Resonant peak splitting for ballistic conductance in magnetic superlattices

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We investigated theoretically the resonant splitting of ballistic conductance peaks in magnetic superlattices. It is found that, for magnetic superlattices with periodically arranged n identical magnetic barriers, there exists a general $(n-1)$-fold resonant peak splitting rule for ballistic conductance, which is the analogy of the $(n-1)$-fold resonant splitting for transmission in n-barrier electric superlattices [R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973)]. [S0163-1829(99)05827-0]

Electron motion in a two-dimensional electron gas (2DEG) subjected to a magnetic field has attracted long-drawn interest, since it provides a variety of interesting and significant information characterizing the behavior of electrons in 2DEG systems. There are a number of papers[1] devoted to the study on quantum transport of a 2DEG in a unidirectional weak sinusoidal magnetic-field modulation with a uniform magnetic-field background, where commensurability effects come into play. This system was recently realized experimentally,[2] and the long-predicted magnetoresistance oscillations[1] resulting from semiclassical commensurability between the classical cyclotron diameter and the period of the magnetic modulation, were observed. Recently, a 2DEG was investigated under the influence of a magnetic step, magnetic well, and magnetic barrier.[3] Electron tunneling in more complicated and more realistic magnetic structures was found to possess wave-vector filtering properties.[4] The studies[5] showed that the energy spectrum of magnetic superlattice (MS) consists of magnetic minibands.

$(n-1)$-fold transmission splitting for n electric-barrier tunneling was first noticed and generalized by Tsu and Esaki in their pioneering paper,[6] and was proved analytically by Lui and Stamp[7] in the electric superlattice (ES) with periodically arranged n identical rectangular barriers. Very recently, Guo *et al.*[8] investigated theoretically the transmission splitting effects in two kinds of magnetic supperlattices (MS) and found no explicit and general resonant peak splitting for transmission in electron tunneling in MS.

We noticed that there is a single conductance peak for electron tunneling through the two-barrier magnetic structure, and two resonant spikes in the triple-barrier structure.[4] We also observed four resonant peaks in the ballistic conductance at low Fermi energies and found that four resonant shoulders can be resolved for a 2DEG modulated by a sinusoidal magnetic field of five periods and a 5-magnetic-step-barrier structure as long as the magnetic strength is strong enough.[9] This urges us to explore whether there is a general resonant peak splitting rule for ballistic conductance in magnetic superlattices (MS). Since ballistic conductance can be derived as the electron flow averaged over half the Fermi surface,[10] the main features of resonant tunneling through magnetic barriers is still preserved for ballistic conductance.[4] It hints that there exists some kind of resonant peak splitting rule for ballistic conductance in magnetic superlattices (MS), which is only dependent on the number of barriers in the magnetic field profile. In the following, we calculated ballistic conductances with the help of transfer matrix method in four kinds of magnetic superlattices.[5] (a) Kronig-Penney magnetic superlattice (KPMS), which is the analogy of the well-known electrostatic Kronig-Penny model. Its magnetic field profile is modeled by the expression $B(x)/B_0 = \gamma\Sigma_{n=-\infty}^{n=+\infty}(-1)^{n+1}\delta(x-nl/2)$, and the vector potential can be taken as $A(x)/A_0 = \gamma\frac{1}{2}\,\mathrm{sgn}[\cos(2\pi x/l)]$ [see Fig. 1(a)]. (b) Step magnetic superlattice (Step MS): $B(x)/B_0 = \gamma\Sigma_{n=-\infty}^{n=+\infty}(-1)^n\theta(x-nl/2)\,\theta[(n+1)l/2-x]$ and $A(x)/A_0 = \gamma\Sigma_{n=-\infty}^{n=+\infty}(-1)^n[x-(2n+1)l/4]\,\theta(x-nl/2)\,\theta[(n+1)l/2-x]$ [see Fig. 1(b)]. (c) Sinusoidal magnetic superlattice (Sinusoidal MS): $B(x)/B_0 = \gamma\sin(2\pi x/l)$ and $A(x)/A_0 = -\gamma 1/(2\pi)\cos(2\pi x/l)$ [see Fig. 1(c)]. (d) Sawtooth magnetic superlattice (Sawtooth MS): $B(x)/B_0 = -\gamma 2/l\Sigma_{n=-\infty}^{n=+\infty}(x-nl)\,\theta[x-(n+1/2)l]\,\theta(x-nl)$ and $A(x)/A_0 = \gamma 1/l\Sigma_{n=-\infty}^{n=+\infty}(x-nl)^2\,\theta[x-(n+1)l/2]\,\theta(x-nl)$ [see Fig. 1(d)]. Here, $\theta(x)$ is the heaviside step function and l is the period of superlattice, γ is a parameter characterizing the

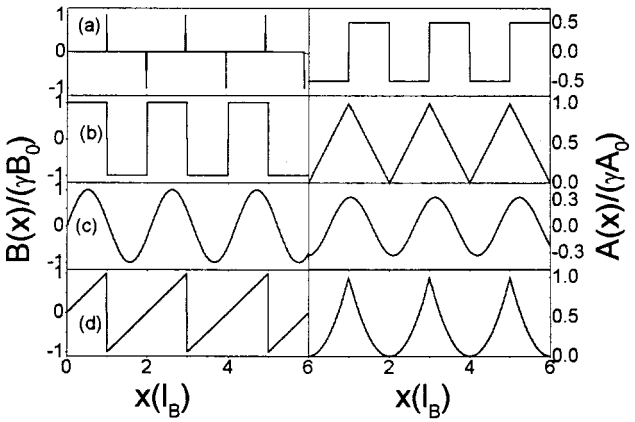

FIG. 1. The magnetic field profiles and the corresponding vector potential about four kinds of magnetic superlattices, where only three periods are plotted.

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magnetic-field strength. The above systems can be performed experimentally.[4,5,11]

For a 2DEG subjected to a periodic magnetic field perpendicular to the 2DEG plane, the corresponding one-electron Hamiltonian reads

$$H=\frac{1}{2m^*}[\mathbf{p}+e\mathbf{A}(x)]^2=\frac{1}{2m^*}\{p_x^2+[p_y+eA(x)]^2\}, \quad (1)$$

where m^* is the effective mass of electron and $A(x)=(0,A(x),0)$ is the vector potential in the Landau gauge. Since $[P_y,H]=0$, the problem is translational invariant along the y direction. Then the wave functions can be written in the form $1/(\sqrt{l_y})e^{ik_y y}\psi(x)$, where k_y is the wave vector in the y direction and l_y the length of the magnetic structure in the y direction. By introducing the magnetic length $l_B=\sqrt{\hbar/eB_0}$ and the cyclotron frequency $\omega_c=eB_0/m^*$, we express the basic quantities in the dimensionless units: (1) coordinates $\mathbf{r}\rightarrow\mathbf{r}l_B$. (2) magnetic field $B(x)\rightarrow B(x)B_0$. (3) the vector potential $A(x)\rightarrow A(x)B_0 l_B$. (4) the energy $E\rightarrow E\hbar\omega_c$. For GaAs and an estimated $B_0=0.1$ T we have $l_B=81.3$ nm, $\hbar\omega_c=0.17$ meV. After some algebras, the following 1D Schrödinger equation for $\psi(x)$ can be obtained[4]

$$\left\{\frac{d^2}{dx^2}-[A(x)+k_y]^2+2E\right\}\psi(x)=0. \quad (2)$$

The function $V(x,k_y)=[A(x)+k_y]^2$ can be interpreted as an effective k_y-dependent electric potential. From this expression we can find out that, electron tunneling in MS is inherently a complicated two-dimensional process, which depends on the electron's wave vectors in the longitudinal and transverse directions of the 2DEG, and thus possesses no general transmission splitting relation as in ES.

For the magnetic structure in region $[0, L=nl]$, we devide it into $M(M\gg 1)$ segments, each of which has width $a=L/M$. The effective potential in each segment can be viewed as constant and then the plane-wave approximation can be taken. In the jth segment, the wave functions may be expressed as

$$\psi(x)=A_j e^{ik_j x}+B_j e^{-ik_j x}, x\in[ja,(j+1)a], \quad (3)$$

where $k_j=\sqrt{2E-[A(ja+a/2)+k_y]^2}$, which may be either real or pure imaginary.

Without any loss of generality, we assume there is no magnetic field in the incident and outgoing regions, then the wave functions can be expressed by plane waves

$$\psi(x)=\begin{cases} e^{ikx}+re^{-ikx}, & x<0, \\ te^{ikx}, & x>L, \end{cases} \quad (4)$$

where $k=\sqrt{2E-k_y^2}$ and r,t are the reflection and transmission amplitudes, respectively.

The match of the wave functions and their derivatives at $x=0$ and $x=L$ yields

$$\begin{bmatrix}1\\ r\end{bmatrix}=\begin{bmatrix}1/2 & 1/(2ik)\\ 1/2 & -1/(2ik)\end{bmatrix}T_M\begin{bmatrix}e^{ikL} & e^{-ikL}\\ ike^{ikL} & -ike^{-ikL}\end{bmatrix}\begin{bmatrix}t\\ 0\end{bmatrix}, \quad (5)$$

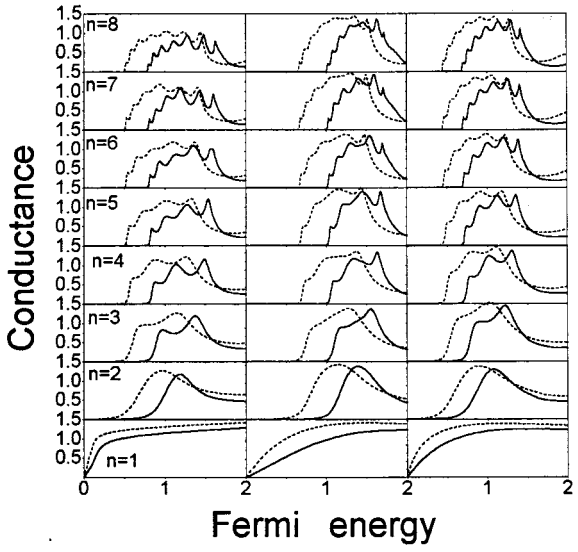

FIG. 2. Ballistic conductances for KPMS, Step MS, and Sinusoidal MS. The left column of the figure corresponds to the KPMS, the middle column to Step MS and the right column to Sinusoidal MS. Here $l=2$, $\gamma=2$ for dashed curves, and $\gamma=2.5$ for solid curves in KPMS and Step MS cases, while $\gamma=4$ for dashed curves and $\gamma=5$ for solid curves in Sinusoidal MS case. n is the number of the magnetic barriers (also the number of magnetic period except for $n=1$ case of half the magnetic period).

where

$$T_M=\begin{bmatrix}T_{11} & T_{12}\\ T_{21} & T_{22}\end{bmatrix}=\prod_{j=1}^{M}T_M^j$$
$$=\prod_{j=1}^{M}\begin{bmatrix}\cos(k_j a) & -\sin(k_j a)/k_j\\ k_j\sin(k_j a) & k_j\cos(k_j a)\end{bmatrix}, \quad (6)$$

where T_M^j is the transfer matrix for the jth segment.

Transmission coefficient $T(E,k_y)$ for electron tunneling through the n-barrier MS can be readily obtained from Eq. (5)

$$T(E,k_y)=t^2=\{1+(T_{11}^2+T_{22}^2+k^2T_{12}^2+T_{21}^2/k^2-2)/4\}^{-1}. \quad (7)$$

With the transmission coefficient, we calculate ballistic conductance from the well-known Landaur-Büttiker formula[10]

$$G/G_0=\int_{-\pi/2}^{\pi/2}T(E_F,\sqrt{2E_F}\sin\theta)\cos\theta d\theta, \quad (8)$$

where θ is the angle between the incidence velocity and the x axis, E_F is the Fermi energy, $G_0=e^2m^*v_F l_y/\hbar^2$, and v_F is the Fermi velocity of electrons.

First, ballistic conductances (in units of G_0) versus incidence energy in Kronig-Penny magnetic superlattice (KPMS) were studied. Our results, shown in the left column of Fig. 2, are calculated for the different number n of mag-

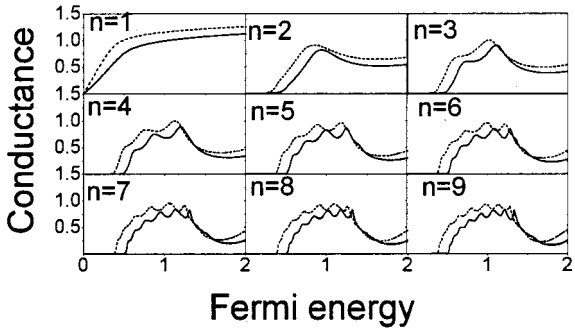

FIG. 3. Ballistic conductance for Sawtooth MS. Here, $l=2$, $\gamma=3$ for dashed curves, and $\gamma=3.5$ for solid curves.

netic barriers (which is also the number of the periods of magnetic superlattice except for $n=1$ case of half the magnetic period). The structure parameters are chosen to be $l=2$, $\gamma=2.5$ for solid curves and $l=2$, $\gamma=2$ for dashed curves. Let us inspect the conductance splitting at the magnetic strength $\gamma=2$. It is obvious that no resonant peak exists in the ballistic conductance for the single-magnetic-barrier case. One resonant peak is seen for double magnetic barriers and one sharper spike along with one resonant shoulder appears for triple-magnetic barriers. With the increase of the number n of magnetic barriers in KPMS, the total number of resonant conductance spikes and shoulders increases along with the resonant peaks and shoulders becoming sharper. As $n\rightarrow\infty$, the peaks will fill in the energy windows of the magnetic minibands continuously as in the periodic ES.[7] By counting the number of resonant peaks and resonant shoulders in n-barrier KPMS, we found that, the number of resonant peaks and resonant shoulders, or the number of resonance splitting equals to $n-1$, which is the number of the magnetic barriers in KPMS. This is the corresponding $(n-1)$-fold resonant peak splitting for ballistic conductance in KPMS, which is similar to the $(n-1)$-fold resonant splitting for transmission in n-barrier ES. The splitting rules for ballistic conductance in KPMS is exactly the same as that for transmission in ES. With the magnetic strength γ increasing, the resonant shoulders become resonant spikes and the resonant peaks are resolved more clearly. More importantly, the $(n-1)$-fold resonant peak splitting for ballistic conductance is unchanged.

To find out the general rules for resonant peak splitting of ballistic conductance in MS of arbitrary magnetic-barrier profile, we calculated ballistic conductances versus incidence energy for Step MS in the middle column of Fig. 2, Sinusoidal MS in the right column of Fig. 2, and Sawtooth MS in Fig. 3. The parameters for the calculated conductances of Step MS are the same as for the KPMS. While the parameters for Sinusoidal MS are set to be $l=2$, $\gamma=4$ for dashed curves and $l=2$, $\gamma=5$ for solid curves. In Fig. 3, $l=2$, $\gamma=3$ are chosen for dashed curves and $l=2$, $\gamma=3.5$ for solid curves. From Figs. 2 and 3, one can also observe clearly resonant splitting of the ballistic conductance peaks in Step MS, Sinusoidal MS, and Sawtooth MS. By checking the number of resonant peaks in ballistic conductances for Step MS, Sinusoidal MS, and Sawtooth MS, we found that the number of resonant conductance peaks in the n-barrier Step MS, Sinusoidal MS, and Sawtooth MS is also $n-1$ (n is the number of magnetic barriers). This indicates the existence of a general $(n-1)$-fold resonant splitting of conductance peaks in MS with n identical magnetic barriers, which is independent of the magnetic-barrier profile. Then, we can generalize this rule as follows: for electron tunneling through magnetic superlattices with periodically arranged n identical magnetic barriers, $(n-1)$-fold resonant peak splitting exists in ballistic conductance within each magnetic miniband. It is a general rule as the $(n-1)$-fold resonant peak splitting for transmission in n-electric-barrier superlattices. For transmission of electron tunneling in magnetic superlattices, there is no such general splitting rule, since it is strongly dependent on the wave vector (momentum) normal to the tunneling direction. It is worth noting that the resonant peaks in ballistic conductances within lower energy minibands will be suppressed and that within higher energy minibands will be resolved gradually by the further-increased magnetic strength.[4]

As is well known, for electron tunneling through electric superlattice, when the incidence energy of electrons coincides with the energy of bound states in potential well, the resonant tunneling occurs (i.e., the transmission is 1). Because of the coupling between the wells via tunneling through the barriers of finite width, the degenerate eigenlevels of the independent wells are split, consequently, these split levels redistribute themselves into groups around their unperturbed positions and form quasibands. This leads to the resonant splitting of transmission. As the number of periods (or the number of barriers) tends to infinity, the locally continuous energy distribution (energy band) is formed. Although electron tunneling in MS is more complicated than in ES due to its dependence on the perpendicular wave vector k_y (Ref. 4), electron tunneling in MS is equivalent to that in ES for a given k_y from the mathematical viewpoint. The resonant tunneling of electrons in MS results from the same physics as ES. We attributed the resonant peak splitting for ballistic conductance in magnetic superlattices to a collective effect of electron's wave-vector-dependent tunneling. Though the number of resonant transmission peaks in electron's tunneling through MS is closely related to the wave vector k_y and may be different for different k_y, on an average, the number of resonant conductance peaks is the same as the number of wells in magnetic vector potential of MS. Since ballistic conductance is derived as the transmission averaged over all the possible wave vectors k_y, it can be viewed as the transmission of the electron's collective tunneling with a characteristic k_y through an average effective potential $V_{aver}(x)$, which has the same number of wells as the magnetic vector potential $A(x)$. This can be clearly seen if we plot the effective potential $V(x,k_y)$ as a function of x and k_y as did by Ibrahim and Peters,[5] the number of the main wells in the effective potential $V(x,k_y)$ really equals to the number of the wells in the magnetic vector potential $A(x)$ on the whole and on average. Because the number of magnetic barriers in MS equals to the number of barriers in the corresponding magnetic vector potential, as can be seen from Fig. 1, $(n-1)$-fold resonant splitting occurs in the ballistic conductance peaks of n-barrier MS.

In summary, we studied the resonant peak splitting effects for ballistic conductance in four kinds of magnetic superlat-

tices of finite periods with identical magnetic barriers. It is found that there is a general $(n-1)$-fold resonant peak splitting rule for ballistic conductance in n-identical-barrier magnetic superlattices, which is the analogy of $(n-1)$-fold transmission splitting in n-barrier electric superlattices.

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[1] P. Vasilopoulos and F. M. Peeters, Superlattices Microstruct. **4**, 393 (1990); D. P. Xue and G. Xiao, Phys. Rev. B **45**, 5986 (1992); F. M. Peeters and P. Vasilopoulos, *ibid.* **47**, 1466 (1993).

[2] H. A. Carmona *et al.*, Phys. Rev. Lett. **74**, 3009 (1995); P. D. Ye *et al.*, *ibid.* **74**, 3013 (1995).

[3] F. M. Peeters and A. Matulis, Phys. Rev. B **48**, 15 166 (1993).

[4] A. Matulis, F. M. Peeters, and P. Vasilopoulos, Phys. Rev. Lett. **72**, 1518 (1994); S. S. Allen and S. L. Richardson, Phys. Rev. B **50**, 11 693 (1994); J. Q. You, Lide Zhang, and P. K. Ghosh, *ibid.* **52**, 17 243 (1995); Yong Guo, Binlin Gu, and Wenhui Duan, *ibid.* **55**, 9314 (1997).

[5] L. S. Ibrahim and F. M. Peeters, Phys. Rev. B **52**, 17 321 (1995); A. Krakovsky, *ibid.* **53**, 8469 (1996).

[6] R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973).

[7] Xue-Wen Liu and A. P. Stamp, Phys. Rev. B **50**, 1588 (1994).

[8] Yong Guo, Bin-lin Gu, Zhi-Qiang Li, Jing-Zhi Yu, and Yoshiyuk Kawazoe, J. Appl. Phys. **83**, 4545 (1998).

[9] J. Q. You and Lide Zhang, Phys. Rev. B **54**, 1526 (1996).

[10] M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

[11] P. D. Ye, D. Weiss, R. R. Gerhardts, M. Seeger, K. Von. Klizing, K. Eberl, and H. Nickel, Phys. Rev. Lett. **74**, 3013 (1995).