Spectral theory of a surface-corrugated electron waveguide: The exact scattering-operator approach

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We apply the *exact surface scattering operator* to solve the problem of scalar (electron or sound) wave propagation through a strip with absolutely soft randomly rough boundaries. This approach is *nonperturbative* in either roughness heights or slopes. We analyzed the roughness-induced dephasing and attenuation of waves both asymptotically and numerically. The analysis proves that the signal is *always* scattered most effectively into the ''*resonant*'' waveguide modes, whose transverse wavelength is comparable to the rms roughness height ζ and whose total number is proportional to ζ^{-1}. According to this *integral resonance rule*, the dephasing dominates over the attenuation and shows a *nonanalytic* (square-root) dependence on the dispersion ζ^2 when $(k\zeta)^2 \ll 1$ (k is the wave number). In the case $(k\zeta)^2 \gg 1$, the dephasing and attenuation may well compete. We predict another two surprising effects: *reentrant transparency* and *increase* of the phase velocity of the wave. [S0163-1829(99)01525-8]

I. INTRODUCTION

An electron wave, which travels through a directed nanodevice (thin film, junction, quantum wire, lead, etc.), experiences inevitable distortion due to scattering by imperfect lateral walls. If the guiding system is long enough, then even atomic-scale boundary perturbations can cause qualitative modification of the electron spectrum, which makes understanding of this phenomenon very important. Clearly, this effect is analogous to dephasing and attenuation of electromagnetic and/or acoustic waves that propagate through rough-bounded waveguide lines.[1,2] Therefore in what follows we will not make any distinction between systems that carry electron or any other type of waves, but we will consider them from a generalized standpoint of their waveguiding efficiency.

In finite systems a waveguide propagation is caused by *multiple* rereflections of a wave from opposite lateral surfaces. Obviously, in rough-bounded channels each reflection is accompanied by a noncoherent scattering of the wave. As a result of the multiple successive scattering events, the unperturbed longitudinal wavenumber k_n of an nth propagating normal mode is modified by a complex amount $\delta k_n = \gamma_n + i(2L_n)^{-1}$, i.e., $k_n \to k_n + \delta k_n$. The real part γ_n is responsible for the roughness-induced correction to the phase velocity, while L_n is the *attenuation length* of the given mode. To preserve the waveguiding properties of the rough-bounded system, the complex phase shift associated with δk_n must remain small over the *cycle length* Λ_n, i.e., over the distance passed by the nth mode between two successive reflections from the rough surface, $|\delta k_n|\Lambda_n \ll 1$. This criterion assures a large number $L_n/\Lambda_n \gg 1$ of the wave rereflections over the attenuation length L_n. The inequality $|\delta k_n|\Lambda_n \ll 1$ together with the obvious relationship $k_n\Lambda_n \gtrsim 1$ imply smallness of δk_n in comparison with the unperturbed wave number k_n.

For surface-corrugated systems, it is usually more plausible to calculate a wave field averaged over some statistical ensemble of reflecting boundaries than the exact (randomly distributed) field itself. If the relief of lateral walls in a waveguide is assumed to be *ergodic*, then an average over the ensemble of all realizations of the random surface is equivalent (in a sense of convergence in probability) to an average over the coordinates of a given realization (see, e.g., Refs. 1–3). The idea behind this statement is that an ergodic random surface has (by definition) the minimal fragment (region) which is statistically equivalent to one of the surface realizations. The statistical properties of a function, defined within that region, are asymptotically independent of the region area. Apparently, the linear dimension of a surface realization is of the order of the longitudinal variation scale $\tilde{R}_c$ of the surface scattering potential. Therefore, a statistical averaging applies only if the spectrum deviation δk_n causes just weak complex phase shift over the realization length $\tilde{R}_c$, $|\delta k_n|\tilde{R}_c \ll 1$.

Thus, we are in a position to write down the necessary and sufficient conditions for a *statistical* approach to the problem of a *waveguide* propagation in a channel with randomly rough walls:

$$|\delta k_n|\Lambda_n \ll 1, \quad |\delta k_n|\tilde{R}_c \ll 1, \tag{1.1}$$

$$\delta k_n = \gamma_n + i(2L_n)^{-1}. \tag{1.2}$$

As far as we know, the problem formulated above was first solved in Refs. 4–6 (see also the book in Ref. 1). The authors[1,4,5] calculated the average scalar field induced by a point source of radiation within a planar waveguide with statistically rough lateral walls. In Ref. 6, quantum electron states in a thin metal film with rough surfaces were analyzed and the roughness-induced residual resistivity was found. The results[1,4–6] were based on the *perturbation* theory in the squared rms height ζ of the boundary irregularities and were claimed to be valid within the so-called *Born approximation* which required smallness of the three parameters, viz. the Rayleigh $(k_z\zeta)^2$, the Fresnel $k\zeta^2/R_c$ parameters, and the squared slope $(\zeta/R_c)^2$ of the surface defects,

$$(k_z\zeta)^2 \ll 1, \quad (\zeta/R_c)^2 \ll 1, \quad k\zeta^2/R_c \ll 1. \tag{1.3}$$

Here k_z is the modulus of the normal (transverse) component of the wave vector $\vec{k}$ and R_c is the mean length of the surface defects. Since the appearance of the papers,[4–6] both classical[7–14] and quantum[15–28] spectral and transport theories for systems with multiple electron-surface scattering have been mainly built within the Born approximation (1.3).

In a series of works,[29–32] the expansion of the scattering amplitude in powers of the mild roughness slopes rather than in the small heights was applied. This made it possible to extend the theory of wave-surface scattering to arbitrary values of the Rayleigh parameter $(k_z\zeta)^2$. However, the other two inequalities of the set (1.3) were still necessary, which put back the analysis of, e.g., steep roughness slopes $[(\zeta/R_c)^2 \gtrsim 1]$ and/or the shadowing effect[1,33] $(k\zeta^2/R_c \gtrsim 1)$. The authors[34,35] applied the results[29] to derive a boundary condition for an electron distribution function at a random metal surface with mild roughness slopes. In Ref. 36, the diffusion classical and quantum transport in films with mildly sloping surface defects was analyzed basing on the boundary condition.[34,35]

We emphasize that the abovementioned approximations are based on perturbative expansions in small *explicit* parameters of the roughness (heights or slopes) and are therefore much more restrictive than the generic conditions (1.1).

In this paper we put forward the approach which is *nonperturbative* in the roughness heights or slopes as well as in any other explicit parameter. It is based instead on the exploitation of the *exact* boundary scattering operator. Due to this fact, we managed to construct the theory of waveguide propagation whose domain of applicability is as wide as that defined by the conditions (1.1). Note that these conditions are equations themselves with respect to the explicit external parameters k, ζ^2, R_c, and n.

We calculate the average Green's function (the average field of a point source) in a planar strip with absolutely soft lateral boundaries, one of which is smooth and the other is randomly rough. Such system is physically equivalent to a waveguide with both boundaries being rough, statistically identical, and not intercorrelated.

In Sec. II we derive a Dyson-type integral equation for the exact (not averaged) Green's function. The effect of the random inhomogeneities is completely incorporated in the integral kernel of the equation, which allows to consider it as the *exact* effective scattering potential. This potential is definitely *nonperturbative* in either roughness heights or slopes. In Sec. III we apply the elegant technique[18,37] to average the equation over the Gaussian ensemble of realizations of the random boundary. The averaged equation turns out to be *solvable analytically* within the assumptions (1.1) due to their being equivalent to the general criteria of *weak* wave-boundary scattering. Consequently, the self-energy can be sought in the first nonvanishing (quadratic) approximation in the *exact* scattering potential (i.e., in the Bourret approximation[38]). Based on the results of Sec. III, we derive in Sec. IV explicit expressions for the real spectrum shift (the dephasing) γ_n and the attenuation length L_n of an nth propagating mode.

In Sec. V we present the analysis of the expressions obtained, which clearly demonstrates the advantages of our *exact scattering operator approach*. The most impressive one is that our method leads to new physical results even in the region of extremely small roughness heights, $(k\zeta)^2 \ll 1$, where the waveguide propagation has been believed to be well studied. In particular, we found a surprising *nonanalytic* (square-root) dependence $\delta k_n \propto \sqrt{\zeta^2}$ of the spectrum deviation on the dispersion ζ^2 of the roughness height, which contrasts the conventional linear law[1,2,4,5,29–32] $\delta k_n \propto \zeta^2$. This nonanalyticity *cannot* be in principle derived perturbatively, because we prove that δk_n is mainly formed by scattering of the primary wave into the ''resonant'' evanescent modes with normal wavelength $2\pi/q_z$ being comparable to the height ζ, $(q_z\zeta)^2 \sim 1$. At the same time, it is usually believed that the condition $(k\zeta)^2 \ll 1$ is sufficient to infer that any ''slipping'' normal mode with $(k_z\zeta)^2 \ll 1$ is mainly scattered into the slipping modes as well, for which $2\pi/q_z \gg \zeta$, or $(q_z\zeta)^2 \ll 1$. This assumption alone allows to replace the exact Dirichlet boundary condition imposed on the randomly rough surface by an approximate impedance one formulated on the averaged (deterministic) boundary. The abovementioned domination of the resonant modes with $(q_z\zeta)^2 \sim 1$ proves groundlessness of such a replacement. Thus, a problem of wave (electron) propagation through a waveguide (quantum strip) with absolutely soft random boundaries *cannot* be reduced to a simpler problem with the smooth random-impedance boundary even for arbitrarily weak perturbations. We note that the spectrum deviation $\delta k_n \propto \sqrt{\zeta^2}$ is nearly real, i.e., $\delta k_n \simeq \gamma_n$, $\gamma_n \gg (2L_n)^{-1}$. This leads to a nontrivial new result that a signal, which propagates through a nearly smooth waveguide, is dephased (chaotized) much earlier (over much shorter distances) than its initial amplitude is decayed. Moreover, if the boundary inhomogeneities are both low and very sharp $(kR_c \ll k\zeta \ll 1)$, then the real spectrum modification γ_n appears to be *negative*, which means that the surface roughness may not only decrease but also increase the phase velocity of a propagating wave.

The other important subject of Sec. V is a practically interesting case of high boundary inhomogeneities, when $(k\zeta)^2 \gg 1$. This case is much more diverse and complicated than that with $(k\zeta)^2 \ll 1$. For a large part it is analyzable only numerically. We reveal that, in contrast to the low-perturbation limit, the imaginary part of δk_n may compete with the real part. Moreover, the situation with $\delta k_n \simeq i(2L_n)^{-1}$ is rather typical. As before, the real spectrum modification γ_n may reverse sign. This happens as $k\zeta$ reaches some threshold value $k\zeta \approx 1.5$–2.5 that is almost independent of the other parameters. Another noteworthy result is the prediction of *reentrant transparency*. This effect can be observed when the parameter $k\zeta$ is fixed at a value of $\gtrsim 2$, while kR_c is being increased. Then the weak-scattering regime (1.1) collapses at some value of kR_c and then restores again at a larger value of kR_c.

Section VI concludes our contribution. The short report preceding this comprehensive paper has been published in Optics Letters.[39]

II. PROBLEM FORMULATION. THE DYSON EQUATION

We consider a 2D strip confined to a region of the xz plane defined by $\xi(x) \leqslant z \leqslant d$, where $\xi(x)$ is a Gaussian random function of the longitudinal coordinate x such that

$$\langle \xi(x) \rangle = 0, \quad \langle \xi(x)\xi(x') \rangle = \zeta^2 \mathcal{W}(|x-x'|). \tag{2.1}$$

The angular brackets denote an average over the ensemble of realizations of the profile function $\xi(x)$. The binary coefficient of correlation $\mathcal{W}(|x|)$ is characterized by the unit amplitude $[\mathcal{W}(0)=1]$ and by the scale R_c of monotonous decrease (correlation radius) which is of the order of the mean length of the boundary irregularities.

The retarded Green's function $\mathcal{G}(x,x';z,z')$ satisfies the equation

$$(\triangle + k^2)\mathcal{G}(x,x';z,z') = \delta(x-x')\,\delta(z-z') \tag{2.2}$$

(k is the wave number) and the boundary conditions which, in the case of absolutely soft walls, have the Dirichlet form

$$\mathcal{G}(x,x';z=\xi(x),z') = 0, \tag{2.3}$$

$$\mathcal{G}(x,x';z=d,z') = 0. \tag{2.4}$$

Now we employ Green's theorem to find out the relation between $\mathcal{G}$ and the Green's function $\mathcal{G}_0$ of the ideally flat strip:

$$\int_V dV(\mathcal{G}\triangle\mathcal{G}_0 - \mathcal{G}_0\triangle\mathcal{G}) = \int_S d\vec{S}(\mathcal{G}\nabla\mathcal{G}_0 - \mathcal{G}_0\nabla\mathcal{G}). \tag{2.5}$$

Integration on the left-hand side (LHS) of Eq. (2.5) is taken over the 2D volume V of the strip and on the RHS over the 1D surface (contour) S. Equation (2.2), which governs both functions $\mathcal{G}$ and $\mathcal{G}_0$, allows to reduce the volume integral to the difference

$$\int_V dV(\mathcal{G}\triangle\mathcal{G}_0 - \mathcal{G}_0\triangle\mathcal{G}) = \mathcal{G}(x,x';z,z') - \mathcal{G}_0(|x-x'|;z,z'). \tag{2.6}$$

The bounding contour S consists of the upper straight line $z=d$, the lower nonuniform boundary $z=\xi(x)$, and the infinitely far ($|x|\to\infty$) contours that connect the opposite strip edges. The integral along the line $z=d$ vanishes due to the boundary condition (2.4). The integrals over the infinitely far contours are suppressed by the rapid oscillations of $\mathcal{G}$ and $\mathcal{G}_0$. Besides, the integral over the lower boundary $z=\xi(x)$ of the first integrand on the RHS of Eq. (2.5) also vanishes due to the boundary condition (2.3). Thus, the RHS of Eq. (2.5) transforms into

$$\int_S d\vec{S}(\mathcal{G}\nabla\mathcal{G}_0 - \mathcal{G}_0\nabla\mathcal{G}) = -\int_{z_s=\xi(x_s)} d\vec{S}\mathcal{G}_0(|x-x_s|;z,z_s)\nabla_s\mathcal{G}(x_s,x';z_s,z'). \tag{2.7}$$

Let us write down the scalar product $d\vec{S}\nabla_s$ explicitly,

$$d\vec{S}\nabla_s = -dx_s\left[\frac{\partial}{\partial z_s} - \frac{d\xi(x_s)}{dx_s}\frac{\partial}{\partial x_s}\right]\Bigg|_{z_s=\xi(x_s)}, \tag{2.8}$$

and substitute it into Eq. (2.7). From Eqs. (2.5)–(2.7) we immediately obtain the required Dyson-type equation:

$$\mathcal{G}(x,x';z,z') = \mathcal{G}_0(|x-x'|;z,z') + \int_{-\infty}^{\infty} dx_s dz_s \mathcal{G}_0(|x-x_s|;z,z_s) \times \hat{\Xi}(x_s,z_s)\mathcal{G}(x_s,x';z_s,z'). \tag{2.9}$$

Here $\hat{\Xi}(x_s,z_s)$ is the *exact effective scattering operator,*

$$\hat{\Xi}(x_s,z_s) = \delta[z_s - \xi(x_s)]\left[\frac{\partial}{\partial z_s} - \frac{d\xi(x_s)}{dx_s}\frac{\partial}{\partial x_s}\right]. \tag{2.10}$$

Note that for the convenience of subsequent averaging we have introduced the additional integration over the transverse coordinate z_s in Eq. (2.9) which is compensated by the delta function in Eq. (2.10).

In an ideal waveguide ($\xi(x)\equiv 0$) the expression (2.9) reduces to $\mathcal{G}=\mathcal{G}_0$ due to vanishing of the unperturbed Green's function $\mathcal{G}_0$ in the integrand of the RHS.

III. AVERAGING AND SOLUTION OF THE DYSON EQUATION

As a consequence of the property (2.1), the averaged Green's function $\langle \mathcal{G}(x,x';z,z') \rangle$ turns out to be uniform and isotropic in the coordinate x:

$$\langle \mathcal{G}(x,x';z,z') \rangle \equiv \overline{\mathcal{G}}(|x-x'|;z,z'). \tag{3.1}$$

To derive the equation for $\langle \mathcal{G}(x,x';z,z') \rangle$ we apply the averaging technique[18,37] to Eq. (2.9), which yields

$$\bar{\mathcal{G}}(|x-x'|;z,z')=\mathcal{G}_0(|x-x'|;z,z')+\int_{-\infty}^{\infty}dx_s\int_{-\infty}^{\infty}dz_s\mathcal{G}_0(|x-x_s|;z,z_s)\langle\hat{\Xi}(x_s,z_s)\rangle\bar{\mathcal{G}}(|x_s-x'|;z_s,z')$$
$$-\int_{-\infty}^{\infty}dx_s dx_s'\int_{-\infty}^{\infty}dz_s dz_s'\mathcal{G}_0(|x-x_s|;z,z_s)\langle\hat{\Xi}(x_s,z_s)\rangle\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')\langle\hat{\Xi}(x_s',z_s')\rangle\bar{\mathcal{G}}(|x_s'-x'|;z_s',z')$$
$$+\int_{-\infty}^{\infty}dx_s dx_s'\int_{-\infty}^{\infty}dz_s dz_s'\mathcal{G}_0(|x-x_s|;z,z_s)\langle\hat{\Xi}(x_s,z_s)\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')\hat{\Xi}(x_s',z_s')\rangle\bar{\mathcal{G}}(|x_s'-x'|;z_s',z'). \quad (3.2)$$

Note that the average of the scattering potential $\hat{\Xi}(x_s,z_s)$ (2.10) is given by

$$\langle\hat{\Xi}(x_s,z_s)\rangle=P(z_s)\frac{\partial}{\partial z_s},\quad P(z_s)=(1/\zeta\sqrt{2\pi})\exp(-z_s^2/2\zeta^2). \quad (3.3)$$

Here $P(z_s)$ is the Gaussian probability density of distribution of the coordinate z_s. Since $\langle\hat{\Xi}(x_s,z_s)\rangle$, $\mathcal{G}_0(|x-x_s|;z,z_s)$, and $\bar{\mathcal{G}}(|x-x_s|;z,z_s)$ are odd functions of z_s [see Eqs. (3.3), (4.1), and (3.13) respectively], then the second (linear in $\hat{\Xi}$) term on the RHS of Eq. (3.2) vanishes. Similarly, the third term can be also shown to vanish. So, we come to the following equation for the average Green's function:

$$\bar{\mathcal{G}}(|x-x'|;z,z')=\mathcal{G}_0(|x-x'|;z,z')+\int_{-\infty}^{\infty}dx_s dx_s'\int_{-\infty}^{\infty}dz_s dz_s'\mathcal{G}_0(|x-x_s|;z,z_s)\times\langle\hat{\Xi}(x_s,z_s)\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')\times\hat{\Xi}(x_s',z_s')\rangle\bar{\mathcal{G}}(|x_s'-x'|;z_s',z'). \quad (3.4)$$

As a matter of fact, Eq. (3.4) is not exact, since it includes only quadratic in $\hat{\Xi}$ term, while the rest of the terms has been cut off. To find out the necessary and sufficient conditions for this truncation we used the ideas proposed in the book.[2] More specifically, we first solved Eq. (3.4) itself (see below), then we substituted the solution back into Eq. (3.4) in place of the function $\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')$ and solved the new *modified* variant of Eq. (3.4). Clearly, the modified equation is more general than the original Eq. (3.4), so that the comparison of their solutions is expected to give the required conditions of applicability of the "quadratic" approximation (3.4). The analysis shows that these conditions are $|\delta k_n|\ll k_n$ and $|\delta k_n|\tilde{R}_c\ll 1$. The latter coincides with the second of the weak-scattering conditions (1.1), while the former follows from the first of Eqs. (1.1) and from the obvious inequality $k_n\Lambda_n\gtrsim 1$.

The statistical uniformity (2.1) makes it effective to apply a longitudinal Fourier transformation to solve Eq. (3.4):

$$\bar{\mathcal{G}}(|x-x'|;z,z')=\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\exp[ik_x(x-x')]\bar{G}(k_x;z,z'), \quad (3.5)$$

$$\mathcal{G}_0(|x-x'|;z,z')=\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\exp[ik_x(x-x')]G_0(k_x;z,z'), \quad (3.6)$$

where k_x is the longitudinal component of the wave vector $\vec{k}$. According to Eq. (3.4), the Fourier transform $\bar{G}(k_x;z,z')$ satisfies the equation

$$\bar{G}(k_x;z,z')=G_0(k_x;z,z')+\int_{-\infty}^{\infty}dz_s dz_s' G_0(k_x;z,z_s)\times\hat{Q}(k_x;z_s,z_s')\bar{G}(k_x;z_s',z'). \quad (3.7)$$

The effective average scattering operator $\hat{Q}(k_x;z_s,z_s')$ is given by

$$\hat{Q}(k_x;z_s,z_s')=\int_{-\infty}^{\infty}dx_s\exp(-ik_x x_s)\langle\hat{\Xi}(x_s,z_s)\times\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')\hat{\Xi}(x_s',z_s')\rangle\exp(ik_x x_s'). \quad (3.8)$$

The integral over x_s in Eq. (3.8) is independent of x_s' due to the statistical uniformity (2.1).

Obviously, Eq. (3.7) could be solved trivially if the integral kernel were degenerate, i.e., more specifically, if the component $G_0(k_x;z,z_s)$ of the kernel were factorable in z and z_s. To analyze a possibility of such factorization, we write down an explicit representation for $G_0(k_x;z,z_s)$:

$$G_0(k_x;z,z_s)=k_z^{-2}g_0(k_x)\sin(k_z z)\sin(k_z z_s)-k_z^{-1}\cos(k_z z)\sin(k_z z_s)\,\theta(z-z_s)-k_z^{-1}\sin(k_z z)\cos(k_z z_s)\,\theta(z_s-z). \quad (3.9)$$

Here k_z is the modulus of the transverse component of the wave vector $\vec{k}$,

$$k_z=k_z(k_x)=(k^2-k_x^2)^{1/2}. \quad (3.10)$$

The first term in Eq. (3.9) is known as the *pole term*. It contains the *pole factor* $g_0(k_x)$,

$$g_0(k_x)=k_z\cot(k_z d). \quad (3.11)$$

Simple poles of $g_0(k_x)$ form the complete set of eigenwave numbers in a flat waveguide:

$$k_z=\pi n/d,\quad n=1,2,3,\ldots. \quad (3.12)$$

We point out that $G_0(k_x;z,z_s)$ has no singularities other than those defined by Eq. (3.12).

As far as the weak-scattering regime (1.1) is concerned, the spectral poles of $\overline{G}(k_x;z,z')$ should lie comparatively close to those of $G_0(k_x;z,z_s)$. At the same time, the poles of $\overline{G}(k_x;z,z')$ *completely* form the integral (3.5). Hence it is sufficient to solve Eq. (3.7) only for those values of k_x that correspond to relatively weak deviation of k_z from the unperturbed spectrum (3.12). Following this conclusion, we replace $G_0(k_x;z,z_s)$ in the integrand of Eq. (3.7) by the first (pole) term of the representation (3.9) which dominates over the rest of the terms if the spectrum is nearly ideal. After this replacement the integral kernel in Eq. (3.7) becomes degenerate and we easily get the solution:

$$\overline{G}(k_x;z,z')\simeq\frac{G_0(k_x;z,z')}{1-g_0(k_x)M(k_x)}, \tag{3.13}$$

where we have retained only the zeroth order term in $\hat{Q}$ in the numerator. The function $M(k_x)$ enters Eq. (3.13) as the self-energy and is related to $\hat{Q}$ by

$$M(k_x)=\int_{-\infty}^{\infty}dz_s dz_s'\frac{\sin(k_z z_s)}{k_z}\hat{Q}(k_x;z_s,z_s')\frac{\sin(k_z z_s')}{k_z}. \tag{3.14}$$

To calculate the correction δk_n to the unperturbed longitudinal wave number k_n,

$$k_n=\sqrt{k^2-(\pi n/d)^2}, \tag{3.15}$$

we need to determine the poles of $\overline{G}(k_x;z,z')$ (3.13), i.e., to solve the dispersion equation

$$g_0^{-1}(k_x)-M(k_x)=0. \tag{3.16}$$

The solution of this equation in the first (linear) approximation in $M(k_x)$ has the form

$$\delta k_n=-\frac{(\pi n/d)^2}{k_n d}M(k_n). \tag{3.17}$$

In fact, the form (3.17) of a relation between δk_n and $M(k_n)$ is common and well known.[1,18] Our improvement lies in the self-energy $M(k_x)$ itself. Indeed, our result (3.14) and (3.8) for $M(k_x)$ is nothing else but a first nonvanishing (quadratic) term in an expansion of $M(k_x)$ in powers of the *exact* scattering operator (2.10). We stress that such an approximation is essentially different from and is much more general than an extensively exploited first term in an expansion of $M(k_x)$ in powers of the dispersion ζ^2.

Concluding this section, we summarize the region of validity for our results. For this purpose we list all the simplifications that were used in deriving Eqs. (3.17) and (3.14). (i) The approximate averaged equation (3.4) is applicable if $|\delta k_n|\tilde{R}_c\ll 1$ and $|\delta k_n|\ll k_n$. (ii) The first (pole) term in the representation (3.9) dominates over the rest two terms if the inequality $|\delta k_n|\Lambda_n\ll 1$ holds. (iii) The perturbative solution (3.17) of the dispersion equation (3.16) was obtained within the limit $|\delta k_n|\tilde{R}_c\ll 1$, $|\delta k_n|\Lambda_n\ll 1$. Thus, we end up with the three inequalities: $|\delta k_n|\tilde{R}_c\ll 1$, $|\delta k_n|\Lambda_n\ll 1$, and $|\delta k_n|\ll k_n$. Since the last one is the direct consequence of the conditions $|\delta k_n|\Lambda_n\ll 1$ and $k_n\Lambda_n\gtrsim 1$, we conclude that the range of validity of our results is confined by the two independent criteria only, viz. by the weak-scattering conditions (1.1).

IV. COMPLEX SPECTRUM SHIFT

To derive an explicit dependence of the complex spectrum deviation δk_n (3.17) on the characteristics of an irregular waveguide (such as k, d, ζ, R_c), we (i) substitute Eq. (3.8) into Eq. (3.14); (ii) replace the operators $\hat{\Xi}(x_s,z_s)$ and $\hat{\Xi}(x_s',z_s')$ by their explicit definition (2.10); (iii) apply the canonical representation for the Green's function $\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')$:

$$\mathcal{G}_0(|x_s-x_s'|;z_s,z_s')=\frac{2}{d}\sum_{n'=1}^{\infty}\sin\left(\frac{\pi n' z_s}{d}\right)\times\sin\left(\frac{\pi n' z_s'}{d}\right)\mathcal{G}_{n'}^{(0)}(|x_s-x_s'|). \tag{4.1}$$

Here $\mathcal{G}_{n'}^{(0)}(|x_s-x_s'|)$ is the subchannel (or mode) Green's function defined by

$$\mathcal{G}_{n'}^{(0)}(|x|)=\int_{-\infty}^{\infty}\frac{dq_x}{2\pi}\frac{\exp(iq_x x)}{(k_{n'}+i0)^2-q_x^2}=\frac{\exp(ik_{n'}|x|)}{2ik_{n'}}; \tag{4.2}$$

(iv) differentiate with respect to z_s and z_s' according to Eq. (2.10); (v) take integrals over z_s and z_s' with the help of the δ functions (2.10); (vi) perform straightforward averaging over the Gaussian ensemble (2.1) of realizations of the random field $\xi(x)$. These six steps yield

$$M(k_x)=\frac{1}{2k_z d}\sum_{n'=1}^{\infty}\left(\frac{\pi n'}{d}\right)\int_{-\infty}^{\infty}dx\exp(-ik_x x)\left\{F_1\left(k_z,\frac{\pi n'}{d};|x|\right)\mathcal{G}_{n'}^{(0)}(|x|)-\left(\frac{\pi n'}{d}\right)^{-1}\frac{\partial}{\partial x}F_2\left(k_z,\frac{\pi n'}{d};|x|\right)\frac{\partial}{\partial x}\mathcal{G}_{n'}^{(0)}(|x|)\right.$$
$$\left.+i\frac{k_x}{k_z}\mathcal{G}_{n'}^{(0)}(|x|)\frac{\partial}{\partial x}F_2\left(\frac{\pi n'}{d},k_z;|x|\right)+i\left(\frac{\pi n'}{d}\right)^{-1}\frac{k_x}{k_z}\frac{\partial^2}{\partial x^2}F_3\left(k_z,\frac{\pi n'}{d};|x|\right)\frac{\partial}{\partial x}\mathcal{G}_{n'}^{(0)}(|x|)\right\}. \tag{4.3}$$

Here the symbols F_1, F_2, F_3 stand for

$$F_1(k_z,q_z;|x|)=(k_z+q_z)^2S(k_z+q_z,k_z+q_z;|x|)-(k_z-q_z)^2S(k_z-q_z,k_z-q_z;|x|), \tag{4.4}$$

$$F_2(k_z,q_z;|x|)=2k_zS(k_z+q_z,k_z-q_z;|x|)-(k_z+q_z)S(k_z+q_z,k_z+q_z;|x|)-(k_z-q_z)S(k_z-q_z,k_z-q_z;|x|), \tag{4.5}$$

$$F_3(k_z,q_z;|x|)=2S(k_z+q_z,k_z-q_z;|x|)-S(k_z+q_z,k_z+q_z;|x|)-S(k_z-q_z,k_z-q_z;|x|); \tag{4.6}$$

$$S(t_1,t_2;|x-x'|)=(t_1t_2)^{-1}\langle\sin[t_1\xi(x)]\sin[t_2\xi(x')]\rangle=(t_1t_2)^{-1}\sinh[t_1t_2\zeta^2\mathcal{W}(|x-x'|)]\exp[-(t_1^2+t_2^2)\zeta^2/2]. \tag{4.7}$$

Now we break up the integral over x in Eq. (4.3) into two parts: from $-\infty$ to 0 and from 0 to ∞. In the former we reverse the sign of the integration variable: $x\to -x$. We next substitute the explicit Green's function $\mathcal{G}_{n'}^{(0)}(|x|)$ (4.2) into the expression obtained and integrate by parts once the second and third terms of Eq. (4.3) and twice the fourth term to get rid of the derivatives acting on the functions F_2 and F_3. This procedure gives us the most convenient for analysis representation for the self-energy $M(k_x)$,

$$M(k_x)=M_1(k_x)+M_2(k_x). \tag{4.8}$$

Here the first term is given by

$$M_1(k_x)=-i\zeta^2\sum_{n'=1}^{\infty}\frac{(\pi n'/d)^2}{k_{n'}d}\left\{\int_0^\infty dx\,\exp[-i(k_x-k_{n'})x]\tilde{\mathcal{W}}(k_x,k_{n'};|x|)+\int_0^\infty dx\,\exp[i(k_x+k_{n'})x]\tilde{\mathcal{W}}(k_x,-k_{n'};|x|)\right\}. \tag{4.9}$$

The second term arises from the integration by parts. It is nothing else but the term outside the integral,

$$M_2(k_x)=\frac{k^2}{2k_z^2d}\sum_{n'=1}^{\infty}[A(k_z,\pi n'/d)+B(k_z,\pi n'/d)]; \tag{4.10}$$

$$A(k_z,q_z)=2S(k_z+q_z,k_z-q_z;0)-S(k_z+q_z,k_z+q_z;0)-S(k_z-q_z,k_z-q_z;0), \tag{4.11}$$

$$B(k_z,q_z)=-\frac{k_zq_z}{k^2}[S(k_z+q_z,k_z+q_z;0)-S(k_z-q_z,k_z-q_z;0)]. \tag{4.12}$$

The function $\tilde{\mathcal{W}}(k_x,q_x;x)$ is the generalized coefficient of correlation,

$$\begin{aligned}\tilde{\mathcal{W}}(k_x,q_x;x)=(4k_zq_z\zeta^2)^{-1}\Bigg\{&\left[(k_z+q_z)^2+(k_z+q_z)(k_x-q_x)\left(\frac{k_x}{k_z}-\frac{q_x}{q_z}\right)-(k_x-q_x)^2\frac{k_xq_x}{k_zq_z}\right]S(k_z+q_z,k_z+q_z;x)\\&-\left[(k_z-q_z)^2+(k_z-q_z)(k_x-q_x)\left(\frac{k_x}{k_z}+\frac{q_x}{q_z}\right)+(k_x-q_x)^2\frac{k_xq_x}{k_zq_z}\right]S(k_z-q_z,k_z-q_z;x)+2(k_x-q_x)\\&\times\left[q_x\frac{k_z}{q_z}-k_x\frac{q_z}{k_z}+(k_x-q_x)\frac{k_xq_x}{k_zq_z}\right]S(k_z+q_z,k_z-q_z;x)\Bigg\},\end{aligned} \tag{4.13}$$

where $q_z=k_z(q_x)$. As $\zeta^2\to 0$, this function reduces to the coefficient of correlation $\mathcal{W}(|x|)$ (2.1) of the underlying random field $\xi(x)$,

$$\tilde{\mathcal{W}}(k_x,q_x;x)\simeq\mathcal{W}(|x|)\quad\text{as}\quad\zeta^2\to 0. \tag{4.14}$$

Now we substitute the self-energy (4.8) into Eq. (3.17) and extract real and imaginary parts of δk_n. The real spectrum shift γ_n has a quite complex structure:

$$\gamma_n=\gamma_n^{(1)}+\gamma_n^{(2)}+\gamma_n^{(3)}, \tag{4.15}$$

$$\gamma_n^{(1)}=2\zeta^2\frac{\pi n/d}{\Lambda_n}\sum_{n'=1}^{n_d}\frac{\pi n'/d}{\Lambda_{n'}}[\tilde{W}_S(k_n,k_{n'})-\tilde{W}_S(k_n,-k_{n'})], \tag{4.16}$$

$$\gamma_n^{(2)}=-2\frac{\pi n/d}{\Lambda_n}M_2(k_n), \tag{4.17}$$

$$\gamma_n^{(3)}=4\zeta^2\frac{\pi n/d}{\Lambda_n}\sum_{n'=n_d+1}^{\infty}\frac{(\pi n'/d)^2}{|k_{n'}|d}\int_0^\infty dx\times\exp(-|k_{n'}|x)\mathrm{Re}[\exp(-ik_nx)\tilde{\mathcal{W}}(k_n,i|k_{n'}|;x)]. \tag{4.18}$$

The expression for the attenuation length L_n^{-1} is much simpler:

$$L_n^{-1}=4\zeta^2\frac{\pi n/d}{\Lambda_n}\sum_{n'=1}^{n_d}\frac{\pi n'/d}{\Lambda_{n'}}\times[\tilde{W}_C(k_n,k_{n'})+\tilde{W}_C(k_n,-k_{n'})]. \tag{4.19}$$

Here integer

$$n_d=[kd/\pi] \tag{4.20}$$

is the number of the propagating normal modes (conducting electron subchannels) in the smooth strip. The cycle length Λ_n for an nth propagating mode is explicitly given by

$$\Lambda_n = 2k_n d/(\pi n/d). \tag{4.21}$$

The functions $\tilde{W}_{S(C)}(k_x, q_x)$ stand for sine and cosine Fourier transforms of $\tilde{\mathcal{W}}(k_x, q_x; x)$, respectively:

$$\tilde{W}_{S(C)}(k_x, q_x) = 2\int_0^\infty dx \sin(\cos)[(k_x - q_x)x]\tilde{\mathcal{W}}(k_x, q_x; |x|). \tag{4.22}$$

Note that the derivation of Eqs. (4.15)–(4.19) relied on two properties of the function $\tilde{\mathcal{W}}(k_x, q_x; |x|)$ (4.13): (i) $\tilde{\mathcal{W}}(k_x, q_x; |x|)$ is real for real arguments k_x and q_x (i.e., for $n' \leqslant n_d$); (ii) $\tilde{\mathcal{W}}(k_n, -k_{n'}; |x|) = \tilde{\mathcal{W}}^*(k_n, k_{n'}; |x|)$ for $n' > n_d$. These properties lead to an essential distinction between γ_n (4.15)–(4.18) and L_n^{-1} (4.19). Specifically, L_n^{-1} is formed by scattering of a given nth propagating mode into propagating waveguide modes with $n' \leqslant n_d$ only, while γ_n has much more complicated structure due to contributions of both propagating ($n' \leqslant n_d$) and evanescent ($n' > n_d$) normal modes.

V. ANALYSIS OF RESULTS

Now we are in a position to analyze the complex spectrum shift δk_n as a function of the explicit external parameters. Since there are as many as four dimensionless parameters, $(k\zeta)^2$, kR_c, kd/π, and n, the complete analysis seems unnecessarily cumbersome. Therefore we concentrate only on those features of δk_n which have been brought in by the exact scattering operator method.

Following this idea, we address the expressions (3.17), (4.3). It is seen that δk_n is, in general, a result of incoherent scattering into *all* normal modes, both propagating and evanescent. Depending on values of the external parameters, this or that group of the scattered modes may dominate in forming δk_n. The careful analysis of Eqs. (3.17), (4.3) shows that the dominating group can be singled out in any given case with the help of the *resonant selection rule* which is the most striking result of our theory. It says that the primary wave is scattered most intensively into so-called *''resonant''* modes, whose normal wavelength $2\pi/q_z$ is comparable to the roughness height ζ, i.e., $(q_z\zeta)^2 \sim 1$. This rule holds true regardless of how small is ζ or what are the values of the other parameters.

The concept of the resonant modes introduces a characteristic integer n_ζ:

$$n_\zeta = [d/\pi\zeta]. \tag{5.1}$$

This number separates the regions of small and large values of the Rayleigh parameter $(q_z\zeta)^2$ for the scattered modes. Indeed, if the number n' of a given scattered mode obeys $n' \ll n_\zeta$, then $(q_z\zeta)^2 = (\pi n'/d)^2\zeta^2 \simeq (n'/n_\zeta)^2 \ll 1$. Otherwise, if $n' \gg n_\zeta$, then $(q_z\zeta)^2 \gg 1$. Obviously, the notation (5.1) is meaningful only if $n_\zeta \gg 1$.

From the resonant selection rule it follows that the dominating scattered modes are always located near the resonance point $n' = n_\zeta$. In other words, there should be the peak of subchannel contributions into the sum (4.3) in a vicinity of the n_ζth subchannel. The detailed analysis of Eqs. (3.17) and (4.3) reveals the existence of the peak and proves its being wide (smooth) enough. The summands in Eq. (4.3) decrease rather slowly (as a power of n') towards the both sides of the resonance peak $n' \sim n_\zeta$, which makes the total number of the contributive (resonant) modes large enough, viz. $\sim n_\zeta$. Therefore the sum (4.3) is formed not by few isolated summands but by a large number of terms with comparable amplitudes. We call such a type of resonance the *integral resonant effect*.

Since $n_d/n_\zeta \simeq k\zeta$ [see Eqs. (4.20) and (5.1)], then, depending on how large is $k\zeta$, the ''resonance point'' n_ζ may be smaller or larger than the number n_d of propagating modes. If the boundary irregularities are low, $(k\zeta)^2 \ll 1$, then $n_d \ll n_\zeta$, i.e., the resonance peak falls into the infinitely wide domain of evanescent modes, for which $n' > n_d$. Since the evanescent modes contribute to the *real* part γ_n of δk_n only, then the resonant selection rule allows to conclude that the spectrum deviation δk_n is nearly real, $\delta k_n \simeq \gamma_n$. The number $\sim n_\zeta$ of the summands contributing to δk_n is much larger than the total number n_d of propagating modes.

Otherwise, in the case of high boundary inhomogeneities, $(k\zeta)^2 \gg 1$, we have $n_\zeta \ll n_d$. The resonant peak is now located among the propagating scattered modes which contribute to both γ_n and L_n. Consequently, depending on the external parameters, the real and imaginary parts of δk_n may compete. Below we will demonstrate that the competition basically relies on the parameter kR_c. If the boundary roughness is small scale, $kR_c \ll 1$, then the spectrum deviation is almost real, $\delta k_n \simeq \gamma_n$. However, if the irregularities are large scale, $kR_c \gg 1$, then the attenuation is much stronger than the dephasing and δk_n is nearly imaginary, $\delta k_n \simeq i(2L_n)^{-1}$. As applied to the high boundary defects, the integral resonance principle says that δk_n is formed by a relatively small fraction $n_\zeta/n_d \ll 1$ of the propagating scattered modes.

To illustrate the most interesting consequences of *the integral resonance rule* and the exact scattering operator approach altogether, we will analyze briefly the limits of low and high boundary inhomogeneities.

A. Low boundary perturbations

We start with a relatively simple and widely used case of *low boundary perturbations*,

$$(k\zeta)^2 \ll 1. \tag{5.2}$$

This definition ensures the smallness of the Rayleigh parameter for both the primary wave $(k_z\zeta)^2$ and the propagating scattered modes $(q_z\zeta)^2$, since $(k_z\zeta)^2 \leqslant (k\zeta)^2 \ll 1$ and $(q_z\zeta)^2 = (\pi n'\zeta/d)^2 \leqslant (\pi n_d\zeta/d)^2 \simeq (k\zeta)^2 \ll 1$ for $n' \leqslant n_d$. Under the conditions $(k_z\zeta)^2 \ll 1$ and $(q_z\zeta)^2 \ll 1$, the generalized coefficient of correlation $\tilde{\mathcal{W}}(k_x, q_x; |x|)$ (4.13) reduces to the asymptotic (4.14). Accordingly, the function $\tilde{W}_C(k_x, q_x)$ (4.22) can be replaced with the spatial spectrum $W(k_x)$ of surface irregularities, i.e., with the Fourier transform of $\mathcal{W}(|x|)$,

$$\tilde{W}_C(k_x, q_x) \simeq W(|k_x - q_x|) \quad \text{as} \quad \zeta^2 \to 0,$$

$$W(k_x)=2\int_0^\infty dx\cos(k_x x)\mathcal{W}(|x|). \tag{5.3}$$

Substituting Eq. (5.3) into Eq. (4.19), we come to the standard perturbative in ζ^2 expression[1,4,5] for the attenuation length L_n:

$$L_n^{-1}=4\zeta^2\frac{\pi n/d}{\Lambda_n}\sum_{n'=1}^{n_d}\frac{\pi n'/d}{\Lambda_{n'}}\times[W(|k_n-k_{n'}|)+W(|k_n+k_{n'}|)]. \tag{5.4}$$

This formula has been analyzed in Refs. 1, 4, and 5 in detail. Here we just need to emphasize that Eq. (5.4) always yields a linear dependence of L_n^{-1} on the dispersion ζ^2, i.e., $L_n^{-1}\propto\zeta^2$.

Within the limit (5.2), the resonance point $n'=n_\zeta$ falls into the interval of evanescent scattered modes, for which $n'>n_d$. Now we trace how this fact affects the real spectrum shift γ_n (4.15). The first term $\gamma_n^{(1)}$ (4.16) is very similar to L_n^{-1} (4.19) and is also made up by the propagating scattered modes only. Therefore, under the assumption (5.2), we can apply for $\widetilde{W}_S(k_x,q_x)$ an asymptotic similar to Eq. (5.3) but with the sine instead of the cosine. As a result, the following interpolation formula for $\gamma_n^{(1)}$ can be derived:

$$|\gamma_n^{(1)}|\sim\frac{(k\zeta)^2}{k}\frac{(kR_c)^2}{1+(kR_c)^2}\frac{\pi n/d}{\Lambda_n}\propto\zeta^2. \tag{5.5}$$

In contrast to $\gamma_n^{(1)}$, the second $\gamma_n^{(2)}$ and third $\gamma_n^{(3)}$ terms of Eq. (4.15) take account of the infinitely large number of the evanescent modes. Let us consider the sum $\gamma_n^{(2)}$ (4.17),

$$\gamma_n^{(2)}=-\frac{k^2}{\pi n}\frac{1}{\Lambda_n}\sum_{n'=1}^{\infty}\left[A\left(\frac{\pi n}{d},\frac{\pi n'}{d}\right)+B\left(\frac{\pi n}{d},\frac{\pi n'}{d}\right)\right]. \tag{5.6}$$

We break it up into two parts: the first, $\gamma_2^<$, consists of the terms with $1\leqslant n'\lesssim n_\zeta$, while the second, $\gamma_2^>$, is made up by the rest of terms with $n'\gtrsim n_\zeta$.

The inequalities (5.2) and $n'\lesssim n_\zeta$ allow to use a small-argument asymptotic for $S(t_1,t_2;0)$ in Eq. (4.11) for $A(\pi n/d,\pi n'/d)$ and in Eq. (4.12) for $B(\pi n/d,\pi n'/d)$. As the consequence, $A\sim-(\pi n/d)^2(\pi n'/d)^2\zeta^6$, while $B\sim(\pi n/d)^2(\pi n'/d)^2\zeta^4k^{-2}$. Thus, $|A/B|\sim(k\zeta)^2\ll1$, so that the terms with $A(\pi n/d,\pi n'/d)$ in $\gamma_2^<$ can be neglected in comparison with those containing $B(\pi n/d,\pi n'/d)$. So, we have

$$\gamma_2^<\sim-\frac{k^2}{\pi n}\frac{1}{\Lambda_n}\sum_{n'=1}^{n_\zeta}B\left(\frac{\pi n}{d},\frac{\pi n'}{d}\right)\sim-\frac{\zeta^4}{\pi n}\frac{(\pi n/d)^2}{\Lambda_n}\sum_{n'=1}^{n_\zeta}\left(\frac{\pi n'}{d}\right)^2\sim-\frac{\pi n/d}{\Lambda_n}\zeta. \tag{5.7}$$

For the other subsum $\gamma_2^>$, the conditions (5.2) and $n'\gtrsim n_\zeta$ yield $(k_z\zeta)^2\ll1\lesssim(\pi n'/d)^2\zeta^2=(q_z\zeta)^2$. This fact allows to substitute a large-argument asymptotic for $S(t_1,t_2;0)$ in Eqs. (4.11) and (4.12). After expanding A and B in a small parameter $(k_z/q_z)^2$ we obtain $A\sim-(\pi n/d)^2(\pi n'/d)^{-4}$, $B\sim(\pi n/d)^2(\pi n'/d)^{-2}k^{-2}$. So, $|A/B|\sim(n_d/n')^2\lesssim(n_d/n_\zeta)^2\sim(k\zeta)^2\ll1$. Thus, we can omit $A(\pi n/d,\pi n'/d)$ again:

$$\gamma_2^>\sim-\frac{k^2}{\pi n}\frac{1}{\Lambda_n}\sum_{n'=n_\zeta}^{\infty}B\left(\frac{\pi n}{d},\frac{\pi n'}{d}\right)\sim-\frac{1}{\pi n}\frac{(\pi n/d)^2}{\Lambda_n}\sum_{n'=n_\zeta}^{\infty}\left(\frac{\pi n'}{d}\right)^{-2}\sim-\frac{\pi n/d}{\Lambda_n}\zeta. \tag{5.8}$$

Combining the asymptotics (5.7) and (5.8), we finally arrive at the result

$$\gamma_n^{(2)}=\gamma_2^<+\gamma_2^>\sim-\frac{\pi n/d}{\Lambda_n}\zeta. \tag{5.9}$$

We see that $\gamma_n^{(2)}$ exhibits a *linear dependence* on the roughness height ζ. This surprising fact can be easily explained and understood. Actually, from Eq. (5.7) it follows that the subchannel contribution $\gamma_{nn'}^{(2)}$ of an n'th waveguide mode into $\gamma_n^{(2)}$ increases as n'^2 for $n'\lesssim n_\zeta$. This increase terminates at $n'\sim n_\zeta$ and is displaced by a power decrease n'^{-2} in accordance with Eq. (5.8). Consequently, there exists the maximum of the subchannel summands in the sum (5.6) that is located around $n'\sim n_\zeta$. This is exactly the region that is referred to by the *resonant selection rule* as the most effective for scattering. The subchannel contributions decrease rather slowly towards the both sides of the maximum. Therefore the number of the resonant modes is quite large, viz. of the order of $n_\zeta\gg n_d\gg1$, which gives rise to the *integral nature* of the resonance. Although the probability of scattering into each of the resonant modes is proportional to ζ^2 for $n'\sim n_\zeta$, their total number $n_\zeta\sim d/\zeta$ makes their total contribution linear in ζ, i.e., nonanalytic (square-root) in the dispersion ζ^2.

In a similar, but more cumbersome technically, manner it can be shown that the third term $\gamma_n^{(3)}$ (4.18) of Eq. (4.15) is also mainly formed by a vicinity of the resonance point $n'=n_\zeta$. This term is well described by the simple interpolation formula

$$\gamma_n^{(3)}\sim\frac{\pi n/d}{\Lambda_n}\frac{\zeta R_c}{\zeta+R_c}. \tag{5.10}$$

From Eqs. (5.10) and (5.9) it follows that $\gamma_n^{(3)}$ is negligible in comparison with $\gamma_n^{(2)}$ if the surface roughness is sharp enough, $\zeta/R_c\gg1$. On the other hand, for mildly sloping irregularities, $\zeta/R_c\ll1$, the contributions $\gamma_n^{(3)}$ and $\gamma_n^{(2)}$ are comparable. So, if we want to deal with arbitrary values of the slope, we should keep the both terms in the sum $\gamma_n^{(2)}+\gamma_n^{(3)}$. At the same time, it is easy to see that under the assumption (5.2) the term $\gamma_n^{(1)}$ is always much less than $\gamma_n^{(3)}$. Thus, the expression (4.15) for γ_n reduces to the following asymptotic:

$$\gamma_n\simeq\gamma_n^{(2)}+\gamma_n^{(3)}\sim-\frac{\pi n/d}{\Lambda_n}\left(1-1.7\frac{R_c}{\zeta+R_c}\right)\zeta. \tag{5.11}$$

Certainly, the numerical factor 1.7 in Eq. (5.11) cannot be obtained by simple adding up Eqs. (5.9) and (5.10). This factor results from the comparison of Eq. (5.11) with the numerical calculations based on the general formulas (4.15)–(4.18). The presence of that factor is very important, because it is responsible for the possibility for γ_n to change the sign.

Obviously, a leading term of Eq. (5.11) is always linear in ζ regardless of the ratio ζ/R_c, i.e., $\gamma_n \propto \zeta$. At the same time, the inverse attenuation length L_n^{-1} (4.19) is proportional to ζ^2 under the condition (5.2), $L_n^{-1} \propto \zeta^2$. These two facts imply that $|\gamma_n| \gg (2L_n)^{-1}$ and therefore the complex spectrum shift δk_n is nearly real, $\delta k_n \simeq \gamma_n$, for low boundary perturbations (5.2). This means that the low surface irregularities cause much more intensive dephasing (chaotization) of the primary wave than damping of its amplitude.

For rather smooth boundary inhomogeneities ($\zeta \ll R_c$) the both terms in Eq. (5.11) are equally significant and the real spectrum shift γ_n is positive. However, in the situation with extremely sharp surface defects ($kR_c \ll k\zeta \ll 1$) the second term in Eq. (5.11) can be omitted and the quantity γ_n becomes *negative*. This unusual effect can be interpreted as the *increase* of the phase velocity of the primary wave.

The last step in analyzing the case (5.2) is to obtain the explicit form of the weak-scattering conditions (1.1). To this end, we substitute the leading term $\zeta(\pi n/d)/\Lambda_n$ for $|\delta k_n|$ into Eqs. (1.1) and apply the asymptotic $\widetilde{R}_c \simeq R_c$ that is valid for $(k\zeta)^2 \ll 1$. As the result, Eqs. (1.1) can be rewritten as

$$(k_z\zeta)^2 \simeq (n/n_\zeta)^2 \ll \min\{1,(\Lambda_n/R_c)^2\}. \qquad (5.12)$$

This inequality is automatically satisfied within Eq. (5.2) if successive reflections of an nth mode from the rough boundary are *not correlated*,[1,4,5] i.e., if $R_c \ll \Lambda_n$. However, if the correlations are *strong* ($\Lambda_n \ll R_c$), then Eq. (5.12) supplements Eq. (5.2) and may even become more restrictive. We emphasize here once again that the roughness slope ζ/R_c may far exceed unity within the domain of validity (5.2), (5.12) of our results.

B. High boundary perturbations

Within a limiting case of *high boundary perturbations*,

$$(k\zeta)^2 \gg 1, \qquad (5.13)$$

the resonant peak $n' \sim n_\zeta$ lies among the propagating scattered modes, $1 \ll n' \sim n_\zeta \ll n_d$. Since those modes contribute to both γ_n and L_n^{-1}, it seems probable that the real γ_n and imaginary $(2L_n)^{-1}$ parts may be equally important in forming δk_n. This conclusion makes it necessary to analyze the whole set of equations (4.15)–(4.19) to understand the behavior of δk_n. In addition to this, the Rayleigh parameter for both falling $(k_z\zeta)^2$ and propagating scattered $(q_z\zeta)^2$ modes can be now in an arbitrary proportion to unity. This suggests a mostly numerical approach to the case (5.13) that is described in the next subsection. In the rest of this subsection we study L_n in strips with *small-scale* boundary irregularities ($kR_c \ll 1$), which is the only limit allowing relatively simple analytical treatment within Eq. (5.13).

It is erroneous to think of the limit $kR_c \ll 1$ as physically trivial. Indeed, if the both inequalities (5.13) and $kR_c \ll 1$ hold simultaneously, then the roughness slope ζ/R_c and the Fresnel parameter $k\zeta^2/R_c$ turn out to be large. Thus, the conditions $kR_c \ll 1 \ll (k\zeta)^2$ define, beyond any doubt, the most interesting and unexplored physical situation.

Let us subdivide the analysis of Eq. (4.19) into cases of "slipping," $(k_z\zeta)^2 \ll 1$, and steeply falling, $(k_z\zeta)^2 \gg 1$, modes. For the former case one can derive the asymptotic for L_n in a similar manner as it was done above, i.e., by breaking up the sum (4.19) into the subsums with $n' \lesssim n_\zeta$ and $n' \gtrsim n_\zeta$. The subchannel contributions $L_{nn'}^{-1}$ can be shown to increase as n'^2 for $n' \lesssim n_\zeta$ and decrease approximately as n'^{-1} for $n_\zeta \lesssim n' \ll n_d$. So, we have again the resonance peak at $n' \sim n_\zeta$, and the total number of the contributive modes to be $\sim n_\zeta$. Within the resonance domain $n' \sim n_\zeta$ the subchannel contributions $L_{nn'}^{-1}$ are almost independent of ζ. As the result, the asymptotic for L_n^{-1} exhibits an unconventional *inversely proportional* dependence on the height ζ. Under the condition (5.13), the inequality $(k_z\zeta)^2 \ll 1$ is satisfied for an anomalously small group of nearly ballistic modes with $(k_z/k)^2 \ll (k\zeta)^{-2} \ll 1$ only. Therefore the domain of realization for the asymptotic $L_n^{-1} \propto \zeta^{-1}$ is quite narrow. The numerical simulations confirm that this asymptotic is hardly observable and is quickly overpowered by the "steep-mode" asymptotic.

The analysis of the steeply falling modes, $(k_z\zeta)^2 \gg 1$, is basically the same as before: we break up the sum (4.19) into two subsums and then treat them separately. The subchannel terms $L_{nn'}^{-1}$ show the usual quadratic in n' increase as $n' \lesssim n_\zeta$. On reaching the resonance region $n' \sim n_\zeta \ll n_d$, the increase is replaced by a rather slow decrease that cannot be described by a simple power law. For a simpler treatment of the region $n' \gtrsim n_\zeta$ we assume that $k_z \sim k$. In contrast to the previous case, $L_{nn'}^{-1}$ exhibit a quadratic dependence on ζ within the resonance interval $n' \sim n_\zeta$. Hence, the integral resonant effect results in a linear dependence of L_n^{-1} on ζ,

$$L_n^{-1} \sim \frac{(k\zeta)(kR_c)}{\Lambda_n} \quad \text{for} \quad kR_c \ll 1 \ll (k_z\zeta)^2 \sim (k\zeta)^2. \qquad (5.14)$$

This asymptotic is rather typical of the high boundary defects (5.13) and clearly seen in the numerical calculations.

C. Numerical results

We start with the attenuation length L_n (4.19). Figure 1 demonstrates the behavior of $\Lambda_n/2L_n$ as a function of $k\zeta$ for several fixed values of kR_c. The thicker curves represent the general formula (4.19), while the thinner curves correspond to the perturbative result (5.4) of the earlier works.[1,4,5] We see that all the thicker curves almost coincide with the corresponding thinner curves for $k\zeta \lesssim 0.2$, while for the larger values of $k\zeta$ the distinctive divergence for all the curves appears. Consequently the perturbative expression (5.4) applies as long as $k\zeta$ is not exceeding the "threshold" value of about 0.2.

It is worth mentioning that the curve with $kR_c = 5$ intersects those corresponding to the smaller values of kR_c. This is a manifestation of the surprising effect of *reentrant transparency* that will be discussed below.

It is indicated in Fig. 1 that the curves with $kR_c \lesssim 0.1$ are governed by the linear in ζ asymptotic (5.14) within a range of large enough values of $k\zeta$. However, this range is

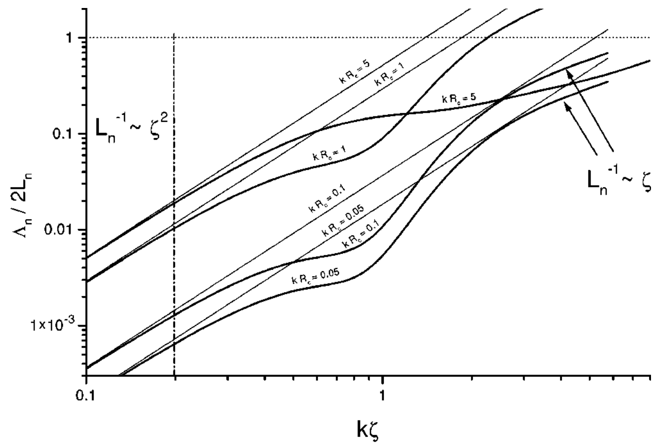

FIG. 1. $\Lambda_n/2L_n$ vs $k\zeta$ (log-log scale) for several fixed values of kR_c and fixed $kd/\pi = 5.5$, $n = 3$. The thicker curves represent the general Eq. (4.19) for L_n, while the thinner curves depict the perturbative result (5.4) of the earlier papers. All the curves to the left of the dash-dot line are proportional to ζ^2, $L_n^{-1} \propto \zeta^2$.

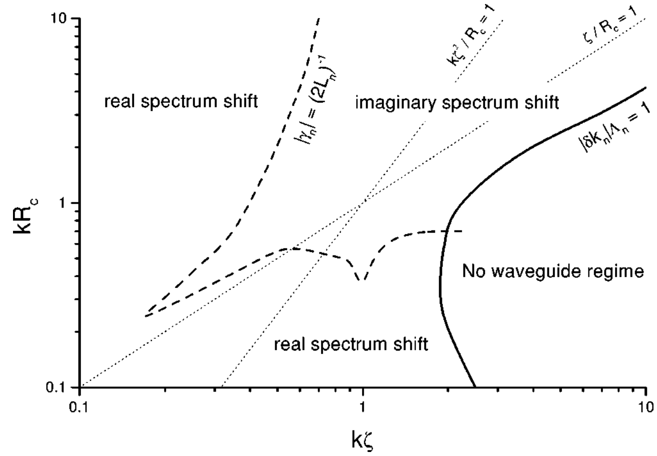

FIG. 3. The plane $k\zeta$-kR_c (log-log scale) divided into the regions where the entire spectrum shift δk_n is nearly real, $|\gamma_n| > (2L_n)^{-1}$, or nearly imaginary, $|\gamma_n| < (2L_n)^{-1}$. The regions where $|\delta k_n|\Lambda_n < 1$ (weak-scattering regime) and $|\delta k_n|\Lambda_n > 1$ (no waveguide propagation) are shown. The figure corresponds to fixed values of $kd/\pi = 5.5$ and $n = 3$.

bounded above with the weak-scattering condition $\Lambda_n/2L_n \ll 1$ and for, e.g., $kR_c = 0.1$ is not very wide, viz. $3.9 \lesssim k\zeta \lesssim 8.5$.

Now we turn to the numerical analysis of the real spectrum shift γ_n (4.15)–(4.18). In Fig. 2 we present the graphs of $\gamma_n\Lambda_n$ vs $k\zeta$ for some fixed values of kR_c. The behavior of γ_n in the domain of low irregularities (5.2) is well described by the asymptotical result (5.11). Therefore we proceed directly to the much more diverse and complicated case of high surface defects (5.13).

The numerical study reveals that, in full accordance with the resonant selection rule, the main contribution to γ_n for $(k\zeta)^2 \gg 1$ is given by terms with $n' \sim n_\zeta \ll n_d$, i.e., by the relatively small subsums of $\gamma_n^{(1)}$ (4.16) and $\gamma_n^{(2)}$ (4.17). The evanescent scattered modes are not resonant and therefore can be neglected.

In Fig. 2 we see a different behavior of the curves corresponding to small $kR_c \lesssim 1$ and large $kR_c \gg 1$. The most striking feature of this difference is the fast increase of γ_n with the increase of $k\zeta$ for $kR_c \lesssim 1$ and $k\zeta \gtrsim 1$, in contrast to the slow decrease for $kR_c \gg 1$ and $k\zeta \gtrsim 1$. Due to this gradual decrease, the curves of γ_n intersect the axis $\gamma_n\Lambda_n = 0$ and γ_n

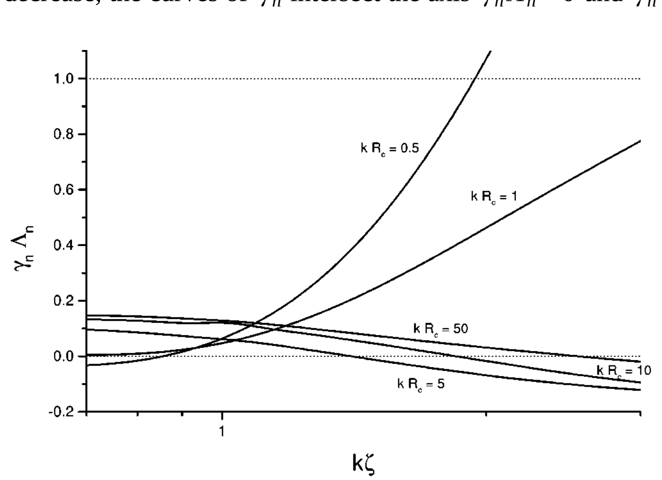

FIG. 2. $\gamma_n\Lambda_n$ vs $k\zeta$ (semi-log scale) for $kd/\pi = 10.5$, $n = 5$ and few fixed values of kR_c.

reverses the sign into negative. This effect occurs at some threshold values of $k\zeta \approx 1.5$–2.5 that are slightly dependent on kR_c. Thus, the high boundary irregularities (5.13), as well as low sharp perturbations ($kR_c \ll k\zeta \ll 1$), may not only decrease but also increase the phase velocity of a propagating wave.

Finally, we present Fig. 3 to illustrate where the real and imaginary parts of the complex spectrum deviation δk_n may compete and what is the domain of validity of our theory. We note that the second of the weak-scattering conditions (1.1) is not so restrictive as the first one for $kR_c \lesssim 100$ and therefore it is not displayed in Fig. 3.

From Fig. 3 it follows that δk_n is nearly real, $\delta k_n \simeq \gamma_n$, in those regions of the $k\zeta$-kR_c plane, where one (or both) of the parameters $k\zeta$ or kR_c are small enough. So, e.g., if $k\zeta$ is fixed at a value less than 0.14, then γ_n always dominates over $(2L_n)^{-1}$, no matter how large is kR_c (at least, up to several hundred). If we fix $kR_c \lesssim 0.2$ instead, then the spectrum shift remains almost real up to $k\zeta \approx 2$, after which the weak-scattering condition fails. In contrast, the imaginary part of δk_n is dominant for a rather corrupted boundary with high and long irregularities simultaneously.

The curve $|\delta k_n|\Lambda_n = 1$ never enters the region of moderate boundary inhomogeneities, where $k\zeta \lesssim 2$. Thus, such a type of defects automatically ensures weak wave-surface scattering. Besides, it is seen that either of the widely used inequalities $\zeta/R_c < 1$ or $k\zeta^2/R_c < 1$ is sufficient for the fulfillment of the weak-scattering requirements (1.1).

Our theory predicts the effect of *reentrant transparency* which can be easily introduced and explained with the help of Fig. 3. Let us start off from a point, say, $k\zeta = 2.1$ on the $k\zeta$ axis vertically upward along the kR_c axis. We first pass the region where the spectrum shift is nearly real and the boundary scattering is still weak. Then we cross the curve $|\delta k_n|\Lambda_n = 1$ and get into the domain where a waveguide type of propagation is no longer possible (i.e., the scattering is not weak). If we keep moving, then we finally cross the curve $|\delta k_n|\Lambda_n = 1$ again, which means that the weak-scattering conditions (1.1) have been restored, i.e., we have *reentered* the regime of waveguide propagation. However, we are now in a region where the spectrum shift δk_n is not real, but close to imaginary. The origin of the reentrant transparency lies in an unconventional nonmonotonic dependence of both γ_n (4.15)–(4.18) and L_n^{-1} (4.19) on kR_c for large enough values of $k\zeta \gtrsim 1$.

VI. CONCLUSION

In concluding, we list the most important consequences of the exact (nonperturbative) scattering operator approach presented in this paper:

(1) It has been believed that the ''absolutely soft'' boundary condition (2.3) can be treated perturbatively whenever the surface deviations $\xi(x)$ are small enough (see, e.g., Refs. 1,2,4–7,15,16,18,20–22). Under the perturbative approach, the *exact* boundary condition (2.3) is expanded into power series in $\xi(x)$ up to linear term inclusive. As the result, one obtains the *impedance* boundary condition formulated on the unperturbed plane $z=0$,

$$\mathcal{G}(x,x';z=0,z')+\xi(x)\frac{\partial}{\partial z}\bigg|_{z=0}\mathcal{G}(x,x';z,z')=0. \quad (6.1)$$

The relief $\xi(x)$ of the rough wall enters Eq. (6.1) as the impedance. The *resonant selection rule*, which has been introduced in this paper, proves that the reduction of Eq. (2.3) to Eq. (6.1) is groundless, no matter how low are the surface irregularities. The main contribution to the complex spectrum shift δk_n (1.2) is always given by the *resonant* scattered modes with $(\pi n'/d)^{-1}\sim\zeta$. This suggests that it is incorrect to apply a perturbative approach in ζ even if ζ is much smaller than the wavelength of the falling wave, $\zeta \ll (\pi n/d)^{-1}$. In other words, a problem of wave propagation through a strip with absolutely soft randomly rough boundaries is, in general, *irreducible* to a simpler problem with the smoothed random-impedance boundary.

(2) Along with the resonance character of forming δk_n, we discovered the *integral nature* of this resonance. Namely, the resonant peak is always smooth enough, so that it includes $\sim n_\zeta \gg 1$ modes with comparable amplitudes [see Eq. (5.1)]. Due to this, a few interesting and unexpected effects can be observed. So, e.g., for low boundary perturbations (5.2) the integral resonance principle results in a linear dependence of the real spectrum shift γ_n (4.15) on ζ, while the inverse attenuation length L_n^{-1} (4.19) shows the standard quadratic law[1,4,5] $L_n^{-1}\propto\zeta^2$. As the consequence, it turns out that $|\gamma_n|\gg L_n^{-1}$ and therefore the entire spectrum deviation δk_n is nearly real, i.e., $\delta k_n \simeq \gamma_n$. This leads to much faster dephasing (chaotization) of the propagating signal in comparison with damping of its amplitude. From the asymptotics $\delta k_n\simeq\gamma_n$ and $\gamma_n\propto\zeta$ it follows that δk_n is a *nonanalytic* (square-root) function of the dispersion ζ^2, or of the binary correlator (2.1): $\delta k_n \propto \sqrt{\zeta^2}$.

(3) It is found that the real spectrum shift γ_n may *reverse the sign* from positive to negative in some domains of the external parameters. In particular, this occurs in the low perturbation limit (5.2) when the parameter kR_c is decreased below $k\zeta$, i.e., when the boundary irregularities are extremely sharp, $kR_c \ll k\zeta \ll 1$. This effect can be interpreted as the roughness-induced increase of the phase velocity of the primary wave.

(4) The *exact scattering operator approach*, which we put forward in this paper, made it possible to analyze (at least, numerically) a quite complicated and practically important situation with high boundary inhomogeneities (5.13). It has been proved that δk_n exhibits a surprising *nonmonotonic* dependence on the parameter kR_c with the maximum being located around $kR_c\sim 1$. Basing on this fact, we predicted *reentrant transparency*: when kR_c is increased from small, $kR_c \ll 1$, up to large, $kR_c \gg 1$, values, the weak-scattering conditions (1.1) are first violated at $kR_c \lesssim 1$ and then restored again after passing a finite ''dark'' region.

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