Stochastic resonance in the Gunn effect

Yuo-Hsien Shiau*
Institute of Physics, Academia Sinica, Taipei 11529, Taiwan, Republic of China
(Received 3 March 1999; revised manuscript received 16 June 1999)

We present a theoretical analysis of stochastic resonance phenomena in *n*-GaAs microwave diodes. When stimulated by external laser beams under a nonlinear feedback control, two stable periodic domain trains can coexist at the same values of the control parameters. If this bistable state is subjected to both modulation and noise, we show that the signal-to-noise ratio will exhibit a maximum at finite noise amplitudes. This analysis may be the first to consider stochastic resonance phenomena in microwave systems.
[S0163-1829(99)04747-5]

I. INTRODUCTION

The mechanism of the Gunn effect in *n*-GaAs has been well known for decades. The main feature is that the oscillating current is associated with the formation of a moving high-field domain.[1,2] This phenomenon is observable at room temperature, implicating great potentials in technological applications, especially in microwave generator. The pioneering works by Segev *et al.*[3] and Subačius *et al.*[4] showed that optical waves can excite multiple Gunn-domain formation in deep-impurity-doped GaAs and semi-insulating GaAs, respectively. The refractive index of GaAs changes via second-order optical nonlinearity, i.e., Pockel's effect, due to the high-field Gunn domain. Therefore, they predicted a new optically nonlinear effect in semiconductors. In our previous paper,[5] we consider the moving multiple Gunn-domain (i.e., we called periodic domain train PDT) generated by the external optical waves in *n*-GaAs with a feedback control. It is interesting to find that different PDT states coexist at same values of control parameters. Especially, the persistent bistability (PB) with nonhysteretic characteristics is observed.

The stochastic resonance (SR) was originally investigated in order to explain the periodic variation of global climate on the earth. SR has since been investigated in different fields including physics, biology, and chemistry.[6] The phenomenon of SR is an enhancement of the response of a nonlinear system which yields to an external weak modulation at finite noise intensities. Therefore, noise can be useful for amplification of weak signals (i.e., modulation). Moreover, noise can be used to control (or select) output responses from the dynamical systems having bistable states. For example, a ring laser[7] can generate modes propagating on a closed orbit either clockwise or anticlockwise. Due to the combination of quasi-white noise and sinusoidal voltage, i.e., modulator, it is possible to switch between the two modes. In this paper we apply modulation and noise into the *previous* feedback loop[5] to investigate the SR in PDT with PB property. From the plot of signal-to-noise ratio (SNR) vs noise intensity D, it shows the maximum value of SNR at the finite D value. It means that the periodic transition between these two PDT states can be enhanced by the increase of noise intensity. Thus, the output microwave emitted from *n*-GaAs can be synchronized with the input modulation.

In Sec. II, we give the nonlinear analysis for the PDT stimulated by the external laser beams. The PDT shows the shock-wave structure which can be derived by the nonlinear Schrödinger equation. In Sec. III, we discuss the mechanisms of the *previous* feedback loop to create the bistable PDT states and PB. In Sec. IV, we input modulation and noise into the *previous* feedback loop to investigate the SR in PDT with PB property. Conclusions are given in Sec. V.

II. SMALL AMPLITUDE ANALYSIS OF PDT

The multiple high-field Gunn domains are triggered by the two optical waves, which are incident on a biased *n*-GaAs with length L. The photon energy is just above the band gap of GaAs, i.e., 1.42 eV, so that electron-hole pairs can be generated by the optical excitation. The intensity $I(x,t)$ of the mixing waves can be described as the moving interference pattern through *n*-GaAs

$$I(x,t)=I_0[1+\overline{m}\cos(Kx+\Omega t)], \tag{1}$$

where Ω is the frequency difference of the two optical waves; $K=2\pi/\Lambda$ is the interference wave number; Λ is the grating period and is much smaller than L; $\overline{m}$ is the modulation depth of the interference grating; and I_0 is the average intensity. The generation-recombination processes include complete thermal ionization of the donor, generation of electron-hole pairs by the optical waves $I(x,t)$, and recombination of electron-hole pairs γ. Therefore, the dynamical equations include the Gauss law, the continuity equations for the electrons and holes, respectively, and the circuit equation

$$\frac{\partial E}{\partial x}=\frac{e}{\epsilon}(n-N_D^*-p), \tag{2}$$

$$\frac{\partial n}{\partial t}=gI(x,t)-\gamma np-\frac{\partial}{\partial x}\left[nv(E)-D_n\frac{\partial n}{\partial x}\right], \tag{3}$$

$$\frac{\partial p}{\partial t}=gI(x,t)-\gamma np+\frac{\partial}{\partial x}\left[p\mu_pE+D_p\frac{\partial p}{\partial x}\right], \tag{4}$$

$$V=\int_0^L E(x,t)dx, \tag{5}$$

where e, ϵ, n, p, N_D^*, L, V, and E are, respectively, elementary charge, static dielectric constant, the free electron density, the free hole density, the effective donor concentration, the sample length, the applied bias, and the electric field. The defenition of N_D^* is $N_D - N_A$, where N_D is the donor concentration and N_A is the acceptor concentration. The symbol g is the generation rate of the electron-hole pairs and γ is the recombination rate of the electron-hole pairs. D_n and D_p denote, respectively, the diffusion coefficient of electrons and holes; μ_p and $v(E)$ are the hole mobility and the electron drift velocity, respectively. Due to the transfer of energetic electrons from high mobility states near the conduction-band edge to low mobility states higher in the band structure, $v(E)$ displays an N-shaped negative differential mobility (NDM).

Before we give the detailed nonlinear analysis of PDT stimulated by the external laser beams, it is worth to discuss the dynamical characteristics in this system. It is intuitive to understand there are two spatiotemporal behaviors in this case. One is the moving interference pattern (MIP) in Eq. (1) to induce the space-charge field (SCF), which has the same phase velocity Ω/K as MIP. The characteristic length of SCF shall be equal to Λ. The other is the traveling high-field domains, i.e., Gunn oscillation, when the applied bias is located in NDM regime. The velocity of high-field domains is $v(E_0)$ $(\equiv v_0)$, where $E_0 = V/L$ and the characteristic length is L. When I_0 is very small, most of the free electrons come from shallow donor impurities. It is not difficult to show that the competitive results between these two spatiotemporal behaviors will create a periodic domain train with a spatial period Λ and the traveling velocity equal to v_0. This mixed behavior from SCF and Gunn domains is obtained via linear stability analysis.[5] From this analysis, we know that the Kroemer's criterion for domain formation in this case shall be written as $-((e/\epsilon)N_D^* v_0^{(1)} + 4\pi^2 D_n/\Lambda^2) > 0$, where $v_0^{(1)} = dv(E)/dE|_{E=E_0}$. In the following, we want to investigate the shock structure of PDT via nonlinear analysis.

When I_0 is very small, we can rearrange the model equations by the following rescaling process for the dynamical variables and parameter in Eqs. (2)–(5).

$$E \rightarrow E, \quad n \rightarrow n, \quad p \rightarrow \overline{\epsilon} p, \quad I \rightarrow \overline{\epsilon} I,$$

where $\overline{\epsilon}$ is a very small value equal to n_1/N_D^*, and $n_1 = gI_0/N_D^*\gamma$ is the free electron density generated by optical intensity, which is much smaller than N_D^*. The small parameter $\overline{\epsilon}$ can distinguish the large variables (i.e., E and n) from small variable p in our theoretical model. When the rescaling variables are introduced into Eqs. (2)–(5), Eqs. (2)–(3) shall be changed to

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon}(n - N_D^*) + O(\overline{\epsilon}) \tag{6}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x}\left[n v(E) - D_n \frac{\partial n}{\partial x} \right] + O(\overline{\epsilon}) \tag{7}$$

combining of Eqs. (6) and (7) and neglecting the small order term $O(\overline{\epsilon})$, we obtain the following equation denoted as the principal equation (PE).

$$\frac{\partial E}{\partial t} = -\frac{1}{\epsilon} e N_D^* v(E) - v(E)\frac{\partial E}{\partial x} + D_n \frac{\partial^2 E}{\partial x^2} + \frac{1}{\epsilon} J_{tot}(t), \tag{8}$$

where $J_{tot}(t)$ is time-dependent current density. If we put Eq. (6) into the Eq. (4) and neglect the small order term $O(\overline{\epsilon^2})$, we get the following slavish equation (SE).

$$\frac{\partial p}{\partial t} = gI(x,t) - \gamma\left(N_D^* + \frac{\epsilon}{e}\frac{\partial E}{\partial x} \right) p + \frac{\partial}{\partial x}\left(p\mu_p E + D_p \frac{\partial p}{\partial x} \right) \tag{9}$$

The physical meanings of PE and SE are quite clear and simple. For PE, it represents the Gunn-domain formation, and the solution of SE is completely determined by the Gunn domains and moving interference pattern.

In order to get the analytic solution in Eq. (8), an ansatz of the electric field, we set

$$E(x,t) = E_0 + \frac{1}{2}[E_s(x,t)e^{i\overline{K}(x - v_0 t)} + E_s^*(x,t)e^{-i\overline{K}(x - v_0 t)}] \tag{10}$$

$E_s(x,t)$ is a new variable which is corresponding to the amplitude of electric-field domains, and $E_s^*(x,t)$ is the complex conjugate of $E_s(x,t)$. $\overline{K}$ $(=2\pi/L)$ is a bulk property of semiconductor. Because of the small $\overline{K}$ value, the spatiotemporal behavior in the dynamical system will be determined by $E_s(x,t)$. When $E_0 \gg |E_s|$, it is reasonable to expand the drift velocity in the neighborhood of E_0 (i.e., small amplitude analysis)

$$v(E) \approx v_0 + v_0^{(1)}(E - E_0) + \frac{v_0^{(2)}}{2}(E - E_0)^2 + \frac{v_0^{(3)}}{6}(E - E_0)^3, \tag{11}$$

where $v_0^{(j)} = d^j v(E)/dE^j|_{E=E_0}$. Substituting Eqs. (10) and (11) into Eq. (8), we find the amplitude equation with the plane-wave factor $\exp i\overline{K}(x - v_0 t)$ is

$$\begin{aligned}\frac{\partial E_s}{\partial t} = & \left(-\frac{e}{\epsilon}N_D^* v_0^{(1)} - D_n \overline{K}^2 \right) E_s + (i2D_n\overline{K} - v_0)\frac{\partial E_s}{\partial x} \\ & - \frac{1}{8}v_0^{(2)}\left(E_s^2 \frac{\partial E_s^*}{\partial x} + 2|E_s|^2 \frac{\partial E_s}{\partial x} \right) + D_n \frac{\partial^2 E_s}{\partial x^2} \\ & - \left(\frac{1}{8}\frac{e}{\epsilon}N_D^* v_0^{(3)} + i\frac{1}{8}v_0^{(2)}\overline{K} \right)|E_s|^2 E_s .\end{aligned} \tag{12}$$

Eq. (12) is a type of Ginzburg-Landau equations. When we consider the value of $v_0^{(2)}$ at the operating point equal to 0 and change the complex variable E_s and x coordinate to the real variable Ψ and the moving coordinate ξ, respectively, we can reduce Eq. (12) to the standard nonlinear Schrödinger equation

$$\frac{d^2\Psi}{d\xi^2} - \frac{eN_D^* v_0^{(1)}}{\epsilon D_n K^2}\Psi - \frac{1}{8}\frac{eE_0^2 N_D^* v_0^{(3)}}{\epsilon D_n K^2}\Psi^3 = 0, \tag{13}$$

where $E_s = \Psi E_0 e^{i\theta}$, $\xi = K(x - v_0 t)$, and θ is a constant. Multiplying Eq. (13) by $d\Psi$ and integrating, we obtain

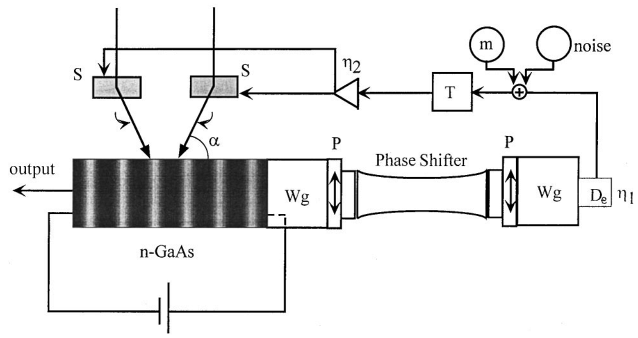

FIG. 1. Schematic illustration of the proposed experimental apparatus to study SR. Please note that the *previous* feedback loop, i.e., without *m* and *noise* in this diagram, can create bistable states.

$$\left(\frac{d\Psi}{d\xi}\right)^2=\Theta+\frac{eN_D^* v_0^{(1)}}{\epsilon D_n K^2}\Psi^2+\frac{1}{16}\frac{eE_0^2 N_D^* v_0^{(3)}}{\epsilon D_n K^2}\Psi^4, \quad (14)$$

where Θ is an integration constant. If there are nonlinear periodic solutions in Eq. (14), then the phase trajectory (Ψ, $d\Psi/d\xi$) of Eq. (14) shall be a closed loop. Therefore, it is quite obvious to see that the only possibility to produce nonlinear periodic solutions is to operate in the NDM regime. When $v_0^{(1)}<0$ and $v_0^{(3)}>0$, the PDT shows the shock-wave structure. We shall notice that there are two properties in PDT. One is the distance between the adjacent domains equal to Λ and the other is the traveling velocity of PDT equal to v_0. Then, the oscillating frequency of the current in the circuit is v_0/Λ and the wavelength λ of the microwaves is equal to $\frac{1}{2}c\lambda_l v_0^{-1}\cos^{-1}\alpha$, where c is the speed of light, λ_l is the wavelength of the laser beams, and α is the incident angle of laser beams. This result is a mixed behavior from semiconductor and optical waves. It shall note that a complete analysis for multiple Gunn-domain formation shall consider boundary condition which was addressed numerically by Luis L. Bonilla *et al.* recently.[8] They think that Gunn oscillation is due to periodic shedding and motion of charge dipole waves at a boundary or a nucleation site. Then, Kroemer's criterion (no relation with boundary condition) may not give quantitatively correct number of Gunn domains in sample. We think it is an interesting and important issue for further analytical study. In the following section, we will use above-mentioned analytical characteristic, i.e., mixed result from semiconductor and optical waves, to propose some feedback loop controlling optical waves, which can create bistable PDT at the same values of the control parameters.

III. BISTABLE PDT STATES AND PB

Figure 1 shows the proposed experimental apparatus. The feedback loop consists of a pair of waveguides Wg, a phase shifter whose fast and slow axes are at 45° to a pair of crossed polarizers P, a detector D_e, i.e., planar-doped barrier diode, with a time response $\tau\sim 10$ ns, a delay line with retardation time $T\sim m$s, modulation current m, stochastic current *noise*, and a pair of acusto-optic scanners S to tune the incident angle α of the laser beams. For the convenience of explaination about the physical mechanisms of Fig. 1 in the following, we firstly neglect the external current sources, i.e., m and *noise*, in this section.

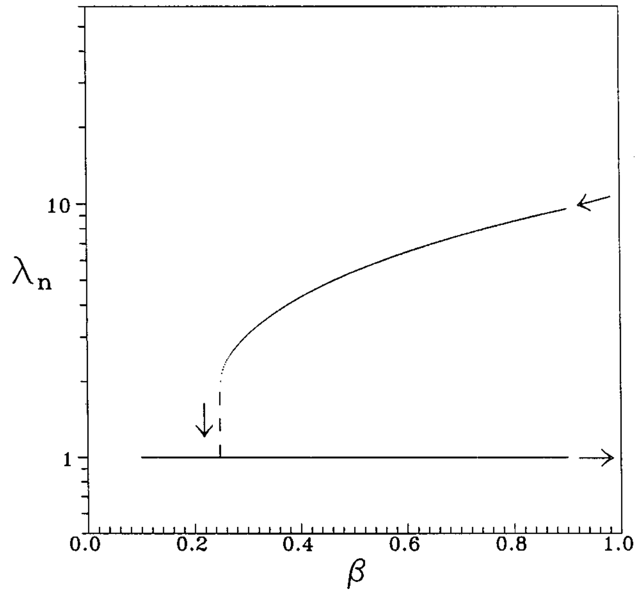

FIG. 2. Illustration of PB via λ_n vs β plot at $A/\lambda_0=1$.

The power of microwave radiation P_m from n-GaAs can be expressed as $E_{rf}^2 v_0^2 R^{-1} c^{-2}\lambda^2$,[9] where E_{rf} and R corresponding to the rf field and resistence in the semiconductor, respectively. P_m is collected by a Wg and modulated by a phase shifter. The phase shifter induces the nonlinear power function $P_m\sin^2(\pi A/\lambda)$ detected by D_e with a gain η_1, where A is the length of the phase shifter. The purpose of D_e is to convert the nonlinear power into an electric current $i(t)$. The current is then retarded by a delay line and enhanced by an amplifier with a gain η_2 to drive S, tuning the incident angle of the laser beams. If the variation of α is linearly proportional to i with a ratio a, it is easy to get the relation between $\lambda(t)$ and $i(t)$: $\lambda(t)=\lambda_0+Bi(t)$, where $B\equiv a\lambda_0\tan\alpha_0$ and λ_0 is the initial wavelength due to the initial incident angle α_0. Therefore, the circuit equation for the experimental apparatus is

$$\tau\frac{d\lambda(t)}{dt}+\lambda(t)=\lambda_0+B\eta_1\eta_2 P_m(t-T)\sin^2\left[\frac{\pi A}{\lambda(t-T)}\right]. \quad (15)$$

Since $T\gg\tau$, the differential term in Eq. (15) can be neglected. Then Eq. (15) can be treated as a difference equation. For the convenience of analysis, we make the difference equation dimensionless

$$\lambda_n=1+\beta\lambda_{n-1}^2\sin^2\left(\frac{\pi A}{\lambda_0}\frac{1}{\lambda_{n-1}}\right), \quad (16)$$

where $\lambda_n/\lambda_0\rightarrow\lambda_n$, $\beta=a\lambda_0^2\eta_1\eta_2\tan\alpha_0 E_{rf}^2 v_0^2/Rc^2$. Equation (16) means that $\lambda(t)$ is kept at a fixed value in each time interval between $(n-1)T$ and nT. Therefore, the wavelength $\lambda(t)$ and output power $P_m(t)$ will show square-wave behavior.

Equation (16) is a highly nonlinear difference equation. It is not diffcult to find that multistability and chaos from Eq. (16), which has been discussed in our previous work.[5] It is worth to point out that PB can be observed when A/λ_0 equal to 1 (Fig. 2). If the β value is increased very slowly, the output wavelength will keep at the lower state and has the same value as the initial wavelength under no feedback control. As far as I know, this kind of result is quite interesting and unique in bistable systems. The natural problem behind PB is how to control the dynamical response from the lower to the higher state because of nonhysteretic characteristic in PB. In the next section, we will use the concept of SR to control the dynamical response with PB property.

IV. SR IN TWO PDT STATES

Now we input the modulating current *m* and the stochastic current *noise* into the feedback circuit (Fig. 1) to study SR. Then, Eq. (16) shall be rewritten as

$$\lambda_n = 1 + \beta\lambda_{n-1}^2 \sin^2\left(\frac{\pi A}{\lambda_0}\frac{1}{\lambda_{n-1}}\right) + h\cos(2\pi f_s n) + \eta_n, \tag{17}$$

where h is the modulation amplitude, f_s is the driving frequency, and η_n satisfies the white noise condition: $\langle \eta_n \rangle = 0$ and $\langle \eta_n \eta_i \rangle = D^2 \delta_{n,i}$. The physical meaning of Eq. (17) can be understood as follows. There are three current sources in the feedback loop of Fig. 1. The first one is a microwave current $\beta\lambda_n^2 \sin^2(\pi A \lambda_0^{-1}/\lambda_n)$ detected by D_e, the second one is a modulating current $h\cos(2\pi f_s n)$, and the third one is a stochastic current η_n. The net current behind the delay line is the combination of these three current sources to drive S tuning the α value.

The SNR value is usually used to verify SR in dynamical systems. The definition of SNR is

$$SNR = \frac{lim_{\Delta f \to 0}\int_{f_s - \Delta f}^{f_s + \Delta f}[S(f) - S_0(f)]df}{S_0(f_s)}, \tag{18}$$

where $S_0(f)$ is the one-sided power-spectrum of the noisy system (i.e., feedback loop with *noise* but without m), $S_0(f_s)$ is the power of the noise at the driving frequency, and the upper term in Eq. (18) is the power of the total response in the neighborhood of the driving frequency.

In Fig. 3, we show that SNR as a function of the noise intensity D at a fixed driving frequency $f_s = 0.01$ and different amplitude of modulation h when $A/\lambda_0 = 1$ and $\beta = 0.27$. It is obvious to see that there is a maximun value occured at finite D values except the case of $h = 0.2$. It means that the time evolution of transition between the two PDT states, i.e., steplike transition, can be synchronized with the input modulation. Besides this resonance effect, some curves (or symbols) with small h values in Fig. 3 show that the extreme value can be found near $D = 0$. It represents no transition between the two PDT states when D is very small. The dynamical system keeps at one of these two states. When D is increased, the SNR value shall be decreased. Therefore, in general case of SR, there are two extreme values that can be observed in the SNR vs D plot. It shall notice that there is no SR in the case with large modulation, i.e., $h = 0.2$. The reason is that the dynamical system shall follow the large modulation in Eq. (17). Therefore, the output signal is very similar with the input modulation. This kind of behavior is easily destroyed by the increasing noise intensities. This is why the SNR value is monotonously decreased when D is increased.

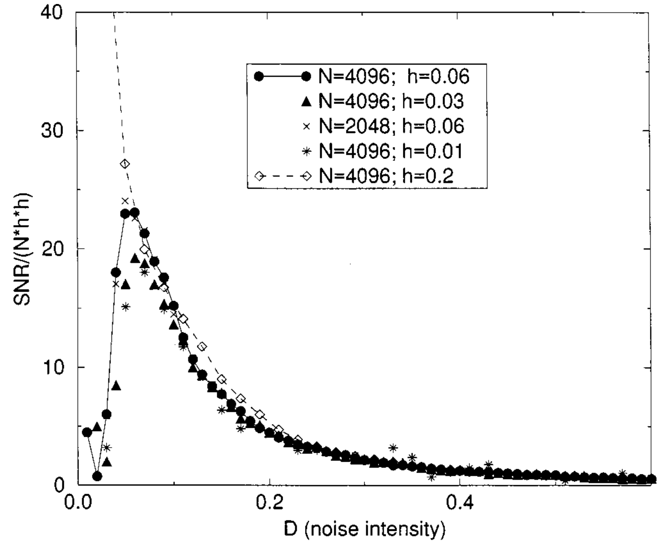

FIG. 3. The SNR value as a function of noise intensity D at different amplitude of modulation. f_s is fixed at 0.01. N is the length of the input λ_n series we investigate.

V. CONCLUSIONS

We study the SR in *n*-GaAs with a nonlinear feedback control. The interference pattern from the external laser beams will trigger PDT and the nonlinear feedback (Fig. 1) without *m* and *noise* will create bistable PDT. With the help of modulating current and stochastic current, the synchronized transition, i.e., SR, between the two PDT states is observed via SNR vs D plot. We think it is the first article to report SR can be found in microwave system, i.e., Gunn diode. This study will broaden the research interests related to the well-known Gunn effect.

I would like to thank Dr. Zoltán Néda for help with numerical calculations. This work was supported in part by the National Science Council of the Republic of China under Contract Nos. NSC 88-2112-M-002-002 and NSC 87-2112-M-001-046.

*Electronic address: hsien@phys.sinica.edu.tw

[1] B. K. Ridley, Proc. Phys. Soc. London **82**, 954 (1963).
[2] J. B. Gunn, Solid State Commun. **1**, 88 (1963).
[3] M. Segev, B. Collings, and D. Abraham, Phys. Rev. Lett. **76**, 3798 (1996).
[4] L. Subačius, V. Gružinskis, E. Starikov, P. Shiktorov, and K. Jarašiunas, Phys. Rev. B **55**, 12 844 (1997).
[5] Y.-H. Shiau, H.-P. Chiang, Y.-C. Cheng, and C.-K. Hu (unpublished).
[6] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998), and references therein.
[7] B. McNamara, K. Wiesenfield, and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988).
[8] L. L. Bonilla, M. Kindelan, and P. J. Hernando, Phys. Rev. B **58**, 7046 (1998).
[9] S. M. Sze, *Physics of Semiconductor Devices* (Wiley, New York 1969).