Effects of field modulation on the Hofstadter spectrum

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We study the effect of spatially modulated magnetic fields on the energy spectrum of a two-dimensional Bloch electron. By taking into account four kinds of modulated fields and using the method of direct diagonalization of the Hamiltonian matrix, we calculate energy spectra with varying system parameters (i.e., the kind of the modulation, the relative strength of the modulated field to the uniform background field, and the period of the modulation) to elucidate that the energy band structure sensitively depends on such parameters: Inclusion of spatially modulated fields into a uniform field leads to the occurrence of gap opening, gap closing, band crossing, and band broadening, resulting in an energy band structure that is distinct from the Hofstadter's spectrum. We also discuss in detail the effect of the field modulation on the symmetries appearing in the Hofstadter's spectrum. [S0163-1829(99)04027-8]

I. INTRODUCTION

The problem of a two-dimensional Bloch electron (2DBE) under a uniform magnetic field has been intensively studied for several decades,[1] and it is well known that the energy spectrum is characterized by the Hofstadter's butterfly showing a fractal nature.[2] Recently, the problem has attracted renewed interest in connection with various phenomena such as the quantum Hall effect,[3] the flux-state model for high-T_c superconductivity,[4] and the mean-field transition temperature of superconducting networks or Josephson junction arrays.[5] Recent advances in submicron technology that make it possible to fabricate any desired microstructures have led to experimental studies that seek to find indications of the Hofstadter's spectrum and its effect on transport and optical properties.[6–9] In parallel with this problem, the problem of 2D electron systems under nonuniform (either disordered[10] or periodic[11–26]) magnetic fields has also been extensively studied, and several interesting characteristics in the energy spectral and transport properties have been elucidated.

Though the problem of a 2DBE under spatially modulated magnetic fields has attracted less attention compared with the problem of a 2D electron gas under spatially modulated magnetic fields,[11–22] it is still an important problem, not only from the viewpoint of theoretical interest but also from the viewpoint of experimental interest, and there have been attempts to solve this problem.[23–26] However, unfortunately, some of the relevant works contain inconsistent results on the energy spectral properties: In Ref. 24, Gumbs and coworkers studied the effect of a one-dimensional sine-modulated (1DSM) field on a 2DBE to argue that the symmetries appearing in the Hofstadter's spectrum break down by the field modulation. Most surprisingly, they also argued that the field modulation leads an additional crisscross pattern like a spiderweb structure onto the Hofstadter's spectrum. However, Oh and coworkers[25] studied the effect of a 1D cosine-modulated (1DCM) field on a 2DBE to elucidate that the occurrence of gap closing and gap opening leads to a different energy spectrum from the Hofstadter's spectrum and that the symmetries appeaing in the Hofstadter's spectrum, except the dual property, still remain despite the field modulation. Meanwhile, Shi and Szeto[26] studied the energy spectrum of a 2DBE under a kind of 2D field modulation to argue that there is no symmetry breaking in the energy spectrum and that the fractal structure remains irrespective of the field modulation.

In this paper we re-examine the problem of a 2DBE under spatially modulated magnetic fields to settle the inconsistency discussed above. In doing this, we take into account four kinds of modulated fields (i.e., 1DSM, 1DCM, 2DSM, and 2DCM fields) in order to obtain rather generic effects of the field modulation on the Hofstadter's spectrum. By means of direct diagonalization of the Hamiltonian matrix, we calculate the energy eigenvalues and examine how the system parameters such as the type of modulation, the relative strength of the modulated field to the uniform field, and the period of the modulation, influence the energy band structure and the symmetry of the Hofstadter's spectrum. Introduction of the field modulation is shown to change the $\vec{k}$ dependence of the energy spectrum drastically, leading to the occurrence of gap opening, gap closing, band crossing, and band broadening, which is the origin of distinctive energy band structure from the Hofstadter's spectrum. Our results indicate that there is no additional spiderweb structure in the energy spectrum, contrary to the result of Ref. 24, and that the field modulation generically breaks the symmetries and the fractal property of the Hofstadter's spectrum.

This paper is organized as follows: In Sec. II we introduce four kinds of magnetic fields and the tight-binding model. In Sec. III we present numerical results on the energy band structure of a 2DBE and its symmetries under these fields. Section IV is devoted to a brief summary.

II. MODULATED MAGNETIC FIELDS AND THE TIGHT-BINDING MODEL

We consider an electron in a 2D square lattice under a spatially modulated magnetic field:

$$\vec{B}=[B_0+B_1(x,y)]\hat{z}, \tag{1}$$

where $B_0(B_1)$ denotes the uniform (modulated) part of an applied magnetic field. Among possible kinds of modulated fields, we pay attention to two kinds of modulated fields. One is the SM field

$$B_1(x,y)=B_x\sin\left(\frac{2\pi x}{T_x}\right)+B_y\sin\left(\frac{2\pi y}{T_y}\right), \tag{2}$$

and the other is the CM field

$$B_1(x,y)=B_x\cos\left(\frac{2\pi x}{T_x}\right)+B_y\cos\left(\frac{2\pi y}{T_y}\right). \tag{3}$$

Here, $B_{x(y)}$ is the strength of the modulated field and $T_{x(y)}$ is the period of the modulation along the $x(y)$ direction. Under the Landau gauge, the vector potential becomes

$$A_x=\frac{B_yT_y}{2\pi}\cos\left(\frac{2\pi y}{T_y}\right),\quad A_y=B_0x-\frac{B_xT_x}{2\pi}\cos\left(\frac{2\pi x}{T_x}\right), \tag{4}$$

for the SM field and

$$A_x=-\frac{B_yT_y}{2\pi}\sin\left(\frac{2\pi y}{T_y}\right),\quad A_y=B_0x+\frac{B_xT_x}{2\pi}\sin\left(\frac{2\pi x}{T_x}\right), \tag{5}$$

for the CM field, respectively.

The tight-binding Hamiltonian describing the motion of an electron in a magnetic field is given by

$$H=-\sum_{ij}t_{ij}e^{i\theta_{ij}}|i\rangle\langle j|, \tag{6}$$

where t_{ij} is the hopping integral between the nearest-neighboring sites i and j, and $\theta_{ij}\equiv(2\pi e/hc)\int_i^j\vec{A}\cdot d\vec{l}$ is the magnetic phase factor. Under the vector potentials given by Eqs. (4) and (5), the magnetic phase factor becomes

$$\theta_{mn;m'n'}=\begin{cases}\pm\theta_m, & (m',n')=(m,n\pm1)\\ \pm\theta_n, & (m',n')=(m\pm1,n)\\ 0, & \text{otherwise},\end{cases} \tag{7}$$

with

$$\theta_m=2\pi m\phi_0-\beta_x\gamma_x\phi_0\cos\left(\frac{2\pi m}{\gamma_x}\right),$$

$$\theta_n=\beta_y\gamma_y\phi_0\cos\left(\frac{2\pi n}{\gamma_y}\right), \tag{8}$$

for the SM field and

$$\theta_m=2\pi m\phi_0+\beta_x\gamma_x\phi_0\sin\left(\frac{2\pi m}{\gamma_x}\right),$$

$$\theta_n=-\beta_y\gamma_y\phi_0\sin\left(\frac{2\pi n}{\gamma_y}\right), \tag{9}$$

for the CM field, respectively. Here $\beta_{x(y)}=B_{x(y)}/B_0$, $\gamma_{x(y)}=T_{x(y)}/a$, and $\phi_0=B_0a^2$, a being the lattice constant. The magnetic flux per unit cell, in units of the flux quantum hc/e, is given by $\phi=(1/2\pi)\Sigma\theta_{ij}=\vec{A}\cdot d\vec{l}=\int\vec{B}\cdot d\vec{S}$.

By means of Eqs. (6) and (7), the tight-binding equation can be written as

$$e^{i\theta_n}\psi_{m+1,n}+e^{-i\theta_n}\psi_{m-1,n}+\lambda(e^{i\theta_m}\psi_{m,n+1}+e^{-i\theta_m}\psi_{m,n-1})=E\psi_{mn}, \tag{10}$$

where $\lambda(\equiv t_y/t_x)$ is the ratio of hopping integrals between the x and y directions, and E is the energy in units of t_x. Here, the wave function is given by $|\psi\rangle=\Sigma_j\psi_j|j\rangle$.

Denoting $R_{x(y)}$ as the periodicity of $\theta_{m(n)}$, the Bloch theorem can be written as

$$\psi_{m+R_x,n}=e^{ik_xR_x}\psi_{mn},\quad \psi_{m,n+R_y}=e^{ik_yR_y}\psi_{mn}, \tag{11}$$

and the first magnetic Brillouin zone (FMBZ) is given by $|k_{x(y)}|\leqslant\pi/R_{x(y)}$. We calculate the energy eigenvalues for all the values of $\vec{k}$ in the FMBZ by directly diagonalizing the Hamiltonian matrix obtained from Eqs. (10) and (11).

III. NUMERICAL RESULTS AND DISCUSSION

In what follows, we assume the modulated field has a square symmetry (i.e., $\beta_x=\beta_y=\beta$ and $\gamma_x=\gamma_y=\gamma$) and consider only the case of the isotropic hopping integral (i.e., $\lambda=1$) for the sake of simplicity. We pay attention to the energy dispersions for $q=2,3$ (with $p=1$) and $\gamma=2,3,4$, since energy dispersions for other values of (q,γ) can be obtained in a similar way. Here p and q denote the numbers (prime of each other) given by $\phi_0=p/q$.

A. Energy spectrum in a uniform magnetic field

In a uniform field ($\beta=0$), θ_n becomes zero. Thus, by means of the translational invariance along the y direction, Eq. (10) reduces to the Harper equation,

$$\psi_{m+1}+\psi_{m-1}+2\cos(\theta_m+k_y)\psi_m=E\psi_m, \tag{12}$$

and there are q eigenvalues for a given value of k_y; the full energy spectrum for an odd q consists of q subbands, while the full spectrum for an even q consists of $(q-1)$ subbands because two central subbands touch at zero energy. It is also well known that the full spectrum has the following symmetries: (i) the dual property between the k_x and k_y directions (let us denote it as S_D), (ii) the symmetry between $-E$ and $E(S_E)$, (iii) the symmetry between $-k_x$ and $k_x(S_X)$, and (iv) the symmetry between $-k_y$ and $k_y(S_Y)$. Here the full energy spectrum means a set of energy eigenvalues obtained by taking into account all the values of $\vec{k}$ in the FMBZ.

Energy dispersions for $q=2$ and 3 are given as follows:

(a) $q=2$: We have $R_x=2$ and the energy dispersion is given by

$$E(\vec{k})=\pm2\sqrt{\cos^2k_x+\cos^2k_y}. \tag{13}$$

There are two subbands and they touch at zero energy as shown in Fig. 1(a), resulting in a single subband structure with $|E|\leqslant2\sqrt{2}$.

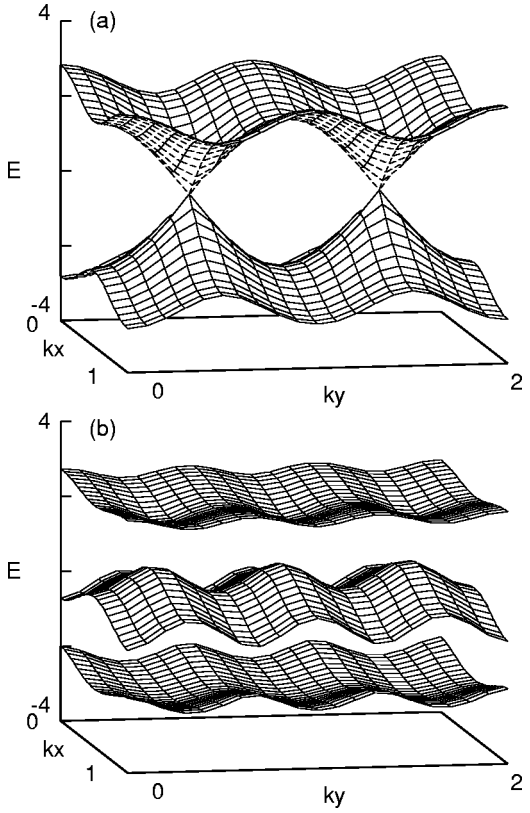

FIG. 1. Energy dispersion in the uniform magnetic field with (a) $q=2$ and (b) $q=3$. The horizontal plane is drawn in units of π.

(b) $q=3$: We have $R_x=3$ and the energy dispersion is given by the equation

$$E^3-6E-2(\cos 3k_x+\cos 3k_y)=0. \tag{14}$$

There is no touching point in the dispersion and the energy spectrum exhibits a three subband structure with $-(1+\sqrt{3})\leqslant E\leqslant -2$, $|E|\leqslant(\sqrt{3}-1)$, and $2\leqslant E\leqslant(1+\sqrt{3})$ as shown in Fig. 1(b).

B. Energy spectrum in 1D modulated magnetic fields

Assuming the field modulation is along the x direction, θ_n is still zero and the tight-binding equation is formally the same as Eq. (12). However, β and γ are introduced in θ_m, and ϕ becomes periodic along the x direction with the period γ:

$$\phi/\phi_0=\begin{cases}1+(\beta\gamma/2\pi)[\cos(2\pi(m+1)/\gamma)-\cos(2\pi m/\gamma)], \\ \text{1DSM} \\ 1-(\beta\gamma/2\pi)[\sin(2\pi(m+1)/\gamma)-\sin(2\pi m/\gamma)], \\ \text{1DCM.}\end{cases} \tag{15}$$

The lattice is called the stripped flux lattice[23] and the energy spectrum can be obtained by diagonalizing the $(R_x\times R_x)$ Hamiltonian matrix.

1. 1DSM field

When $(q,\gamma)=(2,2)$, we have $R_x=2$ and the energy dispersion is given by

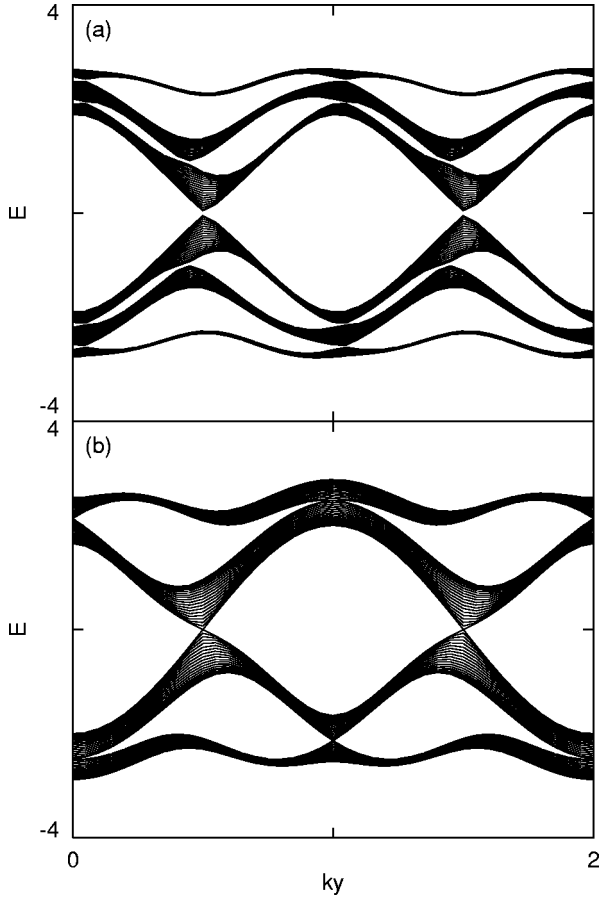

FIG. 2. Plot of E versus k_y in the 1DSM field with $\beta=0.3$, where (a) $(q,\gamma)=(2,3)$ and (b) $(q,\gamma)=(2,4)$. The horizontal axis is drawn in units of π.

$$E(\vec{k})=2\sin\beta\sin k_y\pm 2\sqrt{\cos^2 k_x+\cos^2\beta\cos^2 k_y}. \tag{16}$$

The two subbands touch at $(k_x,k_y)=(\pi/2,\pi/2)$ and $(\pi/2,3\pi/2)$, where $E=\pm 2\sin\beta$. Since the touching points exist irrespective of β, the energy spectrum exhibits a single subband structure as in the case of $\beta=0$. In order to demonstrate the γ dependence of the energy spectrum with $q=2$, we calculate energy spectra for other values of γ and plot some of them in Fig. 2. Even though there are six and four subbands for $(q,\gamma)=(2,3)$ and $(2,4)$, there occurs direct touching between the nearest-neighboring subbands, which leads to a single subband structure of the energy spectrum. In our calculations, we find that the energy band structure is independent of the values β and γ while the total bandwidth slightly changes with varying β.

Figure 3 shows the k_y dependence of the energy spectrum for $(q,\gamma)=(3,2)$, where it can be seen that β plays a key role in forming the band structure. For small values of β, the upper (lower) two subbands touch at some values of k_y, and there exists indirect overlapping (i.e., crossing of subbands at different values of k_y) between the central subbands. Thus, the energy spectrum exhibits a three subband structure [Fig. 3(a)]. However, for a large value of β, there occurs a gap opening between the subbands and the energy spectrum exhibits a six subband structure [Fig. 3(b)]. Further increase of β makes the second and the fourth gaps close, leading to a four subband structure [Fig. 3(c)]. Another example is given

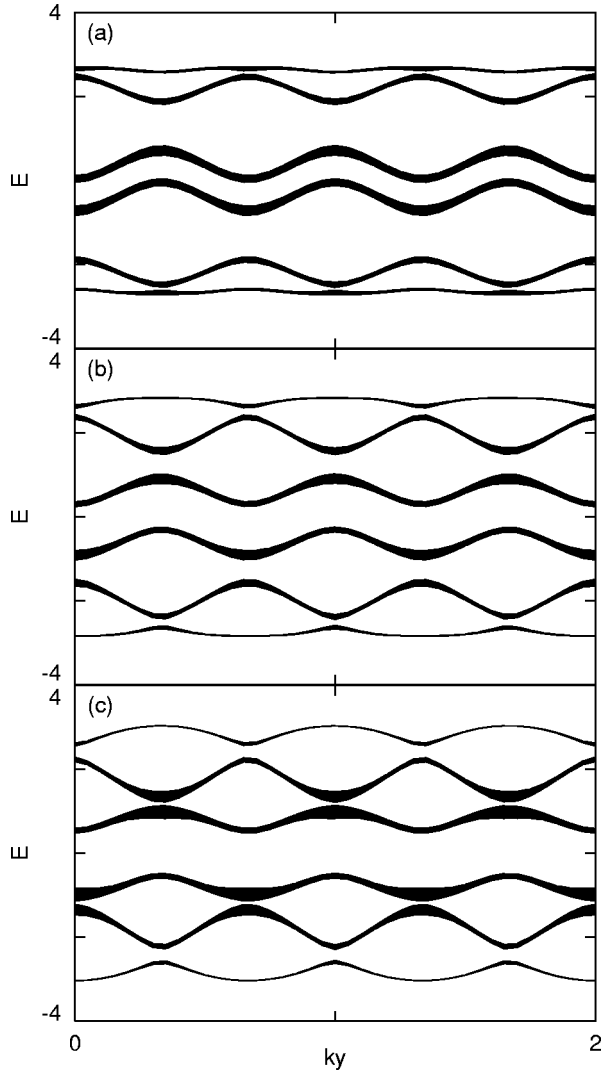

FIG. 3. Plot of E versus k_y in the 1DSM field with $(q,\gamma)=(3,2)$, where (a) $\beta=0.3$, (b) $\beta=0.6$, and (c) $\beta=0.9$.

in Fig. 4, where the k_y dependence of the energy spectrum for $(q,\gamma)=(3,3)$ is shown. For small values of β, the energy spectrum exhibits a three subband structure [Fig. 4(a)]. However, indirect overlapping between subbands occurs as β increases, and the energy spectrum exhibits a single subband structure [Fig. 4(b)]. The occurrence of a gap closing due to indirect overlapping is also found in the case of $(q,\gamma)=(3,4)$.

As for the symmetry of the energy spectrum, Figs. 2 and 3 show that S_E remains under the 1DSM field, which is contrary to the arguments of Ref. 24. The reason for this discrepancy lies in the range of $\vec{k}$ taken into account in discussing the symmetry of the energy spectrum. In Ref. 24, only a particular value of $\vec{k}$ was taken into account, while all the values of $\vec{k}$ in the FMBZ are taken into account in this paper. Here we would like to stress that all the values of $\vec{k}$ in the FMBZ should be considered in order to discuss the symmetry of the energy spectrum. In fact, S_E of the Hofstadter's spectrum holds only when all the values of $\vec{k}$ in the FMBZ are considered. Figures 2 and 3 also show that S_X remains under the 1DSM field while whether S_Y remains or not depends crucially on q and γ. Note that S_D breaks down by introducing 1D field modulation.

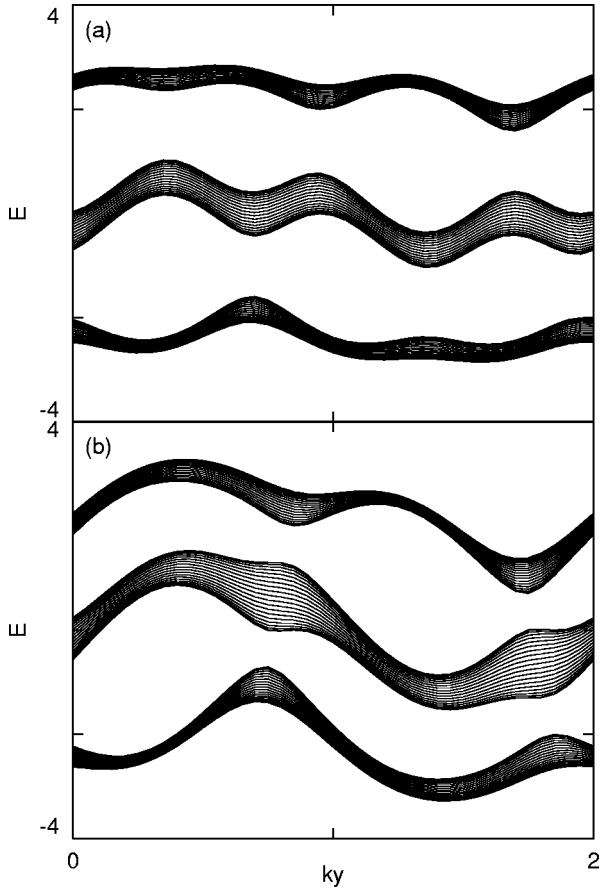

FIG. 4. Plot of E versus k_y in the 1DSM field with $(q,\gamma)=(3,3)$, where (a) $\beta=0.3$ and (b) $\beta=0.9$.

2. 1DCM field

Figure 5 shows the k_y dependence of the energy spectrum for $(q,\gamma)=(2,3)$. In this case, for small values of β, the two central subbands directly touch at two points of k_y and the upper (lower) two subbands indirectly overlap with each other, resulting in a single subband structure [Fig. 5(a)]. However, as β increases, indirect overlapping between the upper (lower) two subbands disappears and the energy spectrum exhibits a three subband structure [Fig. 5(b)]. Further increase of β makes the remaining indirect overlapping disappear, resulting in a five subband structure [Fig. 5(c)]. The β dependence of the energy spectrum for $(q,\gamma)=(2,4)$ is found to be quite different from the case of $(q,\gamma)=(2,3)$. The four subbands directly touch their neighboring subbands, resulting in a single subband structure regardless of β. The only effect of β is to change the energy bandwidth. As for $(q,\gamma)=(3,3)$ and $(3,4)$, similar phenomena to the case of $q=3$ under the 1DSM field (i.e., occurrence of a gap closing due to indirect overlapping) is also found; the energy spectrum exhibits a three (single) subband structure for small (large) values of β. In Table I, we summarize the number of subbands of the energy spectra for the parameters (q,γ,β) we took into account. As for the symmetry of the energy spectrum, we find that S_E, S_X, S_Y remain, while S_D breaks

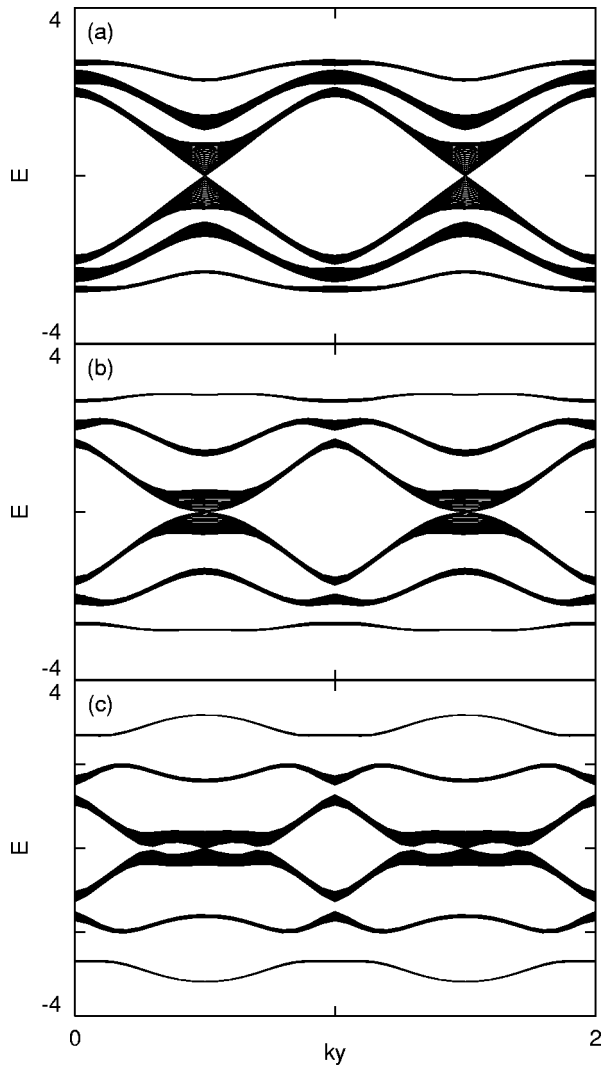

FIG. 5. Plot of E versus k_y in the 1DCM field with $(q,\gamma)=(2,3)$, where (a) $\beta=0.3$, (b) $\beta=0.6$, and (c) $\beta=0.9$.

down under the 1DCM field. Note that S_Y depends on the type of field modulation; it remains (breaks) in the 1DCM (1DSM) field.

Before concluding this subsection, we would like to make two comments. First, we assume γ to be an integer for the sake of simplicity even though it can be an arbitrary real value. Also, we consider only the values of $\gamma \geqslant 3(2)$ in the case of the CM (SM) field because ϕ becomes nonuniform only under these conditions. However, the authors of Ref. 24 violated this condition. They chose $\gamma=1$ in their calculations. But, when $\gamma=1$, ϕ becomes uniform, as can be easily checked by Eq. (15), and the full energy spectrum should be identical to the Hofstadter's spectrum, all of which implies that the results presented in Ref. 24 are erroneous, arising from mistakes in choosing the values of γ and in choosing the values of $\vec{k}$. Second, the tight-binding model we are considering is basically a one-band model. Thus, we focus our attention on energy spectra only for the values of β that are not large since there might be interband mixing between different Landau levels for large values of β.

TABLE I. Number of subbands of the energy spectrum under the 1D modulated fields.

kind		1DSM						1DCM			
q		2			3			2		3	
γ		2	3	4	2	3	4	3	4	3	4
β	0.3	1	1	1	3	3	3	1	1	3	3
	0.6	1	1	1	6	1	1	3	1	1	1
	0.9	1	1	1	4	1	1	5	1	1	1

C. Energy spectrum in 2D modulated magnetic fields

When the field modulation is along both lateral directions, θ_n becomes nonzero and ϕ becomes periodic in both lateral directions with the period γ. The lattice is called the checkerboard flux lattice[23] and the energy spectrum can be obtained by diagonalizing the $(R_xR_y\times R_xR_y)$ Hamiltonian matrix.

1. 2DSM field

When $(q,\gamma)=(2,2)$, the Hamiltonian matrix becomes

$$\begin{pmatrix} 0 & a & b & 0 \\ a^* & 0 & 0 & c \\ b^* & 0 & 0 & d \\ 0 & c^* & d^* & 0 \end{pmatrix}, \tag{17}$$

where $a=-e^{i\beta}-e^{-i(\beta+2k_y)}$, $b=e^{-i\beta}+e^{i(\beta-2k_x)}$, $c=e^{i\beta}+e^{-i(\beta+2k_x)}$, and $d=e^{-i\beta}+e^{i(\beta-2k_y)}$, respectively. By diagonalizing Eq. (17) we obtain the energy dispersion as

$$E(\vec{k})=\pm 2\left|\cos\beta\sqrt{\cos^2k_x+\cos^2k_y}\pm\sin\beta\sqrt{\sin^2k_x+\sin^2k_y}\right|, \tag{18}$$

which is plotted in Fig. 6. Numbering the four subbands in order of lowering energy, one can see that the first (second) and the third (fourth) subbands directly touch at the center of FMBZ regardless of β. There also exists band crossing between the second and the third subbands, which is absent in the case of the 1D field modulation. Equation (18) indicates that the energy spectrum exhibits a single subband structure regardless of β and the total bandwidth changes with varying β. When $(q,\gamma)=(2,3)$, due to direct touching and indirect overlapping between neighboring subbands, the energy spectrum exhibits a single subband structure regardless of β. Meanwhile, when $(q,\gamma)=(2,4)$, the energy band structure sensitively depends on β. For small values of β, due to direct touching between neighboring subbands, the energy spectrum exhibits a single subband structure. However, gap opening between subbands occurs with increasing β, and the energy spectrum exhibits a five subband structure.

When $(q,\gamma)=(3,2)$, even for small values of β, the energy spectrum is quite different from the case of $\beta=0$. The energy spectrum for a small value of β exhibits an eight subband structure [Fig. 7(a)]. And, as β increases, two more gaps are open to yield a ten subband structure [Fig. 7(b)]. Further increase of β makes the energy spectrum exhibit a twelve subband structure [Fig. 7(c)]. The β dependence of the energy spectrum for $(q,\gamma)=(3,3)$ is also different from the case of $(q,\gamma)=(3,2)$: For small values of β, the energy spectrum consists of three subbands as in the case of $\beta=0$.

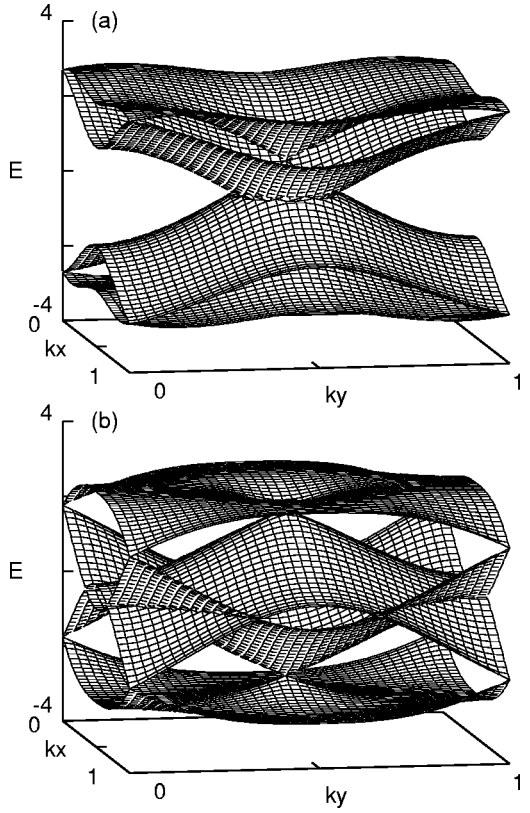

FIG. 6. Energy dispersion in the 2DSM field with $(q,\gamma)=(2,2)$, where (a) $\beta=0.3$ and (b) $\beta=0.9$.

However, gap closing occurs with increasing β and the energy spectrum exhibits a single subband structure. We also find similar gap closing behavior for $(q,\gamma)=(3,4)$.

2. 2DCM field

When $(q,\gamma)=(2,3)$, due to direct touching and indirect overlapping between the subbands, the energy spectrum for small values of β exhibits a three subband structure. However, gap opening occurs between the subbands with increasing β and the energy spectrum exhibits a five subband structure. Also, the widths of gaps broaden with the increase of β. Similar behavior is also found in the case of $(q,\gamma)=(2,4)$, where the energy spectrum exhibits a single (five) subband structure for small (large) values of β. Sensitive dependence of the band structure on β is also found in the case of $q=3$. For the case of $(q,\gamma)=(3,3)$, the energy spectrum is found to exhibit a three subband structure for a small value of β, a single subband structure for an intermediate value of β, and a seven subband structure for a large value of β. Figure 8 shows the β dependence of the energy spectrum for $(q,\gamma)=(3,4)$, where the concurrent occurrence of gap opening and gap closing can be seen. We summarize in Table II the number of subbands of the full energy spectra for the parameters (q,γ,β) taken into account.

The symmetry of the energy spectrum under the 2D modulated fields is more complicated than the cases of the 1D modulated fields. S_E remains for both the 2DSM and 2DCM fields. However, S_X and S_Y depend sensitively on the system parameters. For the parameters considered, we find that S_X and S_Y remain for the cases of $(q,\gamma)=(2,2),(2,4),(3,2)$ in the 2DSM fields, and $(q,\gamma)=(2,4),(3,3)$ in the 2DCM fields, while they break for the cases of $(q,\gamma)=(2,3),(3,4)$ in the 2DSM fields and $(q,\gamma)=(2,3),(3,4)$ in the 2DCM fields. Meanwhile, the energy spectrum in the 2DSM field with $(q,\gamma)=(3,3)$ has a flipped symmetry with respect to $k_{x(y)}=0$. Here the flipped symmetry means that the energy spectrum in the range of $0\leqslant k_{x(y)}\leqslant \pi/R_{x(y)}$ is the same as that in the range of $-\pi/R_{x(y)}\leqslant k_{x(y)}\leqslant 0$ when E is replaced by $-E$. S_D also depends sensitively on the system parameters; it holds (breaks) for the values of (q,γ) that make S_X and S_Y remain (break).

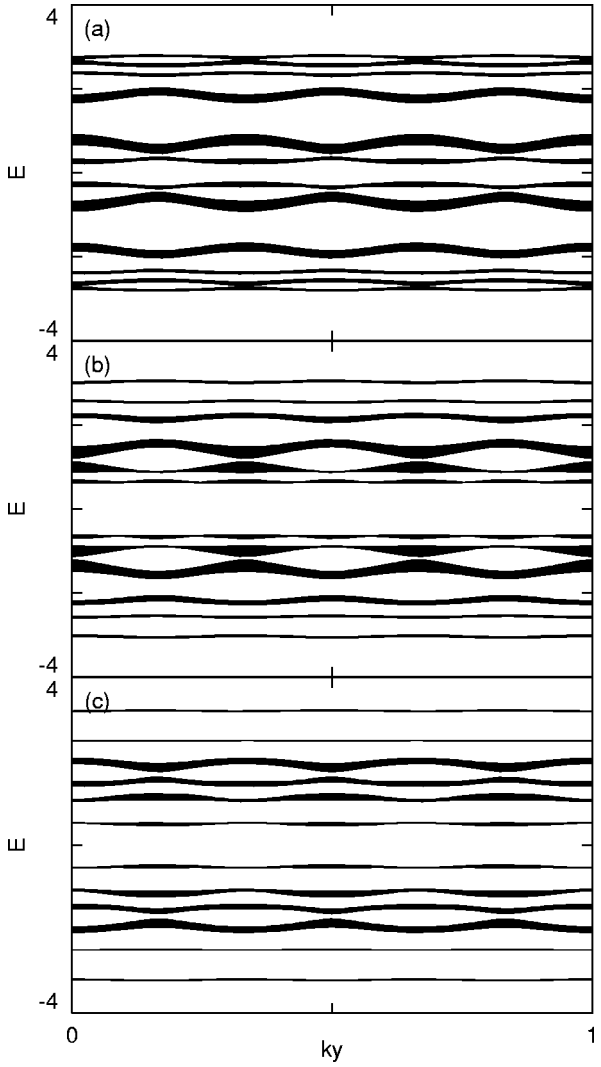

FIG. 7. Plot of E versus k_y in the 2DSM field with $(q,\gamma)=(3,2)$, where (a) $\beta=0.3$, (b) $\beta=0.6$, and (c) $\beta=0.9$.

Before concluding this subsection, we would like to make a few comments. The first is that the above results on $S_{E,X,Y,D}$ are not from a theoretical analysis but from a numerical study. Thus, further study such as the group theoretical analysis[23,26,27] is required in order to deepen the understanding of the symmetry breaking. The second is that the k_x and k_y directions taken into account in discussing $S_{X,Y,D}$ are not the high symmetry directions of the Hamiltonian under the 2D modulated fields. The reason for taking into account these directions, nevertheless, lies in testing how $S_{X,Y,D}$ appeared in the Hofstadter's spectrum is influenced by the field modulation. Since symmetry breaking is generally expected

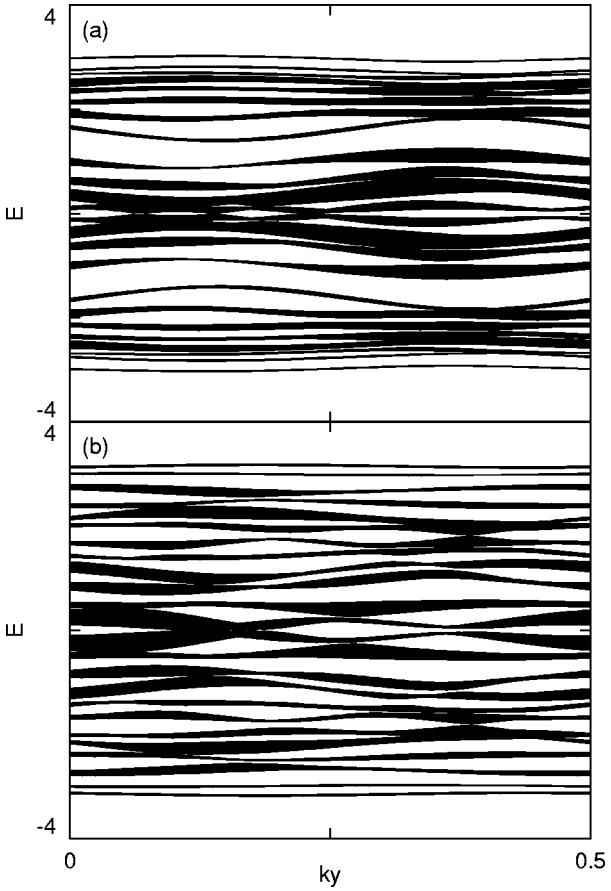

FIG. 8. Plot of E versus k_y in the 2DCM field with $(q,\gamma)=(3,4)$, where (a) $\beta=0.3$ and (b) $\beta=0.6$.

if the symmetry axis is not properly chosen, our result that the breaking of $S_{X,Y,D}$ depends on the system parameters is quite natural. Finally, it may be worthwhile to note the result of Ref. 26, where Shi and Szeto considered a Bloch electron in the magnetic field

$$\vec{B}=[B_0+(-1)^{m-n}B_1]\hat{z}, \tag{19}$$

and found that there is no symmetry breaking in the energy spectrum even though the energy spectrum becomes different from the Hofstadter's spectrum, which seems to be contrary to our results. However, since the directions used in Ref. 26 are the highly symmetric $(k_x \pm k_y)$ directions while the directions used in this paper are the k_x and k_y directions, it may not be easy to compare both results directly. The only thing we can say is that the field modulation given in this paper [Eqs. (1)–(3)] is more generalized than that of Ref. 26 [Eq. (19)]; specifying the periodic magnetic field by a magnetic unit cell, the lattice with Eq. (19) becomes the simple checkerboard lattice [see, for example, Fig. 2(a) of Ref. 23] while the lattice with Eqs. (1)–(3) becomes a generalized checkerboard lattice [see Fig. 2(b) of Ref. 23]. Thus we expect that the latter may present a more generic effect of the field modulation than the former. Further theoretical study on the comparison between the two cases is also required.

TABLE II. Number of subbands of the energy spectrum under the 2D modulated fields.

kind		2DSM						2DCM			
q		2			3			2		3	
γ		2	3	4	2	3	4	3	4	3	4
	0.3	1	1	1	8	3	7	3	1	3	3
β	0.6	1	1	5	10	3	7	5	5	1	7
	0.9	1	1	5	12	3	5	5	5	7	3

IV. SUMMARY

In summary, the effect of spatially modulated magnetic fields on the Hofstadter's spectrum was studied. The occurrence of gap opening, gap closing, band crossing, and band broadening was found to be the generic effect of the field modulation, which leads to an energy band structure that is distinct from the Hofstadter's spectrum. The energy band structure was found to be sensitively dependent on the system parameters, which implies that it may be difficult to detect the direct indication of the Hofstadter's spectrum in experiment since even a tiny change of the magnetic field leads to a very complicated energy band structure. The effect of the field modulation on the symmetries appearing in the Hofstadter's spectrum was also discussed in detail. In this work, we have paid attention to the energy spectral properties only with a rational flux and the isotropic hopping integral. Since an introduction of hopping anisotropy is known to lead to interesting phenomena like band broadening,[28] it may be interesting to study the effect of hopping anisotropy on the energy spectrum of a 2DBE under the modulated magnetic fields.

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