mathbf{k}}^2. \tag{114}$$

Suppose the heavy Fermi surface lies in the lower band, on the Fermi surface,

$$\epsilon_{\mathbf{k}_F}=\frac{v_{1\mathbf{k}}^2}{\lambda},$$
$$E_{\mathbf{k}_F-}^2-E_{\mathbf{k}_F+}^2=-(\lambda^2+v_{1\mathbf{k}}^2)/\lambda \tag{115}$$

so that

$$\Lambda_{\mathbf{k}_F\eta}^2=\frac{4\lambda^2v_1^2(\Phi_{1\mathbf{k}_F}\Phi_{2\mathbf{k}_F})^2}{(\lambda^2+v_{1\mathbf{k}_F}^2)^2}. \tag{116}$$

When we replace the momentum sums by energy integrals, we must remember that the density of quasiparticle states is enhanced by a factor

$$N_{\mathbf{k}_F}^*(0)=N(0)\frac{d\epsilon_{\mathbf{k}}}{dE_{\mathbf{k}-}}\bigg|_{\mathbf{k}=\mathbf{k}_F}=\frac{(\lambda^2+v_{1\mathbf{k}}^2)}{\lambda^2}N(0). \tag{117}$$

The energy scale separating the two regimes is the Kondo temperature for channel one T_{K1}. With these results, approximate expressions for the high and low energy contributions to the composite susceptibility are

$$\frac{\chi_h}{2N(0)}=\int_{T_{K1}}^D\frac{d\epsilon}{\epsilon}\langle\Phi_{2\mathbf{k}}^2\rangle,$$

$$\frac{\chi_l}{2N(0)}=2\int_T^{T_{K1}}\frac{dE}{E}\left\langle\frac{(v_1\Phi_{1\mathbf{k}}\Phi_{2\mathbf{k}})^2}{(\lambda^2+v_{1\mathbf{k}}^2)}\right\rangle\approx 2\int_T^{T_{K1}}\frac{dE}{E}\langle\Phi_{2\mathbf{k}}^2\rangle. \tag{118}$$

The expression for χ_l was simplified by noting that the dominant contribution to the second term occurs in the regions far from the node in the order parameter, where $v_{1\mathbf{k}} \gg \lambda$. The sum of the two expressions then yields

$$\chi_C \approx 2N(0)\left[\ln\left(\frac{D}{T_{K1}}\right)+2\ln\left(\frac{T_{K1}}{T}\right)\right]\langle\Phi_{2\mathbf{k}}^2\rangle. \qquad (119)$$

The Gaussian coefficient of v_2 in the Free energy is thus given by

$$\frac{2}{J_2}-\chi_C = 2N(0)\langle\Phi_{2\mathbf{k}}^2\rangle\left[\frac{1}{g_2}-\ln\left(\frac{D}{T_{K1}}\right)-2\ln\left(\frac{T_{K1}}{T}\right)\right], \qquad (120)$$

where $g_2 = N(0)J_2\langle\Phi_{2\mathbf{k}}^2\rangle$ is the dimensionless Kondo coupling constant for channel two. The first logarithm in this expression describes the renormalization of the coupling constant in channel two down to the energy scale T_{K1}:

$$\frac{1}{g_2(T_{K1})} = \frac{1}{g_2}-\ln\left(\frac{D}{T_{K1}}\right); \qquad (121)$$

the second logarithm describes the subsequent renormalization of g_2 at temperatures below T_{K1}. In a two-channel single impurity model, the second logarithm would be entirely absent because the Kondo effect in channel one cuts off any further renormalization in channel two. Here we see that the constructive interference between the two channels in the lattice actually over-compensates for the Kondo effect in channel one, producing a logarithmic renormalization at low temperatures which is twice as large as at high temperatures. The co-operative Kondo effect thus develops at a temperature which is higher than the Kondo temperature for an isolated channel two. We may rewrite the Gaussian coefficient as

$$\begin{aligned}\frac{2}{J_2}-\chi_C &= \frac{2}{J_2}-2N(0)\left[\ln\left(\frac{D}{T}\right)+\ln\left(\frac{T_{K1}}{T}\right)\right]\langle\Phi_{2\mathbf{k}}^2\rangle \\ &= -2N(0)\left[\ln\left(\frac{T_{K2}}{T}\right)+\ln\left(\frac{T_{K1}}{T}\right)\right]\langle\Phi_{2\mathbf{k}}^2\rangle \\ &= 4N(0)\langle\Phi_{2\mathbf{k}}^2\rangle\ln\left(\frac{T}{\sqrt{T_{K1}T_{K2}}}\right), \end{aligned} \qquad (122)$$

where we have used the definition

$$T_{K2} = De^{-1/[N(0)J_2\langle\Phi_{2\mathbf{k}}^2\rangle]}, \qquad (123)$$

to absorb the coupling constant J_2. In this rough approximation, the composite pairing instability occurs at a temperature

$$T_c \sim \sqrt{T_{K1}T_{K2}}. \qquad (124)$$

When $J_2/J_1 \ll 1$, this same scale sets the size of Δ_0. Figure 8 illustrates the phase diagram calculated numerically for a two channel Kondo lattice with s and d-wave screening channels

$$\Phi_{1\mathbf{k}} = 1 \ (s \text{ channel}),$$
$$\Phi_{2\mathbf{k}} = (\cos k_x - \cos k_y) \ (d_{x^2-y^2} \text{ channel}), \qquad (125)$$

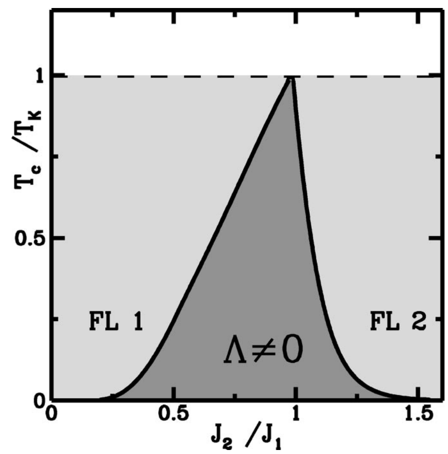

FIG. 8. Phase diagram for a two channel Kondo lattice with s and d-wave screening channels. Temperature is given in units of $T_K = \max(T_{K1}, T_{K2})$. In the shaded region, a co-operative Kondo effect in both channels gives rise to composite pairing and a quasiparticle gap with $s\times d = d$-wave symmetry.

as shown in Fig. 3. As expected, the composite pair instability occurs at the highest temperature when the two channels are most evenly matched.

B. Composite pair instability of the Kondo insulator

Since composite pairing is an intrinsically local process, the presence of a heavy Fermi surface is not a necessary requirement for the formation of the paired state, but in its absence, the instability requires the second-channel coupling constant to exceeds a critical value. This is precisely what happens where the conduction band is half filled, for in this case, the normal state of the Kondo lattice is a ''Kondo insulator,'' with no Fermi surface and a gap to charge excitations. The Kondo insulator is particle-hole symmetric, so in the mean-field theory, $\lambda = 0$, so that the excitation spectrum simplifies to the following form:

$$\omega_{\mathbf{k}\pm} = \sqrt{\left(\frac{\epsilon_{\mathbf{k}}}{2}\right)+v_{\mathbf{k}}^2}\pm\frac{\epsilon_{\mathbf{k}}}{2}. \qquad (126)$$

By minimizing the mean-field Free-energy with respect to variations in v_1 and v_2, and setting $T=0$, we obtain two mean-field equations for the ground state

$$\frac{1}{J_1} = \int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}1}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_{\mathbf{k}})^2}},$$
$$\frac{1}{J_2} = \int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}2}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_{\mathbf{k}})^2}} \qquad (127)$$

(where d is the dimensionality). This composite paired phase will only be stable within a range of J_2, $J_2^* < J_2 < J_2^{**}$. By setting $v_2 = 0^+$, we obtain two parametric equations for the second-order phase boundary between the Kondo insulator and the composite paired state:

$$\frac{1}{J_1} = \int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}1}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_1\Phi_{1\mathbf{k}})^2}}, \qquad (128)$$

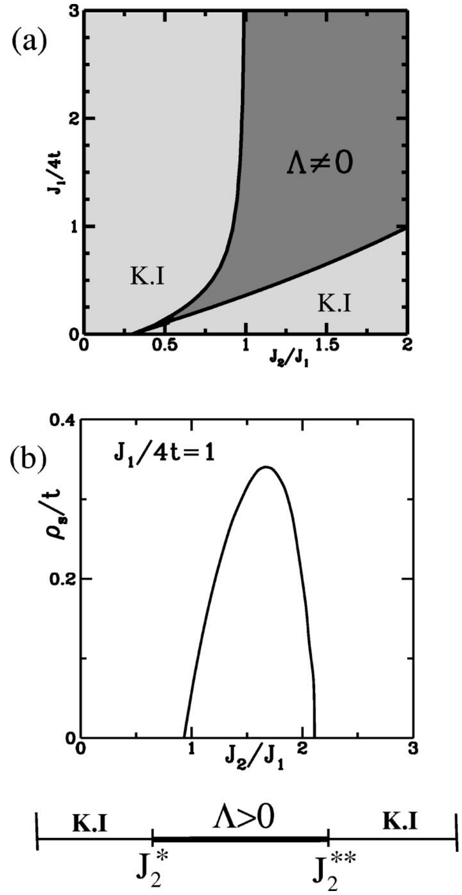

FIG. 9. (a) Phase diagram for two-channel Kondo insulator. KI denotes the Kondo insulating phases. In the intermediate gapless phase both channels participate coherently in the composite pairing process. (b) Showing phase stiffness in the composite paired phase for the case $J_1/4t=1$.

$$\frac{1}{J_2^*}=\int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}2}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_1\Phi_{1\mathbf{k}})^2}}.$$

Beyond the critical value J_2^{**}, the Kondo effect is no longer operative in channel two. By setting $v_1=0^+$, we obtain two parametric equations for the second phase boundary:

$$\frac{1}{J_1}=\int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}1}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_2\Phi_{2\mathbf{k}})^2}},$$

$$\frac{1}{J_2^{**}}=\int \frac{d^dk}{(2\pi)^d}\frac{\Phi_{\mathbf{k}2}^2}{\sqrt{\epsilon_{\mathbf{k}}{}^2+(2v_2\Phi_{2\mathbf{k}})^2}}. \tag{129}$$

We have calculated the phase diagram for a two-dimensional Kondo insulator in two dimensions, with dispersion $\epsilon_{\mathbf{k}}=-2t(c_x+c_y)$ and a Kondo coupling in the s and d channels, as shown in Eq. (125). When $J_2<J_2^*$, the Kondo effect in the s channel leads to Kondo insulator. By contrast, when $J_1\ll J_2$, a ‘‘nodal semimetal’’ forms in channel two, with a gap which vanishes along the d-wave nodes of the form-factor $\Phi_{2\mathbf{k}}$. Figure 9 shows the phase diagram obtained from Eqs. (128) and (129). Notice that the range of J_2 over which the composite paired state is stable, is negligible for small J_1 and J_2, but grows substantially as both J_1 becomes large compared with t.

This composite paired state is interesting, for its quasiparticle spectrum is essentially identical to the Kondo insulator, and furthermore, since $\lambda=0$, there is no anomalous component to the conduction electron self energy: there are no paired conduction electrons in the ground state. Yet despite these similarities, the presence of composite order

$$\Lambda(x)=\frac{2}{J_1J_2}v_1v_2e^{-i\phi(x)}, \tag{130}$$

means that this state is a superconductor, with a finite charge susceptibility. The calculation of the charge susceptibility needs to be carried out subject to the constraint. Expanding the free energy energy to quadratic order in changes in λ and the chemical potential, μ we have

$$F=F_0-\frac{1}{2}[\chi_{\mu\mu}(\delta\mu)^2+2\chi_{\mu\lambda}\delta\mu\,\delta\lambda+\chi_{\lambda\lambda}(\delta\lambda)^2]. \tag{131}$$

The constraint $\partial F/\partial\lambda=0$ implies that

$$\delta\lambda=-\frac{\chi_{\mu\lambda}}{\chi_{\lambda\lambda}}\delta\mu, \tag{132}$$

so that we may write

$$F=F_0-\frac{\chi_C}{2}(\delta\mu)^2, \tag{133}$$

where

$$\chi_C=\chi_{\mu\mu}-\frac{\chi_{\mu\lambda}^2}{\chi_{\lambda\lambda}}. \tag{134}$$

A rather laborious calculation (Appendix C) gives

$$\chi_{\mu\mu}=\sum_{\mathbf{k}}\frac{4v_{\mathbf{k}}^2}{[(\epsilon_{\mathbf{k}})^2+4v_{\mathbf{k}}^2]^{3/2}},$$

$$\chi_{\mu\lambda}=\sum_{\mathbf{k}}\frac{4v_{\mathbf{k}-}^2}{[(\epsilon_{\mathbf{k}})^2+4v_{\mathbf{k}}^2]^{3/2}},$$

$$\chi_{\lambda\lambda}=\chi_{\mu\mu}+\chi_b, \tag{135}$$

where

$$\chi_b=\sum_{\mathbf{k}}\frac{\epsilon_{\mathbf{k}}^2}{[(\epsilon_{\mathbf{k}})^2+4v_{\mathbf{k}}^2]^{3/2}}\left(\frac{2v_{1\mathbf{k}}v_{2\mathbf{k}}}{v_{\mathbf{k}}^2}\right)^2\left(5+\frac{\epsilon_{\mathbf{k}}^2}{v_{\mathbf{k}}^2}\right). \tag{136}$$

Since $|\chi_{\mu\lambda}|\leqslant\chi_{\mu\mu}$ and $\chi_{\lambda\lambda}\geqslant\chi_{\mu\mu}$, χ_C is positive, provided both v_1 and v_2 are finite. In the next section we also confirm that the composite paired state has a finite superfluid phase stiffness, given by

$$\rho_s=\frac{1}{d}\sum_{\mathbf{k}}\frac{(v_1v_2)^2}{v_{\mathbf{k}}^2}\frac{(\Phi_{1\mathbf{k}}\nabla\Phi_{2\mathbf{k}}-\Phi_{2\mathbf{k}}\nabla\Phi_{1\mathbf{k}})^2}{[\epsilon_{\mathbf{k}}^2+(2v_{\mathbf{k}+})^2]^{1/2}}, \tag{137}$$

in the ground-state.

C. Gapped composite paired state

In our simple two dimensional example, $v_{1\mathbf{k}}$ is never zero, so the condition (93) for gapless behavior become

$$\lambda = \frac{v_{1\mathbf{k}}^2}{\epsilon_{\mathbf{k}}},$$

$$v_{2\mathbf{k}} = 0. \tag{138}$$

The modulus $|(v_{1\mathbf{k}}^2/\epsilon_{\mathbf{k}})|$ is smallest at the band-edges, where $|\epsilon_{\mathbf{k}}| = W_{\pm} = 4t \mp \mu$. So if

$$-\left(\frac{v_{1\mathbf{k}}^2}{W_-}\right) < \lambda < \left(\frac{v_{1\mathbf{k}}^2}{W_+}\right), \tag{139}$$

the paired state becomes gapless. Although a small residual Cooper pair density is present, the state is essentially a pure condensate of composite pairs. To understand what happens at the critical values of ζ, it is instructive to examine the underlying Fermi surface, defined by the locus of points

$$\epsilon_{\mathbf{k}} = \frac{v_{1\mathbf{k}}^2 - v_{2\mathbf{k}}^2}{\lambda}. \tag{140}$$

As one approaches the critical value $\zeta = \zeta_1$, this Fermi surface collapses around the zone center, forcing the nodes to mutally annihilate. In going from the region $J_2/J_1 \ll 1$ to the region $J_2/J_1 \gg 1$, λ changes sign, and so that the system must always pass through this region of gapless composite pairing. In the center of this region, where $\lambda = 0$, the state is a pure composite paired state. Once ζ exceeds the value ζ_2, the nodes are reborn at the zone corners and the Fermi-surface reappears along the zone boundary (Fig. 7).

To illustrate these conclusions, we have used the mean-field equations to calculate numerically the two lines where the gap vanishes. In Fig. 1, we summarize the results of these calculations, in a diagram where we have kept $\max(J_1, J_2) = 4t$, and varied the ratio J_2/J_1.

X. SUPERFLUID DENSITY OF THE COMPOSITE PAIRED STATE

To confirm that the composite paired state is superconducting, we need to compute the superfluid density ρ_s. In the London gauge $\nabla \cdot \mathbf{A} = 0$, the supercurrent is given by

$$\mathbf{j}_s = -Q\mathbf{A}, \tag{141}$$

where

$$Q_{ab} = e^2[\rho_s]_{ab} = \frac{\partial^2 F}{\partial \mathbf{A}_a \partial \mathbf{A}_b}. \tag{142}$$

In the presence of an electromagnetic field, the electron kinetic energy *and* form factors acquire a dependence on the vector potential. Using a Nambu notation, we may write

$$\epsilon_{\mathbf{k}} \rightarrow \epsilon_{\mathbf{k} - e\mathbf{A}\tau_3},$$

$$\Phi_{\Gamma\mathbf{k}} \rightarrow \Phi_{\Gamma\mathbf{k} - e\mathbf{A}\tau_3}, \tag{143}$$

so that the hybridization acquires the form

$$\mathcal{V}_{\mathbf{k}}^{\mathbf{A}} = i v_1 \Phi_{1\mathbf{k} - e\mathbf{A}\tau_3} + v_2 \tau_2 \Phi_{2\mathbf{k} - e\mathbf{A}\tau_3}. \tag{144}$$

The appearance of the vector potential in the form factor reflects the fact that the hop and flip motion of electrons around a local moment leads to current flow. In the London gauge $(\nabla \cdot \mathbf{A} = 0)$ we may calculate by compute the second-derivative of the Free energy at fixed values of v_1, v_2, and λ,[29] so the the only important part of the Free energy is the electronic component

$$F_e = -T \sum_{\kappa} \mathrm{Tr} \ln[i\omega_n - \mathcal{H}_{\mathbf{A}}(\mathbf{k})]. \tag{145}$$

To second order in the vector potential, we may write

$$\mathcal{H}_{\mathbf{A}}(\mathbf{k}) = \mathcal{H}(\mathbf{k}) - \mathcal{J}^a A_a + \frac{1}{2} A_a A_b \nabla_{ab}^2 \mathcal{H}(\mathbf{k}) + O(A^3), \tag{146}$$

so expanding the Free energy to second order in $\mathbf{A}$, we obtain

$$F = F_0 + \tfrac{1}{2} Q_{ab} A_a A_b,$$

$$Q^{ab} = T \sum_{\kappa} \{\mathrm{Tr}[\mathcal{G}_\kappa \mathcal{J}_\kappa^a \mathcal{G}_\kappa \mathcal{J}_\kappa^b] + [\mathcal{G}_\kappa \nabla_{ab}^2 \mathcal{H}_\kappa]\}, \tag{147}$$

where

$$\mathcal{G}_\kappa = [i\omega_n - \mathcal{H}(\mathbf{k})]^{-1} \equiv \begin{bmatrix} G & G_{cf} \\ G_{fc} & G_{ff} \end{bmatrix} \tag{148}$$

is the matrix propagator and

$$\mathcal{J}_\kappa = -\nabla_A \mathcal{H}_A(\mathbf{k}) = e \begin{bmatrix} \nabla_{\mathbf{k}} \epsilon_{\mathbf{k}} & \tau_3 \nabla_{\mathbf{k}} \mathcal{V}^\dagger{}_{\mathbf{k}} \\ \nabla_{\mathbf{k}} \mathcal{V}_{\mathbf{k}} \tau_3 & 0 \end{bmatrix} \tag{149}$$

is the current operator. The first and second terms in Eq. (147) correspond to the paramagnetic and diamagnetic components of the stiffness, as in a conventional superconductor. Notice, however, that the current contains anomalous off-diagonal contributions that do not commute with the charge operator τ_3. Unlike a pure BCS superconductor, here the presence of composite pairs affects the current operator.

The diamagnetic contribution to Q^{ab} can integrated by parts, to obtain

$$\sum_{\kappa} \mathrm{Tr}[\mathcal{G}_\kappa \nabla_{ab}^2 \mathcal{H}_\kappa] = -\sum_{\kappa} \mathrm{Tr}[\mathcal{G}_\kappa \mathrm{j}_\kappa^a \mathcal{G}_\kappa \mathrm{j}_\kappa^b], \tag{150}$$

where

$$\mathbf{j}_\kappa = e \nabla_{\mathbf{k}} \mathcal{H}(\mathbf{k}) = e \begin{bmatrix} \nabla_{\mathbf{k}} \epsilon_{\mathbf{k}} \tau_3 & \nabla_{\mathbf{k}} \mathcal{V}^\dagger{}_{\mathbf{k}} \\ \nabla_{\mathbf{k}} \mathcal{V}_{\mathbf{k}} & 0 \end{bmatrix}, \tag{151}$$

so that the full expression for the superfluid stiffness tensor is

$$Q_{ab} = T \sum_{\kappa} \{\mathrm{Tr}[\mathcal{G}_\kappa \mathcal{J}_\kappa^a \mathcal{G}_\kappa \mathcal{J}_\kappa^b] - \mathrm{Tr}[\mathcal{G}_\kappa \mathrm{j}_\kappa^a \mathcal{G}_\kappa \mathrm{j}_\kappa^b]\}. \tag{152}$$

To gain some insight into this equation, it is instructive to evaluate the stiffness for the case of pure composite pairing at half-filling. In this case, the conduction electron propagator commutes with the charge operator τ_3, so contributions to the stiffness which involve the conduction electron component of $\mathcal{G}$ identically vanish. For example, the cross term between $\nabla\epsilon_{\mathbf{k}}$ and $\nabla\mathcal{V}$ is

$$\mathrm{Tr}[G_{cc}\nabla_a\epsilon G_{cf}\nabla_b\mathcal{V}\tau_3 - G_{cc}\nabla_a\epsilon\tau_3 G_{cf}\nabla_b\mathcal{V}]=0. \quad (153)$$

The only surviving terms represent the composite pair stiffness, which can be written in the form

$$Q^C_{ab}=\frac{\mathrm{T}}{2}\mathrm{Tr}[[G_{cf}\nabla_a\mathcal{V},\tau_3][G_{cf}\nabla_b\mathcal{V},\tau_3]+\mathrm{H.\,c.}]. \quad (154)$$

From the Dyson equation,

$$\mathcal{G}(\kappa)=\mathcal{G}_0(\kappa)+\mathcal{G}(\kappa)\begin{bmatrix}0 & \mathcal{V}^\dagger\\ \mathcal{V} & 0\end{bmatrix}\mathcal{G}_0(\kappa), \quad (155)$$

where $\mathcal{G}_0$ is the propagator in the absence of any hybridization, it follows that

$$G_{cf}(\kappa)=G(\kappa)\mathcal{V}^\dagger\frac{1}{\omega-\lambda\tau_3}, \quad (156)$$

so that at half filling ($\lambda=0$)

$$G_{cf}(\kappa)=\frac{1}{\omega(\omega-\epsilon_{\mathbf{k}}\tau_3)-v^2_{\mathbf{k}}}\mathcal{V}^\dagger, \quad (157)$$

and hence

$$[G_{cf}\nabla\mathcal{V},\tau_3]=\frac{1}{\omega(\omega-\epsilon_{\mathbf{k}}\tau_3)-v^2_{\mathbf{k}}}[\mathcal{V}^\dagger\nabla\mathcal{V},\tau_3]. \quad (158)$$

Evaluating the commutator

$$[\mathcal{V}^\dagger\nabla\mathcal{V},\tau_3]=2(v_{1\mathbf{k}}\nabla v_{2\mathbf{k}}-v_{2\mathbf{k}}\nabla v_{1\mathbf{k}})\tau_1, \quad (159)$$

we may then write

$$Q^C=\frac{4e^2T}{d}\sum_\kappa (v_{1\mathbf{k}}\nabla v_{2\mathbf{k}}-v_{2\mathbf{k}}\nabla v_{1\mathbf{k}})^2\ \mathrm{Tr}[G(\kappa)^2], \quad (160)$$

where for convenience, we have assumed an isotropic stiffness $Q^C_{ab}=Q^C\delta_{ab}$. Taking the zero-temperature limit, replacing $i\omega_n\rightarrow i\omega$ and

$$\mathrm{T}\sum_{i\omega_n}\rightarrow\int_\infty^\infty\frac{d\omega}{2\pi} \quad (161)$$

then yields

$$e^2\rho^C_s=\frac{8e^2}{d}\sum_{\mathbf{k}}(v_{1\mathbf{k}}\nabla v_{2\mathbf{k}}-v_{2\mathbf{k}}\nabla v_{1\mathbf{k}})^2 \times\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{(\omega^2+v^2_{\mathbf{k}})^2+\omega^2\epsilon^2_{\mathbf{k}}}. \quad (162)$$

Carrying out the final frequency integral yields the following expression for the composite stiffness

$$e^2\rho^C_s=\frac{e^2}{d}\sum_{\mathbf{k}}\left(\frac{v_1v_2}{v_{\mathbf{k}+}}\right)^2\frac{(\Phi_{1\mathbf{k}}\nabla\Phi_{2\mathbf{k}}-\Phi_{2\mathbf{k}}\nabla\Phi_{1\mathbf{k}})^2}{[\epsilon^2_{\mathbf{k}}+(2v_{\mathbf{k}+})^2]^{1/2}}. \quad (163)$$

This result is quite fascinating, because it confirms that the composite pair condensate has a phase stiffness, even in the absence of an underlying Fermi surface. In a conventional superconductor, the scale of the stiffness is determined by the Fermi energy $e^2\rho_s\sim e^2\epsilon_F/a^2$. Here the size of the stiffness

$$e^2\rho^C_s\sim e^2N(0)\left\langle\left(\frac{v_1v_2}{v_{\mathbf{k}+}}\right)^2\right\rangle\sim e^2T_c/a^2 \quad (164)$$

is determined by the condensation energy. This is a classic example of ''local pair'' condensation.

Away from half filling, the superfluid stiffness contains contributions associated with both the paired conduction electrons, and the composite pairs. To make a clean division of the stiffness into two components, one needs to worry about the various cross terms that appear in the stiffness such as Eq. (153), which do not obviously vanish away from half filling. However, a key observation is that these terms are odd functions of λ, so they are guaranteed guaranteed to vanish provided that the physics is particle-hole symmetric about $\lambda=0$. With this proviso, we can divide the superfluid stiffness into two terms

$$Q=Q^{\mathrm{BCS}}+Q^C, \quad (165)$$

where

$$Q^{\mathrm{BCS}}_{ab}=\frac{T}{2}\sum_\kappa\nabla_a\epsilon_{\mathbf{k}}\nabla_b\epsilon_{\mathbf{k}}\mathrm{Tr}[[iG(\kappa),\tau_3]^2],$$

$$Q^C_{ab}=\frac{\mathrm{T}}{2}\mathrm{Tr}[[G_{cf}\nabla_a\mathcal{V},\tau_3][G_{cf}\nabla_b\mathcal{V},\tau_3]+\mathrm{H.\,c.}] \quad (166)$$

are the BCS and composite pair contributions to the stiffness.

XI. DISCUSSION

In this discussion, we should like to address the results of this paper on two fronts: Alternative theoretical approaches to test and confirm the presence of a co-operative Kondo effect and applications to experiments and the theory of real heavy fermion systems.

A. Alternative theoretical approaches

The main effort of this paper has been to establish the link between co-operative channel interference in the Kondo lattice and composite pairing. Although our key result, the relation between the gauge invariant interference term and composite order

$$\mathcal{V}^\dagger{}_2\mathcal{V}_1=-\frac{J_1J_2}{2}\begin{bmatrix}F^\dagger & \Lambda\\ -\Lambda^\dagger & F^\dagger\end{bmatrix}, \quad (167)$$

does not depend on approximations, the notion that this interference term can develop long-range order relies on various mean-field approximations. There are, however, reasons to be confident in the mean-field solution. First, there are no obvious competing instabilities, such as magnetism. By turning on a J_2, one does not drive the system closer to the antiferromagnetic instability, but rather, simply activates another source of Kondo screening. This should be contrasted with the situation in the alternative model of spin-fluctuation mediated pairing. Here, to attain a transition temperature that is comparable with the heavy fermion bandwidth places the model close to an antiferromagnetic instability, where the competing effects of magnetism make the mean-field theory potentially unreliable.

Nevertheless, the use of mean-field methods inevitably raises questions about our work which motivates us to seek alternative methods to verify the key results. It may be possible to precisely verify our results in both finite size calculations, and in the exactly solvable limit of infinite dimensions. Finite size studies on our model may be facilitated by treating the model as a ladder compound and by using the strong-coupling limit so as to completely eliminate the possibility of antiferromagnetic instabilities.

To formulate our model in a form that is tractable to an exact infinite dimensional study, rather than using explicit form factors in the Kondo interaction, it is better to start with a channel conserving two-channel Kondo lattice, to which a term which destroys channel conservation is then added to activate the channel interference. Suppose one starts out with a two-channel Kondo model, with perfect channel conservation:

$$H^C = \overbrace{\sum_{\mathbf{k}\Gamma\sigma} \epsilon_{\mathbf{k}} c^\dagger{}_{\Gamma\mathbf{k}\sigma} c_{\Gamma\mathbf{k}\sigma}}^{H_0} + \sum_{\Gamma j} J_\Gamma c^\dagger{}_{\Gamma j} \boldsymbol{\sigma} c_{\Gamma j} \cdot \mathbf{S_j}, \tag{168}$$

where

$$c^\dagger{}_{\Gamma j\sigma} = N_s{}^{-1/2} \sum_{\mathbf{k}} c^\dagger{}_{\Gamma\mathbf{k}\sigma} e^{-i\mathbf{k}\cdot\mathbf{R}_j} \tag{169}$$

($\Gamma = 1, 2$) and

$$\epsilon = -2t \sum_{l=1,d} (\cos k_l) - \mu. \tag{170}$$

This model has a perfect U(1) channel symmetry. Suppose one now adds a term to the hopping $H \rightarrow H^C + H'$, where

$$H' = -2\Delta \sum_{l=1,d} (-1)^l \cos k_l (c^\dagger{}_{\mathbf{k}1\sigma} c_{\mathbf{k}2\sigma} + c^\dagger{}_{\mathbf{k}2\sigma} c_{\mathbf{k}1\sigma}), \tag{171}$$

is a hopping term with "d-wave" symmetry that mixes the different channels, breaking the U(1) channel conservation symmetry. This contains no nonlocal interactions, and is thus ideal for a large d treatment. It is easy to see that in the limit of large Δ, it is equivalent to a one band model, with two orthogonal form factors

$$\Phi_{1\mathbf{k}} = 1,$$

$$\Phi_{2\mathbf{k}} = \mathrm{sgn}\left[\sum_l (-1)^l \cos k_l\right]. \tag{172}$$

To see this, note that when Δ is nonzero, the band splits into two components with energies

$$E_{\mathbf{k}\pm} = \epsilon_{\mathbf{k}} \pm 2\Delta \left| \sum_{l=1,3,\ldots} (-1)^l \cos k_l \right|, \tag{173}$$

where

$$H_0 + H' = \sum_{\mathbf{k}\sigma} (E_{\mathbf{k}+} a^\dagger{}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + E_{\mathbf{k}-} b^\dagger{}_{\mathbf{k}\sigma} b_{\mathbf{k}\sigma}),$$

$$a_{\mathbf{k}\sigma} = \frac{1}{\sqrt{2}} (c_{\mathbf{k}1\sigma} + \Phi_{2\mathbf{k}} c_{\mathbf{k}2\sigma}),$$

$$b_{\mathbf{k}\sigma} = \frac{1}{\sqrt{2}} (c_{\mathbf{k}1\sigma} - \Phi_{2\mathbf{k}} c_{\mathbf{k}2\sigma}). \tag{174}$$

At large Δ, one can project out all terms involving the upper band by rewriting the Hamiltonian in terms of the a and b creation operators, and then dropping all terms involving the creation or annihilation operators for the upper band, $a_{\mathbf{k}\sigma}$ or $a^\dagger{}_{\mathbf{k}\sigma}$. When we do this, the interaction becomes

$$H_I = \frac{1}{2N_s} \sum_{\mathbf{k},\mathbf{k}'} J_{\mathbf{k},\mathbf{k}'} b^\dagger{}_{\mathbf{k}} \boldsymbol{\sigma} b_{\mathbf{k}} \cdot S_j e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}_j},$$

$$J_{\mathbf{k},\mathbf{k}'} = (J_1 \Phi_{1\mathbf{k}} \Phi_{1\mathbf{k}'} + J_2 \Phi_{2\mathbf{k}} \Phi_{2\mathbf{k}'}), \tag{175}$$

corresponding to a two-channel Kondo model with two orthogonal matrix elements.

The mean-field theory for this model[30] predicts that that as soon as Δ becomes finite, channel interference will drive the system into a composite paired state, even when $J_1 >> J_2$. This phenomenon should extend all the way out to infinite dimensions, where a precise dynamical mean-field theory treatment of the model becomes possible.

B. Possible consequences of co-operative Kondo behavior for heavy fermion compounds

We conclude with a brief discussion of the general consequences of co-operative channel interference in heavy fermion systems. Our paper has focussed on the superconducting aspects of this problem. Here we should like to put the problem in a more general perspective.

Assuming it becomes possible to verify the theoretical soundness of the co-operative Kondo effect, how could the theory be tested experimentally? Our model suggests a rather intimate relation between the local quantum chemistry of the heavy fermion ion, and the gap symmetry of the order parameter. In heavy fermion compounds, one of the scattering channels is an f channel. Since the two channels must have the same parity, the second channel is in all likelihood another f channel or a p channel.

◇ $f \otimes f$. Candidates: non-Kramers ion, e.g., URu_2Si_2, UBe_{13}.

◇ $f \otimes p$. Candidates: UBe_{13}, UPt_3, and cerium systems, close to quantum critical point.

The first possibility will occur if the Kondo effect involves a non-Kramer's magnetic ion. For example, in the case of URu_2Si_2, there is strong circumstantial evidence that the single-ion physics is dominated by a Kondo effect with a non-Kramers magnetic doublet. The form factors for the two f channels in a tetragonal crystal field are known, and place strong constraints on the symmetry of the putative composite order. In the second case, the number of available p channels is small and the local quantum chemistry will determine the most likely channel for the cooperative pairing process. For example, in hexagonal UPt_3, the most likely second channel is the p_z orbital, which would explain the presence of the node in the basal plane. In principle, cubic UBe_{13} could belong to either category, as this system may also have a non-Kramer's ground state. If, however, the driving force derives from a p channel, molecular orbital theory dictates that the most likely second channel is a p-wave state with normal orientated along the cube diagonals, such as the 111 direction. A gap node normal to this direction could be detected using careful transverse ultrasound measurements.[31]

One of the paradoxical features of heavy fermion superconductors, is that their large entropy of condensation suggests a large superconducting order parameter. Yet Josephson tunneling with a conventional superconductor has not, to date been achieved. Although composite and normal pairs coexist side-by-side in our hypothetical superconductor, the predominantly composite character of the order parameter may help explain why it has proven so interminably difficult to carry out Josephson tunneling into these systems. One way to enhance the Josephson current may be to introduce rare earth or actinide spins into the tunnel junction. Josephson tunneling between a conventional, and composite paired superconductor requires that the addition of a pair is accompanied by a spin-flip. Spin fluctuations of the local moments in the junction may help to catalyse this co-operative process. This is a possibility currently under investigation.

We should like to end with a short note about the nonsuperconducting aspects of the composite Kondo effect. In our key identity

$$\mathcal{V}^\dagger{}_2\mathcal{V}_1 = -\frac{J_1 J_2}{2}\begin{bmatrix} F & \Lambda \\ -\Lambda^\dagger & F^\dagger \end{bmatrix}, \tag{176}$$

we have the possibility of finite diagonal components $F \neq 0$ due to co-operative interference. Unlike composite pairing, such instabilities will require J_1 and J_2 to be of comparable size. There are, to our knowledge two good candidates for this kind of phenomenon:

Orbital magnetism in URu_2Si_2. As mentioned above, this material is a naturally occurring two-channel Kondo lattice, but with strong spin-orbit coupling. One of the long-standing mysteries of this compound, is the appearance of an unidentified magnetic state at 17 K, with a large order parameter which appears to break time-reversal symmetry, but without producing a large magnetic moment.[32,33] One possible way to account for this, is to suppose that the two channels in this compound give rise to a complex order parameter

$$F(x) = F_0 e^{i\mathbf{Q}\cdot\mathbf{x}}, \tag{177}$$

where $\mathbf{Q}$ is commensurate with the lattice. Just as superconducting composite order coexists with a weak BCS order parameter, orbital composite order will coexist with a weak orbital moment. Spin-orbit coupling will then generate a weak magnetic moment.

Ultranarrow gap Kondo insulators CeRhSb and CeNiSn. These Kondo insulators appear to develop gap nodes in their tiny hybridization gap. In a recent paper,[34] we have pointed out that this kind of behavior would arise from the suppression of shape fluctuations, which gives rise to three orbital scattering channels in which the Kondo effect can take place. The resulting interference between the three orbital channels is found to spontaneously generate a crystal field environment that gives rise to a Kondo ''insulator'' with gap nodes. These are both areas of active investigation, which lie outside the scope of this paper.

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APPENDIX A

In this section we apply the SU(2) decoupling scheme originally developed by Affleck *et al.* for the Heisenberg model to the two-channel Kondo problem. This derivation is closer in spirit to the original work by Affleck *et al.*, and differs in detail from the later work by Andrei and Coleman. The approach of Affleck *et al.*, is more explicitly SU(2) symmetric and can be naturally extended to include source terms that couple to the composite order parameter. There are two distinct differences between the approaches: The integration measure over the SU(2) field is ''flat'' and Gaussian coefficient of $\mathrm{Tr}[\mathcal{V}^\dagger{}_\Gamma \mathcal{V}_\Gamma]$ is now $1/J$ rather than $1/2J$, as it was in the earlier work by Andrei and Coleman.

The difference in measure leads to differences in the fluctuations around the mean-field theory, and the mean-field expressions for the Kondo temperature obtained in the 2 methods actually differ by a factor of two in the exponential. We have chosen the approach of Affleck *et al.* because it gives us a much cleaner an symmetric derivation of the final results.

The objective of this section is to show how the interaction between a localized moment $\mathbf{S}$ and the electron local spin density $\sigma_\Gamma = \psi^\dagger \boldsymbol{\sigma} \psi$ can be decoupled in terms of a fluctuating SU(2) field

$$J_\Gamma(\mathbf{S}\cdot\sigma_\Gamma - \tfrac{1}{2}) \rightarrow [\tilde{f}^\dagger \mathcal{V}_\Gamma \tilde{\psi}_\Gamma + \mathrm{H.c.}] + \frac{\mathrm{Tr}[\mathcal{V}^\dagger_\Gamma \mathcal{V}_\Gamma]}{J_\Gamma}, \tag{A1}$$

where V_Γ is directly proportional to an SU(2) matrix g_Γ

$$\mathcal{V}_\Gamma = i V_0^\Gamma g_\Gamma = \begin{bmatrix} V & \Delta \\ \Delta^* & -V^* \end{bmatrix}^\Gamma. \tag{A2}$$

For clarity, all site indices j are omitted from this derivation, but are readily restored later.

Following earlier work, we introduce the following matrix fermions:

$$\mathcal{F}=\begin{bmatrix} f_\uparrow & f_\downarrow \\ f_\downarrow^\dagger & -f_\uparrow^\dagger \end{bmatrix},\quad \Psi_\Gamma=\begin{bmatrix} \psi_{\Gamma\uparrow} & \psi_{\Gamma\downarrow} \\ \psi_{\Gamma\downarrow}^\dagger & -\psi_{\Gamma\uparrow}^\dagger \end{bmatrix}. \tag{A3}$$

By taking the product of these matrix operators with their Hermitian conjugates, we find that

$$\Psi_\Gamma^\dagger\Psi_\Gamma=\underline{1}+\underline{\boldsymbol{\sigma}}^T\cdot\boldsymbol{\sigma}_\Gamma,$$

$$\mathcal{F}^\dagger\mathcal{F}=\underline{1}+2\underline{\boldsymbol{\sigma}}^T\cdot\mathbf{S}, \tag{A4}$$

where $\boldsymbol{\sigma}_\Gamma=\psi^\dagger{}_\Gamma\boldsymbol{\sigma}\psi_\Gamma$ is the electron spin density and $\underline{\boldsymbol{\sigma}}^T$ denotes the transpose of the Pauli spin matrix. Transformations acting to the right of $\mathcal{F}$, $\mathcal{F}\rightarrow\mathcal{F}h$ correspond to physical rotations of the local moment. Transformations acting to the left of $\mathcal{F}$, $\mathcal{F}\rightarrow g\mathcal{F}$ correspond to the local SU(2) transformation, under which the spin operator is explicitly invariant:

$$\mathbf{S}=\frac{1}{4}\mathrm{Tr}[\boldsymbol{\sigma}^T\mathcal{F}^\dagger\mathcal{F}]\rightarrow\frac{1}{4}\mathrm{Tr}[\boldsymbol{\sigma}^T\mathcal{F}^\dagger g^\dagger g\mathcal{F}]=\mathbf{S}. \tag{A5}$$

Multiplying the two equations (A4) together, and taking the trace we obtain

$$\boldsymbol{\sigma}_\Gamma\cdot\mathbf{S}+\tfrac{1}{2}=\frac{1}{4}\mathrm{Tr}[\mathcal{F}^\dagger\mathcal{F}\Psi_\Gamma^\dagger\Psi_\Gamma]. \tag{A6}$$

Anticommuting the conduction electron operator Ψ_Γ to the left through the trace, we then find that

$$J_\Gamma(\boldsymbol{\sigma}_\Gamma\cdot\mathbf{S}-\tfrac{1}{2})=-\frac{J_\Gamma}{4}\mathrm{Tr}[U_\Gamma^\dagger U_\Gamma], \tag{A7}$$

where

$$U_\Gamma=\mathcal{F}\Psi_\Gamma^\dagger=\begin{bmatrix} -a_\Gamma^\dagger & b_\Gamma \\ b_\Gamma^\dagger & a_\Gamma \end{bmatrix} \tag{A8}$$

is an antiunitary matrix and

$$a_\Gamma=\sum_\sigma f^\dagger{}_\sigma\psi_{\Gamma\sigma},$$

$$b_\Gamma=\sum_\sigma \sigma f_{-\sigma}\psi_{\Gamma\sigma}. \tag{A9}$$

Notice that if we expand the above interaction, we obtain

$$H_I=-\frac{J_\Gamma}{4}[a_\Gamma^\dagger a_\Gamma+a_\Gamma a_\Gamma^\dagger+b_\Gamma^\dagger b_\Gamma+b_\Gamma b_\Gamma^\dagger] \tag{A10}$$

showing that it has been decoupled simultaneously in the particle-hole and Cooper channels.

We now apply a Hubbard-Stratonovich procedure to this expression. Formally, we first convert each of the fermionic operators in the interaction to Grassman variables inside a path integral. On each time slice we write

$$e^{-\Delta\tau H_I}=\int D[\mathcal{V}_\Gamma,\mathcal{V}^\dagger{}_\Gamma]e^{-\Delta\tau H_I[\mathcal{V}_\Gamma,\mathcal{V}^\dagger{}_\Gamma]}, \tag{A11}$$

where we have transformed

$$H_I=-\frac{J_\Gamma}{4}\mathrm{Tr}[U_\Gamma^\dagger U_\Gamma]\rightarrow H_I[\mathcal{V}_\Gamma,\mathcal{V}^\dagger{}_\Gamma] \tag{A12}$$

and

$$H_I[\mathcal{V}_\Gamma,\mathcal{V}^\dagger{}_\Gamma]=\frac{1}{2}\{\mathrm{Tr}[\mathcal{V}_\Gamma^\dagger U_\Gamma]+\mathrm{Tr}[U_\Gamma^\dagger\mathcal{V}_\Gamma]\}+\frac{1}{J_\Gamma}\mathrm{Tr}[\mathcal{V}_\Gamma^\dagger\mathcal{V}_\Gamma]. \tag{A13}$$

A priori, $\mathcal{V}$ is a two by two complex matrix. However, if we divide it up into the sum of a unitary and an antiunitary matrix, we find that only the former completely decouples. The residual part of $\mathcal{V}$ is completely antiunitary, and has the form

$$\mathcal{V}_\Gamma=\begin{bmatrix} V & \Delta \\ \Delta^* & -V^* \end{bmatrix}_\Gamma, \tag{A14}$$

where there are only two independent complex parameters. This is a significant simplification. Notice that $\mathcal{V}_\Gamma$ is directly proportional to an SU(2) matrix

$$\mathcal{V}_\Gamma=iV_0^\Gamma g_\Gamma\ \ (V_0^\Gamma=\sqrt{|V_\Gamma|^2+|\Delta_\Gamma|^2}). \tag{A15}$$

The measure of integration for each time slice is then simply

$$D[\mathcal{V}_\Gamma,\mathcal{V}^\dagger{}_\Gamma]=dV_\Gamma dV_\Gamma^* d\Delta_\Gamma d\Delta_\Gamma^*. \tag{A16}$$

As our final step, we now reduce the decoupled interaction to a more manageable two-component notation. Writing

$$\tilde{f}^\dagger=(f^\dagger{}_\uparrow,f_\downarrow),$$

$$\tilde{\psi}^\dagger{}_\Gamma=(\psi^\dagger{}_{\Gamma\uparrow},\psi_{\Gamma\downarrow}), \tag{A17}$$

then H_I reduces to the form

$$H_I=[\tilde{f}^\dagger\mathcal{V}_\Gamma\tilde{\psi}_\Gamma+\mathrm{H.c.}]+\frac{1}{J_\Gamma}\mathrm{Tr}[\mathcal{V}_\Gamma^\dagger\mathcal{V}_\Gamma], \tag{A18}$$

which is the form quoted in the main text.

APPENDIX B

The purpose of this section is to establish the direct relationship

$$\mathcal{V}^\dagger{}_2\mathcal{V}_1=-\frac{J_1J_2}{2}\begin{bmatrix} F^\dagger & \Lambda \\ -\Lambda^\dagger & F^\dagger \end{bmatrix}, \tag{B1}$$

where

$$F=\psi^\dagger{}_2\boldsymbol{\sigma}\psi_1\cdot\mathbf{S},$$

$$\Lambda=\psi_2(-i\sigma_y)\boldsymbol{\sigma}\psi_1\cdot\mathbf{S} \tag{B2}$$

represent the single composite order in the particle-hole, and particle-particle channels, respectively. In order to establish this identity, we introduce a source term into the Lagrangian which couples to the gauge invariant matrix product $\mathcal{V}_2\mathcal{V}_1$, writing

$$H_I=\sum_\Gamma \left\{[\tilde{f}^\dagger \mathcal{V}_\Gamma \tilde{\psi}_\Gamma + \text{H.c.}] + \frac{1}{J_\Gamma}\text{Tr}[\mathcal{V}_\Gamma^\dagger \mathcal{V}_\Gamma]\right\} + \text{Tr}[\mathcal{V}_2^\dagger \mathcal{V}_1 \alpha + \text{H.c.}], \quad \text{(B3)}$$

where the source term $\alpha=\alpha_o+i\vec{\alpha}\cdot\boldsymbol{\tau}$ is a unitary matrix, with four real coefficients.

We shall now invert the Hubbard Stratonovich transformation, with the source terms in place. We begin by rewriting the Gaussian term in the interaction to obtain

$$H_I=\sum_\Gamma \tfrac{1}{2}[\text{Tr}[\mathcal{V}_\Gamma^\dagger U_\Gamma]+\text{H.c.}]+\text{Tr}[\mathcal{V}_\Gamma^\dagger \mathcal{V}_{\Gamma'} m_{\Gamma'\Gamma}], \quad \text{(B4)}$$

where

$$m_{\Gamma\Gamma'}=\begin{bmatrix} \frac{1}{J_1} & \alpha \\ \alpha^\dagger & \frac{1}{J_2}\end{bmatrix}. \quad \text{(B5)}$$

When we carry out the Gaussian integral over $\mathcal{V}_\Gamma$, the transformed Hamiltonian now becomes

$$H_I=-\frac{1}{4}\text{Tr}[U^\dagger{}_\Gamma U_{\Gamma'}(m^{-1})_{\Gamma'\Gamma}], \quad \text{(B6)}$$

where

$$m^{-1}=\frac{J_1J_2}{(1-|\alpha|^2J_1J_2)}\begin{bmatrix} \frac{1}{J_2} & -\alpha \\ -\alpha^\dagger & \frac{1}{J_1}\end{bmatrix}. \quad \text{(B7)}$$

When we expand this to linear order in α we obtain

$$H_I=-\sum_\Gamma \frac{J_\Gamma}{4}\text{Tr}[U^\dagger{}_\Gamma U_\Gamma]+\frac{J_1J_2}{4}\text{Tr}[U^\dagger{}_2 U_1\alpha+\text{H.c.}]. \quad \text{(B8)}$$

Inserting $\mathcal{F}^\dagger\mathcal{F}=1+2\boldsymbol{\sigma}^{\text{T}}\cdot\mathbf{S}$ into this expression, and making the observation that

$$\text{Tr}[\alpha\Psi^\dagger{}_2\Psi_1+\text{H.c.}]=0 \quad \text{(B9)}$$

we can rewrite

$$\text{Tr}[U^\dagger{}_2U_1\alpha+\text{H.c.}]=2\,\text{Tr}[\alpha\Psi_2(\boldsymbol{\sigma}^T\cdot\mathbf{S})\Psi^\dagger{}_1+\text{H.c.}] \quad \text{(B10)}$$

so that the final form of the interaction with the source term is

$$H_I=-\sum_\Gamma \frac{J_\Gamma}{4}\text{Tr}[U^\dagger{}_\Gamma U_\Gamma] + \frac{J_1J_2}{2}\text{Tr}[\alpha\Psi_2(\boldsymbol{\sigma}^T\cdot\mathbf{S})\Psi^\dagger{}_1+\text{H.c.}]. \quad \text{(B11)}$$

Comparing coefficients of α in Eq. (B3) with Eq. (B11), we obtain the following identity:

$$\mathcal{V}^\dagger{}_2\mathcal{V}_1=\frac{J_1J_2}{2}[\Psi^\dagger{}_2(\boldsymbol{\sigma}^{\text{T}}\cdot\mathbf{S})\Psi^\dagger{}_1]. \quad \text{(B12)}$$

This expresses, in a compact form, the relationship between the interchannel interference and the composite order. To complete the job, we now expand the right-hand side. We first write

$$[\Psi_2(\boldsymbol{\sigma}^T\cdot\mathbf{S})\Psi^\dagger{}_1]=-[\Psi_2(\sigma_y\boldsymbol{\sigma}\sigma_y\cdot\mathbf{S})\Psi^\dagger{}_1], \quad \text{(B13)}$$

where we have replaced $\boldsymbol{\sigma}^{\text{T}}=-(\sigma_y\boldsymbol{\sigma}\sigma_y)$. To make the expansion, it it is convenient to write

$$\Psi_2=\begin{pmatrix}\psi_2^T\\ \psi^\dagger{}_2(i\sigma_y)\end{pmatrix}, \quad \Psi^\dagger{}_1=(\psi_1^*,-i\sigma_y\psi_1), \quad \text{(B14)}$$

where ψ_2^T is the row spinor formed by taking the transpose of the column spinor ψ_2, and $\psi_1^*=(\psi^\dagger{}_1)^T$ is the column spinor formed by taking the transpose of $\psi^\dagger{}_1$. Multiplying out the matrices, we obtain

$$-\frac{2}{J_1J_2}\mathcal{V}^\dagger{}_2\mathcal{V}_1=\begin{bmatrix}\psi_2^T\sigma_y\boldsymbol{\sigma}\sigma_y\psi_1^* & \psi_2^T(-i\sigma_y\boldsymbol{\sigma})\psi_1\\ \psi^\dagger{}_2(\boldsymbol{\sigma}i\sigma_y)\psi_1^* & \psi_2^\dagger\boldsymbol{\sigma}\psi_1\end{bmatrix}\cdot\mathbf{S}$$
$$=\begin{bmatrix}\psi_1^\dagger\boldsymbol{\sigma}\psi_2 & \psi_1(i\sigma_y\boldsymbol{\sigma})\psi_2\\ \psi^\dagger{}_2(\boldsymbol{\sigma}i\sigma_y)\psi_1^* & \psi_2^\dagger\boldsymbol{\sigma}\psi_1\end{bmatrix}\cdot\mathbf{S}$$
$$=\begin{bmatrix}F & \Lambda\\ -\Lambda^\dagger & F^\dagger\end{bmatrix}. \quad \text{(B15)}$$

Substituting Eq. (B15) into Eq. (B12) we obtain the quoted result.

APPENDIX C

The purpose of this section is to evaluate the susceptibilities associated with the expansion of the mean-field Free energy about the pure composite paired state

$$F=F_0-\frac{1}{2}[\chi_{\mu\mu}(\delta\mu)^2+2\chi_{\mu\lambda}\delta\mu\,\delta\lambda+\chi_{\lambda\lambda}(\delta\lambda)^2]. \quad \text{(C1)}$$

By integrating over the Gaussian fluctuations in λ to impose the constraint on the f charge, we can use these susceptibilities to compute the physical charge susceptibility

$$\chi_C=\chi_{\mu\mu}-\frac{(\chi_{\mu\lambda})^2}{\chi_{\lambda\lambda}}. \quad \text{(C2)}$$

To compute the susceptibilities we expand the Hamiltonian about the half-filled state

$$\mathcal{H}=\mathcal{H}_0-\delta\mu\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}+\lambda\begin{bmatrix}0 & 0\\ 0 & \tau_3\end{bmatrix}. \quad \text{(C3)}$$

The electronic part of the Free energy is given by

$$F_e=-T\sum_\kappa \text{Tr}\ln[i\omega_n-\mathcal{H}(\mathbf{k})]. \quad \text{(C4)}$$

Expanding this to second-order then gives

$$\chi_{\mu\mu}=-T\sum_\kappa \text{Tr}\left(\mathcal{G}_\kappa\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}\mathcal{G}_\kappa\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}\mathcal{G}_\kappa\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}\right), \quad \text{(C5)}$$

$$\chi_{\mu\lambda}=T\sum_{\kappa}\,\mathrm{Tr}\left(\mathcal{G}_\kappa\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}\mathcal{G}_\kappa\begin{bmatrix}0 & 0\\ 0 & \tau_3\end{bmatrix}\right),$$

$$\chi_{\lambda\lambda}=-T\sum_{\kappa}\,\mathrm{Tr}\left(\mathcal{G}_\kappa\begin{bmatrix}0 & 0\\ 0 & \tau_3\end{bmatrix}\mathcal{G}_\kappa\begin{bmatrix}0 & 0\\ 0 & \tau_3\end{bmatrix}\right).$$

At half-filling, the electron propagator can be written

$$\mathcal{G}(\kappa)=\begin{bmatrix}\omega\tilde{G} & -iv_{\mathbf{k}}\tilde{G}g^\dagger\\ iv_{\mathbf{k}}g\tilde{G} & g(\omega-\epsilon_{\mathbf{k}}\tau_3)\tilde{G}g^\dagger\end{bmatrix}_\kappa, \tag{C6}$$

where $v_{\mathbf{k}}=\sqrt{v_{1\mathbf{k}}^2+v_{2\mathbf{k}}^2}$,

$$\tilde{G}(\kappa)=\frac{1}{\omega(\omega-\epsilon_{\mathbf{k}}\tau_3)-v_{\mathbf{k}}^2}, \tag{C7}$$

and

$$g=\frac{1}{v_{\mathbf{k}}}[v_{1\mathbf{k}}-iv_{2\mathbf{k}}\tau_2]. \tag{C8}$$

The Green function $\mathcal{G}$ has poles at $\pm E_{\mathbf{k}\eta}$ where

$$E_{\mathbf{k}\eta}=\frac{\epsilon_{\mathbf{k}}}{2}+\eta\sqrt{\left(\frac{\epsilon_{\mathbf{k}}}{2}\right)^2+v_{\mathbf{k}}^2}\;(\eta=\pm). \tag{C9}$$

Expanding the electron propagator about its poles, we write

$$\tilde{G}(\kappa)=\sum_{\eta=\pm}G_\eta(\kappa)\frac{\eta}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}\tau_3,$$

$$G_\eta(\kappa)=\frac{1}{\omega-E_{\mathbf{k}\eta}\tau_3}. \tag{C10}$$

Inserting this into the full propagator, we can write it in the form

$$\mathcal{G}(\kappa)=\sum_\eta\begin{bmatrix}c_{\mathbf{k}\eta}^2G_\eta & -ic_{\mathbf{k}\eta}s_{\mathbf{k}\eta}G_\eta\tau_3g^\dagger\\ ic_{\mathbf{k}\eta}s_{\mathbf{k}\eta}g\tau_3G_\eta & s_{\mathbf{k}\eta}^2gG_\eta g^\dagger\end{bmatrix}_\kappa$$

$$=\sum_\eta\zeta_{\mathbf{k}\eta}\otimes G_\eta(\kappa)\zeta^\dagger{}_{\mathbf{k}\eta}, \tag{C11}$$

where

$$\zeta_{\mathbf{k}\eta}=\begin{pmatrix}c_{\mathbf{k}\eta}\\ is_{\mathbf{k}\eta}g_{\mathbf{k}}\tau_3\end{pmatrix} \tag{C12}$$

is the eigenvector corresponding to the quasiparticle with energy $E_{\mathbf{k}\eta}$. The quantities

$$c_{\mathbf{k}\eta}^2=\frac{E_{\mathbf{k}\eta}}{E_{\mathbf{k}\eta}-E_{\mathbf{k}-\eta}}=\frac{1}{2}\left[1+\eta\frac{\epsilon_{\mathbf{k}}}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}\right],$$

$$s_{\mathbf{k}\eta}^2=\frac{E_{\mathbf{k}\eta}-\epsilon_{\mathbf{k}}}{E_{\mathbf{k}\eta}-E_{\mathbf{k}-\eta}}=\frac{1}{2}\left[1-\eta\frac{\epsilon_{\mathbf{k}}}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}\right],$$

$$c_{\mathbf{k}\eta}s_{\mathbf{k}\eta}=\frac{v_{\mathbf{k}}}{E_{\mathbf{k}\eta}-E_{\mathbf{k}-\eta}}=\eta\frac{v_{\mathbf{k}}}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}} \tag{C13}$$

describe the admixture between the conduction and f electrons. The matrix elements of the charge operators appearing inside the susceptibilities are

$$\zeta_{\mathbf{k}\eta}{}^\dagger\begin{bmatrix}\tau_3 & 0\\ 0 & 0\end{bmatrix}\zeta_{\mathbf{k}\eta'}=c_{\mathbf{k}\eta}c_{\mathbf{k}\eta'}\tau_3,$$

$$\zeta_{\mathbf{k}\eta}{}^\dagger\begin{bmatrix}0 & 0\\ 0 & \tau_3\end{bmatrix}\zeta_{\mathbf{k}\eta}=s_{\mathbf{k}\eta}s_{\mathbf{k}\eta'}\tau_3g_{\mathbf{k}}^2$$

$$=s_{\mathbf{k}\eta}s_{\mathbf{k}\eta'}(\cos\phi_{\mathbf{k}}\tau_3+\sin\phi_{\mathbf{k}}\tau_2), \tag{C14}$$

where $C_{\mathbf{k}}=v_{\mathbf{k}-}^2/v_{\mathbf{k}}^2$ and $S_{\mathbf{k}}=2v_{1\mathbf{k}}v_{2\mathbf{k}}/v_{\mathbf{k}}^2$. The expressions for the susceptibilities can now be written

$$\chi_{\mu\mu}=-T\sum_{\kappa,\eta,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3],$$

$$\chi_{\mu\lambda}=T\sum_{\kappa,\eta,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3]C_{\mathbf{k}},$$

$$\chi_{\lambda\lambda}=-T\sum_{\kappa,\eta,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3C_{\mathbf{k}}^2+G_\eta\tau_2G_{\eta'}\tau_2S_{\mathbf{k}}^2],$$

where we denote $G_\eta\equiv G_\eta(\kappa)$ and vanishing cross terms between τ_3 and τ_2 have been dropped. We now evaluate the Matsubara sums in these expressions, and take the zero-temperature limit. Key results that we use are

$$-T\sum_{\imath\omega_n,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3]c_\eta^2c_{\eta'}^2$$

$$=2\sum_{\eta'}\frac{f_{\mathbf{k}\eta'}-f_{\mathbf{k}\eta}}{E_{\mathbf{k}\eta}-E_{\mathbf{k}\eta'}}c_\eta^2c_{\eta'}^2\to\frac{2c_\eta^2s_\eta^2}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}},$$

where $f_{\mathbf{k}\eta}\equiv1/(e^{\beta E_{\mathbf{k}\eta}}+1)$ denotes the Fermi function. Similarly,

$$T\sum_{\imath\omega_n,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3](cs)_\eta(cs)_{\eta'}\to\frac{2c_\eta^2s_\eta^2}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}},$$

$$T\sum_{\imath\omega_n,\eta'}\mathrm{Tr}[G_\eta\tau_3G_{\eta'}\tau_3]s_\eta^2s_{\eta'}^2\to\frac{2c_\eta^2s_\eta^2}{\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}.$$

Lastly, there is one anomalous term

$$-T\sum_{\imath\omega_n,\eta,\eta'}\mathrm{Tr}[G_\eta\tau_2G_{\eta'}\tau_2]s_\eta^2s_{\eta'}^2\to\sum_\eta\frac{s_\eta^4}{|E_{\mathbf{k}\eta}|}$$

$$=\frac{v_{\mathbf{k}}^2+\epsilon_{\mathbf{k}}^2}{v_{\mathbf{k}}^2\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}.$$

Putting these results together, we obtain

$$\chi_{\mu\mu}=\sum_{\mathbf{k}}\frac{4v_{\mathbf{k}}^2}{(\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2)^{3/2}},$$

$$\chi_{\mu\lambda}=\sum_{\mathbf{k}}\frac{4v_{\mathbf{k}-}^2}{(\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2)^{3/2}},$$

$$\chi_{\lambda\lambda}=\sum_{\mathbf{k}}\frac{4v_{\mathbf{k}}^2}{(\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2)^{3/2}}\left(\frac{v_{\mathbf{k}-}^2}{v_{\mathbf{k}}^2}\right)^2+\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}^2+\epsilon_{\mathbf{k}}^2}{v_{\mathbf{k}}^2\sqrt{\epsilon_{\mathbf{k}}^2+4v_{\mathbf{k}}^2}}\left(\frac{2v_{1\mathbf{k}}v_{2\mathbf{k}}}{v_{\mathbf{k}}^2}\right)^2. \tag{C15}$$

We can also rewrite $\chi_{\lambda\lambda}$ in the following form:

$$\chi_{\lambda\lambda}=\chi_{\mu\mu}+\chi_b,$$

$$\chi_b=\sum_{\mathbf{k}}\frac{\epsilon_{\mathbf{k}}^2}{[(\epsilon_{\mathbf{k}})^2+4v_{\mathbf{k}}^2]^{3/2}}\left(\frac{2v_{1\mathbf{k}}v_{2\mathbf{k}}}{v_{\mathbf{k}}^2}\right)^2\left(5+\frac{\epsilon_{\mathbf{k}}^2}{v_{\mathbf{k}}^2}\right). \tag{C16}$$

Since $\chi_{\lambda\lambda}\geqslant\chi_{\mu\mu}$ and $|\chi_{\mu\lambda}|\leqslant\chi_{\mu\mu}$, the the final result for the charge susceptibility is then guaranteed to be positive when $v_1v_2\neq 0$:

$$\chi_C=\chi_{\mu\mu}-\frac{(\chi_{\mu\lambda})^2}{\chi_{\mu\mu}+\chi_b}>0\ (v_1v_2\neq 0). \tag{C17}$$

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