Engine cycle of an optically controlled vacuum energy transducer

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An idealized system composed of two parallel, semiconducting boundaries separated by an empty gap of variable width is considered. A gedanken experiment is discussed to show that, in general, the total work done by the Casimir force along a closed path that includes appropriate transformations does not vanish. It is shown that, in the limit of an engine cycle bringing the two boundaries to a relatively small distance, positive net exchange of energy associated with the Casimir force field could quite possibly be achieved. Viable technological implementations of this idealized system are analyzed in some quantitative detail, in particular, in the case of doped and undoped c-Si boundaries. For the purpose of direct experimentation, measurements with both macroscopic and microelectromechanical devices are suggested. A full theoretical and experimental study of systems of this kind on every scale will greatly contribute to a much deeper understanding of the nature of the Casimir force and associated concepts, including the possible manipulation of semiconducting nanostructures and the noninvasive optical characterization of semiconducting samples. In the event of no other alternative explanations, one should conclude that major technological advances in the area of endless, by-product free-energy production could be achieved. [S0163-1829(99)05345-X]

I. INTRODUCTION

In the last 50 years, the study of Casimir effects has evolved from an esoteric effort dealing with ''one of the least intuitive consequences of quantum electrodynamics''[1] to a very active and multifaceted research area. This is documented in extensive theoretical and experimental reviews dealing with this topic, its relationship to the structure of the quantum vacuum, as well as applications to other areas, such as cosmology and elementary particle physics (see, for instance Refs. 2–14 and references therein).

The increasing awareness of the potential impact of the Casimir force in the experimental arena can even be traced at the semantic level. For instance, references to the Casimir force as a ''tiny force,'' ''hardly easy to measure,''[7,8] are now found alongside others where the possibility is examined that such force ''could conceivably compete with the gravitational force in experiments to measure the gravitational constant G where the interacting masses are extremely close.''[15,16] In fact, experimental sensitivity in Casimir force experiments conducted with different techniques has improved from $\sim$100% in earlier attempts[17] to $\sim$1–5% over an ever widening range of distances[18–20] (see also Ref. 21 for possible macroscopic distance experimentation).

Research has not only focused on a range of inorganic materials, some of which will be discussed below, but also on the promise that ''the Lifschitz theory of van der Waals forces should continue to reveal a rich diversity of unexplained qualitative features peculiarly pertinent to biological systems.''[22,23]

Just as impressive is the growing attention paid to Casimir effects within the context of microelectromechanical system (MEMS) engineering. In this case, the Casimir force has been invoked to explain, at least in part, the unexpected latching of micromachined membranes and cantilevers to nearby planes (stiction).[24] Furthermore, the possibility has been explored to manufacture micromechanisms in which Casimir forces actually play a role in actuating MEMS components.[24,25]

Perhaps the most controversial subject related to the Casimir force has been that of manipulating energy associated to quantum vacuum fluctuations of the electromagnetic field.[26]

Despite the finding that such an unsettling notion is found not to contradict the basic laws of thermodynamics,[27] one of the most often cited contributions to the search for a device ''to extract energy from the vacuum'' highlighted a fundamental difficulty inherent to the very nature of the Casimir force. In Ref. 28, it was suggested that the work done by the Casimir force on two equally charged conducting leaves could be recovered as electrical energy. Although this is, in principle, possible, it was pointed out that, in analogy to hydroelectric energy production, the conservative nature of the Casimir force would require that any device so used be discarded after energy conversion.

More recently, that same author has discussed a ''Casimir vacuum energy extraction cycle,'' based on the cyclical manipulation of the dimensions of an idealized Casimir cavity[29] (see also Ref. 30), which would appear to allow for the endless extraction of vacuum energy, and has also suggested the possible use of MEMS technology for future experimentation in this area.

It must be pointed out that it appears to be quite common in the published literature on this particular subject to attribute any energy that might be ''extracted'' to quantum fluctuations of the electromagnetic field. This, however, corresponds to only one possible view of the origin of the Casimir force. As is well known, several theories exist that start from quite different points of view and all obtain what appear to be the same experimental predictions on the Casimir force. Therefore, it is important at the onset to state that any discussion of the implications of the findings described here will only be complete if carried out by comparing the ultimate meaning of energy ''extraction'' or ''exchange'' within

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these other alternative schemes, so far thought to be equivalent to the zero-point energy approach.

In this paper, a transducer of vacuum energy, based on an oscillating boundary with variable optical properties, is introduced. A gedanken experiment is then carried out using such a device to demonstrate that, in the present implementation, the total work done by the Casimir force along an appropriate engine cycle does not in fact vanish. This allows one to circumvent all difficulties of principle previously described in the literature, and to point to a feasible experiment to probe the regime in which ''vacuum energy'' appears to become available.

In Sec. II, the basic physical ideas behind our gedanken experiment are presented, along with their logical consequences. In Sec. III quantitative estimates of the energy output are obtained. In the final section, a discussion of the results is presented along with conclusions concerning the direction of future theoretical and experimental work in this area.

II. A CASIMIR FORCE ENGINE CYCLE

In 1948, Casimir proved that the effect on the vacuum energy of the electromagnetic field between two perfectly conducting (pc), neutral, parallel planes is to give rise to an attractive force, since referred to as the Casimir force. The magnitude of the force per unit area was found to be[31]

$$F_{\text{Cas,pc}}(s)=\frac{\pi^2}{240}\frac{\hbar c}{s^4}, \tag{2.1}$$

where h is Planck's constant, c the speed of light *in vacuo*, and s is the separation between the two conducting surfaces (cgs units will be used throughout unless otherwise specified). Given the divergent behavior of this quantity as $s \to 0$, it is reasonable to expect that this force will affect or even dominate the dynamics of two very close surfaces. In fact, the Casimir force between two planes at a distance of ~ 0.1 μm is equivalent to the electrostatic force between the same two plates in the presence of a potential difference ~ 100 mV. If the distance decreases to ~ 10 nm, with all idealizations still valid, the equivalent Casimir pressure rises to ~ 1 atm.

The most important point for what follows here is that closer examination of the problem reveals that the Casimir force between two boundaries also depends on a variety of additional parameters, which are quite relevant in realistic situations. These include, for instance, the exact geometry involved, which may cause the Casimir force to be either attractive or repulsive, the temperature of the conductors, their optical properties, the roughness of the surfaces, and imperfections in their parallelism (Refs. 8, 11, 32, and 1, and references therein).

A. The engine cycle

In order to show the motivation for the present paper, let us consider the idealized system shown in Fig. 1. In such a system, a material of dielectric properties ϵ_2 fills the space to the right of a fixed boundary. A gap of dielectric properties ϵ_3 separates this plane surface from a moving boundary facing it at a variable distance s, to the left of which space is filled with another material of dielectric properties ϵ_1. For the purposes of this discussion and for the quantitative estimates to follow, we shall assume that the two semi-infinite dielectric slabs have the same dielectric properties ($\epsilon_1=\epsilon_2=\epsilon$) and that vacuum fills the gap ($\epsilon_3=1$).

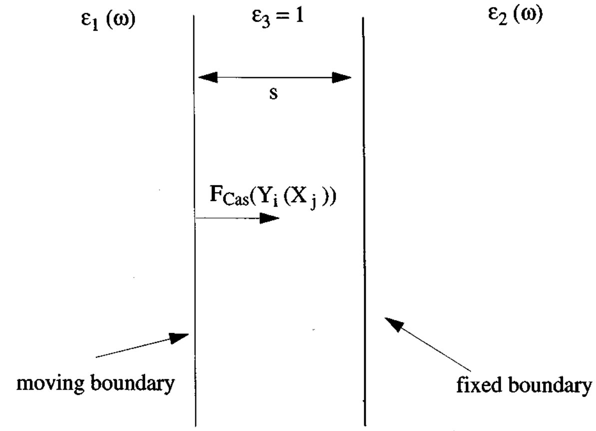

FIG. 1. Idealized Casimir force system.

Let us consider the Casimir force $F_{\text{Cas}}(s)$ between the two boundaries. In the case of realistic materials, this force will not just depend on the distance s, as in Eq. (2.1), but also, indirectly, on every physical parameter Y_i, which determines the dielectric properties of the materials involved. For instance, such parameters as the concentration of free carriers or the location and strength of any absorption bands will play a role in determining the value of the Casimir force, $F_{\text{Cas}}(s;Y_i)$, with $i=1,2,\ldots$. In turn, such quantities will in general be affected by appropriate environmental parameters X_j, $j=1,2,\ldots$, such as the absolute temperature or the radiation density (this is in addition to the intrinsic dependance of the Casimir force on the temperature, as discussed later). Thus, for any given geometry, the Casimir force will in general be a function of the kind

$$F_{\text{Cas}}=F_{\text{Cas}}(s;Y_i(X_j)). \tag{2.2}$$

Let us now analyze the following cycle, indicated in Fig. 2, where the absolute value of the Casimir force is schematically shown as a function of the distance between the two boundaries, s. The cycle might start at point A, with a first transformation ending at point B. During this first transformation, all the variables $Y_i(X_j)$ affecting the Casimir force,

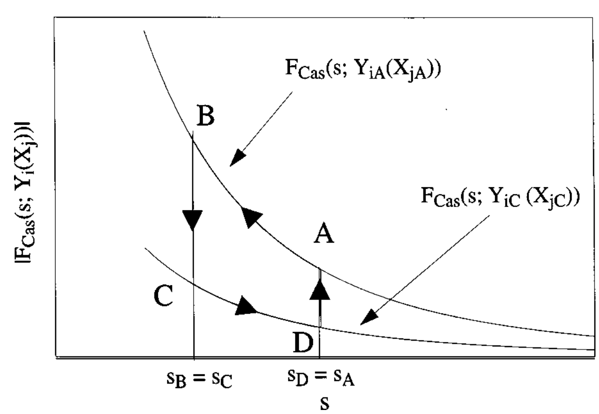

FIG. 2. Schematic view of the idealized engine cycle.

excluding of course, the distance, are assumed constant and equal to their initial values, $Y_{iA}(X_{jA})$. The work done by the Casimir force as the distance between the boundaries changes from s_A to s_B, with $s_A > s_B$, can be written as

$$W_{\mathrm{Cas,AB}} = \int_{s_A}^{s_B} F_{\mathrm{Cas}}(s; Y_{iA}(X_{jA}))ds. \tag{2.3}$$

Once point s_B is reached, and while the moving boundary is held at rest, a transformation is applied to all variables Y_i and X_j, so that the value of the Casimir force at $s_B = s_C$ in general will change to

$$F_{\mathrm{Cas}}(s_C) = F_{\mathrm{Cas}}(s_B; Y_{iC}(X_{jC})). \tag{2.4}$$

For instance, this may correspond to an amount of energy $W_{BC} > 0$ transferred from the system to a lower temperature heat reservoir.

At this point, the distance between the two boundaries is varied back to its initial value s_A, while all other parameters are held constant; the work done by the Casimir force will of course be

$$W_{\mathrm{Cas,CD}} = -\int_{s_B}^{s_A} F_{\mathrm{Cas}}(s; Y_{iC}(X_{jC}))ds. \tag{2.5}$$

Finally, the $Y_i(X_j)$ parameters are restored to their initial values, corresponding to one further energy exchange $W_{\mathrm{DA}} < 0$, which might, for instance, represent heat transferred to the system from a higher temperature heat reservoir.

Let us now define the total energy available from this cycle as

$$W_{\mathrm{tot}} \equiv (W_{\mathrm{DA}} - W_{\mathrm{BC}}) + W_{\mathrm{Cas}}, \tag{2.6}$$

where

$$W_{\mathrm{Cas}} \equiv \oint F_{\mathrm{Cas}}[s; Y_i(X_j)]ds \tag{2.7}$$

is the total mechanical work done by the Casimir force over its closed path, that is, the area enclosed by the curve representing the cycle in Fig. 2.

The central issue raised by this paper deals with the following question: is there any reason that, in general, the quantity W_{tot} should always vanish?

B. Thermodynamical considerations

In order to gain a deeper perspective on this issue, let us consider an observer at point P carrying out measurements of the total mass energy M_{tot} in a three-dimensional volume V of space containing all the relevant components of the above engine, including the two heat reservoirs. For simplicity, we shall assume the observer to be at a remote distance r from the system, and we shall neglect the presence of the gravitational field of the Earth as well as any self-gravitational effects among the different parts of the engine.

As is well known,[33] for an appropriate choice of the gauge, both the Newtonian gravitational potential U_{grav} and the metric component g_{00} of a weakly gravitating system are uniquely determined by the total mass-energy of the field source. That is, $U_{\mathrm{grav}} = \frac{1}{2}(g_{00} - \eta_{00}) = -\mathrm{GM}_{\mathrm{tot}}/r + O(1/r^3)$, where η_{00} is the flat space metric component and G is the gravitational constant. In principle, such a far-away observer can monitor the total mass energy present in the volume V by studying the Keplerian motion of a test particle in the gravitational field of the engine or by using a sensitive torsion balance.[33]

In the case of the idealized Casimir force system under consideration, the total mass energy M_{tot} will be determined by the sum of the engine contribution and of the renormalized Casimir energy of the force field between the two boundaries. The renormalized Casimir energy $\mathcal{E}_{\mathrm{Cas}}$ is the integral over the volume between the two boundaries of the renormalized energy density ϵ_{Cas}, defined as $\epsilon_{\mathrm{Cas}} = \langle T_{00}\rangle_{\mathrm{Cas}} = \lim_{\alpha\to 0}[\langle 0|T_{00}|0\rangle_\alpha - \langle 0_M|T_{00}|0_M\rangle_\alpha]$, where $\langle 0|T_{00}|0\rangle_\alpha$ and $\langle 0_M|T_{00}|0_M\rangle_\alpha$ are the nonrenormalized (infinite) vacuum expectation values of the 00 component of the energy-momentum tensor in the presence of the boundaries and in unbounded Minkowski space, respectively. The limit operation in this definition requires the choice of a cutoff function $f(\omega,\alpha)$ dependent upon both the oscillator frequency ω and the index α, but whose exact form is inconsequential to the final estimate of ϵ_{Cas} (Refs. 6, 11 and 34) (here we are not dealing with the issue of the effects of vacuum energy on the curvature of space on a cosmological scale and its relationship to astronomical determinations of the cosmological constant).[7,35]

Let us now consider the $D\to A$ and $B\to C$ quasistatic transformations in the cycle above. Independently of the mechanisms responsible for the transfers of energy W_{DA} and W_{BC}, it is evident that such transformations will not, by themselves, result in a change of the total mass energy contained in the volume V as measured by the observer at P. However, the Casimir force F_{Cas} between the two boundaries and the renormalized Casimir energy $\mathcal{E}_{\mathrm{Cas}}$, related to each other by $F_{\mathrm{Cas}}(s; Y_i(X_j)) = -\partial\mathcal{E}_{\mathrm{Cas}}(s; Y_i(X_j))/\partial s$, will both be affected by such energy transfers via the effect these have on the $Y_i(X_j)$ parameters. This has very important operational consequences, as the energy-momentum tensor $\langle T_{00}\rangle_{\mathrm{Cas}}$, which determines the value of ϵ_{Cas} as we have seen above, directly enters Einstein's field equations. In other words, "The absolute value of the vacuum energy is, in principle, a measurable quantity, because it gravitates."[34] In our case, we can change such absolute value by just causing energy to flow from a location to another inside the volume V.

For instance, in the case analyzed in detail in this paper, the optical properties of semiconducting boundaries are altered by a flux of radiation or by a temperature change, both corresponding to energy transfers. Although the flow of energy between the heat reservoirs and the boundaries does not have any measurable mass-energy effects at point P, the Casimir energy changes are directly observable by the far-away observer.

This interesting phenomenon can be intuitively visualized in terms of the vacuum-field radiation pressure interpretation of the Casimir force.[36] In such approach, the virtual photons of the quantum vacuum outside the space between the two parallel boundaries exert a classical radiation pressure, which pushes the two surfaces together. At the same time, the virtual photons in the volume between the two boundaries exert an outward radiation pressure. Although each of these two pressures is infinite in value, their difference in the case of perfectly reflecting surfaces can be proved to be equal to the expression of the Casimir force per unit area at Eq. (2.1). Furthermore, the reason that the net force per unit area is attractive in the case of this geometry can be understood in terms of the boundary conditions, which cause only a discrete number of modes of the vacuum field within the space between the two surfaces to be able to contribute to the outward radiation pressure.[36]

Although such interpretation is very appealing, it is important to stress a remarkable difference between the general application of the concept of radiation pressure to a typical gas of real photons as opposed to the ''gas of virtual photons'' considered here. In the case of real photons, the radiation pressure on an imperfectly conducting wall due to a plane wave of frequency ω with an angle of incidence θ is given by $P_{\omega,\mathrm{rad}}=u_{\omega,\mathrm{rad}}(1+R_\omega)\cos^2\theta$, where R_ω is the reflectivity of the wall at the given frequency and *the energy density* $u_{\omega,\mathrm{rad}}$ *is given*.[37] A change in the reflectivity of the surface will affect the radiation pressure on the wall but not the energy density of the gas of real photons.

On the other hand, in the Casimir force case, for any given boundary conditions, *the normalized energy density of the radiation field of virtual photons is drastically affected by the dielectric properties of all media involved* via the source-free Maxwell equations. Thus, according to this interpretation, the change observed at P in the total mass energy within the volume V containing the engine is due to the variation of the energy density of the virtual photon gas caused by the manipulation of the optical properties of the boundaries during the $D\rightarrow A$ and $B\rightarrow C$ quasistatic transformations.

The $A\rightarrow B$ and $C\rightarrow D$ quasistatic transformations are carried out by means of a slightly unbalanced external force doing work on the moving boundary. In the implementation described below, such external force is electrostatic and, ultimately, both $W_{\mathrm{Cas,AB}}$ and $W_{\mathrm{Cas,CD}}$ correspond to quasistatic energy exchanges between a battery and the local electric field (Sec. IV A). As such, these transformations do not cause any observable change in M_{tot}.

In conclusion, the net change in the total mass-energy contained in the volume V measured by the observer at P at the end of the cycle will be $\Delta M_{\mathrm{tot}}=W_{\mathrm{Cas}}/c^2$ (again for simplicity we have neglected throughout both gravitational and electromagnetic energy possibly radiated away from the system during each transformation).

The virtual photon gas interpretation of the Casimir force offers a further perspective on the question at the end of Sec. II A by means of an analogy with the first law of thermodynamics, $\Delta U=Q-W$, where, as usual, ΔU is the internal energy change, Q is the heat exchanged, and W is the work done by the system during a particular transformation (notice that, in our particular geometry, the ''Casimir pressure'' always acts opposite to the gas pressure of classical thermodynamics. Consequently, $W_{\mathrm{Cas}}>0$ in our case even though the cycle we just described corresponds to a clockwise path in Fig. 2).

As the $A\rightarrow B$ and $C\rightarrow D$ quasistatic transformations are adiabatic $(Q=0)$, it is natural to draw a parallel between the renormalized Casimir energy and the internal energy changes $(\mathcal{E}_{A,C}-\mathcal{E}_{B,D}\leftrightarrow\Delta U)$, and between the work done by the Casimir force and the thermodynamical work $(W_{\mathrm{Cas}}\leftrightarrow W)$. However, the two isochoric ($V=$const) $D\rightarrow A$ and $B\rightarrow C$ transformations, which evidently correspond to renormalized Casimir energy changes, are not directly related to the flow of a quantity that one might identify as ''heat'' in the sense of classical thermodynamics.[38] It is certainly true that energy, for instance in the form of heat or of electromagnetic radiation, is transferred between the heat reservoirs and the boundaries to alter their dielectric properties. However, this energy does not flow to the virtual photon gas; rather, it is the change in the optical properties of the boundaries that acts as ''temperature difference'' and makes the existence of the engine cycle possible.

Thus, describing the problem of the Casimir force in terms of radiation pressure, despite its evident intuitive advantage, presents us with a new challenge, that is, to develop a self-consistent thermodynamical description for a quantum gas whose internal energy is not only determined by appropriate state variables, but also by the dielectric properties of the containing boundaries. Despite this present apparent limitation in implementing this point of view, the above reasoning definitely shows that the mechanical energy, which appears to become available at the end of the cycle, and is measured by the far-away observer, is, from this standpoint, simply a fraction of the internal energy associated with the quantum fluctuations of the electromagnetic field, that is, the virtual photon gas.

Evidently, both the above general reasoning and the exact amount of mechanical energy W_{Cas} are completely independent of the exact value of the total energy available, W_{tot}. However, for both scientific and engineering purposes, one is clearly interested in engine cycles in which $W_{\mathrm{tot}}>0$. Whether this can be achieved in practice ultimately depends on the efficiency with which a fraction of the energy W_{DA} can be recovered before the energy W_{BC} is lost to the lower temperature reservoir. For instance, in the most inefficient cycle possible ($W_{\mathrm{BC}}=|W_{\mathrm{DA}}|$), one needs $W_{\mathrm{Cas}}\geqslant|W_{\mathrm{DA}}|$ to achieve or cross the break-even point; however, in more efficient cycles, this requirement can be considerably relaxed. In this paper, it will be shown that the break-even point can be reached even in the most inefficient case, that is, in the very conservative assumption that all the energy acquired by the system at the higher temperature reservoir cannot be recovered.

Finally, it is helpful to compare and to contrast the example we just considered with another situation, such as, for instance, that of a hydroelectric plant.[28] In such case, it is quite evident from the conservative properties of the gravitational force that, under no circumstances, could one transport some water back up to the top of the water falls without expending exactly an amount of energy equal to the kinetic energy obtained from that same water as it went through the turbines at the bottom of the falls (neglecting dissipation). In particular, it is useful to remember that a remarkable amount of experimentation has so far consistently shown that the gravitational constant *does not* depend on the chemical properties of the materials involved, nor on their temperature (Ref. 15, and references therein).

On the other hand, unlike the gravitational case, the Casimir force depends on a variety of physical parameters that can be changed if a specific energy price is paid. When such penalty is different in absolute value than the total work done by the Casimir force in a closed cycle, a net amount of energy associated to the Casimir force field is available for transformation. For this reason, likening the exchange of energy associated to the Casimir force to hydroelectric energy production is inappropriate.

One further comment is in order concerning whether the Casimir force should properly be considered as conservative. Strictly speaking, as the total work done by this force on a closed path does not always vanish, the only possible answer appears to be negative. On the other hand, the ultimate reason that this happens is not its particular analytical form nor, at least in this quasistatic example, any velocity dependence (this latter point shall be again mentioned at the end of Sec. IV B); this is instead due to the role played by a variety of additional optical, geometrical, and thermodynamical variables. When such additional parameters are held constant, the Casimir force is strictly conservative in the classical sense. When they are changed, however, it is possible to identify closed paths along which the total work done by this force does not vanish.

III. THE CALCULATION OF THE CASIMIR FORCE

Once the existence of the issue exposed by our gedanken experiment is established, it is of interest to determine realistic values for the quantities involved in Eq. (2.7). This is necessary in order to test the self-consistency of the assumptions made during the calculations, to determine an appropriate setting for experimental follow-up, and to obtain an estimate of the energy possibly available for possible practical exploitation.

It is important to mention at the onset that accurate calculations of the Casimir force for any realistic material are nontrivial. As will be recalled shortly, this involves a two-dimensional improper integral requiring knowledge of the dielectric function describing the optical properties of the two boundaries over the entire frequency range.

As, until the recent past, experimentation in this area was relatively inaccurate, several simplifying assumptions and approximations have been typically used in the literature to force results out of expressions that were understandably defined as ''unwieldly.'' These approximations have been due to three different causes: (1) simplifications of the form of the exact expression for the Casimir force; (2) lacking or incomplete data on the needed properties of the materials used; (3) simplified form of the interpolations and extrapolations of the available data.

In the following subsections, the basic results of Casimir force theory are recalled. Then, in view of the greatly enhanced experimental accuracy reached in this field, we describe the efforts made in this paper to produce more realistic estimates of the Casimir force.

In particular, we shall concentrate on the calculation of the Casimir force between two crystalline Si (c-Si) boundaries with the free-carrier concentration as the primary parameter Y_i. In this case, the radiation flux or the temperature can serve as our X_j parameters, as they both can drastically affect the free-carrier concentration. This corresponds to optically or thermally controlled implementations of the engine cycle.

A. The Lifshitz integral

In what follows, we shall consider the exact expression for the Casimir force between two semi-infinite boundaries separated by a third dielectric. In practice, this corresponds to assuming that the thickness of the semiconducting substrates be much larger than the skin depth of the plasma of free carriers in the semiconductor, $\sim\omega_\rho/c$, where ω_ρ is the plasma frequency.

Furthermore, we shall assume that the absolute temperature of the system be always $T \ll \hbar c/k_B s$, where k_B is the Boltzmann constant and s is the distance between the two semiconducting boundaries.[8] For our purposes, s will always be smaller than ~ 1 μm, and this condition becomes $T \ll 2-3\times10^3$ K, which is amply satisfied in all regimes we shall consider below. This allows us to neglect the explicit dependence of the Casimir force on the temperature.

Thus, let us consider two parallel, semi-infinite media with dielectric function $\epsilon(\omega)$, separated by a vacuum gap of thickness s at a temperature that can be assumed to be 0 K. By carrying out the change of variable $x=2\xi s/c$ in this particular case, the full expression for the Casimir force per unit area[1,8,39] becomes equivalent to that found by Lifshitz:[40]

$$F_{\mathrm{Cas}}(s) = -\frac{\hbar c}{32\pi^2 s^4} I_{\mathrm{Lif}}(s), \tag{3.1}$$

where $I_{\mathrm{Lif}}(s)$ shall be referred to as the Lifshitz integral, defined as

$$I_{\mathrm{Lif}}(s) \equiv \int_1^\infty dp\, p^2 \int_0^\infty dx\, x^3 \left(\left[\frac{1}{Xe^{xp}-1} \right] + \left[\frac{1}{Ye^{xp}-1} \right] \right), \tag{3.2}$$

and the following definitions apply:

$$X \equiv \left(\frac{S+\epsilon p}{S-\epsilon p} \right)^2, \quad Y \equiv \left(\frac{S+p}{S-p} \right)^2, \quad S^2 \equiv p^2 - 1 + \epsilon, \tag{3.3}$$

where $\epsilon = \epsilon(i\xi)$ is the dielectric function along the imaginary frequency axis.

The Lifshitz integral cannot be calculated in closed form in terms of elementary functions, except in the case of perfect conductors. In this limiting case ($\epsilon \to +\infty$), it can be shown that its value converges to the result found by Casimir [Eq. (2.1)].

In the past, approximate analytical expressions for the Lifshitz integral have been found, for instance, in the case of imperfect conductors ($c/\omega_p s \ll 1$), dielectrics with small separations and Dirac deltalike absorption features, and rarefied media (Refs. 8, 11, and 39–41 and references therein).

Further, recent activity in the area of analytical approximations of the Lifshitz integral has focused on the case of nondispersive, dilute dielectric media, partly motivated by the need to assess the relevance of the Casimir effect to the phenomenon of sonoluminescence.[42–45]

Also, semiempirical relationships have been introduced to fit the general dependence of the Casimir force on the distance s to experimental data by means of more or less sophisticated power-law expansions.[24,25,39,46,47]

In this study, the Lifshitz integral has been calculated numerically, without any analytical approximations on the integrand, by means of a two-dimensional Romberg integration of an appropriately modified expression.[48] In order to illustrate the procedure adopted, let us first consider the case of ideal conductors. In this limit, Eq. (3.1) becomes

$$F_{\mathrm{Cas}}(s) = -\frac{\hbar c}{16\pi^2 s^4}\int_1^\infty dp\, p^2 \int_0^\infty dx\, x^3 \frac{1}{e^{xp}-1}. \quad (3.4)$$

Let us first consider the improper integration in the p variable. By setting $t=e^{-xp}$ ($x=$const), one finds

$$F_{\mathrm{Cas}}(s) = -\frac{\hbar c}{16\pi^2 s^4}\int_0^\infty dx \int_0^{e^{-x}} dt \frac{\ln^2 t}{1-t}. \quad (3.5)$$

In order to deal with the improper integration in the x variable, let us now set $e^{-x}=u$. This yields

$$F_{\mathrm{Cas}}(s) = -\frac{\hbar c}{16\pi^2 s^4}\int_0^1 \frac{du}{u} \int_0^{u} dt \frac{\ln^2 t}{1-t}. \quad (3.6)$$

Finally, in order to eliminate the mild singularity at $t=0$, we make the further change of variable $t=v^2$, which yields the final expression

$$F_{\mathrm{Cas}}(s) = -\frac{\hbar c}{16\pi^2 s^4}\int_0^1 \frac{du}{u} \int_0^{\sqrt{u_0}} dv\, 2v \frac{\ln^2(v^2)}{1-v^2}. \quad (3.7)$$

The numerical integration of the expressions multiplying the constant factor was checked against the exact analytical result of $\pi^4/15$.

In order to obtain another check on this approach and of the code developed to implement it, we dealt with the case of imperfect conductors, whose dielectric constant can be approximated as $\epsilon(\omega) \approx 1-\omega_p^2/\omega^2$. In order to use Eq. (3.1), we must first evaluate this expression in the upper-half of the imaginary axis ($\omega \to i\xi$). Then, by calculating the auxiliary variables S, X, and Y, and by implementing the changes of variable $\xi=-(c/2s)\ln u$; $p=\ln(v^2)/\ln u$ ($u=$const), we obtain an expression that can be integrated numerically. Although this case cannot be solved analytically, it is possible to find approximate expansions in series of $\delta/s=c/\omega_p s$.[8,11] In fact, such expansions have represented the basis for data analysis even in the most recent experiments on the Casimir force.[18–20]

To second order, one finds

$$F_{\mathrm{Cas}}(s) \approx F_{\mathrm{Cas,pc}}\left(1-\frac{16}{3}\frac{\delta}{s}+24\frac{\delta^2}{s^2}\right). \quad (3.8)$$

The results of the numerical integration in this case are shown in Figs. 3 and 4 where they are compared to the first- and second-order expansions for a plasma frequency $\omega_p = 1.0\times 10^{16}\,\mathrm{s}^{-1}$. As can be seen, the numerical result agrees with the first-order approximation for relatively very large distances, but departs from it as the distance decreases.[49,50] On the other hand, the second-order approximation remains accurate for even shorter distances, before becoming unreliable.

B. The dielectric function

As the dielectric function, along with the distance between the two boundaries, uniquely determines the behavior

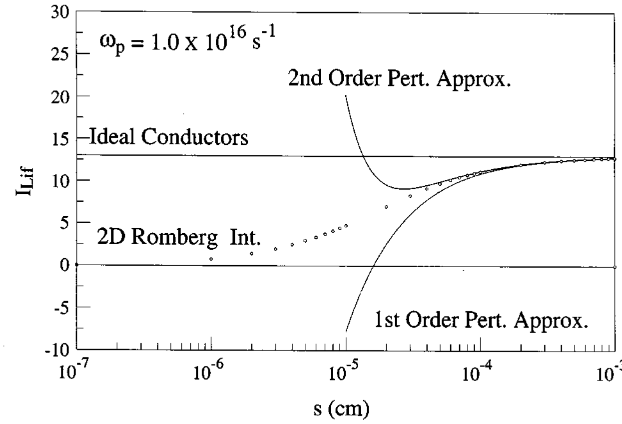

FIG. 3. The Lifshitz integral for an imperfect conductor in the 100 Å–10 μm range, for $\omega_p = 1.0\times 10^{16}\,\mathrm{s}^{-1}$, as calculated by numerical integration, and by means of analytical first- and second-order expansions. As expected, the analytical expansions fail as $c/\omega_p s \sim 1$ while all approaches converge to the ideal conductor value of $2\pi^4/15$ in the $c/\omega_p s \to 0$ limit.

of the Lifshitz integral, data on this quantity over a wide region of the electromagnetic spectrum (0–10 eV) are a necessary step for our calculations.

Historically, this has represented a weak link in the chain leading to accurate estimates of the Casimir force. In fact, all authors involved in calculations of this kind both critically needed and lamented the absence of published information and had to develop alternative schemes to interpolate and extrapolate from whatever information was available at the time (see, for instance, Refs. 41, 51, 23, 22, 52, 53, 50, and 47). In the case of Si, early data in[54] for c-Si were used to evaluate the force between a plate of crystalline Si and a layer of amorphous Si (a-Si) evaporated on a lens of borosilicate glass, despite the significant differences in the optical properties of these two types of silicon.[47]

In the case of several semiconductors, including Si, this lack of data has been filling quite satisfactorily in the last 20 years with accurate measurements of the dielectric function now available over a range of temperatures and doping concentrations. Given the choice of information available and

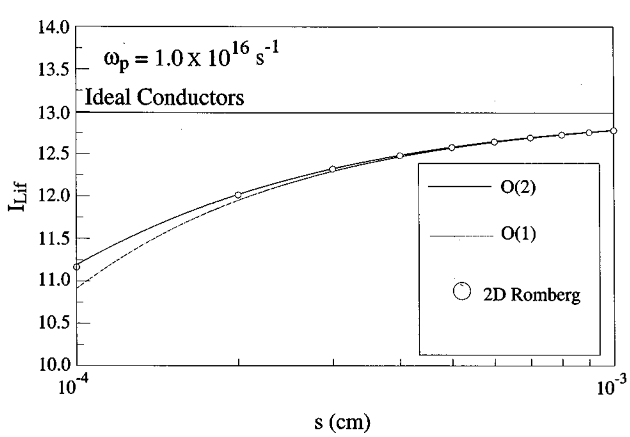

FIG. 4. The behavior of the Lifshitz integral in the 1–10 μm range.

the particular goals of this paper, we have chosen to concentrate on crystalline silicon, both undoped and doped with donor phosphorus [Si(P)].

Because of the Kramers-Krönig relations, the absorption properties of a sample uniquely determine the complex dielectric function $\tilde{\epsilon}=\epsilon_1+i\epsilon_2$ $(i=\sqrt{-1})$. The absorption mechanisms in a semiconductor we shall be concerned with here are band transitions, impurities, lattice vibrations, and free carriers. When deciding on how to account for each of these processes, two issues need to be addressed: (1) Is the absorption contribution from a particular mechanism relatively important when compared to the others? (2) Does the absorption from this particular mechanism appreciably contribute to the value of the Lifshitz integral?

Concerning impurities, the absorption coefficient for photons with energy close to the impurity ionization energy is approximated in Ref. 55 (see also Ref. 56) as

$$\alpha\approx 8.3\times 10^{-17} m N_I / m^* n_r E_d , \tag{3.9}$$

where m is the mass of the free electron, m^* is the effective mass of the electron, n_r is the index of refraction, N_I is the impurity atom concentration, and E_d is the impurity ionization energy, in electron volts. By using typical values for all parameters, one finds $\alpha\sim 10^{-16}N_I$, also quoted in Ref. 41. As the absorption coefficient due to band absorption or free carriers is typically $\sim 10^3-10^5$ or larger, it is clear that impurity absorption can play a significant role only in the case of ultraheavily doped samples. Although, in what follows, we shall deal with such situations, this effect will be neglected, as the frequencies at which the absorption becomes important ($\sim$30 μm) are in the far infrared, well past the exponential cutoff in Eq. (3.1).

The typical frequencies of lattice vibrations for Si are located in the far infrared ($\sim$10–30 μm), largely independently of impurity concentration and lattice defects.[57] Again because of the exponential cutoff in the expression for the Lifshitz integral, and because the absorption coefficients are relatively small ($\sim$10–30 cm^{-1}) compared to free-carrier absorption in the same frequency range, they do not contribute appreciably to the value of the Lifshitz integral and shall not be considered here.

Extensive ellipsometric data on the contribution of interband transitions to the complex pseudodielectric function for pure c-Si at room temperature in the (1.5–6.0) eV range have appeared in Ref. 58 This study provided a thorough update to the measurements in Ref. 54, typically used as the basis for the calculations of the Casimir force between semiconductors in Refs. 41, 47 (the term *pseudo* refers to small effects of uncharacterized surface layers that might affect the published values).

Interestingly, these authors have repeatedly emphasized the importance of assessing the actual relationship between measurements of the dielectric function and the intrinsic bulk properties of the materials used. Such issues as, for instance, cleaning procedures, surface roughness, and contamination have direct bearing on the true value of the Casimir force, both because sample quality enters the idealized expression at Eq. (3.1) via the optical properties, and because it may directly cause departures from the idealized geometry. Consideration of these factors in Casimir force experimentation is still at a very early stage.[19,20]

TABLE I. The best-fit parameters for the five-oscillator, critical point (CP) interband dielectric function of c-Si. The frequency ω_j is given in units of 10^{15} s^{-1}.

Oscillator	f_j	ω_j	Γ_j	S_j
1	1.859 725	5.124 442	0.037 431	−0.239 371
2	5.989 242	5.727 373	0.145 954	−0.115 290
3	2.726 762	6.481 675	0.046 672	0.032 613
4	0.404 491	7.959 576	0.063 017	−0.142 377
5	9.977 181	8.083 606	1.655 511	0.329 895
h_0	0.795 033			

Experimental data on the effects of heavy doping on these absorption spectra appeared in Refs. 59–62. The dependence of the complex dielectric function on the absolute temperature was studied in Refs. 63 and 64. In the present study, these effects have been neglected, although the approach we describe can easily accommodate these data as will be discussed later.

The choice of the analytical description of the interband complex dielectric function is critical to the calculation of the Lifshitz integral. If, because of its very form, the analytical expression of the dielectric function does not lend itself to a good fit of the experimental data, the reliability of the numerical calculation will ultimately be affected.

In the past, in order to simplify the evaluation of the rather complicated Lifshitz integral, the band spectrum of the substances considered in Casimir force studies has been approximated by means of Dirac delta functions,[41] of rectangular functions,[47] and by Lorentz resonance functions.[41,23,51,52,53,50]

Very good representations of the needed complex interband dielectric functions have been obtained over the years by means of several *ab initio* physical approaches.[65–74] Although such results are grounded in physically well-understood interpretations of all quantities involved, none has resulted in expressions with the needed degree of accuracy over the entire energy spectrum.

Consequently, motivated by a practical need, several approximations schemes have been developed to accurately fit the experimental data over the whole range of available measurements by generalizing the traditional Lorentz model.[75] The parameters that appear in these representations do not necessarily have an immediate physical interpretation, although this approach achieves very high goodness of fit[76–78] while closely satisfying the Kramers-Krönig consistency requirements.[79,80]

In our case, we have adopted a five-oscillator, critical point (CP) description for c-Si, according to the following extension of the harmonic oscillator model:[77]

$$\tilde{\epsilon}_{\mathrm{CP}}(\omega)=h_0+\sum_j \frac{f_j e^{iS_j\pi}}{1-\left(\dfrac{\omega}{\omega_j}+i\Gamma_j\right)^2}, \tag{3.10}$$

where the appropriate parameters are shown in Table I (Ref. 81) (notice the slightly different form of this equation with respect to the one appearing in Ref. 77).

The last contribution to the total dielectric function we shall consider is that due to free carriers. As is well known, the important features of free-carrier absorption can be accounted for by means of the classical Drude model of electrical conductivity,[82–84,75,85] which predicts a dielectric function of the kind

$$\tilde{\epsilon}_{\rm fc}(\omega)=\epsilon_\infty-\frac{\omega_p^2}{\omega(\omega-i/\tau)}, \tag{3.11}$$

where τ is the relaxation time. In general, experience shows a satisfactory agreement with the prediction of the Drude model that the free-carrier absorption coefficient $\alpha_{\rm fc}\propto 1/\lambda^2$ for semiconductors in which $\omega\tau\gg 1$ and the mean electron energy $\langle E\rangle \ll \hbar\omega$, usually at submillimeter wavelengths.[86,87]

At shorter wavelengths, the dependence of the absorption coefficient on the wavelength becomes more complicated,[88,85] and a full quantum treatment is necessary to develop a theory based on first principles for all relevant carrier scattering mechanisms. This analysis, contained in an extensive body of published literature, has been relatively successful in quantitatively describing the departure from the classical model shown by the experimental data at middle infrared wavelengths for a number of semiconductors, including n-doped Si.[89–92,56,93–104] Alternatively, a generalized Drude approximation has also been successfully used to develop a first-principle theory of electron scattering by assuming a frequency dependent relaxation time $\tau=\tau(\omega)$.[105]

In what follows, we shall adopt the classical Drude model both because of its relatively good success in fitting the doped Si data over a wide frequency range[106–110] and because of its relative ease of implementation. Furthermore, persisting disagreements between theory and experiments even in the case of n-doped Si (Ref. 105) indicate that the payoffs of introducing a much more complicated quantum model may be past the point of diminishing returns. Perhaps, the best course of action to further improve the present treatment may very well be to fit data of the specific experimental sample under study by means of generalized semiempirical expressions, in analogy with the approach used to describe the interband spectrum above.

Finally, the total complex dielectric function used in the Lifshitz integral is given by substituting $\omega\to i\xi$ in the sum of the interband and free-carrier contributions just discussed:

$$\tilde{\tau}_{\rm tot}(i\xi)=\frac{\omega_p^2}{\xi^2+\xi/\tau}+h_0+\sum_j \frac{f_j e^{iS_j\pi}}{1+\left(\dfrac{\xi}{\omega_j}+\Gamma_j\right)^2}, \tag{3.12}$$

where it is assumed that the high-frequency limit of the free-carrier dielectric function match the low-frequency limit of the interband term.

C. Numerical results

We carried out numerical calculations of the Casimir force for both high resistivity ($\omega_p=0$, case 0) c-Si and for three different carrier concentrations considered in Ref. 110: (1) $n_e=0.011\times10^{19}\,\mathrm{cm}^{-3}$, (2) $n_e=0.52\times10^{19}\,\mathrm{cm}^{-3}$, (3) $n_e=10\times10^{19}\,\mathrm{cm}^{-3}$. Furthermore, we considered the hyperheavy doped case (4), with $n_e=3.8\times10^{21}\,\mathrm{cm}^{-3}$.

The Lifshitz force is determined by the product of the inverse of the fourth power of the distance between the two semiconducting boundaries by the Lifshitz integral, as shown

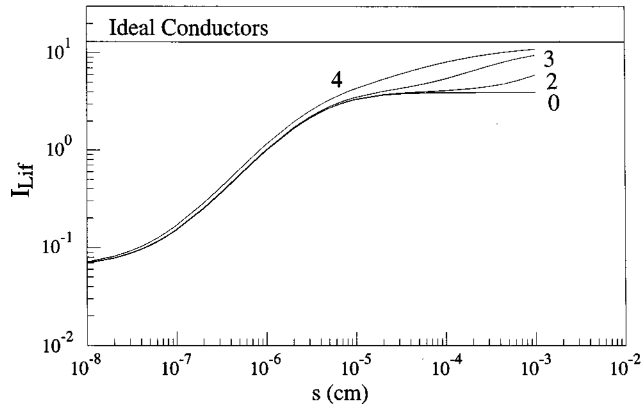

FIG. 5. The behavior of the Lifshitz integral for c-Si in the 10^{-4}–$10^2\,\mu$m range, both for the high resistivity material (0) and for different carrier concentrations (2–4). The curve corresponding to case (1) is not shown as it does not significantly depart from the baseline case.

in Eq. (3.1). Figure 5 shows the results of the numerical calculation of the Lifshitz integral as a function of the distance for the above doping levels.

As expected from both analytical and physical considerations, the value of this quantity undergoes a substantial increase for distances larger than the wavelengths where the dielectric function of Si shows significant absorption structure.

In the cases of nonvanishing carrier concentration, the Lifshitz integral shows a departure from the baseline case, which is proportional in magnitude to carrier concentration and becomes evident at distances of the order of the characteristic wavelength of plasma oscillations. Unlike the high resistivity case, in which the Lifshitz integral approaches a value always smaller than the ideal $2\pi^4/15$ as the distance increases, if carriers are present the Lifshitz integral always behaves as that of an imperfect conductor in this same limit.

As a consequence of the existence of an upper bound to the value of the Lifshitz integral, the Casimir force decreases approximately as the inverse of the fourth power of the distance for relatively larger distances (retarded regime). For small separations, one recovers the dependence on the inverse of the third power of the distance characteristic of the unretarded van der Waals interaction between dielectrics (Fig. 6).

After having computed the Casimir force for all cases, the effect of the free carriers was estimated from the quantity $\Delta F_{\rm Cas}(s)=F_{{\rm Cas},i}(s)-F_{\rm Cas,0}(s)$, which is shown in Fig. 7. As a term of comparison, it is useful to notice that, for all distances of interest in what follows, such a perturbation of the Casimir force is usually much larger than that due to finite temperature corrections for $T\sim10^2\,$K. That is, $\Delta F_{\rm Cas}(s)/F_{\rm Cas,0}\gg\frac{16}{3}(k_BTs/\hbar c)$.[42] Similarly, this change is also much larger than the second-order radiative correction to the Casimir force, which is $\approx\alpha\lambda_c/s$, where α is the fine structure constant and λ_c is the Compton wavelength of the electron.[111–114]

Finally, $\Delta F_{\rm Cas}(s)$ was integrated between a minimum distance $s_{\rm min}$ and a maximum distance $s_{\rm max}=10\,\mu$m by means of a spline interpolation over the computed force data points. This integral corresponds to the quantity $W_{\rm Cas}$ at Eq. (2.7),

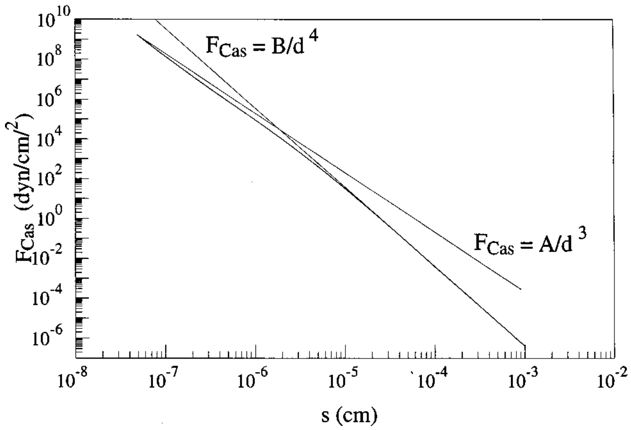

FIG. 6. The Casimir force for high resistivity c-Si. Also shown are the asymptotic behaviors in the nonretarded and retarded regimes.

that is, the total work done by the Casimir force over a closed cycle including as branches the Casimir force of the particular carrier concentration considered and that for the high resistivity case, as discussed in Sec. II. The results are shown in Fig. 8 for the four carrier concentrations and for three different values of the minimum distance, 1 Å, 5 Å, and 10 Å. As can be seen, the Casimir force field energy transfer lies in the $\sim 10^{-3} - 10^{2}$ erg/cm^2 range depending on the values of the $(s_{\min}, n_e)$ variables.

IV. CYCLE ANALYSIS

In order to understand the implications of the results reported in the previous section, we must be able to provide an order of magnitude for the other terms in Eq. (2.6). As a substantial change in the number of free carriers in a semiconductor can be achieved by means of an array of different techniques, here we shall focus on a particular technological realization of the cycle described in Sec. II.

In general, any implementation of the moving boundary system studied so far with the addition of a means to transform the available part of the potential energy associated to the Casimir force field into electrical energy will be referred to as a transducer of vacuum energy. In what follows, we

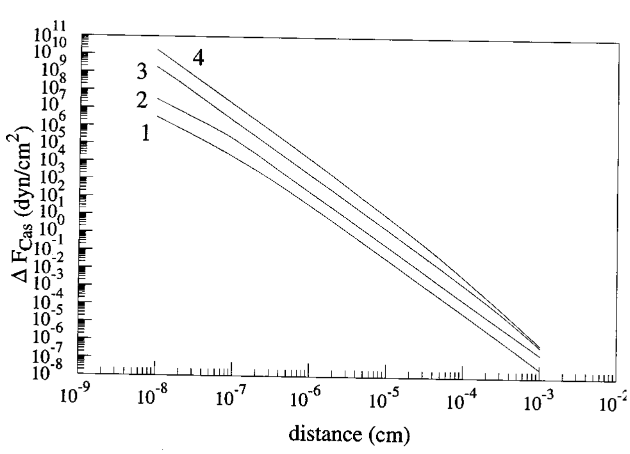

FIG. 7. The effect of free carriers on the Casimir force for the four doping levels considered.

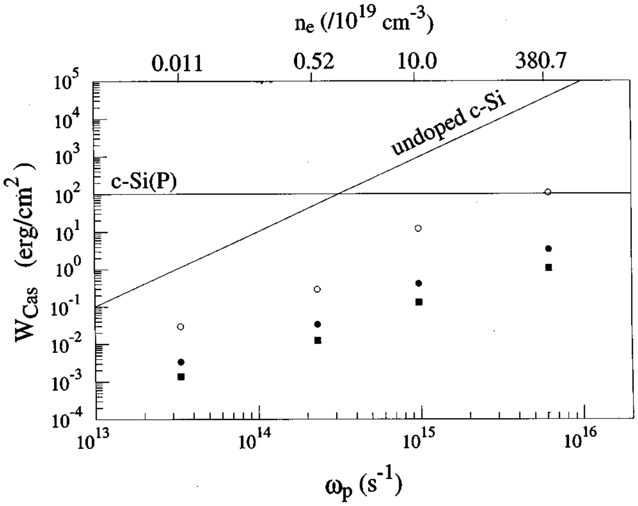

FIG. 8. The net work done by the Casimir force over a closed cycle compared to the energy required to achieve the free carrier concentration indicated, W_{DA} (solid lines), for both doped and undoped c-Si, as a function of the plasma frequency. Open circles, filled circles, and filled squares refer to $s_B = s_C = 1$ Å, 5 Å, and 10 Å, respectively ($s_A = s_D = 10\ \mu$m).

shall focus on an idealized, optically controlled transducer of vacuum energy, designated as an optically controlled idealized vacuum energy transducer (OC-IVET). It shall become immediately evident that other possibilities, based, for instance, on thermal control (TC-IVET), can be considered without complications in complete analogy with the reasoning below.

As can be seen from Figs. 9–12, a semiconducting membrane (SCM) and a semiconducting plane (SCP), respectively, play the role of the moving and fixed semiconducting boundaries considered above. However, the top side of the moving membrane is made conducting and it faces an upper conducting plane, labeled (CP2). Similarly, the opposite side of the fixed boundary is made conducting, and it faces a bottom conducting plane, labeled (CP1). The semiconduct-

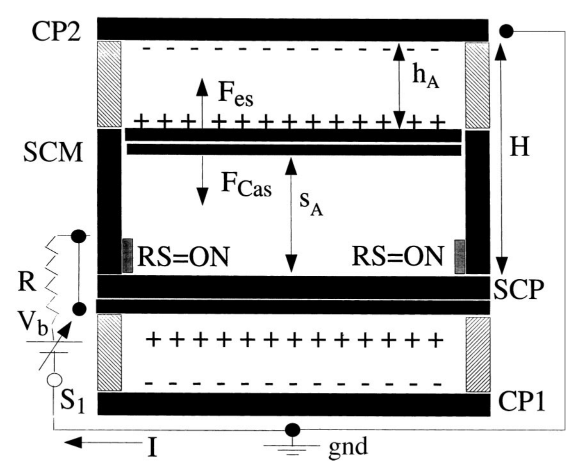

FIG. 9. A schematic view of the optically controlled idealized vacuum energy transducer (OC-IVET) and of the forces involved during the $D \rightarrow A$ transformation. The radiation sources are on and the carrier concentration is increasing (darker tone). Consequently, positive charges must be flowing as indicated in order to increase the electrostatic force balancing the Casimir force.

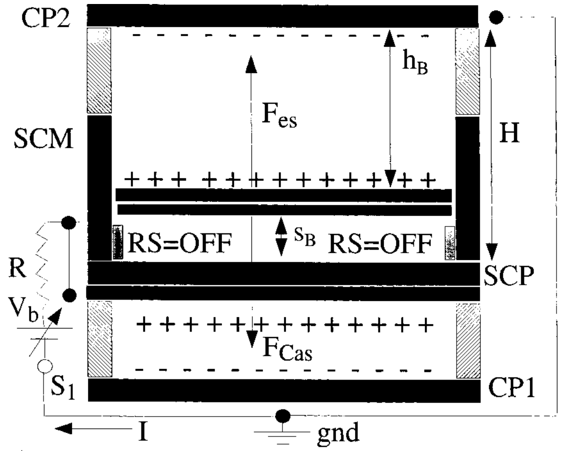

FIG. 10. During the $A \rightarrow B$ transformation, charge continues to flow as indicated to maintain the SCM in quasistatic equilibrium as the SCM-SCP distance decreases.

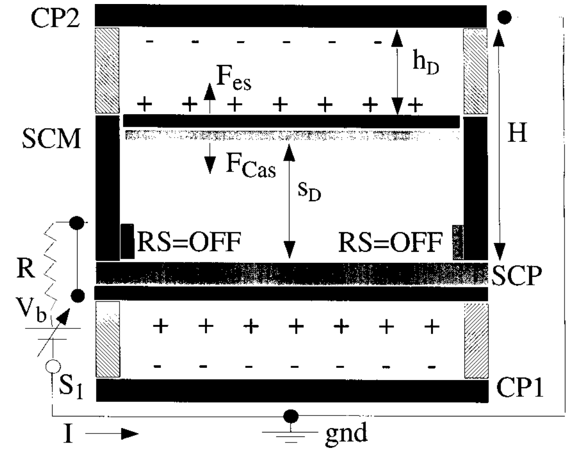

FIG. 12. The $C \rightarrow D$ transformation is completed when the SCM reaches its maximum distance from the SCP while the free carrier concentration is at a minimum. The increasing distance between the SCM and the SCP causes the current to continue flowing as shown.

ing membrane and plane on the one hand, and the two conducting planes independently on the other, are maintained at the same potentials by appropriate contacts. However, if switch S_1 is closed, a bidirectional power supply can apply a potential difference V_b between the (SCM-SCP) and the (CP1-CP2) subsystems. Also indicated are monochromatic radiation sources (RS) tuned to the impurity ionization energy.

In the case of microelectromechanical system (MEMS) engineering designs, the above subsystems can be included within an integrated structure, in close analogy with already existing implementations for the optical control of semiconductor microactuators[115–117] (Fig. 13).

A. Energy conversion cycle

In order to describe the conversion of potential energy associated to the Casimir force into electrical energy, we shall consider in detail the four conducting surfaces CP2, conducting (upper) side of SCM, conducting (lower) side of SCP, and CP1 of the OC-IVET. We shall assume that, at any time, the value of the potential difference V_b between the (SCM-SCP) and the (CP1-CP2) subsystems is such as to maintain the semiconducting membrane either in equilibrium or in quasistatic equilibrium during the cycle. We shall neglect the effects of mutual electrostatic and mechanical interactions between the semiconducting and conducting layers in what follows. That is, for instance, we shall not consider the possible effects on free-carrier concentration due to the additional electric fields caused by the voltage source and to mechanical stresses.

As is seen from a straightforward application of the electrostatic theory of a system of N conducting planes[118,119] to our specific case, the excess charge due to the potential difference V_b will equally distribute on the facing sides of the CP2-SCM and SCP-CP1 conducting surfaces with no charges appearing on the facing sides of the SCM-SCP subsystem, as indicated in Figs. 9–12.

The free body diagram for the moving membrane, also shown in Figs. 9–12, includes the Casimir force between the two semiconducting boundaries and the electrostatic force between the upper conducting plane and the semiconducting membrane. The Casimir force between the upper conducting plane and the moving membrane is neglected in the assumption that $H \gg s$; as the carrier concentration and the temperature of the conducting surfaces is assumed to be constant, removing this assumption would not change our conclusions.

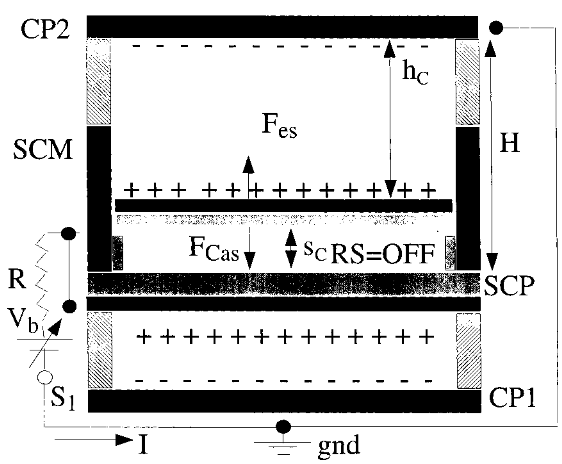

FIG. 11. The free carrier concentration decreases (lighter tone) while the SCM-SCP distance is held constant during the $B \rightarrow C$ transformation. This causes the current to reverse its direction.

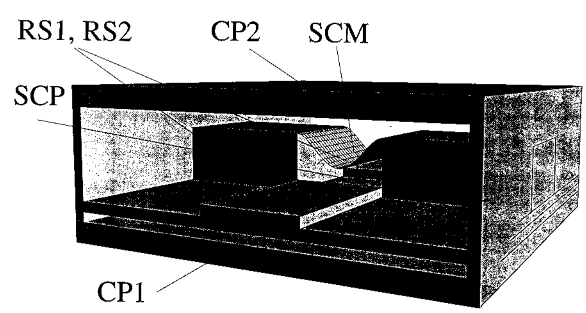

FIG. 13. A possible MEMS implementation of the OC-IVET. The plane, semiconducting membrane (SCM) is replaced by a flexible membrane whose shape is determined by the combined action of elastic, electrostatic, and Casimir forces.

With reference to our general discussion in Sec. II, we shall assume that, at some time before the beginning of the cycle for this OC-IVET, switch S_1 was closed and a potential difference V_b was applied while the membrane was at a distance s_D from the semiconducting plane. We shall also assume that this state, corresponding to point D in Fig. 2, be a position of mechanical equilibrium for the system. Also, as the distance between the SCP and the CP1 does not change during the cycle, we shall immediately conclude that the total energy change over a cycle due to the interaction of these two elements vanishes and we shall not consider it any further.

The radiation sources (RS) are then turned on and, as we have seen in detail, the illumination of the facing sides of the semiconducting media will cause an increase of the Casimir force between the two boundaries, corresponding to a transformation towards point A in the cycle.

We shall assume that the increase in illumination of the semiconducting surfaces by the radiation sources, and the corresponding Casimir force increase, take place as a succession of very small changes matched by correspondingly small increases of the potential difference V_b so that the moving membrane remains at rest during this sequence of transformations leading the system from point D to point A of the cycle (Fig. 9).

The infinitesimal change in the electrostatic energy $U_{\rm es}$ of the field in the CP2-SCM region of space during the $D\rightarrow A$ transformation is $dU_{\rm es}=[\partial(\frac{1}{2}CV_b^2)/\partial V_b]dV_b=CV_b dV_b$, where C is the capacitance of the CP2-SCM subsystem, $C=\epsilon_0 A/h$, A is the CP2-SCM surface area, and $h=H-s$ (mks units are used in this subsection).[120]

The corresponding decrease in battery energy due to the transfer of charge dq is $dU_b=-V_b dq=-CV_b dV_b$. Thus, for each of these infinitesimal transformations, $dU_{\rm es}=-dU_b$ and the change in the total energy is $dU_{\rm tot}\equiv dU_{\rm es}+dU_b=0$. A completely analogous conclusion can be reached for the $B\rightarrow C$ transformation, provided that the carriers recombine at a ''slow'' rate and the potential difference is decreased by a sequence of infinitesimal transformations. Consequently, there is no net change in the total energy of the system for both the $D\rightarrow A$ and the $B\rightarrow C$ transformations (Figs. 9 and 11).

In the case of $A\rightarrow B$ and the $C\rightarrow D$ transformations, the infinitesimal change in electrostatic energy is due to a variation of both capacitance and potential (Figs. 10 and 12). This can be written as

$$\begin{aligned} dU_{\rm es}&=\frac{\partial}{\partial V_b}\left(\tfrac{1}{2}CV_b^2\right)dV_b+\frac{\partial}{\partial C}\left(\tfrac{1}{2}CV_b^2\right)dC\\ &=CV_b dV_b+\tfrac{1}{2}V_b^2 dC. \end{aligned} \tag{4.1}$$

In order to proceed, we shall assume that the electrostatic force between the SCM and the CP2 be equal and opposite to the Casimir force during this quasi-equilibrium transformation. That is, $F_{\rm es}(h)=\frac{1}{2}\epsilon_0 E^2 A=\frac{1}{2}\epsilon_0 (V_b/h)^2 A=F_{\rm PP}(h)$. This yields $V_b=[(2h^2/\epsilon_0 A)F_{\rm PP}(h)]^{1/2}$ with a total charge on each conducting surface equal to $q=\sigma A=[(2\epsilon_0 A)F_{\rm PP}(h)]^{1/2}$.

By writing $dV_b=(\partial V_b/\partial h)dh$, $dC=(\partial C/\partial h)dh$, and $dq=(\partial q/\partial h)dh$, it is immediate to prove that

$$dU_{\rm tot}\equiv\frac{\partial}{\partial h}(U_{\rm es}+U_b)dh=-F_{\rm PP}(h)dh=dW_{\rm Cas}, \tag{4.2}$$

where $dW_{\rm Cas}$ is the infinitesimal work done by the Casimir force.

By combining the result for this kind of transformation with that for constant-position transformations, one can conclude that the total energy change over a closed cycle is

$$\Delta U_{\rm tot}=W_{\rm Cas}\equiv-\oint F_{\rm Cas}(h;Y_i(X_j))dh, \tag{4.3}$$

where the unimportant $s\rightarrow h$ change of variable was explicitly indicated and $ds=-dh$.

As $\Delta U_{\rm tot}=\Delta U_{\rm es}+\Delta U_b$, and as the net electrostatic energy change over a closed cycle vanishes ($\Delta U_{\rm es}=0$), the total energy change found above must appear as battery energy ($\Delta U_b=W_{\rm Cas}$). In other words, at the end of every closed cycle, *the battery is charged by an energy amount equal to the net work done by the Casimir force over that same cycle*. This energy is then available for use by connecting the battery to a load. Of course, by adopting a reverse cycle (counterclockwise direction in Fig. 2), one will find that the battery becomes discharged by an amount $-|W_{\rm Cas}|$. In the more realistic case of nonquasistatic transformations, some energy made available by the cycle will be dissipated on the load R and it will appear as Joule heating of such element [Figs. (9–12)].

B. Time scales and total energy budget estimates

In order to obtain an order of magnitude for the total energy $W_{\rm DA}$ expended to achieve a free-carrier concentration increase, we shall assume that the radiation sources are active for a time Δt_γ relatively larger than the free-carrier recombination time, $\tau_{\rm fc}$. This allows for the equilibrium concentration $n_e=J_{\omega_d}(1-R_{\omega_d})\alpha\tau_{fc}/\hbar\omega_d$ to be reached,[47] where J_{ω_d} is the radiation flux, R is the reflectivity of the material, α is the absorption coefficient, and ω_d is the ionization frequency of the donor impurities. This yields the following expression for the energy $W_{\rm DA}$:

$$W_{\rm DA}=J_{\omega_d}\Delta t_\gamma=\frac{n_e}{(1-R_{\omega_d})\alpha\tau_{\rm fc}}\hbar\omega_d\Delta t_\gamma. \tag{4.4}$$

By substituting the approximate expression for the absorption coefficient of impurities at Eq. (3.9), and by using $R_{\omega_d}\approx 0.3$, $n_r\approx 3.5$, $m^*/m\approx 0.32$, (Refs. 121 and 109) $E_d({\rm Si(P)})=\hbar\omega_d\approx 0.044\,{\rm eV}$, we find the numerical estimate:

$$W_{\rm DA}\approx 10^2\frac{n_e}{N_d}\frac{\Delta t_\gamma}{\tau_{\rm fc}}\ \ {\rm erg/cm^2}, \tag{4.5}$$

where the impurity concentration has been set equal to the donor atom concentration N_d.

In order to provide a term of comparison, the quantity at Eq. (4.4) was also estimated in the case of intrinsic absorption for pure c-Si. By using $\hbar\omega=4.34\,{\rm eV}$, $R=0.7$, $\alpha=2.2\times 10^6\ {\rm cm}^{-1}$,[122] we find

$$W_{\rm DA}\approx 10^{-17}n_e\ \ {\rm erg/cm^2}. \tag{4.6}$$

Figure 8 shows a comparison between the value of the Casimir force field energy available for transformation with the estimates provided by Eqs. (4.5) and (4.6), in the simplifying assumption that the contribution to the Casimir force due to intrinsic holes may be neglected. As can be seen, there seems to be a net amount of energy available in the case of hyperdoped c-Si for minimum interboundary distances of the order of ~ 1 Å.

Also, it appears that using doped silicon is advantageous only in the high-carrier concentration regime, whereas using pure Si allows one to obtain higher values of W_{tot} for lower carrier concentrations.

The potential implications of these findings are undoubtedly stimulating, as they show that the gedanken experiment previously described exposes a situation with very practical, and not only theoretical energy consequences. For this reason, a substantial effort will have to be focused on strengthening the quantitative reliability of the cycle calculations presented in this paper. Among the many points that may have relevance in the technological implementation of the ideal transducer described above, we shall mention the following four.

The first is that, although the theory of Casimir forces has been used for interboundary distances ~ 1 Å in the past, for instance in the context of particle adhesion and chemisorption theories,[41,123] corrections to the predictions obtained here in such regimes are expected[47] and have been dealt with in the case of metals and dielectrics.[124–126] Notice also that we neglected the effects due to both gravitational and Pauli repulsion interactions between the two boundaries as they may affect membrane dynamics but not the overall energy budget because of their conservative nature.

The second comment is that, as one can verify directly, the plasma of free carriers produced in the case of high-carrier concentration silicon is located in a surface layer thinner than the wavelength of the oscillations of that plasma. This has already been discussed in similar experimental situations[50,47] and a complete analysis of the problem will require an extension of the Lifshitz integral to multiple layer systems, as in the case of water-hydrocarbon films.[51]

Third, one must consider the time scales involved in the idealizations made above. In order to achieve such ideal performance, one would need to move the semiconducting membrane by distances ~ 10 Å within the free-carrier recombination time, that is, $\tau_{\mathrm{fc}} \sim 10^{-14}$ s in the highly doped case.[127,128,109] This is, relatively speaking, a much higher average speed than that achieved by membranes than can vibrate over ~ 1 μm at frequencies $\sim 10^4$ Hz. From this point of view, the free-carrier lifetime in the undoped case appears to be much more manageable, falling in the $\sim 10^{-4}$ s range.[47] In connection with the time scale problem there is that of electromagnetic losses, which was not addressed here.

Finally, it is important to notice that the comparison made in Fig. 8 does not include the value of the energy released back into the environment, for instance because of carrier recombination, W_{BC}. The author cannot identify any reason of principle that would forbid one from collecting a fraction of this energy, in the form of heat or recombination radiation, to further improve the overall energy budget for the system. Such improvement would, in turn, allow one to achieve a positive net energy exchange at larger interboundary distances, so facilitating possible engineering implementations.

Although these comments, as well as those relating to any other idealization made herein, all appear to be technologically relevant, they do not represent, it seems, obstacles in principle to manipulating a net positive energy associated to the Casimir force. In other words, it appears quite likely at this stage that, given the multiple infinity of the parameter space in which the above considerations could be applied, appropriate dimension and time scales, materials, environmental conditions, and techniques could be identified to take technological advantage of this effect.

Evidently, if the present approach is to succeed in the future, research will have to be carried out to identify, for instance, semiconductors allowing for the appropriate doping levels, impurity ionization energies, and recombination times to minimize the energy price to be paid to take maximum advantage of the nonconservative nature of the Casimir force. However, it must be stressed that even transducer of vacuum energy working below the break-even point would be of potentially great commercial interest because of their possible use to dramatically improve on solar power conversion.

It may also be useful to look back and compare the transducer of vacuum energy concept in general and the OC-IVET in particular to other systems described in the literature in connection to potentially feasible, innovative technological proposals in Casimir force research. As already pointed out, use of the Casimir force to compress same-charge leaves to transfer electrical energy was suggested in Ref. 28. However, as the Casimir force was explicitly considered independent of any parameter other than the interboundary distance, no closed engine cycle attaining a net energy exchange could be proposed. Microelectromechanical system (MEMS) technology was described Refs. 24 and 25 to achieve submicron fabrication of Casimir force-driven devices, but not to exchange net energy. The only experimental results on controlling the van der Waals force between two semiconducting surfaces by means of radiation were given in Ref. 47. Once again, however, Casimir force field energy exchange was not mentioned as a possible implication of experiment nor theory.

Furthermore, two important corollaries of the above results appear to have received little or no explicit mention despite their stimulating technological promise. The first is the possibility of achieving optical control of Casimir force actuated devices, in close analogy with already existing technologies for the control of semiconductor microactuators.[115–117] As the Casimir force acts on components on any scale, this technology could allow for the direct dynamical manipulation and control of semiconducting nanostructures.

The latter implication is that, as the Casimir force depends directly and appreciably on the optical properties of the materials involved, an inverse Casimir-force-based approach can be developed to explore such properties in an entirely noninvasive way. Potentially, the complex dielectric function of a sample could be probed in real time by means of dynamical or static Casimir force measurements.

Finally, it is important to notice that there is no relationship between the application of the Casimir force discussed above, which is fundamentally static, and others, which are instead based on dynamical Casimir effects.

These would include, for instance, earlier, and recently refuted speculations about the possible relevance of the dynamical Casimir effect in the interpretation of sonoluminescence experimental data.[129,130,42–45]

Also, because of the assumption of a quasistatic regime, we have neglected any dependence of the Casimir force on the relative velocity of the two boundaries, which is expected to manifest itself as an ''electromagnetic vacuum viscosity.''[131] For the same reason, we have not considered the effects of the predicted creation of real photons between the two boundaries due to both their relative motion and to the change of their optical properties (Ref. 132, and references therein). In fact, it is important to recall the proposal to simulate the highly accelerated motion of a boundary with respect to another by causing a sudden change in the conductivity of a semiconducting boundary at rest by means of a femtosecond laser pulse.[133] However, even though these ''dissipative'' phenomena are not included here, it is clear that the existence of dynamical Casimir effects contributes an additional perspective to the issue, introduced at the end of Sec. II, of whether the Casimir force should be considered as fundamentally nonconservative.

V. CONCLUSIONS

This paper has primarily dealt with the following three important issues: (1) is it possible to design a closed engine cycle at the end of which the net energy transferred to a system does not equal the net work done by the Casimir force field? (2) If the answer to the first question is affirmative, is it in principle that the work done by the Casimir force be larger than the absolute value of the net energy transferred to the system? (3) Finally, if the answer to the last question is also affirmative, do the orders of magnitude of all physical quantities involved point towards a situation that may be of practical, technological interest?

The gedanken experiment discussed in Sec. II has provided a very general framework to address the first issue. The strength of the conclusions one can draw from such logical reasoning lies with its independence on the particular system involved or on any assumptions on the origins of the Casimir force.

The fundamental reason for this is that the Casimir force, and other closely related concepts,[45,134] such as the Casimir-Polder[135] or the van der Waals forces,[136] all depend on quantities that can, at an energy expense, be altered. The question is, therefore, always well posed as to whether the energy to effect such transformations equals the net work done by a Casimir force cycle that includes them, or equivalently, whether W_{tot} in Eq. (2.6) must necessarily vanish.

Furthermore, the issue exposed remains independently of whether the Casimir force is seen, for instance, within a source theory or as associated to quantum vacuum fluctuations; likewise, the question applies in the case of any subatomic, atomic, microscopic, or macroscopic system considered. For instance, one can conceive of processes of excited (Rydberg) atom scattering by a surface whose possible final outcome is a net increase in the total translational kinetic energy of the scattered atom at the expense of the potential energy of their mutual dispersive force interaction (see below).

In the second part of this paper, we quantitatively implemented the idealized engine cycle of Sec. II by using two boundaries of semiconducting c-Si, whose optical properties are made vary by means of appropriate radiation sources.

In order to improve on previous estimates of the Casimir force in this case, we adopted a mathematically much more sophisticated description of the band transitions spectrum, compatible with the most recently published data, and computed the full Lifthitz integral numerically without any integrand simplifications.

Most of the needed improvements on the results so obtained, such as, for instance, a multilayer approach and the exact shape of the free-carrier absorption spectrum, can probably be dealt with effectively only within a specific experimental situation. Furthermore, effects such as that of surface roughness on the value of the Casimir force can only be assessed once the scale of the device is established.

The exploratory calculations conducted both in the case of donor phosphorus doping and of pure silicon show that, for very high free-carrier concentrations and for minimum boundary distances of the order of the interatomic distance, it is certainly possible that the net work done by the Casimir force exceed the total energy expended on the transducer. The orders of magnitude involved indicate that the total work done by the Casimir force can be as high as $\sim 10^2$ erg/cm^2/cycle. At a rate of 10^4cycles/s, this corresponds to a power per unit area of $P_{\mathrm{Cas}} \sim 1$ kW/m^2.

A very stimulating question concerns the possibility of manufacturing transducers of vacuum energy approximating the ideal one here described by making use of micromachining technology. This approach would allow one to prepare, for instance, silicon structures such as microcantilever beams and microbridges, which have a natural oscillation frequency of the order of the free-carrier lifetime in the same materials. For instance, from elementary elasticity theory,[137] one can readily show that the equivalent free vibration frequency of a bulk silicon cantilever with width, length, and thickness equal to 50 μm, 100 μm, and 5 μm, respectively, is $\sim 10^4$ Hz (Ref. 138) the same quantity we just used to calculate the Casimir power; *in vacuo*, quality factors as high as $\sim 10^4$ can be obtained.

By using our estimates, we find that the total Casimir power for this cantilever ($\sim 5 \times 10^{-10}$ W) would establish a potential difference ~ 1 mV across a 1-kΩ load. It is also interesting to obtain the ratio of the average heat capacity of this cantilever to the Casimir power obtained above. This yields $C/P_{\mathrm{Cas}} \sim 10^{-2}$ s/K, where the room-temperature value of the specific heat [0.18 cal/(g K)] was used.[139]

On the other hand, besides the already discussed need to identify a greater number of promising semiconducting materials and of developing more sophisticated models for the ideal case, we are still far from fully understanding the dynamics of flexible membranes under the combined action of Casimir and electrostatic forces.[24,25] For this reason, it is certainly too soon to quantitatively predict what fraction of the net work done by the Casimir force one would be able to transform into electric energy with devices of this kind.

In closing, it may be useful to mention some of the related issues on which research by the present author is focusing. This effort deals with a full exploration of all questions raised by the gedanken experiment here described and of its proposed implementations. This includes the development of a full thermodynamical theory of Casimir force systems and its relationship to both relativistic quantum field theory and, in general, to presently existing theories of the Casimir force. The effects of modeling will be taken into more detailed account, especially in the context of more realistic, dynamical systems, such as those mentioned in the context of MEMS engineering.

It is now clear that an object such as the OC-IVET, or any other equivalent implementation of the transducer of vacuum energy proposed herein, can be used as both a static and dynamical device to carry out the real time measurement of a number of important properties of the boundary materials. An inverse application of this concept promises to achieve the Casimir force-based dynamical control of microstructures and nanostructures. Furthermore, a very stimulating possibility is being explored to use these Casimir force systems as probes of such forces as gravitation in previously unaccessible regimes, where both gravity and quantum field theory play a simultaneous role.[140–143]

Evidently, as Casimir effects play a role in systems on every scale, there is great interest in using the concepts so far described in situations where the net energy available for transformation may be much larger than that estimated herein. In this context, the present approach is being applied to both atomic and subatomic systems.

Finally, it is useful to recall further examples of advanced computational techniques and applications of the Casimir effect to the electromagnetic field and other types of fields, and even, in some cases, relations between them.[29,34,144–151] These works, among others, can represent the foundation for the extension of the gedanken experiment here discussed to entirely different domains as well as to include other fundamental interactions in nature.

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