Local antiferromagnetism in high-T_c cuprate superconductors and the magnetic phases of U-based heavy-fermion compounds

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We investigate the possibility of the local induction of the antiferromagnetism (AF) in high-T_c superconductors and U-based heavy fermion compounds. In the former case, assuming the *d*-wave superconductivity in the bulk system, we find that AF can locally appear around surfaces and impurities and can coexist with the *d*-wave superconductivity. The induced AF is *enhanced* near the surface by the presence of superconductivity. Furthermore, the zero-energy bound state near the (110) surface of $d_{x^2-y^2}$-wave superconductors is split by the local AF, as was observed in $YBa_2Cu_3O_7$. We also show in a model that this phenomenon is mathematically equivalent to (1) the surface $s+id$-wave superconductivity, which has been also proposed in the high-T_c cuprates, and (2) the local AF in the unconventional spin-density wave, which has been proposed as a candidate for the magnetism in URu_2Si_2. In particular, the case (2) can explain the recent experiment for URu_2Si_2, which observes that the temperature at which the antiferromagnetic moment appears is lower than the phase-transition temperature. [S0163-1829(99)01846-9]

I. INTRODUCTION

The $d_{x^2-y^2}$-wave superconductivity is the most promising pairing state in the high-T_c cuprates. In this anisotropic superconductivity, the order parameter can be suppressed near surfaces within the order of the superconducting correlation length. Then, a second-order parameter that is insensitive to the surfaces can appear near the boundaries, even if it is completely suppressed in the bulk system. Indeed, a recent tunneling experiment for $YBa_2Cu_3O_7$ observes a split of the zero-bias conductance peak (ZBCP) at low temperatures,[1] which implies the local induction of the second-order parameter.[2]

As a candidate for the second-order parameter, *s*-wave superconductivity has been studied so far.[2–5] In this case, the $s+id_{x^2-y^2}$-wave state is realized near the (110) surface, if a weak *s*-wave interaction is assumed. Then, it is shown that the zero-energy bound state that appears near the surface in the pure $d_{x^2-y^2}$-wave superconductivity has a finite energy as observed experimentally.[2,5] However, the origin of the assumed *s*-wave interaction is unclear. Since the cuprates has a strong on-site Coulomb *repulsion* at copper sites, the ordinary on-site pairing is not favorable.[6] Although the other kind of symmetry, such as the d_{xy}-wave one, has been also proposed, a reliable source interaction has not been clarified yet.

The second-order parameter is not necessarily a kind of superconductivity. In particular, the high-T_c superconductor is known to have a strong on-site Coulomb repulsion at copper sites, and the superconducting phase is near the antiferromagnetic one. Even in the superconducting phase, a strong antiferromagnetic spin correlation still remains.[7] In addition, it has been clarified that the antiferromagnetic spin correlation is enhanced by nonmagnetic impurities in (1) the normal state of a two-dimensional Hubbard model,[8] and (2) various Heisenberg spin systems.[9–11] Then, we can expect that, instead of the superconductivity, antiferromagnetism (AF) may be locally induced as the second-order parameter near surfaces and impurities in the high-T_c cuprates. We also mention that a phenomenological Ginzburg-Landau theory being based on the SO(5) theory predicts the local AF in vortex cores.[12] Motivated by these circumstances, we investigate the local induction of AF in inhomogeneous $d_{x^2-y^2}$-wave superconductors. We microscopically show that it is really possible, and furthermore the induced AF is enhanced by the superconductivity. It is also shown that the AF causes a split of ZBCP as observed experimentally.

Besides the high-T_c cuprates, the second-order parameter is also expected in the magnetic phases of the U-based heavy fermion compounds UPt_3 and URu_2Si_2. Their extremely small antiferromagnetic moments (0.02–$0.04\ \mu$B) imply that AF may not be the dominant order parameter but a subdominant one. Indeed, a recent neutron-scattering experiment for URu_2Si_2 shows that the temperature at which the ordered moment appears ($T_{\rm moment}$) is lower than the phase-transition temperature $T_c=17.5$ K.[13]

A possible state of this curious magnetism is the *d*-wave spin-density wave (SDW), which is just the SDW version of the *d*-wave superconductivity.[14–16] This novel SDW can explain various anomalies that are observed in this magnetism.[16] On the other hand, since the *d*-wave SDW does not show any magnetic ordering, the presence of the tiny moment itself cannot be explained by this state only. However, If AF can be locally induced as a second-order parameter, it may explain the weak AF. In this paper, we prove by using a particle-hole transformation that the surface states in the *d*-wave superconductivity ($s+id$ and AF$+d$) and the local AF in the *d*-wave SDW are mathematically equivalent to one another. Thus the local AF can be really induced in the *d*-wave SDW. Then we find that it can explain the observed difference between $T_{\rm moment}$ and T_c.

This paper is organized as follows: We present our formulation in Sec. II and show results for the $d_{x^2-y^2}$-wave superconductivity in Sec. III. We discuss the mechanism of the local AF and relation to various surface states in Sec. IV where the local induction of AF in the d-wave SDW is also discussed. Finally, we summarize our conclusions in Sec. V.

II. FORMULATION

A. Model Hamiltonian

We start with the case of the high-T_c superconductivity. We consider the simple model on a square lattice, which is described by the Hamiltonian

$$H=-t\sum_{\langle ij\rangle,\sigma} c_{i\sigma}^{\dagger}c_{j\sigma}+V_s\sum_i n_{i\uparrow}n_{i\downarrow}-V_d\sum_{\langle ij\rangle} n_{i\uparrow}n_{j\downarrow}+H_{\mathrm{imp}}$$
$$-\mu\sum_{i\sigma} n_{i\sigma}, \tag{2.1}$$

where $c_{i\sigma}^{\dagger}$ is an electron creation operator at the ith site, and $n_{i\sigma}=c_{i\sigma}^{\dagger}c_{i\sigma}$. In the first term in Eq. (2.1), t is the nearest neighbor hopping, and the summation $\langle ij\rangle$ is taken over nearest-neighbor sites. In the following, we put $t=1$ as the unit of energy. The second and the third terms describe the on-site Coulomb $(V_s>0)$ and the nearest-neighbor pairing $(V_d>0)$ interactions, respectively. In this paper, we consider the half-filling so that we put the chemical potential $\mu=V_s/2$. Then the V_s term favors AF, while the $d_{x^2-y^2}$-wave superconductivity is realized by the V_d one.[17]

A surface is described by arranging nonmagnetic impurities with a large potential energy. It is given by $H_{\mathrm{imp}}=u\Sigma_{i\in\mathrm{surface},\sigma}n_{i\sigma}$. In our calculations, we put $u=10^5$.

Let us apply the mean-field theory to Eq. (2.1). Introducing the superconducting order parameter $\Delta_d^{ij}=-V_d\langle c_{j\downarrow}c_{i\uparrow}\rangle$, and the antiferromagnetic one $\Delta_s^i=-(V_s/2)\langle(n_{i\uparrow}-n_{i\downarrow})\rangle$, we obtain the mean-field Hamiltonian as

$$H_{\mathrm{MF}}=-t\sum_{\langle ij\rangle,\sigma} c_{i\sigma}^{\dagger}c_{j\sigma}+\sum_i \Delta_s^i(n_{i\uparrow}-n_{i\downarrow})$$
$$+\sum_{\langle ij\rangle}\Delta_d^{ij}(c_{i\uparrow}^{\dagger}c_{j\downarrow}^{\dagger}+h.c)+H_{\mathrm{imp}}. \tag{2.2}$$

The phase of Δ_d^{ij} can be chosen arbitrarily in the present case so that we have put it real. Equation (2.2) has a bilinear form so that it can be diagonalized by an appropriate unitary transformation.[18]

It is convenient to introduce the magnitude of Δ_s,

$$\tilde{\Delta}_s(x,y)=-(-1)^{x+y}\frac{V_s}{2}\langle n_{i\uparrow}-n_{i\downarrow}\rangle, \tag{2.3}$$

where $\mathbf{i}=(x,y)$ is the position vector of the ith site. (We put the lattice constant unity.) We also define the $d_{x^2-y^2}$-wave order parameter at the ith site by

$$\tilde{\Delta}_d(x,y)=\frac{1}{4}(\Delta_d^{\mathbf{i},\mathbf{i}+\mathbf{e}_x}+\Delta_d^{\mathbf{i},\mathbf{i}-\mathbf{e}_x}-\Delta_d^{\mathbf{i},\mathbf{i}+\mathbf{e}_y}-\Delta_d^{\mathbf{i},\mathbf{i}-\mathbf{e}_y}), \tag{2.4}$$

where $\mathbf{e}_\nu(\nu=x,y)$ expresses a unit vector in the ν direction. (In the homogeneous case, the $d_{x^2-y^2}$-wave order parameter satisfies $\Delta_d^{\mathbf{i},\mathbf{i}+\mathbf{e}_x}=\Delta_d^{\mathbf{i},\mathbf{i}-\mathbf{e}_x}=-\Delta_d^{\mathbf{i},\mathbf{i}+\mathbf{e}_y}=-\Delta_d^{\mathbf{i},\mathbf{i}-\mathbf{e}_y}$.) We use these two quantities in the following figures.

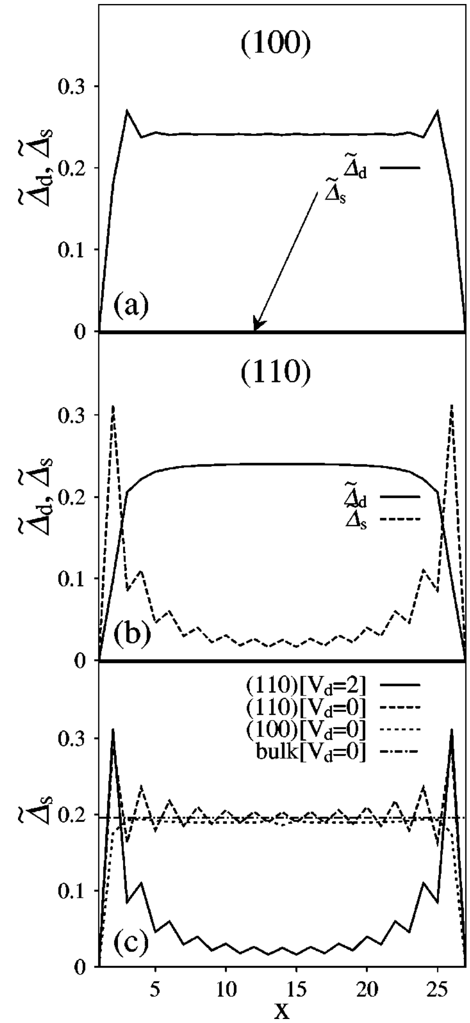

FIG. 1. Spatial variations of $\tilde{\Delta}_s$ and $\tilde{\Delta}_d$ at $T=0.01$ $(t=1)$. The figure is at $y=1$. In this case, the impurity site is at $x=1$ (left edge), and $x=27$ (right edge), which is equivalent to $x=1$ due to the periodicity. (a) (100) surface. (b) (110) surface. (c) $\tilde{\Delta}_s$ for various cases.

B. Numerical calculation

We determine the spatial variation of the two order parameters self-consistently. We put $V_d=2$ and $V_s=1.5$ so that the transition temperature of the $d_{x^2-y^2}$ superconductivity is higher than that of AF. In this case, the latter is completely suppressed by the former in the homogeneous system. The system size is 26×26, and the periodic boundary condition is imposed. We have ascertained by examining larger sizes that the finite size effect is weak in the following results. The (100) surface and the (110) one are, respectively, described by putting the impurities at $x=1(y=1,2,\ldots,26)$ and $y=x$.

III. RESULTS

A. Surface antiferromagnetism in $d_{x^2-y^2}$-wave superconductivity

Figure 1 shows the local induction of AF near the (100) and the (110) surfaces in the $d_{x^2-y^2}$-wave superconductivity. Near the (100) surface, AF is still suppressed completely. On

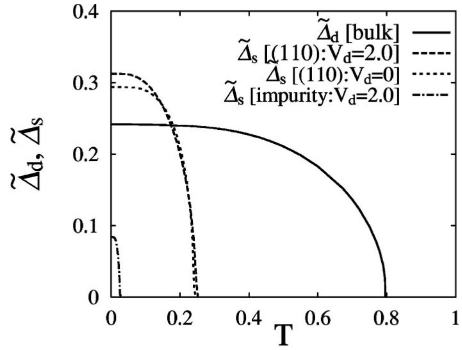

FIG. 2. Temperature dependence of $\tilde{\Delta}_d$ in the bulk system and $\tilde{\Delta}_s$ at the nearest site to the (110) surface. In the figure, ''impurity'' is $\tilde{\Delta}_s$ at the nearest site to one impurity.

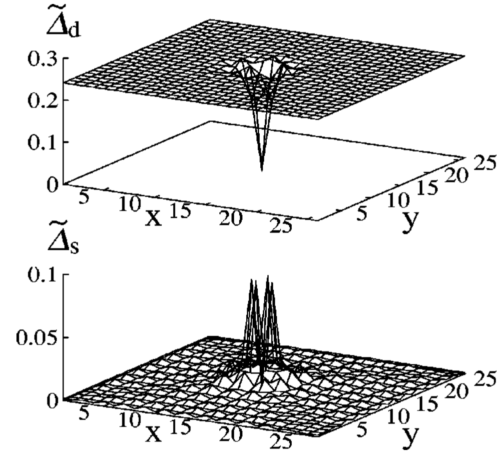

FIG. 3. Spatial variations of $\tilde{\Delta}_s$ and $\tilde{\Delta}_d$ around one impurity at $(x,y)=(13,13)$. We put $T=0.01$.

the other hand, it appears near the (110) surface. Since the superconducting correlation length is short in the present case (~ 4),[21] the region where the *d*-wave order parameter is suppressed is not so different between the (100) and the (110) cases. Thus we find that the suppression of the *d*-wave state itself is not a dominant reason why AF is induced.

Since the surface AF is inhomogeneous, a finite magnetization appears near the surface. We also find in Fig. 1(c) that $\tilde{\Delta}_s$ at the nearest site to the surface ($x=2$) is larger than the ones in the cases when the superconductivity is absent ($V_d=0$). [Although the difference between the cases of (110) ($V_d=2.0$) and (110)($V_d=0$) is not clear in Fig. 1(c), $\tilde{\Delta}_s(x=2)=0.312$ in the former, while 0.294 in the latter.] Namely, in contrast to the bulk system, AF is *enhanced* by the superconductivity near the (110) surface. We will discuss this enhancement later.

Figure 2 shows the temperature dependence of $\tilde{\Delta}_s$ at the nearest site to the (110) surface. This figure shows that the local AF does not appear just below the transition temperature of the superconductivity. It appears at the temperature that is slightly higher than the transition temperature of AF in the case when the superconductivity is absent. This result also indicates that the superconductivity assists the local induction of AF.

B. Local antiferromagnetism around an impurity

The local AF also appears around a single impurity as shown in Figs. 2 and 3. In Fig. 3, the direction of the spin at the four peaks in $\tilde{\Delta}_s$ is the same. Thus, although the impurity itself is nonmagnetic, it looks having a finite magnetic moment by the induced AF. When we take into account the effect of the magnetic field, a circulating current should flow so as to screen this local moment.

IV. ANALYSES

A. Mechanism of the local antiferromagnetism and its enhancement by the $d_{x^2-y^2}$-wave superconductivity

First of all, we present an intuitive explanation about the mechanism of the local AF: In the $d_{x^2-y^2}$-wave superconductivity, it is known that the zero-energy bound states are induced near the (110) surface[19,20] and an impurity with $u \to \infty$.[21] These bound states make a peak structure at $\omega=0$ in the superconducting density of states (DOS) near the surface and the impurity.[22] Thus AF can appear by using this induced DOS. Since the peak intensity is stronger near the (110) surface than near the single impurity, the induced AF is stronger in the former case, as shown in Fig. 2. On the other hand, since the zero-energy bound state is absent in the case of the (100) surface, DOS near the (100) surface is simply suppressed below the energy gap as in the homogeneous case. As a result, AF is not induced around the (100) surface.

We note that, although the fact of the enhancement of the antiferromagnetic spin correlation is the same among the present case and the previous ones that were mentioned in the introduction,[8–11] their mechanisms are different from one another. (1) In the case of the normal state of a two-dimensional Hubbard model,[8] the local DOS is enhanced near an impurity by the Friedel oscillation; the enhanced DOS increases the antiferromagnetic spin correlation. (2) In the case of the Heisenberg spin systems,[9–11] impurities locally suppress the ''resonating-valence-bond''; the suppression leads to the enhancement of the singlet spin-spin correlation around the impurities, which brings about the enhancement of the antiferromagnetic spin correlation.[9]

Next, we show how the surface AF is enhanced by the superconductivity more analytically. For this purpose, we neglect the spatial variation of the order parameters for simplicity. In this case, $H_{\rm MF}$ can be written as

$$H_{\rm MF}=\sum_{\mathbf{p}} \Psi_{\mathbf{p}}^\dagger(\varepsilon_p\rho_3+\Delta_d^{\mathbf{p}}\rho_1\tau_3+\Delta_s^0\rho_1\tau_1)\Psi_{\mathbf{p}}+H_{\rm imp}, \tag{4.1}$$

where $\Psi_{\mathbf{p}}^\dagger=(c_{\mathbf{p}\uparrow}^\dagger, c_{-\mathbf{p}-\mathbf{Q}\downarrow}, c_{-\mathbf{p}\downarrow}, c_{\mathbf{p}+\mathbf{Q}\uparrow}^\dagger)$ with $\mathbf{Q}=(\pi,\pi)$, $\Delta_d^{\mathbf{p}}=2\tilde{\Delta}_d(\cos p_x-\cos p_y)$, $\Delta_s^0=|\Delta_s|<\tilde{\Delta}_d$, $\varepsilon_p=-2(\cos p_x+\cos p_y)$, while both ρ_i and τ_j express the Pauli matrices: Their multiplication forms a 4×4 matrix as, for example,

$$\rho_1\tau_3=\begin{pmatrix} 0 & \tau_3 \\ \tau_3 & 0 \end{pmatrix}. \tag{4.2}$$

The one electron thermal Green function in the absence of the surface is given by

$$G_0(p)=\frac{1}{i\omega-\varepsilon_p\rho_3-\Delta_d^{\mathbf{P}}\rho_1\tau_3-\Delta_s^0\rho_1\tau_1}, \tag{4.3}$$

where ω represents the fermion Matsubara frequency. The Green function in the presence of the (110) surface (G) can be obtained by summing up all the scattering diagrams by $H_{\rm imp}$.[22] Introducing new coordinates $X=(x-y)/\sqrt{2}$ and $Y=(x+y)/\sqrt{2}$, which are, respectively, perpendicular and parallel to the (110) surface, and corresponding momenta $P_x=(p_x-p_y)/\sqrt{2}$ and $P_y=(p_x+p_y)/\sqrt{2}$, we find

$$G(X,X',P_y)=G_0(X,X',P_y)-G_0(X,0,P_y)\times\frac{1}{G_0(0,0,P_y)}G_0(0,X',P_y), \tag{4.4}$$

where

$$G_0(X,X',P_y)=\sum_{P_x} e^{iP_x(X-X')}G_0(P). \tag{4.5}$$

We have put $u\to\infty$ in calculating Eq. (4.4).

Equation (4.4) shows that the pole of $G_0(0,0,P_y,i\omega\to\nu+i\delta)^{-1}$ gives a bound state (if it is lower than the energy gap). When AF is absent ($\Delta_s^0=0$), we obtain a bound state at $\nu=0$ (zero-energy bound state), which corresponds to ZBCP. This state splits to be $\nu=\pm\Delta_s^0$ when $\Delta_s^0\neq 0$. Namely, ZBCP is splitted by AF as observed experimentally.

Recalculating the antiferromagnetic order parameter by using Eq. (4.4), we obtain

$$\Delta_s^{\rm recalc.}=\Delta_s^0+\frac{V_sT}{2}\sum_{P_y,\omega}\frac{\Delta_s^0}{|v_X|\Omega}e^{-2(\Omega/|v_X|)X}\times\frac{\Delta_d^{\mathbf{P}}[\Delta_d^{(P_x,P_y)}-\Delta_d^{(-P_x,P_y)}]}{\omega^2+\Delta_s^{0^2}}, \tag{4.6}$$

where $\Omega=\sqrt{\omega^2+\Delta_s^{0^2}+\Delta_d^{\mathbf{P}^2}}$, and v_X is the X component of the Fermi velocity. In Eq. (4.6), $\mathbf{P}$ in $\Delta_d^{\mathbf{P}}$ is put on the Fermi surface. Because $\Delta_d^{(-P_x,P_y)}=-\Delta_d^{(P_x,P_y)}$ is satisfied [$\Delta_d\propto\sin(P_x/\sqrt{2})\sin(P_y/\sqrt{2})$], the second term is always positive. Thus, when Δ_d is finite, AF is enhanced near the (110) surface by the superconductivity. On the other hand, the second term is absent in the case of (100) surface because of $\Delta_d^{(-p_x,p_y)}=\Delta_d^{(p_x,p_y)}$. Thus such an enhancement does not occur near the (100) surface.

Since the zeros of the denominator in the second term in Eq. (4.6), $\omega^2+\Delta_s^{0^2}=0$ $(\leftrightarrow\nu^2-\Delta_s^{0^2}=0)$, correspond to the bound states, these low-energy states are expected to play a role in the local induction of AF. Since the bound state adds a finite intensity in the superconducting DOS near the (110) surface, AF can appear by using this additional intensity as discussed in the intuitive explanation. We mention that this enhancement is also obtained in the surface $s+id$-wave superconductivity.[5]

B. Relation to other surface states

In this subsection, we prove by using particle-hole transformations that the present model is mathematically equivalent to the surface $s+id$-wave state and the local AF in the d-wave SDW.

1. Local $s+id$-wave superconductivity

First of all, we divide the system into two sublattices A and B. After rewriting Eq. (2.2) in the summation of nearest-neighbor pairs, we apply the following transformation to each (ij) pair ($i\in A, j\in B$):

$$A\begin{cases} c_{i\uparrow}=\dfrac{1}{\sqrt{2}}(a_{i\uparrow}+ia_{i\downarrow}^\dagger),\\[2ex] c_{i\downarrow}^\dagger=-\dfrac{1}{\sqrt{2}}(a_{i\uparrow}^\dagger+ia_{i\downarrow}),\end{cases}$$

$$B\begin{cases} c_{j\uparrow}=[Q(a_{j\uparrow}-ia_{j\downarrow}^\dagger)-R(a_{j\uparrow}^\dagger-ia_{j\downarrow})]e^{-i\phi},\\ c_{j\downarrow}^\dagger=-[Q(a_{j\uparrow}-ia_{j\downarrow}^\dagger)+R(a_{j\uparrow}^\dagger-ia_{j\downarrow})]e^{-i\phi},\end{cases}$$

where

$$Q=\frac{1}{\sqrt{2}}\frac{t}{\sqrt{t^2+\Delta_d^{ij^2}}},$$

$$R=\frac{1}{\sqrt{2}}\frac{\Delta_d^{ij}}{\sqrt{t^2+\Delta_d^{ij^2}}},$$

$$e^{i\phi}=\frac{t+i\Delta_d^{ij}}{\sqrt{t^2+\Delta_d^{ij^2}}}. \tag{4.7}$$

Then Eq. (2.2) is transformed into

$$H_{\rm MF}\to -t\sum_{\langle ij\rangle,\sigma}a_{i\sigma}^\dagger a_{j\sigma}+\sum_i\overline{\Delta_s^i}(a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger-a_{i\downarrow}a_{i\uparrow})+\sum_{\langle ij\rangle}\Delta_d^{ij}(a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger+\text{H.c.})+H_{\rm imp}, \tag{4.8}$$

where $\overline{\Delta_s^i}=i(-1)^{x+y}\Delta_s^i=-(V_s/2)\langle c_{i\downarrow}^\dagger c_{i\uparrow}^\dagger-c_{i\uparrow}c_{i\downarrow}\rangle$ is the imaginary s-wave order parameter. On the other hand,

$$\Delta_d^{ij}=-\frac{V_d}{4}\left[\frac{\Delta_d^{ij^2}}{t^2+\Delta_d^{ij^2}}\langle a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger+a_{j\uparrow}^\dagger a_{i\downarrow}^\dagger+\text{H.c.}\rangle-\frac{t\Delta_d^{ij}}{t^2+\Delta_d^{ij^2}}\langle a_{i\uparrow}^\dagger a_{j\uparrow}+a_{i\downarrow}^\dagger a_{j\downarrow}+\text{H.c.}\rangle\right], \tag{4.9}$$

where we have dropped expectation values that are equal to zero. Since we can prove the relation

$$\Delta_d^{ij}\langle a_{i\uparrow}^\dagger a_{j\uparrow}+a_{i\downarrow}^\dagger a_{j\downarrow}+\text{H.c.}\rangle=-t\langle a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger+a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger+\text{H.c.}\rangle, \tag{4.10}$$

Eq. (4.9) turns out to be the $d_{x^2-y^2}$-wave superconducting order parameter. [We derive Eq. (4.10) in the Appendix.] Equation (4.8) is just the Hamiltonian for the $d+is$-wave superconductivity that is also obtained from Eq. (2.1) with $V_s < 0$ within the mean-field theory. This mapping means that we may regard Figs. 1–3 as those for the local induction of the s-wave superconductivity. (Although transformed $H_{\rm imp}$ has a different form from the nonmagnetic impurity scattering, the difference is irrelevant when we put $u \to \infty$.)

2. Local antiferromagnetism in the d-wave SDW

When we apply the transformation

$$a_{i\uparrow} = \frac{1/\sqrt{2}}{(b_{i\uparrow} + i b_{\downarrow})},$$

$$a_{i\downarrow} = \frac{(-1)^{x+y}/\sqrt{2}}{(b_{i\downarrow}^\dagger - i b_{\uparrow}^\dagger)}, \tag{4.11}$$

to Eq. (4.8), we obtain

$$H_{\rm MF} \to -t \sum_{\langle ij\rangle,\sigma} b_{i\sigma}^\dagger b_{j\sigma} + \sum_i \Delta_s^i (n_{i\uparrow} - n_{i\downarrow})$$

$$+ \sum_{\langle ij\rangle} \overline{\Delta}_d^{ij} (b_{i\uparrow}^\dagger b_{j\uparrow} - b_{i\downarrow}^\dagger b_{j\downarrow}) + H_{\rm imp}, \tag{4.12}$$

where $\overline{\Delta}_d^{ij} = i(-1)^{x+y} \Delta_d^{ij} = -(V_d/4) \langle b_{i\uparrow}^\dagger b_{j\uparrow} - b_{i\downarrow}^\dagger b_{j\downarrow} - \text{H.c.}\rangle$ is the $d_{x^2-y^2}$-wave SDW order parameter.[14–16] Equation (4.12) is the Hamiltonian for the AF+ d-wave SDW system, which is obtained from Eq. (2.1) with $V_d < 0$. Thus, we find that Figs. 1–3 also describe the local induction of AF in the d-wave SDW.

Figure 2 shows that the temperature at which the antiferromagnetic moment appears is lower than the phase-transition temperature of the d-wave SDW, as recently observed in URu_2Si_2. Since defects and impurities should exist to some extent in real systems, it can be expected that AF is locally induced around them. Thus, the d-wave SDW plus locally induced AF is a possible scenario for the curious magnetism in URu_2Si_2.

V. SUMMARY

We have studied the local AF that is induced in the $d_{x^2-y^2}$-wave superconductivity and the d-wave SDW. In both cases, AF can be induced around the (110) surfaces and impurities. Since they do not appear near the (100) surface where the low-energy bound state is absent, it is expected that AF is induced by using the additional density of states introduced by the bound state.

The local AF in the d-wave superconductivity can explain the recent tunneling experiment for $YBa_2Cu_3O_7$, which observes a split of the ZBCP. On the other hand, the local AF in the d-wave SDW gives a possible explanation for the curious magnetism in URu_2Si_2, which shows a difference between $T_{\rm moment}$ and T_c. We have also proved in our model that these two phenomena and the local $s+id$-wave state are mathematically equivalent to one another.

We finally discuss the stabilization of the local AF against fluctuations. When we take into account the stack of the CuO_2 layers and their coupling in high-T_c cuprates, the surface ordering can be regarded as a two-dimensional AF. Then it is possible that an anisotropy stabilizes the surface AF overwhelming fluctuations. As for AF around impurities, in real materials, impurities and defects should be distributed more or less in three-dimensional space. Then we can expect that, when the density of impurities and defects is high to some extent, the local AF around them can connect with one another, which stabilizes three-dimensional AF. (The ordering is, however, inhomogeneous.) Thus, although the effect of fluctuations should be considered in the next step, taking into account the above situations, we expect that the present AF within the simple mean-field theory can still survive fluctuations. We also mention that a recent neutron experiment on $YBa_2Cu_3O_{6+x}$ shows that Zn doping enhances the antiferromagnetic spin fluctuations in the low-energy region.[23] Since the enhancement in the low-energy region indicates that the system approaches the antiferromagnetic phase by the doping, this experiment is consistent with the present calculation.

After finishing this work, the author was aware of a similar work by Honerkamp *et al.*[24] Although they also reach the same conclusion for the local AF in the $d_{x^2-y^2}$-wave superconductivity, they do not study the local AF in the d-wave SDW in URu_2Si_2 and the relation among other surface states that we have proved in this paper.

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APPENDIX A: PROOF OF EQ. (4.10)

We write the hopping between ith and jth sites as t_{ij} in Eq. (4.8), and then apply the following transformation to each site:

$$a_{i\uparrow} = \cos\theta_i \alpha_{i\uparrow} \sin\theta_i \alpha_{i\downarrow}^\dagger,$$

$$a_{i\downarrow} = \cos\theta_i \alpha_{i\downarrow} - \sin\theta_i \alpha_{i\uparrow}^\dagger. \tag{A1}$$

Then the Hamiltonian is transformed as

$$H_{\rm MF}(\{t_{ij}\},\{\Delta_d^{ij}\},\{\overline{\Delta}_s^i\}) \to H_{\rm MF}(\{t'_{ij}\},\{\Delta'^{ij}_d\},\{\overline{\Delta}_s^i\}), \tag{A2}$$

where

$$\mathbf{A}'_{ij} \equiv \begin{pmatrix} t'_{ij} \\ \Delta'^{ij}_d \end{pmatrix} = \begin{pmatrix} \cos(\theta_i+\theta_j) & \sin(\theta_i+\theta_j) \\ -\sin(\theta_i+\theta_j) & \cos(\theta_i+\theta_j) \end{pmatrix} \begin{pmatrix} t_{ij} \\ \Delta_d^{ij} \end{pmatrix}. \tag{A3}$$

When we consider the partition function $Z(H_{\mathrm{MF}})$, it is invariant under the transformation Eq. (A1). Since the transformation can be regarded as a rotation in (t_{ij},Δ_d^{ij}) space as shown in Eq. (A3), the invariance means that Z is spherical in this space. Thus Z must be a function of $\mathbf{A}_{ij}\cdot\mathbf{A}_{ij}=t_{ij}^2+\Delta_d^{ij2}$, which gives

$$\Delta_d^{ij}\left(\frac{\partial Z}{\partial t_{ij}}\right)=t\left(\frac{\partial Z}{\partial \Delta_d^{ij}}\right). \tag{A4}$$

We then obtain

$$\begin{aligned}\Delta_d^{ij}\langle a_{i\uparrow}^{\dagger}a_{j\uparrow}+a_{i\downarrow}^{\dagger}a_{j\downarrow}+\mathrm{H.c.}\rangle\\ &=-\Delta_d^{ij}\left(\frac{\partial Z}{\partial t_{ij}}\right)_{t_{ij}=t}=-t\left(\frac{\partial Z}{\partial \Delta_d^{ij}}\right)_{t_{ij}=t}\\ &=-t\langle a_{i\uparrow}^{\dagger}a_{j\downarrow}^{\dagger}+a_{i\uparrow}^{\dagger}a_{j\downarrow}^{\dagger}+\mathrm{H.c.}\rangle.\end{aligned} \tag{A5}$$

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