Coupled phonon-ripplon modes in a single wire of electrons on the liquid-helium surface

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The coupled phonon-ripplon modes of the quasi-one-dimensional electron chain on the liquid helium surface are studied. It is shown that the electron-ripplon coupling leads to the splitting of the collective modes of the wire with the appearance of low-frequency modes and high-frequency optical modes starting from threshold frequencies. The effective masses of an electron plus the associated dimple for low-frequency modes are estimated and the values of the threshold frequencies are calculated. The results obtained can be used in experimental attempts to observe the phase transition of the electron wire into a quasiordered phase. [S0163-1829(99)02144-X]

One of the most distinctive features of the quasi-two-dimensional (Q2D) classical electron system localized on liquid helium surface is that it exhibits liquid-solid-like phase transition at low-enough temperature T and high-electron densities n_s when the interaction coupling $\Gamma = e^2\sqrt{\pi n_s}/T \simeq 140$. The peculiarities of the transition leading to the formation of 2D Wigner solid (WS) are widely discussed both theoretically and experimentally for Q2D surface electrons.[1] Due to their interaction with liquid surface oscillations (ripplons), electrons with 2D wave vector $\mathbf{k}$ are coupled to ripplons with $\mathbf{q}=\mathbf{G}+\mathbf{k}$, where $\mathbf{G}$ belongs to the 2D reciprocal lattice, and crystallize in a triangular lattice whose elementary cell contains the electron and the self-induced surface deformation (dimple). As a consequence, the spectrum of collective modes of the WS on the helium surface differs significantly from the collective mode spectrum of an ideal 2DWS of electrons decoupled from ripplons[2] and splits into low-frequency and high-frequency modes.[1,3,4] Low-frequency longitudinal and transverse modes are acousticlike and are related to in-phase motion of the electron and associated dimple. High-frequency optical modes start from ω_α at $\mathbf{k}=0$ and correspond to the relative oscillations of electrons and dimples. The threshold values ω_α are determined by the strength of the electron-ripplon interaction and are significantly larger than typical frequencies of ripplons involved in electron-ripplon scattering processes. This allows us to average the Hamiltonian of the electron-ripplon interaction only over high-frequency modes. Note that the observation of resonances in the absorption spectrum at peculiar coupled phonon-ripplon mode frequencies leads, for the first time, to the evidence of the electron WS over the liquid helium surface.[5]

Recently there has been an increasing interest in investigating quasi-one-dimensional (Q1D) electron systems over liquid helium.[6–8] In such systems the electrons are confined not only in the direction normal to the liquid surface (where the 2D electron system takes place) but also by a lateral constriction, which is provided by geometric or electrostatic means, leading to the creation of a classical wire system (low-density regime) similar to quantum wires in semiconductor heterostructures. The confinement potential can be, within a good accuracy, approximated by a parabolic potential. Figure 1 illustrates an example of the realization of the single electron wire by using a bent foil that provides the helium surface with a curvature radius $R \sim 10^{-3}-10^{-4}$ cm.[9] If the holding electric field $E_\perp$ along the z axis is $\sim 10^3$ V/cm, the electron wave function for the motion across the channel becomes oscillatorylike with frequency $\omega_0^2 = eE_\perp/mR$. It is essential to emphasize that, despite the additional restriction in the electron motion, the electron-ripplon interaction can still be considered in the same manner as in 2D case.[10]

The possibility that a linear chain of electrons, with period a, may undergo a phase transition into an ordered state is not obvious as in 2D. It is well known that there is no true long order in a strictly 1D infinite system. However, one can expect long-enough regions of a quasiordered electron chain in these Q1D electrons for very low temperatures.[11] Based on this assumption, the spectrum of longitudinal and transverse phonons was obtained in the case of the decoupled electron chain from the solution of equations of motion in the harmonic approximation.[11,12] The dispersion laws for longitudinal (ω_l) and transverse (ω_t) branches are given by[12]

$$\omega_l^2 = \omega_e^2; \quad \text{and} \quad \omega_t^2 = \omega_0^2 - \frac{\omega_e^2}{2}, \tag{1}$$

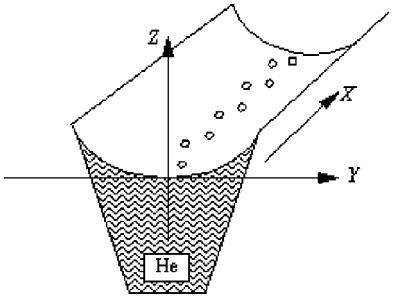

FIG. 1. Structure of the single classical electron wire over liquid helium.

where the frequency ω_e, for $|k_x a| \ll 1$, can be written as

$$\omega_e^2 = \frac{4e^2}{ma^3} \sum_{n \geqslant 1} \left(\frac{1-\cos nak_x}{n^3} \right) \simeq \frac{2e^2}{ma} k_x^2 \ln\left(\frac{1}{|k_x a|} \right).$$

Here k_x is the 1D wave vector along the wire. As it can be seen from Eq. (1), the transverse branch is stable only if the lateral confinement exists. As it was shown in Ref. 12, the stability holds for $E_\perp > 7\zeta(3)eR/2a^3$, where $\zeta(x)$ is the Riemann's zeta-function. For $a = 10^{-4}$ cm and $R = 5 \times 10^{-4}$ cm, one has $E_\perp > 300$ V/cm. The spectrum of longitudinal oscillations of the classical Q1D itinerant electron system was also studied by Sokolov and Studart[13] using the random-phase approximation. The results show qualitatively similar dispersion laws as in Eq. (1) when a in the argument of the logarithm is replaced by the scale of the electron localization across the wire. No electron-ripplon coupling was taken into accounted in Refs. 10–12, even though, as stated previously, such a coupling should influence, in a crucial way, the mode spectrum of the Q2DWS.[5]

In this Brief Report, we discuss the possibility of observation of the transition into a quasiordered state of the classical Q1D electron system over liquid helium by calculating the coupled phonon-ripplon modes. We show that the formation of an ordered state in the single electron wire should be accompanied by drastic changes in the mode spectrum in comparison with that of the decoupled WS wire [Eq. (1)]. New unusual branches appear which can be checked experimentally.

The Hamiltonian of the electron wire is given by

$$\hat{H} = \hat{H}_e + \hat{H}_r + \hat{H}_{er}, \tag{2}$$

where the N-electron Hamiltonian for small displacements from the equilibrium positions $\{x_l^{(0)};0;0\}$, $l = 1,2,\ldots,N$, can be written as

$$\hat{H}_e = \frac{m}{2} \sum_{l=1}^{N} (\dot{u}_{xl}^2 + \dot{u}_{yl}^2 + \omega_0^2 u_{yl}^2) + \frac{e^2}{2a^3} \sum_{l \neq j} \left[\frac{(u_{xl}-u_{xj})^2}{|x_l^{(0)}-x_j^{(0)}|^3} - \frac{(u_{yl}-u_{yj})^2}{2|x_l^{(0)}-x_j^{(0)}|^3} \right]. \tag{3}$$

Here u_{xl} and u_{yl} are the electron displacements along x and y directions, respectively. In terms of the Fourier-transformed phonon displacements ζ_{xk} and ζ_{yk}, the Hamiltonian can be expressed as

$$\hat{H}_e = \frac{m}{2} \sum_k \left[|\dot{\zeta}_{xk}|^2 + |\dot{\zeta}_{yk}|^2 + \omega_e^2 |\zeta_{xk}|^2 + \left(\omega_0^2 - \frac{\omega_e^2}{2} \right) |\zeta_{yk}|^2 \right]. \tag{4}$$

The Hamiltonian for free ripplons is given in the standard form

$$\hat{H}_r = \frac{1}{2} \sum_{\mathbf{q}} \left(\frac{\rho}{q} \right) (|\dot{\xi}_{\mathbf{q}}|^2 + \omega_q^2 |\xi_{\mathbf{q}}|^2), \tag{5}$$

where $\xi_{\mathbf{q}}$ is Fourier-transformed displacement of the liquid surface from the equilibrium position at $z = 0$, and $\omega_q^2 \simeq (\alpha/\rho)q^3 + \tilde{g}q$. The helium bulk density and the surface tension coefficient are denoted ρ and α, respectively, and $\tilde{g}$ is the gravity acceleration.

The Hamiltonian $\hat{H}_{er}$ describes the electron-ripplon interaction and can be written as

$$\hat{H}_{er} = \frac{1}{\sqrt{S}} \sum_{l=1}^{N} \sum_{\mathbf{q}} \xi_{\mathbf{q}} V_{\mathbf{q}} \exp(-q^2\langle u_f^2 \rangle/4) e^{iq_x x_l^{(0)}} e^{(q_x u_{xl} + q_y u_{yl})}, \tag{6}$$

where S is the surface area, $V_{\mathbf{q}}$ is the electron-ripplon interaction averaged over the ground state electron wave function along the z direction.[14] The Debye-Waller factor $\exp(-q^2\langle u_f^2 \rangle/4)$, which appears in Eq. (6), comes from taking into account the ''smearing'' of the electron positions due to high-frequency modes and corresponds to the contribution $\langle u_f^2 \rangle$ in the electron root-mean-square (RMS) displacements from the fast modes. So, u_{xl} and u_{yl} in Eq. (6) should be considered as the electron displacements at low enough velocities. Expanding the exponential function in Eq. (6) up to quadratic terms and making the Fourier transform of the electron displacements one can finally arrive to the following expression for the total Hamiltonian:

$$H = \sum_k \left\{ \frac{1}{2} \sum_{\mathbf{q}g} \left(\frac{\rho}{q} \right) (|\dot{\xi}_{\mathbf{q}}|^2 + \omega_q^2 |\xi_{\mathbf{q}}|^2) \delta_{q_x;g+k_x} + \frac{m}{2} \left[|\dot{\zeta}_{xk_x}|^2 + |\dot{\zeta}_{yk_x}|^2 + (\omega_e^2 + \omega_{dx}^2) |\zeta_{xk_x}|^2 + \left(\omega_0^2 + \omega_{dy}^2 - \frac{\omega_e^2}{2} \right) |\zeta_{yk_x}|^2 \right] + i\sqrt{n_s} \sum_{\mathbf{q}g} V_{\mathbf{q}} \times \exp(-q^2\langle u_f^2 \rangle/4)(q_x \zeta_{x,-k_x} + q_y \zeta_{y,-k_x}) \xi_{\mathbf{q}} \delta_{q_x;g+k_x} \right\}. \tag{7}$$

Here, $\delta_{\alpha\beta}$ is the Kronecker symbol, $g = (2\pi/a)n$ is the 1D reciprocal-chain vector with $n = 0, \pm 1, \pm 2, \ldots, \pm N/2$. As it is seen in Eq. (7), the phonon displacements $\zeta_{x(y)k_x}$ are coupled to ripplons with $q = \sqrt{(g+k_x)^2 + q_y^2}$. The frequencies ω_{dx} and ω_{dy} are

$$\omega_{dx}^2 = \sum_{gq_y} C_{gq_y} \omega_{gq_y}^2 \quad \text{and} \quad \omega_{dy}^2 = \sum_{gq_y} C_{gq_y} \left(\frac{q_y}{g} \right)^2 \omega_{gq_y}^2, \tag{8}$$

where the coefficients

$$C_{gq_y} = \frac{n_s V_{gq_y}^2 g^2 \exp\left[-\frac{(g^2+q_y^2)\langle u_f^2 \rangle}{4} \right]}{m[\alpha(g^2+q_y^2) + \rho\tilde{g}]\omega_{gq_y}^2}$$

are dimensionless parameters related to the strength of the electron-ripplon coupling. The double index gq_y in Eq. (8) means that g substitutes q_x in the quantities depending on $q = \sqrt{q_x^2 + q_y^2}$.

The dispersion laws of coupled phonon-ripplon oscillations can be obtained by solving the equations of motion for

the Hamiltonian given in Eq. (7). In the long wave limit, $k_x \ll \sqrt{g^2+q_y^2}$, the dispersion equations can be written as

$$Z_l\omega^2-\omega_e^2=0 \quad \text{and} \quad Z_t\omega^2-\omega_0^2+\frac{\omega_e^2}{2}=0, \tag{9}$$

where

$$Z_l=1+\sum_{gq_y} C_{gq_y}\frac{\omega_{gq_y}^2}{\omega_{gq_y}^2-\omega^2} \quad \text{and}$$

$$Z_t=1+\sum_{gq_y} C_{gq_y}\left(\frac{q_y}{g}\right)^2\frac{\omega_{gq_y}^2}{\omega_{gq_y}^2-\omega^2}.$$

One can easily see that the above equations have solutions that differ substantially from the phonon modes given by Eq. (1). We consider both low and high-frequency regions of the spectrum. In the limit of $\omega^2 \ll \omega_{gq_y}^2$, the phononlike solutions of Eq. (9) can be given by

$$\tilde{\omega}_l \simeq \left(\frac{m}{M_l}\right)^{1/2}\omega_l \quad \text{and} \quad \omega_t \simeq \left(\frac{m}{M_t}\right)^{1/2}\omega_t, \tag{10a}$$

which are formally equivalent to the modes expressed in Eq. (1), but depend on the effective masses of the electron plus dimple instead on the free electron mass as it follows:

$$M_l=m\left[1+\sum_{gq_y} C_{gq_y}\right] \quad \text{and} \quad M_t=m\left[1+\sum_{gq_y} C_{gq_y}\left(\frac{q_y}{g}\right)^2\right]. \tag{10b}$$

Note that the longitudinal mode is gapless and the spectrum is similar to that of low-frequency modes in the coupled 2DWS over liquid helium at same q. However, in contrast to the 2D case, the dispersion of longitudinal and transverse modes shows different effective masses. For large holding fields, where $V_{gq_y} \simeq eE_\perp$, and summing over q_y, analytical expressions for the effective masses are obtained as

$$M_l \simeq m\left[1+\frac{n_l e^2 E_\perp^2 \rho \langle u_f^2\rangle^2}{4\sqrt{\pi}\alpha^2 m}\sum_{g>0} g^2 \times \exp(-g^2\langle u_f^2\rangle/2)\Psi\left(\frac{5}{2};3;\frac{g^2\langle u_f^2\rangle}{2}\right)\right], \tag{11a}$$

$$M_t \simeq m\left\{1+\frac{n_l e^2 E_\perp^2 \rho \langle u_f^2\rangle}{4\sqrt{\pi}\alpha^2 m}\left[\frac{1}{\sqrt{\pi}}\Psi\left(2;2;\frac{\kappa_c^2\langle u_f^2\rangle}{2}\right) + \sum_{g>0}\exp(-g^2\langle u_f^2\rangle/2)\Psi\left(\frac{5}{2};2;\frac{g^2\langle u_f^2\rangle}{2}\right)\right]\right\}. \tag{11b}$$

Here, $k_c=(\rho\tilde{g}/\alpha)^{1/2}$ is the helium capillary constant, $n_l=N/L_x$ is the linear electron density along the wire with L_x the system length, and $\Psi(a;b;x)$ is the degenerated hypergeometric Tricomi's function. Equations (11) were obtained under the condition $k_c \ll g_1$.

In order to estimate the magnitudes of the effective masses we need characteristic values of a, $E_\perp$, and $\langle u_f^2\rangle$. The relationship between n_l and $E_\perp$ is much more complicated than its 2D counterpart, and we consider here $a \sim n_l^{-1} \sim 1/\sqrt{n_s}=10^{-4}$ cm and the high-field limit of $E_\perp = 3000$ V/cm, because we neglected the contribution of the polarization interaction between the electron and the liquid helium in Eqs. (11). The RMS contribution from the fast modes $\langle u_f^2\rangle$ can be treated as a fitting parameter as in Ref. 3 or can be calculated rigorously in a self-consistent procedure.[4] Unfortunately there is no experimental data up to now and a self-consistent calculation of $\langle u_f^2\rangle$ in the Q1D case is cumbersome. On the grounds of the results of Ref. 4, we choose $\langle u_f^2\rangle=\hbar/(2m\omega_{dx})\coth(\hbar\omega_{dx}/2T)$ and considering the regime where $2T \ll \hbar\omega_{dx}$, we find best self-consistency by taking $\sqrt{\langle u_f^2\rangle} \simeq 10^{-1}a \ll a$. Furthermore, this value also satisfies well the condition $\sqrt{\langle u_f^2\rangle} \ll R=5\times10^{-4}$ cm.

For realistic values of $a=10^{-4}$ cm and $E_\perp = 3000$ V/cm, $M_l=2.3\times10^4\ m$, which is near two orders of magnitude larger than the effective mass in the low-frequency mode in the 2DWS for $n_s \gtrsim 10^8$.[1] M_t is significantly larger than M_l due to the large contribution of $g=0$. Note the κ_c-dependent argument of Tricomi's function in the first term in square brackets of Eq. (10b). This contribution is formally related with transverse oscillations as the whole electron-dimple chain when $k_x \to 0$ and the interparticle distance becomes very large ($g \to 0$). Such uniform oscillations yield large M_t, and $M_t \to \infty$ for $k_c=0$. However, $k_c \neq 0$ leads to very large, even though finite, value of M_t when the condition $k_c^{-1} \gg a$ plays the role of an effective cutoff in the divergent contribution of the $g=0$ term in Eq. (11b). On the other hand, all electron positions are equivalent under longitudinal displacements of the whole electron chain and, because of this, the term with $g=0$ does not contribute to the longitudinal oscillations of the chain at very small k_x. So, M_l is significantly smaller than M_t. For the same values of a and $E_\perp$, we obtain $M_t \simeq 1.1\times10^{11}\ m$ such that the threshold frequency $\tilde{\omega}_0=(m/M_t)^{1/2}\omega_0$ of the transverse mode $\tilde{\omega}_t$ is strongly softened and decreases more than 5 orders of magnitude in comparison with $\omega_0 \simeq 10^{11}\ \text{s}^{-1}$ for $E_\perp = 3000$ V/cm and $R=5\times10^{-4}$ cm.[15] From these estimates, one can conclude that the electron-ripplon coupling in the WS wire is stronger than in the 2DWS.

In the limit of high frequencies, $\omega^2 \gg \omega_{gq_y}^2$, the solutions of Eq. (9) are two optical branches

$$\omega_{tr(l)}^2 \simeq \omega_{dx}^2+\omega_e^2 \quad \text{and} \quad \omega_{tr(t)}^2 \simeq \omega_{dy}^2+\omega_0^2-\frac{\omega_e^2}{2}, \tag{12}$$

where the frequencies ω_{dx} and ω_{dy}, given by Eq. (8), can be expressed, in the strong-field limit of $V_{gq_y} \simeq eE_\perp$, as

$$\omega_{dx}^2 \simeq \frac{n_l e^2 E_\perp^2}{\alpha m}\sum_{g>0} g[1-\text{erf}(\sqrt{g^2\langle u_f^2\rangle/2})], \tag{13a}$$

$$\omega_{dy}^2 \simeq \frac{n_l e^2 E_\perp^2}{\alpha m\sqrt{2\pi\langle u_f^2\rangle}}\sum_g \exp(-g^2\langle u_f^2\rangle/2)-\omega_{dx}^2. \tag{13b}$$

We remind that in the Q2DWS both optical branches start from the same threshold frequency ω_α at $q=0$. For $a=10^{-4}$ cm and $E_\perp=3000$ V/cm, the threshold frequencies

of the Q1D electron chain are $\omega_{dx} \simeq 7.1\times10^{9}\ \mathrm{s}^{-1}$ and $\omega_{dy} \simeq 7.5\times10^{9}\ \mathrm{s}^{-1}$. We observe that ω_{dy} is significantly smaller than ω_0 for the same $E_\perp$ and the threshold frequency of $\omega_{tr(t)}$ in Eq. (12) practically coincides with ω_0. So, this mode branch is almost unaffected by the electron-ripplon coupling [see Eq. (1)].

Besides phononlike modes given by Eqs. (11) and (12), Eq. (9) also has solutions which represent optical modes whose dispersion laws starting from frequencies which are only rather lower than ω_{gq_y} at $k_x=0$. By increasing k_x, the frequencies of these modes practically reach ω_{gq_y}. So these modes should be considered as quasi-ripplonic oscillations which do not contribute to the electron dynamics and, in particular, to the electron RMS displacement.

One should emphasize that the values of g and q_y that mainly contribute to the results shown above satisfy the condition g and $q_y \lesssim (\langle u_f^2 \rangle)^{-1/2}$ due to the exponential cutoff entering into the sums in Eqs. (8)–(11). We have also shown that $L_y < R$.[15] For $L_y = 10^{-4}$ cm, $(q_y)_{\min} = 2\pi/L_y > 10^{4}\ \mathrm{cm}^{-1}$ and taking into account that $(g)_{\min}$ is of the same order of magnitude, we obtain, for $(\langle u_f^2 \rangle)^{1/2} \lesssim 10^{-5}$ cm and for $a = 10^{-4}$ cm, the values $\omega_{gq_y} \sim 10^{7} - 10^{8}\ \mathrm{s}^{-1}$ for the frequencies of ripplons participating in the electron-ripplon coupling, which are significantly smaller than the values of the threshold frequencies. Furthermore, the low-frequency transverse mode $\tilde{\omega}_t$ given by the second of Eqs. (10a), reaches a value much lower than $10^{8}\ \mathrm{s}^{-1}$. The maximum value of the longitudinal mode $\tilde{\omega}_l$ can be estimated by extrapolating Eq. (10a) for $k_x = g_1 = 2\pi/a$ with the value of ω_e given by Eq. (1). The result is $3.06\times10^{8}\ \mathrm{s}^{-1}$ and becomes smaller for $k_x \to 0$, where Eqs. (10) are valid. These results justify the *a priori* assumption for taking the asymptotic limits $\omega^2 \ll \omega_{gq_y}^2$ and $\omega^2 \gg \omega_{gq_y}^2$, in the long wavelength limit, for the calculation of the dispersion laws of Eqs. (10a) and (12). However one should note that using $\omega^2 \ll \omega_{gq_y}^2$ for calculating mode spectrum $\tilde{\omega}_l$ may become questionable with increasing k_x because in this case the mode frequency can be of the same order of magnitude of ω_{gq_y}. In such a condition, $\tilde{\omega}_l$ should be calculated from a numerical solution of the first of Eqs. (9). One can also point out that the splitting of low- and high-frequency modes $\omega_{tr(l)}$ and $\omega_{tr(t)}$ is rather well-pronounced supporting the introduction of a Debye-Waller factor to take into account the smearing of electron positions from fast modes.

In conclusion, we have calculated the coupled electron-phonon mode spectrum of the single quasicrystalline wire on the liquid helium surface. We have shown interesting features of the spectrum, which makes it particularly different from its 2D counterpart. Because the strong coupling of the phonons of the WS wire with the oscillations of the helium surface, the spectrum of collective modes changes drastically in comparison with the decoupled wire electron system. The low-frequency (high-frequency) modes have frequencies significantly smaller (larger) than characteristic frequencies of ripplons, which contribute to the electron-ripplon scattering. Low-frequency modes have same dispersion as that of modes of the decoupled electron wire but with large effective masses different for longitudinal and transverse oscillations. The values of these effective masses are much larger than the effective mass in the Q2DWS that means that the electron-ripplon coupling in Q1D case should be stronger. High-frequency longitudinal and transverse modes start from different threshold frequencies. The value of the threshold frequency for the transverse mode is slightly affected by the electron-ripplon coupling and is close to that of decoupled electron wire. In our opinion, experimental observation of the predicted modes should give strong evidence of the phase transition of the classical electron wire to a quasiordered state.

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