# Hopping conduction in uniaxially stressed Si:B near the insulator-metal transition

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Using uniaxial stress to tune the critical density near that of the sample, we have studied in detail the low-temperature conductivity of $p$-type Si:B in the insulating phase very near the metal-insulator transition. For all values of temperature and stress, the conductivity collapses onto a single universal curve, $\sigma(S,T) = AT^{1/2}F[T^*(S)/T]$. For large values of the argument, the scaling function $F[T^*(S)/T]$ is well fit by $\exp[-(T^*/T)^{1/2}]$, the exponentially activated form associated with variable-range hopping when electron-electron interactions cause a soft Coulomb gap in the density of states at the Fermi energy. The temperature dependence of the prefactor, corresponding to the $T$ dependence of the critical curve, has been determined reliably for this system, and is $\propto T^{0.5}$. We show explicitly that neglecting the prefactor leads to substantial errors in the determination of the $T^*$'s and the critical exponents derived from them. The conductivity is not consistent with Mott variable-range hopping, $\exp[-(T^*/T)^{1/4}]$, in the critical region, nor does it obey this form for any range of the parameters. Instead, the conductivity of Si:B is well fit by $\sigma = AT^{1/2}\exp[-(T^*/T)^{\alpha}]$ for smaller argument of the scaling function, with $\alpha = 0.31$ related to the critical exponents of the system at the metal-insulator transition. [S0163-1829(99)15027-6]

## I. INTRODUCTION

Hopping conductivity of localized electrons in disordered insulators was a subject of considerable controversy two decades ago. For noninteracting electrons, the problem was first addressed by Mott,[1,2] who showed that below any microscopic energy scale, a tradeoff between the exponential thermal activation due to the difference in energy between the initial and final electron states on the one hand, and the exponential factor associated with the spatial overlap between the two (localized) states on the other, leads to an optimal conductivity at low temperatures of the form

$$\sigma \propto \sigma_0(T)\exp[-(T_0/T)^{1/4}] \tag{1}$$

in three dimensions, where

$$T_0 \propto 1/[N(E_f)a^3]. \tag{2}$$

The prefactor $\sigma_0(T)$ is a weak function of the temperature, and is usually assumed constant. In the above equation, $N(E_f)$ is the (constant) one-electron density of states at the Fermi level, and $a$ is the (linear) size of the localized electronic wave function. This expression, known as Mott's variable-range hopping (VRH), was put on a rigorous footing using a percolation formalism.[3–5] Many different materials appeared to agree well with the Mott formula,[6] providing experimental confirmation of Mott's ideas.

The applicability to real disordered insulators was, however, challenged by a number of theorists,[7,8] because of the presence of Coulomb interactions between electrons. A key step in understanding the role of electron interactions was put forward by Efros and Shklovskii (ES),[9] who showed that a self-consistent Hartree treatment of the long-range $(1/r)$ Coulomb interactions in an insulator leads to a soft gap in the one-electron density of states at the Fermi level, resulting in a depletion of low-lying excitations. This, in turn, leads to a much lower conductivity at low temperatures of the form

$$\sigma \propto \exp[-(T_0'/T)^{1/2}], \tag{3}$$

where

$$T_0' \propto e^2/\epsilon a. \tag{4}$$

Here $e$ is the electronic charge, $\epsilon$ is the dielectric constant of the semiconductor, and $a$ is the linear size of the localized electronic state.

Considerable activity on the issue ensued in the years following, during which various materials were shown to obey either the Mott form [Eq. (1)] or the ES form [Eq. (3)]. A crossover with decreasing temperature from Mott to ES variable-range hopping was observed in CdSe:In (Ref. 10) and CdTe:Cl.[11] This was attributed[10,12] to hopping energies that were larger than the gap energy at high temperature (Mott hopping) and smaller than the gap at low $T$ (ES hopping). A crossover with dopant concentration was found in $n$-GaAs (Ref. 13) and Si:P,[14] where Mott hopping was claimed for samples near the metal-insulator transition when the Coulomb gap has a small energy width, and ES hopping prevails deeper in the insulating phase where electron-electron interactions are stronger and the hopping electrons probe the gap. Although variable-range-hopping exponents have been found that deviate from these values, it is found that strong electron interactions yield a hopping exponent of $\frac{1}{2}$ while weak interactions (compared with hopping energies) give rise to exponent $\frac{1}{4}$. This has given rise to the expectation that Mott variable-range hopping will always be ob-

 

served near the metal-insulator transition as electron screening increases and the Coulomb gap collapses approaching the metallic phase.[13,15]

In this paper we report measurements of the hopping conduction in insulating Si:B very near the transition to the metallic phase. By applying a compressive uniaxial stress along the [001] direction using a pressure cell described elsewhere,[16] we have driven a sample of Si:B from the metallic phase into the insulating phase, and mapped out the conductivity as a function of applied stress ($S$) and temperature ($T$) in the range $0.05\,\mathrm{K} < T < 0.75\,\mathrm{K}$. We find[17] that the conductivity over this entire temperature range for stress values varying by about 40% on either side of the critical stress $S_c$ is described accurately by the scaling form

$$\sigma(S,T) = \sigma_c(T) f[\Delta S T^{-1/z\nu}], \tag{5}$$

where $\sigma_c(T)$ is the conductivity at the transition, $\Delta S = (S_c - S)$ is the difference between the stress and its critical value (i.e., the control parameter), $\nu$ is the critical exponent that characterizes the divergence of the correlation length, $\xi \propto (\Delta S)^{-\nu}$, and $z$ is the dynamical exponent that describes the divergence of the time scale, $\tau \propto \xi^z$. By defining a stress-dependent temperature scale $T^* \propto (\Delta S)^{z\nu}$, and noting from our previous work[17] that $\sigma_c(T) \propto T^{1/2}$, we may rewrite the above equation in the insulating phase as

$$\sigma(S,T) = A T^{1/2} F[T^*/T]. \tag{6}$$

This equation fully describes the conductivity of Si:B on the insulating side in the vicinity of the metal-insulator transition. We present below a detailed analysis of the temperature dependence of the conductivity of the insulating branch in the critical region near the transition.

## II. EXPERIMENTAL RESULTS AND DISCUSSION

A bar-shaped $8.0 \times 1.25 \times 0.3\,\mathrm{mm}^3$ sample of Si:B was cut with its long dimension along the [001] direction. The dopant concentration, determined from the ratio of the resistivities[16] at 300 K and 4.2 K, was $4.84 \times 10^{18}\,\mathrm{cm}^{-3}$. Electrical contact was made along four thin boron-implanted strips. Uniaxial compression was applied to the sample along the long [001] direction using a pressure cell described elsewhere.[16] Four-terminal measurements were taken at 13 Hz (equivalent to dc) for different fixed values of uniaxial stress at temperatures between 0.05 and 0.75 K. Resistivities were determined from the linear region of the $I$-$V$ curves.

For two Si:B samples with different dopant concentrations that are metallic in the absence of stress, Fig. 1 shows the resistivity at 4.2 K normalized to its zero-stress value as a function of uniaxial stress. With increasing stress, the resistivity initially increases rapidly (by several orders of magnitude) and then decreases gradually above several kilobar. This is in marked contrast with Si:P, which exhibits little change at small stress values, and then shows a similar decrease in resistivity at larger stresses. This can be understood as follows. The acceptor state in Si:B has a fourfold degeneracy in the unstressed cubic phase which is lifted by relatively small uniaxial stress into two doublets (each retaining only the Kramers degeneracy); this initially drives Si:B to be more insulating. By contrast, the sixfold valley degeneracy

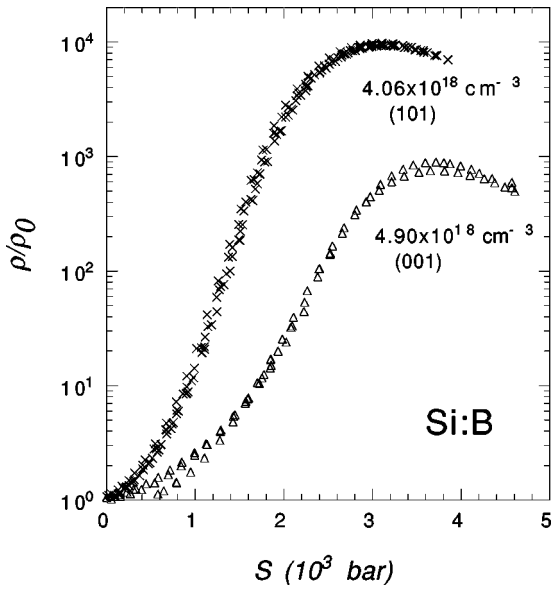


FIG. 1. For two Si:B samples with different dopant concentrations, as labeled, the resistivity at 4.2 K normalized to its zero-stress value is shown as a function of uniaxial stress along the directions indicated.

(on top of the required Kramers or spin degeneracy) of an effective-mass donor in Si has already been removed (even in zero stress) by the central-cell potential of the phosphorus dopants.[18] Such contrasting behavior is due in part to wave-function anisotropy[19] and in part to degeneracy in the presence of electron correlations,[20] whose effects have been separately considered for the case of effective-mass donor systems.

The conductivity is shown as a function of temperature on a log-log scale in Fig. 2 for different uniaxial stresses for which the sample is in the insulating phase. Based on a detailed analysis published elsewhere,[17] the critical stress for

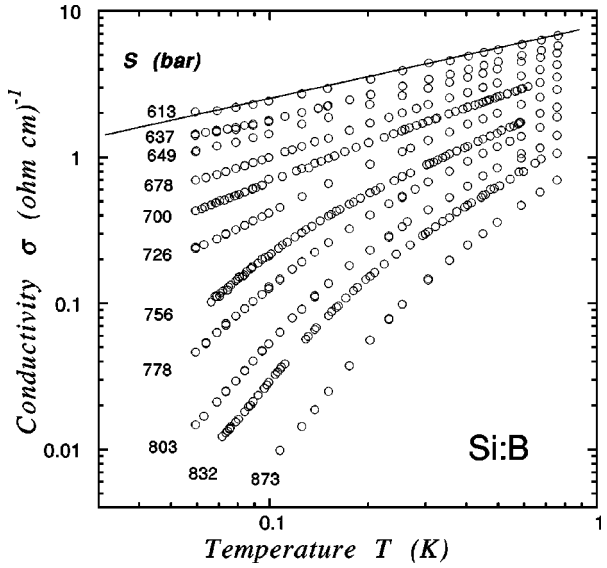


FIG. 2. The conductivity as a function of temperature on a double logarithmic scale for various values of uniaxial stress that place the sample on the insulating side of the metal-insulator transition. The critical stress, $S_c$, is 613 bar.

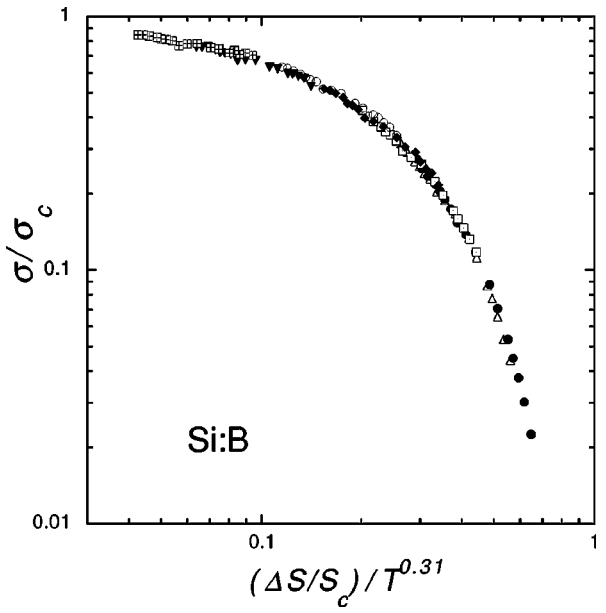


FIG. 3. On a log-log scale, the normalized conductivity, $\sigma/\sigma_c$, as a function of the scaling variable $[(\Delta S)/S_c]/T^{1/z\nu}$, with $z\nu = 3.2$. Here $\Delta S=(S-S_c)$, where $S_c$ is the critical stress.

this sample was determined to be $S_c = 613$ bar. The critical curve is a straight line on this scale, with the conductivity $\sigma_c \to 0$ at $T \to 0$, following a power law, $\sigma \propto T^{0.5}$.

The conductivity $\sigma(S,T)$ normalized by the critical conductivity $\sigma_c(T)$ is shown in Fig. 3 as a function of the scaling variable, $\Delta S/T^{1/z\nu}$, where $z\nu = 3.2$ has been chosen so that the data for all values of stress and all measured temperatures collapse onto a single universal curve, as predicted by Eq. (1). The resulting scaling function fully describes the temperature dependence of the conductivity in the insulating phase in the vicinity of the transition.

As discussed earlier, the conductivity in the insulating phase is expected to exhibit variable-range hopping at low temperature of the form

$$\sigma(S,T) \propto \sigma_0(T) \exp[-(T^*/T)^{\alpha}], \tag{7}$$

with $\alpha = \frac{1}{4}$ when the density of states is a slowly varying function of energy [Mott-VRH (Refs. 1 and 2)], and $\alpha = \frac{1}{2}$ [ES-VRH (Ref. 9)] when hopping energies are comparable with or smaller than electron interactions, forming a soft ''Coulomb'' gap at the Fermi level. While these analyses have been done for the strongly localized regime (deep in the insulating phase), arguments have been advanced[21] as to why such behavior persists in the insulating phase even close to the transition, provided the temperature is low enough that the localized character of the phase becomes evident.

However, it is not clear whether the hopping conduction is included in the scaling part of the conductivity near the metal-insulator transition. It has been suggested[22] that this is the case for a quantum Hall transition in two-dimensional electron gases in the presence of a strong perpendicular magnetic field, but the experimental evidence for this is not unambiguous. For the metal-insulator transition in three dimensions, it is clear from Eq. (4) that for VRH to be part of the scaling description, we must have $\sigma_0(T) = \sigma_c \propto T^{1/2}$, $f[T^*/T] \propto \exp[-(T_0/T)^{\alpha}]$, and $T^* \propto T_0 \propto (\Delta S)^{z\nu}$.

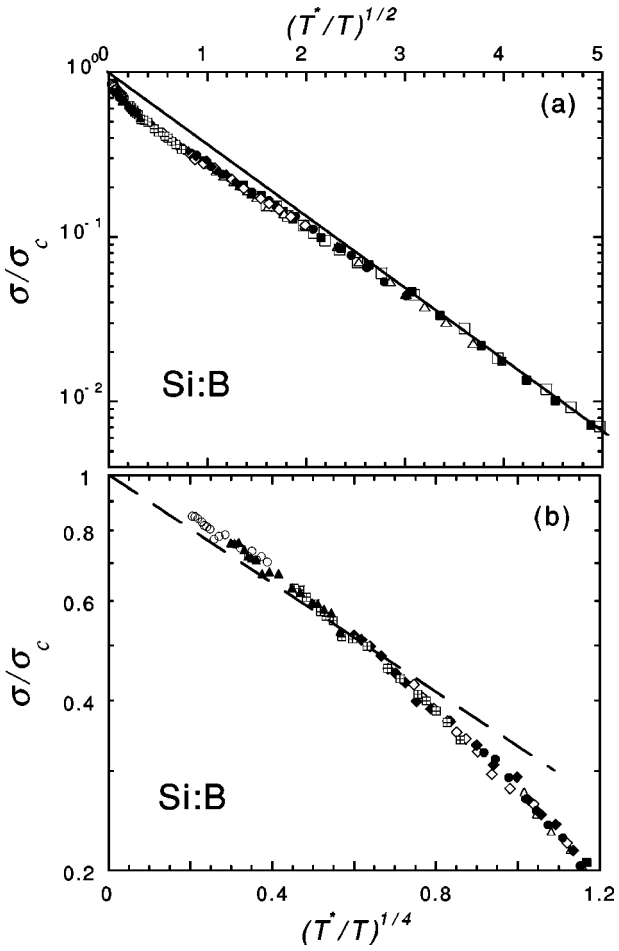


FIG. 4. (a) The normalized conductivity, $\sigma/\sigma_c$, on a logarithmic scale as a function of $(T^*/T)^{1/2}$; here $T^* \propto (\Delta S)^{z\nu}$; (b) the normalized conductivity, $\sigma/\sigma_c$, on a log scale as a function of $(T^*/T)^{1/4}$.

To test if the scaling description contains either form of hopping conduction, we plot the conductivity normalized to the conductivity of the critical curve, $\sigma/\sigma_c$, on a log scale as a function of $(T^*/T)^{1/2}$ in Fig. 4(a) and as a function of $(T^*/T)^{1/4}$ in Fig. 4(b). As can be clearly seen in Fig. 4(a), the experimental data for $(T^*/T)^{1/2} > 2.8$ $(T^*/T > 8)$ lie on a straight line passing through the origin, indicating that the conductivity crosses over to an ES-VRH form within the scaling region at large but experimentally accessible values of the argument of the scaling function, with a temperature-dependent prefactor given by the critical curve, namely

$$\sigma(T) \propto T^{1/2} \exp[-(T^*/T)^{1/2}]. \tag{8}$$

Deviations are evident for $T^*/T \leq 8$. For such small arguments of the exponential factor, it has been argued for the insulator that hopping energies may be comparable to or larger than the energy width of the Coulomb gap; in this regime a crossover has been observed in some systems, albeit within a limited range,[10,11] to Mott VRH with an exponent $\frac{1}{4}$ rather than $\frac{1}{2}$. However, it is clear from the consistently downward curving plot in Fig. 4(b) and the fact that the curve must pass through the upper left corner that Mott

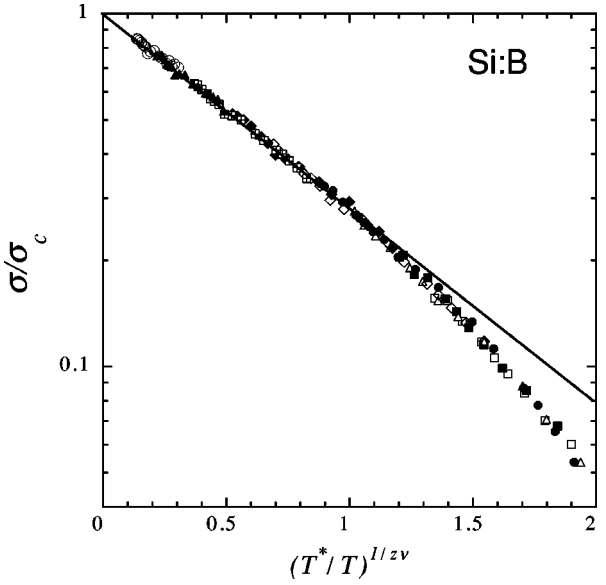


FIG. 5. The normalized conductivity, $\sigma/\sigma_c$, on a logarithmic scale as a function of $(T^*/T)^{1/z\nu}$ with $z\nu=3.2$.

hopping does not provide an adequate description of the conductivity of uncompensated Si:B in the critical region for any range of $T^*/T$.

What, then, is the form of the scaling function for $T^*/T \leqslant 8$? In a scaling description of a continuous phase transition, the singular behavior of the system in the vicinity of the transition is embodied in nontrivial but universal exponents, as well as ratios and combinations of variables which have nonanalytic form at the approach to the transition, but with scaling functions that are themselves analytic functions of these ratios or combinations. Consequently, we would expect the scaling function $f$ in Eq. (5) to be analytic in its argument around the origin. Given that $f(0)=1$, and that we expect it to decrease monotonically to its asymptotic value $f(\infty)=0$ on the insulating side, a reasonable choice for $y \geqslant 0$ is $f(y)=\exp(-\gamma y)$, suggesting that

$$\sigma(T) \propto T^{1/2} \exp[-(T^*/T)^{1/z\nu}], \tag{9}$$

i.e., the normalized conductivity $\sigma/\sigma_c$ should yield a straight line when plotted on a semilogarithmic scale versus $(T^*/T)^{1/z\nu}$. That this is the case is shown in Fig. 5. Attempts to fit the scaling function by a power-law form yield a reasonable fit over a much smaller range of the parameters. This suggests that the scaling function is better described by an exponential than any single power law; indeed, it fits over much of the range of the argument of the scaling function before it crosses over to ES-VRH [Fig. 4(a)]. By combining data for the temperature-dependent conductivity for a number of values of uniaxial stress, we have thus been able to map the scaling function for a large range of its argument. We have established that for Si:B, the conductivity in the insulating phase in the scaling region appears to be equal to a prefactor given by the power-law behavior of the critical conductivity, multiplied by an exponential function of $(T^*/T)$ raised to a power $\alpha$ which equals $1/z\nu$ for small argument and $\frac{1}{2}$ for large values of the argument, corresponding to ES-VRH.

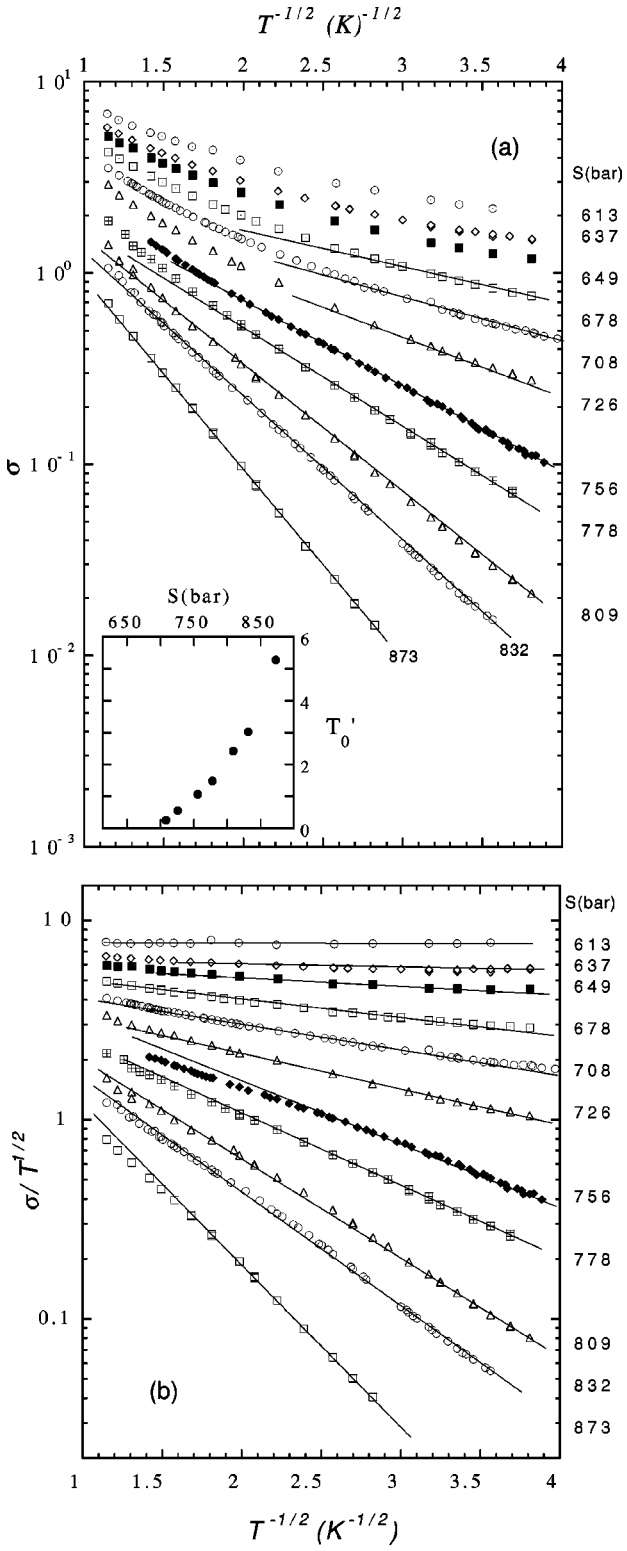


FIG. 6. (a) The conductivity $\sigma$ on a log scale as a function of $T^{-1/2}$; the inset shows $T_0'$ derived from the slopes in the main figure plotted as a function of stress. (b) On a log scale, the conductivity normalized by the temperature dependence of the prefactor, $(\sigma/T^{1/2})$, as a function of $T^{-1/2}$.

When analyzing the conductivity of the insulator in the VRH regime, the temperature-dependent prefactor is very often omitted because its weak (power-law) dependence is negligible compared to the strong temperature dependence of the exponential term. This is certainly justified deep in the

insulator; however, its neglect is questionable in the critical regime, where fits to the ES-VRH form have been used to extract critical exponents pertaining to the insulator-metal transition. We now show explicitly that omission of this term near the transition in our data leads to significant errors in the determination of $T^*=T_0'$ and the critical exponents derived from them.

Applying the usual analysis for ES hopping which assumes a constant, temperature-independent prefactor, we plot $\sigma$ on a log scale versus $T^{-1/2}$ in Fig. 6(a). A reasonably good fit (i.e., straight line) is obtained for the higher values of stress; not unexpectedly, deviations become progressively more pronounced as the transition is approached. Except very near the transition, the conductivity appears to be well-described by ES-VRH, for which

$$T_0' \propto 1/(\epsilon\xi) \propto (\Delta S)^{\alpha}. \tag{10}$$

Here $\xi$ is the correlation length which diverges with exponent $\nu$, and $\epsilon$ is the dielectric constant which diverges with exponent $\zeta$, so that $\alpha=\xi+\zeta$. The inset to Fig. 6(a) shows a plot of $T_0'$ derived from this analysis versus $S$, yielding $\alpha=2.8$. Since $\alpha$ plays the same role as $z\nu$, which was found to equal 3.2 in the earlier analysis, neglect of the temperature dependence of the prefactor gives rise to an error on the order of 15% in the determination of the critical exponents. For comparison and completeness, we plot in Fig. 6(b) the correct form, $\sigma/T^{1/2}$ versus $T^{-1/2}$. Inclusion of the prefactor, as in Fig. 6(b), provides a much better fit over a wider range to ES-VRH than does the neglect of the prefactor, as in Fig. 6(a). Moreover, it yields a different value for the critical exponent $1/z\nu$. We caution, however, that comparison with the full scaling curve of Fig. 3 reveals that (smaller) deviations occur here due to departures from the ES hopping form as the transition is approached. We suggest that a reliable determination of $T_0'$ and of the critical exponent $\alpha$ requires a full scaling analysis of the conductivity, and cannot be obtained from the individual curves.

Given our findings that $\alpha=z\nu=3.2=(\nu+\zeta)$ and $\mu=1.6$, and assuming again that $\nu=\mu$, we obtain $\zeta=\nu=1.6$. This implies that the correlation length and the dielectric constant diverge with the same exponent, contrary to the simple expectation that $\zeta=2\nu$. Similar behavior has been reported in CdSe:In (Ref. 10) as well as some diluted magnetic semiconductors.[23]

## III. SUMMARY

To summarize, we have shown that scaling provides an excellent description of the conductivity near the metal-insulator transition in uniaxially stressed Si:B. Based on data at many values of stress and temperature, the scaling function in the insulating phase yields particularly reliable determinations of the conductivity in the critical region. It is found that the conductivity expected for variable-range hopping in the presence of Coulomb interactions, in the form predicted by Efros and Shklovskii, is part of the scaling description in the insulating phase for large values of the scaling argument (i.e., temperatures $T$ an order of magnitude lower than the characteristic temperature $T^*$). For lower values, a clear deviation is seen, and seems to be well fit by an exponentially activated form with an exponent $1/z\nu$, which is found to be 0.31 for Si:B. It would be of interest to see if similar behavior is found in other systems near the metal-insulator transition, and to check if some of the earlier crossovers seen from ES to Mott VRH could be manifestations of the same effect. We have also examined the errors associated with analysis of conductivity data based on individual curves and neglecting the temperature dependence of the prefactor.

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