Nonstationary Josephson effect for superconductors with spin-density waves

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The nonstationary Josephson I^1, interference I^2, and quasiparticle J current amplitudes through symmetrical and nonsymmetrical (ns) tunnel junctions involving partially dielectrized (partially gapped) superconductors with spin-density waves (SDW's) and described by the dielectric order parameter Σ were calculated. Riedel-type singularities and jumps for all currents are determined by superconducting order parameter Δ, as well as Σ. It was shown that each current for symmetrical junctions, made up of *thermodynamically* identical SDW superconductors (with $|\Sigma_{\rm left}|=|\Sigma_{\rm right}|$), can exhibit three possible current-voltage characteristics (CVC's) depending on the relationship between the electrodes' Σ signs in the course of experiment: (i) one symmetrical CVC for genuinely symmetrical configuration (s) when the dielectric order parameters $\Sigma_{\rm left}$ and $\Sigma_{\rm right}$ have the same sign for both electrodes and (ii) two nonsymmetrical ones for symmetrical junction in the state of broken symmetry (bs) when $\Sigma_{\rm left}$ and $\Sigma_{\rm right}$ are opposite in sign. The actual setup choice is at random. Thus, the symmetrical junctions can serve as phase-sensitive indicators for the SDW's. The ns junctions which include SDW and ordinary BCS superconductors are also studied. The current amplitudes $I_{ns}^{1,2}$ and J_{ns} are asymmetrical functions of the voltage V and depend on the sign of Σ. Our calculations reveal main features appropriate to CVC's of tunnel junctions involving the heavy-fermion SDW superconductor URu_2Si_2.
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I. INTRODUCTION

Soon after the discovery of the Bardeen-Cooper-Schrieffer (BCS) superconductivity mechanism, which was originally applied to the s-wave spin-singlet Cooper pairing,[1] the problem of the possible compatibility between some kind of magnetic ordering and superconductivity came into being.[2,3]

Conditions for the coexistence between superconductivity and antiferromagnetism are much more favorable than in the ferromagnetic case.[3–5] Really, on the scale of the Cooper pair radius, i.e., the correlation length, the average magnetic induction (the acting magnetic field) is zero in antiferromagnetics. That is why s-wave superconductivity can survive here. Moreover, it has been long ago suggested that the spin fluctuation exchange in antiferromagnetics may constitute the genuine pairing mechanism in this case (see, e.g., Ref. 4). The antiferromagnetic rare-earth-based superconductors are quite numerous and seem to be spin-singlet ones of the s-wave type: RMo$_6$S$_8$ (R=Gd, Tb, Dy, Er), RRh$_4$B$_4$ (R=Nd, Sm, Tm), R(Rh$_{1-x}$Ir$_x$)$_4$B$_4$ (R=Ho, Tb), ErMo$_6$Se$_8$,[3,4] RNi$_2$B$_2$C (R=Sc, Y, Th, Lu, Tm, Ho, Dy, Er).[6] But in the ternary families RMo$_6$S$_8$, RMo$_6$Se$_8$, RRh$_4$B$_4$ and their pseudoternary derivatives, or in borocarbides, magnetic properties are determined by the interacting rare-earth ions with partially filled f levels, whereas superconductivity takes its origin from the joint itinerant electron system of all atoms.

On the other hand, there are metals with an antiferromagnetism of the spin-density wave (SDW) type, i.e., generated by the spin susceptibility divergence at the definite wave vector **Q** below the Néel temperature, T_N.[5,7–9] This logarithmic divergence is the consequence of the Fermi surface (FS) nesting. So, the SDW state is a close relative to the charge-density-wave (CDW) state. The latter is also a result of the FS nesting below the structural phase transition temperature T_d. Two types of the low temperature phase are possible, namely, the excitonic phase induced by electron-hole Coulomb interaction, or the Peierls insulator (for electron-phonon interaction).[10–12]

In SDW metals superconductivity emerges, if at all, in the same electron system modulated by SDW's. Cooper and antiferromagnetic dielectric pairings compete here for the FS. It is precisely in this manner that magnetic correlations try to destroy superconductivity, as opposed to the ferromagnetic case discussed above. The result of the competition may be disastrous for superconductivity if the electron spectrum dielectrization (gapping) becomes complete. As an example, one should mention the majority of Bechgaard salts (TMTSF)$_2X$, where X=ReO$_4$, PF$_6$, AsF$_6$, TaF$_6$, SbF$_6$, SO$_3$, NO$_3$, at the ambient pressure,[7,13,14] organic superconductors (DMET)$_2$Au(CN)$_2$, (MDT-TTF)$_2$Au(CN)$_2$,[7] and oxide families with high critical temperatures, T_c, La$_{2-x}$[Ca(Ba,Sr)]$_x$CuO$_{4-y}$ and YBa$_2$Cu$_3$O$_{7-y}$ for nonoptimal doping concentrations x and y (''parent'' SDW state).[15]

Subtle relationships between relevant and not yet fully recognized parameters can lead to the partial SDW dielectrization. Then a substance remains metallic down to $T=0$ and the s-type superconducting order parameter develops both on the dielecrtization-free nonnested (nd) and dielectrized (d) FS sections.[16–25] The phases with coexisting superconductivity and SDW's are observed in (TMTSF)$_2$ClO$_4$ at ambient pressure,[14] in the related organic superconductor (TMTSF)$_2$PF$_6$ at pressure 6 kbar,[26] Cr$_{1-x}$Re$_x$ ($x>0.18$) alloys,[8,27] the compounds RRh$_2$Si$_2$ (R=La, Y),[28] the heavy-fermion superconductors URu$_2$Si$_2$,[29,30] UR_2Al$_3$ (R=Ni, Pd),[31,32] Laves-phase compound CeRu$_2$,[33] in alloys RNi$_2$B$_2$C (R=Ho$_{1-x}$Dy$_x$, Lu$_{1-x}$Dy$_x$, Er),[34] and possibly in compound YbBiPt.[35]

There is no evidence thus far that the SDW superconducting state with the complete FS dielectrization occurs in the substances mentioned above or any other objects. Nevertheless, much effort has been put forth to describe such a state theoretically.[36,37] The authors often claim that one can easily extend the corresponding results to the partial dielectrization case.[36] But it is not true because in the fully dielectrized (perfectly nested) system there are no free carriers left for superconducting transition to happen. The only way out is to introduce the intrinsic doping shift $\delta\mu$ of the chemical potential into the primordial electron spectrum.[11] After such a modification the Fermi level is no longer within the dielectric gap but lies in the conduction or valence band depending on the $\delta\mu$ sign. In our opinion, the experimental data are more adequately described by the partial dielectrization picture[16–25] originally brought into being by Bilbro and McMillan[38] for CDW superconductors or normal metals.

In the CDW case a great body of information supports the concept of a metal which loses below T_d the nested FS sections as a free carrier source (see details in Refs. 16,19,39). Much less experimental data are available for itinerant SDW superconductors. But the same idea seems to remain valid here too.[29,30]

In this paper we carry on an investigation of the partially dielectrized SDW *s*-wave superconductors on the basis of the Bilbro-McMillan model,[38] i.e., accepting the spin-triplet dielectric gapping to make an adverse effect on Cooper pairing by the FS distortion. The heavy-fermion compound URu_2Si_2 serves as the reference object because the relevant model parameters have been measured for it (although the available results scatter quantitatively[29,30]).

As was mentioned above, the theory of thermodynamic and electrodynamic properties of SDW superconductors is well elaborated.[16–21] On the other hand, in order to reveal the specific features of these materials the powerful methods of tunnel current-voltage characteristic (CVC) measurements[40] would be of fundamental importance. Unfortunately, their theoretical background for SDW superconductors has not yet been established. The only exceptions are calculations of the quasiparticle CVC $J(V)$ and conductivity $G^{\rm diff}(V)=dJ/dV$ in the complete dielectrization scheme[37] and the temperature dependences of the critical stationary Josephson current.[20]

Here we consider the more general case of the nonstationary Josephson effect in tunnel junctions with one or both electrodes being partially dielectrized SDW superconductors. The amplitudes of the Josephson I^1, interference pair quasiparticle I^2, and quasiparticle J currents through tunnel junctions as the functions of the bias voltage V are calculated. The analysis is similar to that for CDW superconductors[39] (hereafter Ref. 39 will be denoted as paper I). In the absence of superconductivity the results for $J(V)$ are identical with CVC's analyzed in Ref. 41. We should note that CVC's for antiferromagnetic superconductors are much more involved than their counterparts for CDW superconductors due to the more complicated character of the former substances' quasiparticle spectrum.

The plan of the article is the following. In Sec. II we formulate the problem of superconductivity in a partially dielectrized metal with SDW's. In Sec. III the general dependences of the tunnel currents $I^{1,2}$ and J between SDW superconductors on the bias voltage V are discussed. In Sec. IV this treatment is applied to the most important special cases of symmetrical (S_Σ-I-S_Σ) and nonsymmetrical (S_Σ-I-$S_{\rm BCS}$) junctions (ns). Here S_Σ denotes SDW superconductor, $S_{\rm BCS}$ is an ordinary BCS superconductor, and I is an insulator. Section V is devoted to the detailed analysis of the CVC feature points for *s*, bs, and ns junctions and contains the illustrative calculations. The discussion of the available experimental data can be found in Sec. VI. Since the approach is similar to that of paper I, we give references to paper I where needed.

II. PARTIALLY DIELECTRIZED SDW SUPERCONDUCTORS

The model Hamiltonian of the partially gapped (partially dielectrized) SDW superconductor has the form[23,24,38,39,41]

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{\rm MF}. \tag{1}$$

Here

$$\mathcal{H}_0=\sum_{i=1}^{3}\sum_{\mathbf{p}\alpha}\xi_i(\mathbf{p})a^\dagger_{i\mathbf{p}\alpha}a_{i\mathbf{p}\alpha} \tag{2}$$

is the free-electron Hamiltonian. The operator $a^\dagger_{i\mathbf{p}\alpha}$ ($a_{i\mathbf{p}\alpha}$) is the creation (annihilation) operator of a quasiparticle with a quasimomentum $\mathbf{p}$ and spin projection $\alpha=\pm\frac{1}{2}$ from the ith FS section. Namely, $i=1$ and 2 for the nested sections where the electron spectrum is degenerate

$$\xi_1(\mathbf{p})=-\xi_2(\mathbf{p}+\mathbf{Q}), \tag{3}$$

$\mathbf{Q}$ being the SDW vector, while $i=3$ for the rest of the FS where the dispersion relation for elementary excitations is described by the different function $\xi_3(\mathbf{p})$.

The mean-field term $\mathcal{H}_{\rm MF}$ of the Hamiltonian (1)

$$\mathcal{H}_{\rm MF}=H_{\rm BCS}+H_{\rm SDW} \tag{4}$$

is the sum of the BCS term

$$\mathcal{H}_{\rm BCS}=-\Delta\sum_{i=1}^{3}\sum_{\mathbf{p}}a^\dagger_{i\mathbf{p}\uparrow}a^\dagger_{i,-\mathbf{p},\downarrow}+\text{H.c.}, \tag{5}$$

which leads to superconductivity, and the SDW term

$$\mathcal{H}_{\rm SDW}=-2\Sigma\sum_{i=1}^{2}\sum_{\mathbf{p}\alpha}\alpha a^\dagger_{i\mathbf{p}\alpha}a_{i,\mathbf{p}+\mathbf{Q},\alpha}+\text{H.c.} \tag{6}$$

describing the electron-hole spin-triplet excitonic pairing (cf. with $\mathcal{H}_{\rm CDW}$ term in paper I). The dielectric order parameter Σ emerges on the nested FS sections, so the summation in Eq. (6) is carried out over them only. On the other hand, the *single* superconducting order parameter Δ appears on the whole FS. As was explained in paper I, the quantity Σ can be taken as independent of Δ phenomenological function of T. Hereafter we assume the pinning of SDW's (see Sec. III), so the order parameter Σ is real and can be of either sign.[41,42] A possible imaginary component of the spin-triplet dielectric order parameter would mean the emergence of spin-current density waves.[10] We do not know any indications for such a phenomenon to exist, so this interesting opportunity is left beyond the scope of the article.

In all SDW superconductors that are discussed here the inequality $T_N>T_c$ (or even $T_N\gg T_c$) holds. Therefore, the exact form of the function $\Sigma(T)$ is not crucial for the determination of $\Delta(T)$ dependence. Moreover, the calculations of Machida[23] showed that below T_c the existence of the superconducting gap Δ stabilizes the magnitude of Σ at a certain constant level in much the same fashion as for CDW superconductors.[38] Taking into account the above-mentioned circumstances we did not make the self-consistent calculations of $\Delta(T)$ and $\Sigma(T)$ in this paper. Instead, we suggested for $\Sigma(T)$ two possibilities. According to the first one, Σ is a trivial constant. The second function $\Sigma(T)$ chosen here is the BCS dependence[1] inherent, e.g., to the SDW spin-triplet excitonic insulator.[9,11] As for the experiment, muon spin rotation data[7,43] show that in $(TMTSF)_2PF_6$ the T dependence of the normalized dielectric order parameter deviates from the BCS curve, being much steeper. The other metal URu_2Si_2, which undergoes a transition into the SDW state, demonstrates an almost BCS shape of $\Sigma(T)$, that was revealed by the point-contact measurements,[44–47] although the results of Ref. 47 seem to be closer to those for $(TMTSF)_2PF_6$.[43] But we should note that due to the nonsymmetrical character of the CVC's for URu_2Si_2 and the anisotropy of $\Sigma(T)$, its extraction from the data is rather ambiguous.

As might be expected, these two possibilities appeared to be almost indistinguishable for CVC's in the voltage range $|eV|\leqslant\Delta$, e being the elementary charge, although detectable for voltages of the order of $|\Sigma|$. Given these facts, we performed most calculations using the BCS curve for $\Sigma(T)$ because the specific choice is not important from the conceptual point of view.

Beyond Δ and Σ, there is another essential characteristic of the partially dielectrized superconductor. Let $N(0)$ be the total electron density of states on the FS. It is the sum of the densities of states for dielectrized and nondielectrized parts of the FS,

$$N(0)=N_d(0)+N_{\mathrm{nd}}(0). \tag{7}$$

The ratio

$$\nu=\frac{N_{\mathrm{nd}}(0)}{N_d(0)} \tag{8}$$

was introduced[38] to characterize the gapping degree of the metal. It may vary from 0 (the case of complete dielectrization) to infinity when the FS gapping is absent.

The T dependence of the superconducting gap Δ for partially dielectrized SDW superconductor can be easily found from our theory developed earlier,[17,18,20,21] using the function $\Sigma(T)$ discussed above and appropriate values of the control parameter ν.

The normal $\mathsf{G}_{ij}^{\alpha\beta}(\mathbf{p};\omega_n)$ and anomalous $\mathsf{F}_{ij}^{\alpha\beta}(\mathbf{p};\omega_n)$ Matsubara Green's functions corresponding to the Hamiltonian (1) can be found from the Dyson-Gor'kov equations obtained earlier.[16,18–21,42] They are matrices in the space which is the direct product of the spin space and the isotopic space of the FS sections.[16,18,19] The explicit expressions for $\mathsf{G}_{ij}^{\alpha\beta}$ and $\mathsf{F}_{ij}^{\alpha\beta}$ will be given elsewhere.

For CDW superconductors the Green's function $\mathsf{F}_{is}\equiv 0$, whereas for SDW ones it is no more true, resulting in the increase of the component numbers for tunnel currents against the CDW case. Both these circumstances lead to much more complex structure of CVC's for tunnel junctions including SDW superconductors.

To calculate the tunnel currents we need temporal Green's functions $F(\omega)$ and $G(\omega)$ rather than the temperature ones. They are obtained in the conventional manner.[48,49]

III. TOTAL CURRENT THROUGH THE TUNNEL JUNCTIONS

To calculate the total tunnel current I through the junction we use the conventional tunnel Hamiltonian approach,[40,48] according to which the Hamiltonian has the form

$$\mathcal{H}_{\mathrm{tun}}=\mathcal{H}+\mathcal{H}'+\mathcal{T}. \tag{9}$$

The left- and right-hand-side electrodes of the junction are described in Eq. (9) by the terms $\mathcal{H}$ and $\mathcal{H}'$, respectively, which coincide with the Hamiltonian (1) with an accuracy of notations. Hereafter primed entities including subscripts and superscripts correspond to the right-hand side of the junction. The tunnel term $\mathcal{T}$ is of the form

$$\mathcal{T}=\sum_{i,i'=1}^{3}\sum_{\mathbf{pq}'\alpha}\mathsf{T}_{\mathbf{pq}'}^{ii'}a_{i\mathbf{p}\alpha}^{\dagger}a_{i'\mathbf{q}'\alpha}+\mathrm{H.c.}, \tag{10}$$

where $\mathsf{T}_{\mathbf{pq}'}^{ii'}$ are the tunnel matrix elements. The general expression for $I(T)$ obtained in the lowest order of the perturbation theory in $\mathcal{T}$ is a sum of functionals depending on temporal Green's functions $F(\mathbf{p},\tau)$ and $G(\mathbf{p},\tau)$, where τ denotes time (see paper I). The Green's functions $F(\mathbf{p},\tau)$ and $G(\mathbf{p},\tau)$, integrated over $\mathbf{p}$ variable, are connected to $F(\omega)$ and $G(\omega)$ by Fourier transformation. Making the assumptions (see discussion in paper I) that (i) all matrix elements $\mathsf{T}_{\mathbf{pq}'}^{ii'}$ are equal and not influenced by the existence of Δ and Σ, in the spirit of the standard Ambegaokar-Baratoff approach,[40] and (ii) the current I is independent of the relative spatial orientation of the junction plane and the SDW vector $\mathbf{Q}$, we introduce the universal tunnel resistance R:

$$R^{-1}=4\pi e^2 N(0)N'(0)\langle|\mathsf{T}|^2\rangle_{\mathrm{FS}}. \tag{11}$$

Here angular brackets $\langle\cdots\rangle_{\mathrm{FS}}$ imply averaging over the FS. Then, on the basis of the Green's function set and using the adiabatic approximation $V^{-1}dV/d\tau\ll T_c$ for the ac bias voltage $V(\tau)\equiv V_{\mathrm{right}}(\tau)-V_{\mathrm{left}}(\tau)$ across the Josephson junction, we obtain the nine-term expression for the total current I through the junction made up of the SDW superconductors, which is a generalization of that for the BCS-superconductor case[48]

$$I[V(\tau)]=\sum_{i=1}^{9}[I_i^1(V)\sin 2\phi+I_i^2(V)\cos 2\phi+J_i(V)], \tag{12}$$

where $\phi=\int^{\tau}eV(\tau)d\tau$, $I^1=\Sigma_{i=1}^{9}I_i^1$ is the Josephson current, $I^2=\Sigma_{i=1}^{9}I_i^2$ is the interference pair-quasiparticle current, and $J=\Sigma_{i=1}^{9}J_i$ is the quasiparticle current. The explicit expressions for $I_i^{1,2}$ and J_i are cumbersome and will be given elsewhere.

The phases ϕ and ϕ' of the superconducting order parameters are, as usual,[40] considered free, with their difference $\phi_{\mathrm{diff}}=\phi'-\phi$ obeying the above-given Josephson relationship connecting it to the bias voltage. On the contrary, when obtaining Eq. (12), we made a suggestion of the strong SDW pinning by lattice defects or impurities, so their phases χ and χ' on either side of the junction are fixed. In the absence of pinning, it is known from the fundamental generic models of the dielectric pairing, e.g., the Hubbard model describing the electron-electron interactions,[7,9] that the phase of the SDW (and consequently the phase χ of the order parameter $\Sigma \equiv |\Sigma| e^{i\chi}$) is arbitrary.

Pinning prevents SDW sliding in quasi-one-dimensional compounds for small electric fields, whereas for large ones various coherent phenomena of the Josephson type, e.g., Shapiro steps on CVC's, become possible.[7] For excitonic insulators the behavior is more complicated. In particular, the phase χ is fixed by Coulomb interband matrix elements (linking FS sections 1 and 2) corresponding to two-particle transitions V_2, and by the interband electron-phonon interaction described by the constant $\lambda_{\text{el-ph}}$.[10,11,50] Moreover, the excitonic transitions due to the finite values of V_2 and $\lambda_{\text{el-ph}}$ are always of the first order although close to the second order transitions.[50] The contribution from the single-particle Coulomb interband matrix elements V_3, which connect three particles from, say, FS section 1 and one particle from FS section 2, or vice versa, results in even more radical consequences. Namely, the self-consistency equation for the order parameter Σ becomes nonhomogeneous, with the right-hand side proportional to V_3. This leads to the fixation of the phase χ.[51]

With due regard of all these factors, we have calculated the quasiparticle current $J(V)$ between two partially dielectrized normal metals (here the results for SDW's and CDW's are identical, as was mentioned before).[41] The expressions obtained comprise the particular case of Eq. (12). On the other hand, if the phase fixation causes are neglected, the quasiparticle current between two Peierls insulators involves the term proportional to $\cos(\chi'-\chi)$.[52,53] We note that for the case of full gapping ($\nu=\nu'=0$) our results for symmetrical junctions agree with those of Ref. 52 when the oscillating term is averaged out. On the contrary, the expressions for tunnel conductances from Ref. 53 cannot be reduced to ours.

The equivalence of the dielectrized FS sections 1 and 2 in the SDW superconductor reduces the number of Green's functions for each electrode. Namely, they are $F_d(\omega), F_{\text{nd}}(\omega), F_{is}(\omega), G_d(\omega), G_{\text{nd}}(\omega)$, and $G_{is}(\omega)$. All other Green's functions vanish. Thus, to obtain each tunnel current amplitude $I^{1,2}$ or J between two different partially gapped superconductors only three Green's functions for each electrode are needed, namely, $F_d(\omega), F_{\text{nd}}(\omega)$, and $F_{is}(\omega)$ to calculate $I^{1,2}$ and $G_d(\omega), G_{\text{nd}}(\omega)$, and $G_{is}(\omega)$ to calculate J.

IV. CURRENT-VOLTAGE CHARACTERISTICS

Below we shall confine ourselves to symmetrical (i.e., when both electrodes are identical SDW superconductors) and nonsymmetrical (i.e., when one electrode is an ordinary BCS superconductor) junctions. At the same time, our results obtained for CDW superconductors and normal metals show that a junction commonly assumed as symmetrical may not always reveal the corresponding behavior.[54]

A. *s* junctions S_Σ-I-S_Σ

The symmetrical S_Σ-I-S_Σ junction is the ideal limiting case when the difference between the relevant thermodynamic parameters characterizing both electrodes can be considered negligible. For such *s* junctions $\nu=\nu'$, $\Sigma=\Sigma'$, $\Delta=\Delta'$, and

$$F_d=F'_d,\quad F_{\text{nd}}=F'_{\text{nd}},\quad F_{is}=F'_{is},$$

$$G_d=G'_d,\quad G_{\text{nd}}=G'_{\text{nd}},\quad G_{is}=G'_{is}. \tag{13}$$

Then the total current (12) through the junction may be decomposed into four different components. The relevant current amplitudes $I^{1,2}_{si}$ and J_{si} can be deduced easily. For them the usual symmetry relations hold. For *s*-junctions CVC's of all three currents do not depend on the sign of Σ.

B. bs junctions S_Σ-I-$S_{-\Sigma}$

For formally symmetrical junctions involving identical SDW superconductors an alternative opportunity may be realized. Namely, the *symmetry breaking* can take place, i.e., the left-hand partially gapped electrode possessing, say, a positive dielectric order parameter $\Sigma>0$ and the right-hand one having a negative parameter $\Sigma'=-\Sigma<0$, or vice versa. In both cases the junction is nonsymmetrical in reality, although $|\Sigma|=|\Sigma'|$ and all macroscopical properties of each separated electrode are *identical* due to the thermodynamical equivalence of SDW superconductors with equal Δ's and $|\Sigma|$'s.[18,23] However, if the junction concerned is a part of the electric circuit, it will serve as a phase-sensitive indicator of the symmetry breaking between the electrodes.[54] Such a phenomenon comprises a new macroscopical manifestation of the symmetry breaking in many-body systems. The corresponding CVC's are substantially different from their genuinely symmetrical counterparts. Really, for this state of a junction $\nu=\nu'$, $\Delta=\Delta'$, $\Sigma=-\Sigma'$,

$$F_d=F'_d,\quad F_{\text{nd}}=F'_{\text{nd}},\quad G_d=G'_d,\quad G_{\text{nd}}=G'_{\text{nd}}, \tag{14}$$

but

$$F_{is}=-F'_{is},\quad G_{is}=-G'_{is}. \tag{15}$$

Then the total current I_{bs} consists of six different terms. For two of them (which are absent in the *s* case) the dependence on the voltage polarity is inverse. It affects crucially the total current amplitudes $I^{1,2}_{\text{bs}}(V)$ and $J_{\text{bs}}(V)$, making them neither symmetrical nor antisymmetrical in V. Therefore, the CVC's for total currents in the bs case depend on the voltage polarity. Furthermore, with changing Σ sign, the different V-polarity branches are interchanged. These phenomena are analogous to the polarity and the Σ-sign dependences of CVC's for the ns junction.

One should bear in mind that a spontaneous symmetry breaking in isolated bulk SDW superconductors (or normal metals) has no observable consequences since thermodynamical properties of such objects are the same for any sign of Σ.[16,18,19,21] The existence of two electrically connected pieces of SDW superconductors makes the symmetry breaking *macroscopically observable*. Fluctuations act here as a driving force promoting selection between four possible states: S_Σ-I-S_Σ,$S_{-\Sigma}$-I-$S_{-\Sigma}$,S_Σ-I-$S_{-\Sigma}$, and $S_{-\Sigma}$-I-S_Σ. The first two equivalent possibilities correspond to the genuinely symmetrical case (s), whereas the last two potentialities represent different alternatives of the broken-symmetry case (bs). Thus, we obtain for a symmetrical junction a discrete set of states corresponding to various possible combinations of the dielectric order parameter signs in the electrodes. The statistical weight of the s-state is twice that for each of the bs ones.

The frustrated junction between SDW (CDW) superconductors above or below T_c, but of necessity below T_N (T_d), can be treated as a discrete analog, with respect to the relative phase difference, of the Josephson junction. It is radically different, however, from the SDW counterpart of the phase-coherent weak link between two Peierls insulators with sliding CDW's considered by Artemenko and Volkov.[52] Unlike these authors, we assume the pinning of the Σ and Σ' phases, therefore ruling out coherent effects. Nevertheless, the junction concerned feels the difference or coincidence between the dielectric order parameter signs. Thus, the symmetry breaking in the symmetrical junction serves as a detector of the order-parameter phase multiplicity in electrodes. This is also common to nonsymmetrical junction.

C. ns junctions S_Σ-I-S_{BCS}

In the nonsymmetrical case $N_d'(0)=0$, so, according to Eq. (8), $\nu'=\infty$. Then, only the following Green's functions are inherent to this junction: F_d, F_{nd}, F_{is}, G_d, G_{nd}, G_{is}, $F'=F_{\mathrm{BCS}}$, and $G'=G_{\mathrm{BCS}}$, where F_{BCS} and G_{BCS} are the Green's functions of the BCS superconductor. Then only three terms for each current amplitude $I^{1,2}_{\mathrm{ns}i}$ and $J_{\mathrm{ns}i}$ survive.

The main difference between SDW and CDW superconductors (cf. with paper I) is the loss of CVC symmetry not only for quasiparticle but also for Josephson and interference currents. That is, all three relevant CVC's in ns junctions including SDW superconductors depend on the voltage polarity, contrary to the well-known polarity independence for nonsymmetrical junctions involving different BCS superconductors.[40]

V. RESULTS OF CALCULATIONS

For ordinary BCS superconductors the CVC's $I^{1,2}(V)$ and $J(V)$ possess logarithmic singularities (the so-called Riedel peaks) and discontinuities at certain gap-determined biases.[40,48,55] The character and magnitudes of peculiarities for different kinds of currents are correlated according to the Kramers-Kronig relations between them.[40,55] The CVC's for CDW superconductors are much more involved (see paper I) due to the presence of two gaps, superconducting and dielectric, in the quasiparticle spectrum. Nevertheless, the case of SDW superconductors is even more complicated. First, there are *two* "effective combined gaps" $|D_\pm|=\Delta\pm\Sigma$ of the quasiparticle spectrum. It results in the doubling of all Σ-governed peculiarity points in comparison with the CDW case. Second, for the s and bs junctions, taking into account the *linear* dependence of $D_\pm$ on Δ, the possibility of various combinations of $|D_\pm|$ and Δ for current components involving dissimilar FS sections leads to the superposition of singularities of different types (jumps and logarithmic peaks) at the same biases, the situation never occurring for junctions involving CDW superconductors, not to say about BCS ones. Although originating from different terms, they jointly produce the as yet unfamiliar CVC features for each current. The characteristic features can be evaluated for arbitrary temperatures. In all figures below we mark logarithmic peaks, which are hardly seen at the chosen scale, by single arrows. Double arrows mark the positions of tiny jumps.

TABLE I. Types and positions of CVC peculiarities inherent in components of currents through s and bs junctions. [a]

$\lvert eV\rvert$	Type [b]	Position [c]	Components [d]
2Δ	1	2Δ	4
$2\lvert D_\pm\rvert$	1	$2(\Sigma\pm\Delta)$	1,2,5
$H_+=\lvert\lvert D_+\rvert+\lvert D_-\rvert\rvert$	1	2Σ	1,2
$H_-=\lvert\lvert D_+\rvert-\lvert D_-\rvert\rvert$ [e]	2	2Δ	1,2,5
$M_+=\lvert D_+\rvert+\Delta$	1	$2\Delta+\Sigma$	3,6
$M_-=\lvert D_-\rvert+\Delta$	1	Σ	3,6
$N_+=\lvert\lvert D_+\rvert-\Delta\rvert$ [e]	2	Σ	3,6
$N_-=\lvert\lvert D_-\rvert-\Delta\rvert$ [e]	2	$\lvert 2\Delta-\Sigma\rvert$	3,6

[a]In the s case the choice $\Sigma>0$ is made. In the bs case $\Sigma>0$ in the lhs electrode and $\Sigma<0$ in the rhs one. See details in the text.

[b]Type 1 corresponds to logarithmic singularities for Josephson current components and jumps for interference and quasiparticle current components. Type 2 corresponds to jumps for Josephson current components and logarithmic singularities for interference and quasiparticle current components. See details in the text.

[c]For $\Delta<\Sigma$.

[d]Components 5 and 6 are inherent in currents through bs junctions only.

[e]For $T\neq 0$.

A. s junctions S_Σ-I-S_Σ

For this kind of junction CVC's do not depend on the sign of Σ. The locations and types of all possible peculiarities for current components are listed in Table I in the case $\Sigma>0$. Type 1 corresponds to logarithmic singularities proportional to $Y_+(D_1,D_2,eV)$ for Josephson current components and jumps $\delta I(eV)\equiv I(eV+0)-I(eV-0)$ proportional to $Z_+(D_1,D_2)$ for interference and quasiparticle current components. Type 2 corresponds to jumps proportional to $Z_-(D_1,D_2)$ for Josephson current components and logarithmic singularities proportional to $Y_-(D_1,D_2,eV)$ for interference and quasiparticle current components. Here

$$Z_\pm(D_1,D_2)=\sqrt{D_1D_2}\left|\tanh\frac{D_1}{2T}\pm\tanh\frac{D_2}{2T}\right|, \tag{16}$$

$$Y_\pm(D_1,D_2,eV)=Z_\pm(D_1,D_2)\ln\frac{(D_1+D_2)}{|eV-|D_1\pm D_2||}, \tag{17}$$

with parameters $D_{1,2}$ being specific for each peculiarity. The explicit expressions will be published elsewhere. Below we confine our analysis to the positive voltage branch.

It should be noted that for $T\neq 0$ the form of the main characteristic features appropriate to currents at $T=0$ are distorted by thermally excited quasiparticles in two ways: (i) by deformation of the existing CVC peciliarities and (ii) by appearance of new ones. At the same time, for substances to which we have restricted ourselves, the inequality $|\Sigma|>\Delta$ holds good, so that $H_-=2\Delta$ and $N_+=M_-=|\Sigma|$. Then, for *finite temperatures* the positions of some new logarithmic singularities coincide with those of certain jumps and vice versa, due to the mixing of contributions from different spin-split states and various FS sections. For instance, when studying CVC's of symmetrical junctions for voltages in the neighborhood of the *superconductivity-determined* feature point $eV=2\Delta$, one should bear in mind a possible influence of the electron-hole correlations, although the respective gap $|\Sigma|$ is usually much larger than Δ.

The CVC's depend on two dimensionless parameters, specific for each substance: ν and $\sigma(T)\equiv|\Sigma|/\Delta_0$, where Δ_0 is the superconducting gap at $T=0$ in the absence of the dielectrization ($\nu\rightarrow\infty$). Similar to paper I, the dependence $\Sigma(T)$ is chosen to be of the BCS type, the choice being not crucial because $T_N\gg T_c$ in the objects concerned. So, the values of $\Sigma(T)$ at any given T are determined by the parameter $\sigma_0\equiv\Sigma_0/\Delta_0$, where $\Sigma_0\equiv|\Sigma(T=0)|$.

In Fig. 1(a) the normalized total Josephson current $i_s^1\equiv I_s^1 eR/\Delta_0$ (below all current amplitudes and their components normalized in the same manner are denoted by the same letters in the lower case with the same indices) is shown versus dimensionless bias $x\equiv eV/\Delta_0$ for $\sigma_0=1.5$ and $\nu=1$, and for two values of normalized temperature $t\equiv T/T_{c0}=0$ (dashed curve) and 0.2 (solid curve). Here $T_{c0}=\gamma\Delta_0/\pi$ is the critical temperature of the SDW superconductor in the absence of the dielectrization and $\gamma=1.7810\ldots$ is the Euler constant. The ratios $t^*\equiv T/T_c$ of T to the actual critical temperature are 0 ($t=0$) and 0.816 ($t=0.2$). The procedure of calculating the quantities $T_c(\nu,\sigma_0)$ and $\Delta(\nu,\sigma_0,t)$, which is used throughout the paper, was described elsewhere.[16,18,20,21] As one can see, there are four positive (at $eV=2|D_\pm|$, 2Δ, and M_+) and two negative (at $eV=H_+$ and M_-) logarithmic singularities of $i_s^1(x)$. For $t\neq0$ a positive jump at $eV=N_+$ and a negative one at $eV=N_-$ emerge. However, the former, inherent to the term i_{s3}^1, cannot be traced against the background of the singularity at $eV=M_+$ (numerically equal to N_+), stemming from the same current term. Thus, here the peculiar interference between different kinds of features is taking place due to the spin splitting in the SDW state. One should also bear in mind that the steps in I_{s1}^1 and I_{s2}^1 at $eV=H_-$ completely compensate each other.

The dimensionless quasiparticle current $j_s\equiv J_s eR/\Delta_0$ is shown in Fig. 1(b). The jumps of J_{s1} and J_{s2} at $eV=2|D_\pm|$ and H_+ compensate each other, so that only three positive steps at $eV=M_\pm,2\Delta$ remain. There are also two detectable positive logarithmic singularites at $eV=N_\pm$ for $T\neq0$, whereas finite-T singularities at $eV=H_-$ are too tiny to be observed.

The quasiparticle conductances $g_s^{\rm diff}\equiv dj_s/dx$ are displayed in Fig. 1(c).[56] The linear divergences clearly seen at the voltages corresponding to the logarithmic singularity points of j_s, in reality have to be reduced to smooth features due to the averaging over the spread of Δ and/or Σ. On the other hand, a rich variety of jumps found here should reveal themselves in experiment. A finite-T zero-bias logarithmic

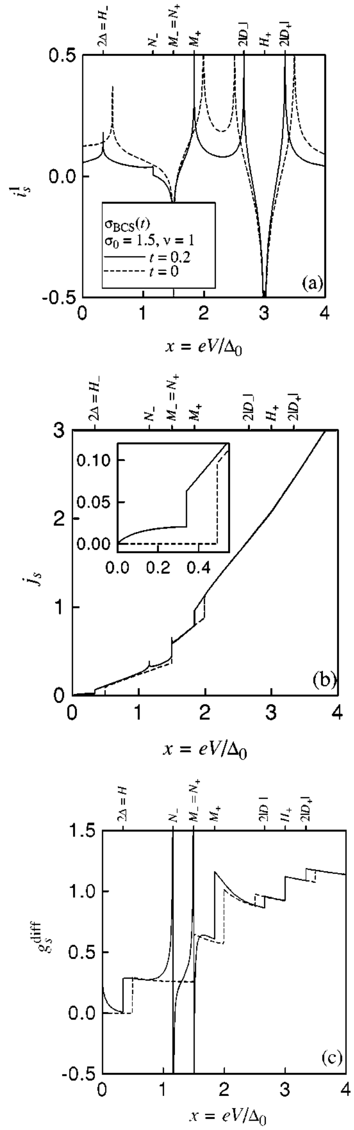

FIG. 1. CVC's of dimensionless nonstationary Josephson $i_s^1\equiv I_s^1 eR/\Delta_0$ (a) and quasiparticle $j_s\equiv J_s eR/\Delta_0$ (b) current amplitudes and quasiparticle conductance $g_s^{\rm diff}\equiv dj_s/dx$ (c) through the symmetrical S_Σ-I-S_Σ tunnel junction between SDW superconductors for various dimensionless temperatures $t=T/T_{c0}$. Here e is the elementary charge, R is the junction resistance in the normal state, Δ_0 is the superconducting gap at $T=0$ in the absence of the dielectrization, $T_{c0}=\gamma\Delta_0/\pi$, $\gamma=1.7810\ldots$ is the Euler constant, $x=eV/\Delta_0$, V is the applied voltage, σ_0 is the value of $\sigma=\Sigma/\Delta_0$ at $T=0$, $\Sigma(T)$ is the dielectric order parameter, $\nu=N_{\rm nd}(0)/N_d(0)$, $N_{{\rm nd}(d)}(0)$ is the electron density of states at the nondielectrized (dielectrized) Fermi surface section. The peculiarity positions for the curve corresponding to $t=0.2$ are marked on the top axes. The inset shows the scaled-up details of the relevant curves.

singularity predicted for BCS superconductors[48] is readily visible. It is driven by the density of states peaks near the edges of the gaps Δ and $|D_\pm|$.

B. ns junctions S_Σ-I-$S_{\rm BCS}$

The locations and types of the ns junction CVC's peculiarities are listed in Table II. For the sake of definiteness we assume that $\Sigma>0$. If $\Sigma<0$ the different polarity CVC branches change places.

TABLE II. Types and positions of CVC peculiarities inherent in components of currents through ns junctions.

$\lvert eV\rvert$	Type [a]	Components
$K_{\pm}=\lvert D_{\pm}\rvert+\Delta_{\mathrm{BCS}}$	1	1,2
$K=\Delta+\Delta_{\mathrm{BCS}}$	1	3
$L_{\pm}=\lvert\lvert D_{\pm}\rvert-\Delta_{\mathrm{BCS}}\rvert$ [b]	2	1,2
$L=\lvert\Delta-\Delta_{\mathrm{BCS}}\rvert$ [b]	2	3

[a]Type 1 corresponds to logarithmic singularities for Josephson current components and jumps for interference and quasiparticle current components. Type 2 corresponds to jumps for Josephson current components and logarithmic singularities for interference and quasiparticle current components. See details in the text.
[b]For $T\neq 0$.

In Fig. 2 the bias dependences of the normalized total currents $i^1_{\mathrm{ns}}\equiv I^1_{\mathrm{ns}}eR/\Delta_0$ and $j_{\mathrm{ns}}\equiv J_{\mathrm{ns}}eR/\Delta_0$ are shown for various values of the ratio $\epsilon_0\equiv\Delta_{\mathrm{BCS}}(T=0)/\Delta_0$. One can clearly see the main Riedel-like logarithmic singularities and jumps, and the absence of definite symmetry for all currents.

C. bs junctions S_{Σ}-I-$S_{-\Sigma}$

For this kind of junction CVC peculiarities are located at the same voltages as in the s case (see Table I).

The interplay between singularities and jumps in junctions with broken symmetry is more interesting than in genuinely symmetrical ones. Together with the bias polarity dependence, it makes the resulting picture rather intricate.

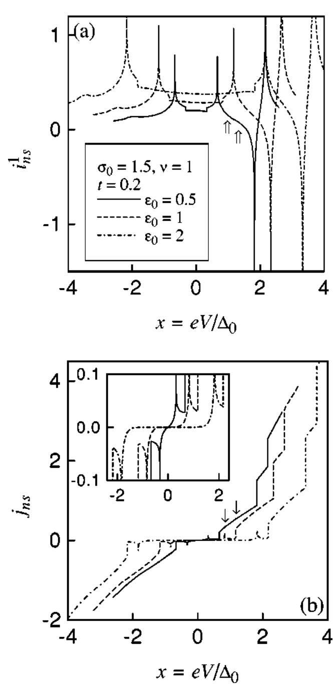

FIG. 2. CVC's of dimensionless nonstationary Josephson $i^1_{\mathrm{ns}}\equiv I^1_{\mathrm{ns}}eR/\Delta_0$ (a) and quasiparticle $j_{\mathrm{ns}}\equiv J_{\mathrm{ns}}eR/\Delta_0$ (b) current amplitudes through nonsymmetrical S_{Σ}-I-S_{BCS} junctions, where S_{BCS} is an ordinary BCS superconductor with the gap Δ_{BCS}, for different $\epsilon_0\equiv\Delta_{\mathrm{BCS}}(T=0)/\Delta_0$. Single arrows indicate the positions of logarithmic singularities and double arrows indicate the positions of discontinuities which are hardly seen on a scale selected. The insets show the scaled-up details of the relevant curves.

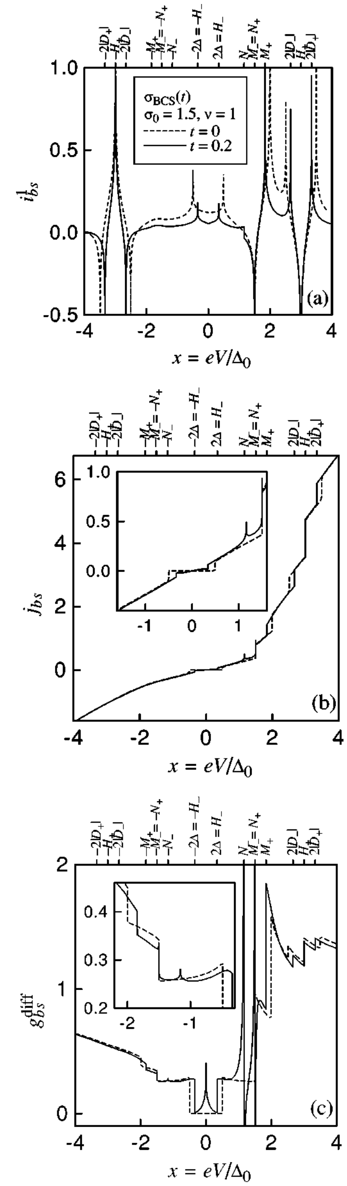

FIG. 3. CVC's of dimensionless nonstationary Josephson $i^1_{\mathrm{bs}}\equiv I^1_{\mathrm{bs}}eR/\Delta_0$ (a) and quasiparticle $j_{\mathrm{bs}}\equiv J_{\mathrm{bs}}eR/\Delta_0$ (b) current amplitudes and quasiparticle conductance $g^{\mathrm{diff}}_{\mathrm{bs}}\equiv dj_{\mathrm{bs}}/dx$ (c) through the formally symmetrical tunnel junction with broken symmetry (S_{Σ}-I-$S_{-\Sigma}$) between SDW superconductors for various t. The peculiarity positions for the curve corresponding to $t=0.2$ are marked on the top axes. The insets show the scaled-up details of the relevant curves.

For definiteness, we shall consider below the broken symmetry state with $\Sigma=-\Sigma'>0$. Such a state will be denoted as the bs+ state. The CVC's for the other possible state (bs−) with $\Sigma=-\Sigma'<0$ can be easily obtained from the symmetry relations.

In Fig. 3(a) the CVC for the dimensionless Josephson current $i^1_{\mathrm{bs}}\equiv I^1_{\mathrm{bs}}eR/\Delta_0$ is shown for different t. One can see that the singularities at $\lvert eV\rvert=2\lvert D_{\pm}\rvert$ and H_+ exist although they are compensated for the sum of ''conventional'' components I^1_{bs1} and I^1_{bs2}. However, the corresponding pattern fragment has an unusual antisymmetrical dependence on the voltage polarity. The singularities at $eV=M_{\pm}$ are amplified by the broken symmetry component I^1_{bs5}, whereas they are fully compensated for $eV=-M_{\pm}$. As for the Riedel singularity of the BCS-like term I^1_{bs4}, it remains the same as in the s case.

The finite temperature jumps at $|eV|=H_-$ are almost fully determined by the ''conventional'' components $I^1_{\mathrm{bs1,2}}$ because for the chosen parameter values the jump $\delta I^1_{\mathrm{bs5}}$ $\sim\Delta/|\Sigma|$ is much smaller. However, since $H_-=2\Delta$, these features are unobservable against the background generated by the BCS term. The negative step at $eV=N_-$, resulting both from $\delta I^1_{\mathrm{bs3}}$ and $\delta I^1_{\mathrm{bs6}}$, is seen in the figure. At the same time, for $eV=-N_-$ these terms compensate each other. The same is true for the feature points $eV=\pm N_+$. Yet, the overall positive jump at $eV=N_+=\Sigma$ occurs at the same bias $eV=M_-=\Sigma$ as the negative logarithmic singularity, and therefore becomes unobservable.

The dimensionless quasiparticle current $j_{\mathrm{bs}}\equiv J_{\mathrm{bs}}eR/\Delta_0$ shown in Fig. 3(b) is especially nonsymmetrical. The contributions at $eV=|D_{\pm}|,H_+,M_{\pm}$ are combined to make jumps, while they are mutually compensated for corresponding negative biases. Also a small superconductivity-driven jump is present for $|eV|=2\Delta$.

The finite-T singularities at $|eV|=H_-$ are too tiny to be noticed in the figure against the jump δJ_{bs4}. On the contrary, the singularities at $eV=N_{\pm}$ are pronounced. At the same time, for negative biases $eV=-N_{\pm}$ their contributions cancel out.

All the compensations discussed for the quasiparticle current do not concern its derivatives, so the conductance versus voltage curves can be a valuable source of information for the case of broken symmetry. The curves $g^{\mathrm{diff}}_{\mathrm{bs}}(x)\equiv dj_{\mathrm{bs}}/dx$ for the same set of parameters are shown in Fig. 3(c). The curves remain nonsymmetrical, however, some structure can be seen also for negative voltages and the logarithmic singularity at zero bias, appropriate to symmetrical junctions,[48] manifests itself clearly in the bs case too.

For a specific symmetrical tunnel junction S_{Σ}-I-$S_{\Sigma'}$ with $|\Sigma|=|\Sigma'|$ any of the three above-mentioned possibilities (s, bs+, and bs−) can be realized, in principle. In reality, one can imagine a number of accessory factors to make a certain state preferable. Thus, a choice of the actual experimental CVC would be made for given external conditions and the electrical or thermal prehistory of the junction. Far below T_N (and below T_c for antiferromagnetic superconductors) the fluctuation-induced switching between states with different Σ signs is impossible because it would require large energy connected with the SDW rearrangement. Of course, heating above T_N and subsequent cooling may result in another CVC if the possible states are almost degenerate energetically.

VI. DISCUSSION

The heavy-fermion compounds are the most probable objects to be described by the presented theory. For instance, CVC's are *asymmetrical* for break junctions (symmetrical in essence) made of superconducting UNi_2Al_3.[32] It is reconciled with our theory. However, in order to propose any quantitative comparison with experiment, one should know the superconducting and dielectric gaps as well as the control parameter ν.

The most studied now partially dielectrized SDW superconductor is the heavy-fermion compound URu_2Si_2. Here the actual $T_c\approx1.3-1.5$ K and $T_N\approx17-17.5$ K, according to different sources.[29,30,57] There are, however, substantial discrepancies for the parameters Σ and ν inferred from specific heat measurements, namely, $\Sigma\approx115$ K and $\nu\approx0.4$, according to Ref. 29, or $\Sigma\approx129$ K and $\nu\approx1.5$, according to Ref. 30. Another investigation of thermal properties also leads to $\Sigma\approx115$ K.[57]

Tunnel and point-contact measurements of URu_2Si_2 conductivity both in the symmetrical and nonsymmetrical setup have been carried out recently.[45–47,58] The respective CVC's clearly demonstrated gaplike peculiarities disappearing above T_N, thus being the manifestation of the SDW-related partial dielectric gapping. Below T_c superconducting gap features were also seen at voltages associated with T_c's by the BCS relationship. Usually, such experiments give an opportunity to obtain 2Σ value directly as a voltage difference between two humps (tunnel method), or valleys (point-contact technique) of the curves $G^{\mathrm{diff}}(V)$. However, in this case the CVC's for junctions URu_2Si_2-I-M or URu_2Si_2-C-M, where C denotes constriction, are highly nonsymmetrical. It agrees qualitatively with our theory but the quantitative comparison is hampered. Direct tunnel or point-contact studies lead to strikingly different values of Σ as compared to those cited above, e.g., $\Sigma\approx68$ K.[46] At the same time, these experiments may be regarded as an evidence of the electron spectrum partial gapping in URu_2Si_2. That is why our theory is actual, but the input parameters should be taken from the bulk measurements of electron conductivity, magnetic susceptibility, heat capacity, or thermal expansion.

It also turned out that the broken symmetry scenario has already been realized for URu_2Si_2 point homocontacts.[47] In agreement with our theory, the CVC asymmetry is smaller for homocontacts than for heterocontacts. Moreover, together with symmetrical CVC's, it often happens that the Σ-determined peculiarities of the experimental dV/dJ curves are more pronounced either on the positive or negative V branches. It correlates well with our classification of formally symmetrical junctions as of s, bs+, or bs− types.

We should note that the cited tunnel and point-contact measurements for junctions involving URu_2Si_2 were carried out for single crystals, whereas our summation procedure of all possible tunnel currents between different FS sections implies a certain direction averaging. However, the gap features and the general appearance, e.g., of the dV/dJ versus V dependences[47] are very similar for directions along the c axis or normal to it. It is so because some kind of averaging is inevitably present in such experiments. In this manner, our approach is reconciled with the experimental data.

The dimensionless conductances $g^{\mathrm{diff}}_{\mathrm{ns}}(V)$ are shown in Fig. 4 for a nonsymmetrical tunnel junction URu_2Si_2-I-Pb ($T_c\approx7.2$ K of Pb is larger than T_c of URu_2Si_2). One can see that the CVC's are highly asymmetrical with fine structures of logarithmic singularities and jumps.[59] All possible sample inhomogeneities or direction averaging would wipe out most of the information leaving the major features. The asymmetrical character of CVC's seems, however, insensitive to such modifications. And really, the observed tunnel spectra of URu_2Si_2-Al_2O_3-Al

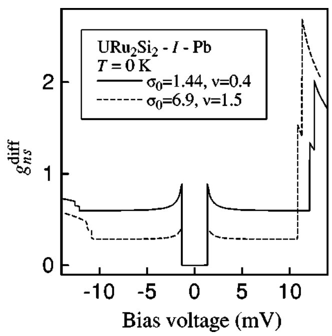

FIG. 4. Conductances $g_{\mathrm{ns}}^{\mathrm{diff}}(V)$ of the URu_2Si_2-I-Pb junction for the sets of parameters from Ref. 29 (solid curve) and Ref. 30 (dashed curve).

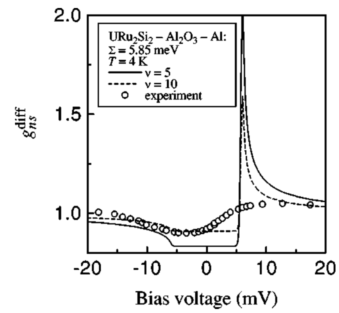

FIG. 5. Dependences $g_{\mathrm{ns}}^{\mathrm{diff}}(V)$ for the ns junction URu_2Si_2-Al_2O_3-Al in the normal state taken from the experiment (Ref. 46) and calculated for various ν's.

junctions[46] and point-contact spectra for contacts URu_2Si_2-Ag,[45] URu_2Si_2-Au,[46] URu_2Si_2-M ($M=$Zn, NbTi),[58] and URu_2Si_2-M ($M=$Fe, Ag, Cu),[47] reveal substantial dependence on voltage polarity.

As for the direct confirmation of our theory by experiment, unfortunately, the most intriguing majority of available data are obtained by point-contact spectroscopy for which only the location of the feature points can be compared with our predictions. The only tunnel measurements are made for nonsymmetrical junction URu_2Si_2-Al_2O_3-Al.[46] But the CVC presented there was obtained for $T=4$ K, i.e., well above T_c's both for URu_2Si_2 and Al (1.19 K). So, both electrodes were in the normal state. Figure 5 shows the relevant experimental data together with our calculations. The general trend is reproduced indeed. Note that the much larger fitting values of ν are required in comparison with those from thermal measurements.[29,30,57] Smearing of the peculiarities may be attributed to the averaging inherent in such kind of experiments.

One sees that the relationship between T_N and T_c in URu_2Si_2 is not favorable to observe many characteristic features of CVC's. Thus, a quest of a proper SDW superconductor remains on agenda. It would be interesting also to measure the nonstationary Josephson CVC's which have not been measured so far for the objects concerned. In this connection we call attention to related experiments for high-T_c cuprates.[60] There due to large values of the superconducting gap the nonstationary Josephson current generates nonequilibrium phonons with the energy 2Δ corresponding to the Riedel singularity. They are revealed in the quasiparticle current and observed as peaks of $G^{\mathrm{diff}}(V)$. Such a possibility can be realized for URu_2Si_2 with its unexpectedly large Σ or any other suitable SDW (or CDW) superconductor. The favorable circumstance here is the existence of coherent current components with combined ''gaps'' determined both by Δ and Σ, as is shown in this paper.

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