Resonant transmission of normal electrons through Andreev states in ferromagnets

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Giant oscillations of the conductance of a superconductor-ferromagnet-superconductor Andreev interferometer are predicted. The effect is due to the resonant transmission of normal electrons through Andreev levels when the voltage V applied to the ferromagnet is close to $2h_0/e$ (h_0 is the spin-dependent part of the electron energy). The effect of bias voltage and phase difference between the superconductors on the current and the differential conductance is presented. These effects allow a direct spectroscopy of Andreev levels in the ferromagnet. [S0163-1829(99)04545-2]

Recently a high sensitivity of the conductance of mesoscopic systems to the superconductor phase difference ϕ has been observed and theoretically considered in superconductor–normal conductor–superconductor structures ($S/N/S$ structures) (see, e.g., the review paper by Lambert and Raimondi[1]). This effect arises because of a quantum interference of quasiparticles due to Andreev scattering at two (or more) N-S interfaces. This is caused by the fact that the phase of the superconducting condensate is imposed on the quasiparticle wave function in the normal metal. One of the manifestations of the quantum interference is giant oscillations of the conductance of the normal metal as a function of the phase difference between the superconductor predicted in Refs. 2,3.

A single electron in a normal metal with energy below the superconductor energy gap Δ cannot penetrate into the superconductor. However, under Andreev reflection at an N-S interface two electrons with nearly opposite momenta and spins leave the normal metal to create a Cooper pair in the superconductor; hence the incident electron is transformed into a hole with the opposite direction of the spin. The spin flip does not effect the interference pattern of the nonmagnetic normal metal because all energy levels are doubly degenerate with respect to spin. In ferromagnets, however, this degeneracy is lifted due to the interaction of the electron spin with the ferromagnet's spontaneous moment (below we refer to it as the exchange-interaction energy h_0), and electrons with opposite spins occupy different energy bands (Fig. 1). In this case, the change of spin direction associated with Andreev scattering shifts the reflected quasiparticle from one band to the other. The latter influences the quantum interference. The Josephson current in a superconductor-ferromagnet-superconductor ($S/F/S$) structure was investigated in Refs. 4–6; transport properties of F/S junctions were investigated in Refs. 7–15; experiments on the boundary resistance of an $S/F/S$ system were reported in Ref. 16, and phase-coherent effects in the conductance of a ferromagnet contacting a superconductor were observed in Ref. 17. In this paper we predict giant oscillations in the conductance of an $S/F/S$ heterostructure in which the ferromagnet part is separated from the reservoirs of normal electrons with potential barriers (''beam splitters'') of low transparency, $t_r \ll 1$, see Fig. 2.[18]

In the case of Andreev reflections, the paramagnetic effect essentially modifies the interference pattern in the ferromagnetic region. The momentum of an electron with spin up/down $p^e_\uparrow/p^e_\downarrow$ and the momentum of the reflected hole with the spin down/up $p^h_\downarrow/p^h_\uparrow$ are (see Ref. 7).

$$p^{(e)}_{\uparrow\downarrow}=\sqrt{p_F^2+2m(E\pm h_0}, \quad p^{(h)}_{\downarrow\uparrow}=\sqrt{p_F^2-2m(E\pm h_0)}, \tag{1}$$

where E is the energy of the incident electron measured from the Fermi level ϵ_F, p_F is the Fermi momentum, and m is the electron mass.

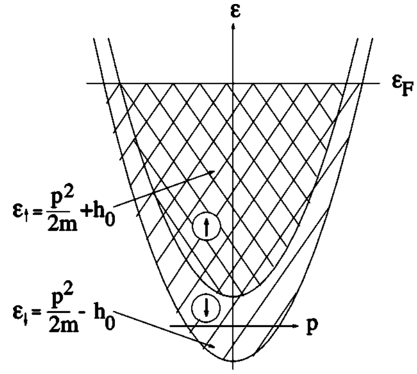

FIG. 1. Energy bands for electrons with opposite spins.

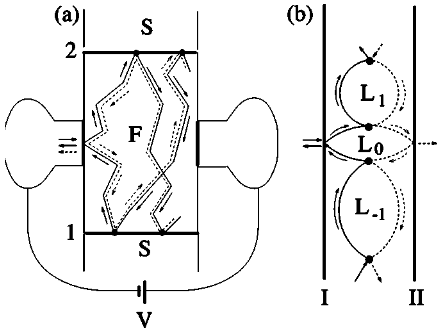

FIG. 2. (a) The geometry of the system under consideration and a classical path contributing to the resonant part of the conductance. Thick lines indicate potential barriers. (b) Schematic representation of the path in (a) through which the resonant transmission occurs: an incident electron tunnels through potential barrier I moves along a 1D chain of scatterers at F/S interfaces (dots) where Andreev + normal scattering takes place, and is reflected back through the first barrier I as an electron and transmitted through the second barrier II as a hole. Semiclassical electron and hole paths are shown with full and dashed lines, respectively. The 1D chain is disordered as the lengths of the sections between successive scattering events at different F/S interfaces L_i; $i=0,\pm 1,\ldots$ are randomly distributed.

From Eq. (1) it follows that in contrast to the nonmagnetic case, near the Fermi level ($E\approx 0$) the electron and the hole momenta in the ferromagnet are different, and for large enough h_0 (usually h_0 is greater than the Thouless energy) the interference effects are absent due to the destructive interference. This fact demonstrates the conflict between superconductivity and magnetic ordering in $S/F/S$ structures. However, interference effects in the ferromagnet can exist albeit at some finite voltage V applied between the reservoirs. If the energy $|E|\approx h_0<|\Delta|$ the change of the quasiparticle momentum under Andreev reflection is small [see Eq. (1)], while the velocity changes its sign, and an essential cancellation of the phase gain along trajectories including electron-hole transformations at the superconducting boundaries takes place. At $|E|=h_0$ any such a classical trajectory is closed (in this case, under Andreev reflection the electron and hole momenta are equal and hence the reflected quasiparticle is sent exactly back along the classical path of the incident quasiparticle), and this cancellation is complete at $\phi=\pi(2l+1)$, $l=0,\pm 1,\ldots,$ irrespective of the geometry and the length of the trajectory.[19] From here it follows that at $||E|-h_0|\ll E_{Th}$ (E_{Th} is the Thouless energy) and ϕ close to odd multiples of π, such paths take part in the constructive interference resulting in resonant transmission through Andreev levels. In our calculations of the probability amplitude of the electron-hole reflection back to the reservoir of the electron injection and the electron-electron transmission to the other one, we use the approach developed by us in Ref. 20 assuming the motion of quasiparticles inside the ferromagnet to be semiclassical. The new class of two-dimensional (2D) magnetic semiconductors with large dielectric constants and small effective masses[21] very well satisfy the condition of the semiclassical motion $\alpha=r_s p_F/\hbar\gg 1$ (r_s is the screening length in the ferromagnet) with $\alpha=10$–10^2. Within this approach one can find the wave function of the scattered quasiparticles by mapping the incident wave along classical paths determining the phase of rapid oscillations $\Theta=S/\hbar$ as a classical action $S=\int p\,dl$ along the path. A typical classical trajectory of this kind for an incident electron that undergoes a number of Andreev and normal reflections at F-S boundaries is shown in the insert of Fig. 2 (solid and dashed lines are for electronic and hole paths, respectively). The electron-hole transmission along this trajectory is similar to the resonant transmission of an electron through a two-barrier system (schematically shown in Fig. 2) in which the incident electron tunnels through a potential barrier I (solid line I), moves along a one-dimensional chain of scatterers (black dots in Fig. 2 representing Andreev and normal reflections at F/S interfaces), and then is reflected back as an electron through potential barrier I and transmitted through potential barrier II as a hole. L_i ($i=0,\pm 1,\ldots$) is the length of the quasiparticle path between two successive scatterings at F/S interfaces, which is the distance between the neighboring scatterers for the 1D chain of Fig. 2. The paths L_i are uncorrelated and hence the chain of Fig. 2 is a 1D system with random distances between the scatterers. In the same way as in Ref. 20, it can be shown that due to the above-mentioned phase compensation the motion of the quasiparticle in this chain is reduced to the conventional quantum motion of an electron with energy $|E|-h_0$ (but having the Fermi velocity $\sim v_F$) in the 1D disordered chain of centers of backscatterings where the backscattering amplitude is the probability amplitude of the Andreev reflection $r_A^{(1,2)}$ and the amplitude to pass to the next section of the chain is the probability amplitude of the normal reflection $r_N^{(1,2)}$ at F/S interfaces 1 and 2 (the probability amplitudes r_A and r_N are given in Refs. 7, 22, and 23). In this situation, for $E\neq h_0$ the phase gains between successive backscatterings are random, and quasiparticle localization takes place. For $|r_N^{(1,2)}|\ll 1$ (Ref. 24) and $t_r\ll 1$ a sharp resonant transmission occurs between points I and II through discrete energy levels (of the Andreev-type) that correspond to the quasiparticle states localized around the section of the electron injection. Matching amplitudes of the electron and hole semiclassical wave functions in every section of propagation between scattering points (dots in Fig. 2) and taking into account the phase gains along the paths between them show the probability of electron-hole resonant transmission through an energy level E_α (Ref. 25) to be of the Breit-Wigner form, $T(E,\alpha)\propto t_r^2/\{[(E-E_\alpha)\tau_0]^2/\hbar^2+bt_r^2\}$, where τ_0 is the time of motion along the path of the length L_0 in the section of injection, constant $b\sim 1$.

The total electron-hole transmission probability $T_{eh}(E)$ is a sum of $T(E,\alpha)$ with respect to the starting points of the semiclassical trajectories inside the reservoir separated by distances of the order of λ_F. These trajectories meet different ''random'' sets of impurities, and hence their path lengths and the times of quasiparticle propagation along them are randomly distributed. Therefore, the summation over the starting points is equivalent to averaging the transmission probability with respect to realizations of times τ_i(τ_i is the time of propagation along section i) (see Refs. 3 and 20). It seems reasonable to assume the propagation times τ_i to be uncorrelated. Under this assumption, as is shown in Ref. 20, the total transmission probability $T_{eh}(E)$ is proportional to the density of localized states in the 1D disordered

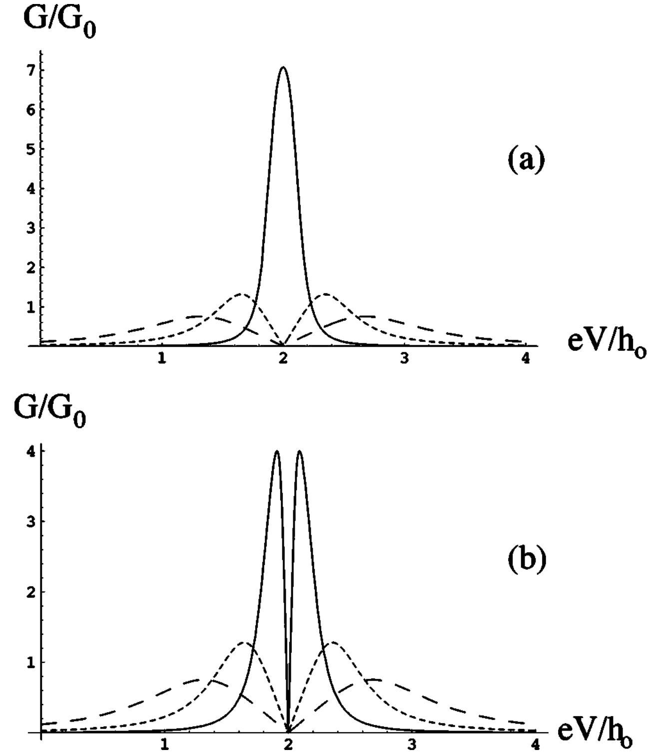

FIG. 3. Normalized differential conductance $G=dI/dV$ of the $S/F/S$ structure for $|r_N^{(1)}|=|r_N^{(2)}|=0.1$ and $|r_N^{(1)}|=0.05$, $|r_N^{(2)}|=0.1$ shown in (a) and (b), respectively, at phase differences $\phi=\pi$ (full line), $\phi=1.1\pi$ (dotted line), and $\phi=1.2\pi$ (dashed line); $G_0=(\sqrt{2}e^2/h)N_\perp t_r$.

chain of Fig. 2, and using the Lambert formula[26] one gets the transport current at temperature $t_r \ll kT/E_{Th} \ll |r_N^{(1,2)}|$ as

$$I=(t_r N_\perp e/h)E_{Th}\sum_{\uparrow,\downarrow}\int_{-eV/2}^{eV/2}\langle \nu_{r\,\text{and}}^{(\uparrow,\downarrow)}(E)\rangle dE \qquad (2)$$

(here and below we assume $t_r \ll (|r_N^{(1)}|+|r_N^{(2)}|)/2 \ll 1$). In Eq. (2), $N_\perp=S/\lambda_F^2$, S is the F/S contact area, λ_F is the electron wave length, $\langle \nu_{\uparrow,\downarrow}^{rand}(E)\rangle$ is the density of states for a quasiparticle with the spin up (↑) or down (↓) averaged with respect to the configurations of τ_n.

In order to get an analytical result we assume the distribution $P(\tau)$ for the propagation times to be of the Lorentzian form $P(\tau)=\gamma/\pi[(\tau-\overline{\tau})^2+\gamma^2]$ ($\overline{\tau}=L_S^2/D$ and L_S is the distance between the superconductors) that, for the configuration of Fig. 2, permits to find the density of states exactly. Using Eq. (2) one finds the resonant phase-sensitive part of the differential conductance of the system $G=dI/dV$ to be

$$G=\frac{\sqrt{2}e^2}{h}N_\perp t_r|\overline{V}| \times\left\{\frac{\sqrt{[4\overline{V}^4+\epsilon_a^4][4\overline{V}^4+\epsilon_b^4]}+\epsilon_a^2\epsilon_b^2-4\overline{V}^4}{[4\overline{V}^4+\epsilon_a^4][4\overline{V}^4+\epsilon_b^4]}\right\}^{1/2}, \qquad (3)$$

where $\overline{V}=(eV/2-h_0)/E_{Th}$ is the dimensionless applied voltage measured from h_0, $\epsilon_{a,b}=[\delta\phi^2+(|r_N^{(1)}|\pm|r_N^{(2)}|)^2]^{1/2}\delta\phi$ is the minimal value of $|\phi-\pi(2l+1)|$, $l=0,\pm 1,\pm 2,\ldots$. While writing Eq. (3) we took $\gamma=\overline{\tau}$, and assumed $|h_0-eV/2|\ll E_{Th}$. Equations (2) and (3) describe the current and differential conductance at $t_r \ll kT/E_{Th} \ll |r_N^{(1,2)}$ for both the magnetic $h_0\neq 0$ and nonmagnetic $h_0=0$ cases.

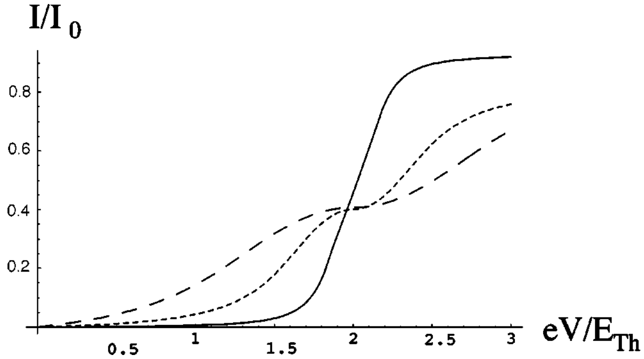

FIG. 4. Normalized current-voltage characteristics for phase differences $\phi=\pi$ (full line), $\phi=1.1\pi$ (dotted line), and $\phi=1.2\pi$ (dashed line) shown for $|r_N^{(1)}|=0.05$, $|r_N^{(2)}|=0.1$, and $h_0=E_{Th}$; $I_0=(\sqrt{2}e^2/2\pi\hbar)N_\perp t_r(2h_0/e)$.

Numerical results for the conductance and the current based on Eq. (3) are shown in Figs. 3 and 4. They demonstrate a high sensitivity of the conductance and the nonlinear current-voltage characteristics to both the superconductor phase difference ϕ and voltage V.

At odd multiples of π and $|r_N^{(1)}|=|r_N^{(2)}|$ there is a symmetry between the clockwise and counterclockwise motions of electron-hole pairs in the ferromagnet, and the energy level $|E|=h_0$ is degenerate (see above).[27] Under this condition the maximum of the resonant transmission through Andreev levels is at $eV/2=h_0$, and a resonant peak in the conductance is observed [Fig. 3(a)]. Even a small deviation of ϕ from an odd multiple of π will repel Andreev levels from h_0 that splits the conductance peak.

In a more realistic experimental case when $|r_N^{(1)}|\neq|r_N^{(2)}|$ the symmetry is broken, and Andreev levels are repelled from the level h_0 (see Ref. 3), the shift being proportional to $\delta r_N=\|r_N^{(1)}|-|r_N^{(2)}\|$. As a result the resonant peaks of the conductance are split [see Fig. 3(b)]. At low voltages, far from $2h_0/e$, we have a resonant tunneling of quasiparticles through separate Andreev levels, and the current level is low. When $eV/2\approx h_0$ and $\phi=\pi$, Andreev levels concentrate near h_0, and we have simultaneous resonant transport through the whole number of $N_\perp$ states resulting in a jump of the current $\Delta I=\|r_N^{(1)}|+|r_N^{(2)}\|G_{max}h_0/2e$ (G_{max} is the maximal value of the conductance). When ϕ deviates from π the number of Andreev levels concentrated near h_0 is decreasing that results in a decrease of the sensitivity of the current to the voltage.

We note here that the curve for the differential conductance G as a function of eV repeats the density of Andreev states in the diffusive ferromagnet permitting a direct spectroscopy of the Andreev levels by conductance and current measurements.

In conclusion we have demonstrated a pronounced possibility for spectroscopy of Andreev states in ferromagnets at energies even greater than the Thouless energy. The paramagnetic effect determines sharp peaks in the conductance as a function of the superconductor phase difference ϕ and the applied voltage V near $\phi=\pi(2l+1)$, $l=0,\pm 1,\pm 2,\ldots,$ and $V=2h_0/e$, respectively. This phenomenon is a convenient tool for the Andreev level spectroscopy, and enables applications, e.g., as a double-gate ferromagnet transistor and a logical AND-element described in Ref. 29.

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[25] In our case, the transparencies of the barriers are low and the dispersion in the distribution of times of motion inside the wells is of the order of the mean value $\overline{\tau}$. Hence, the localization radius is $\sim\overline{\mathcal{L}}=v_F\overline{\tau}$, and for a given time configuration there is only one resonant level in the energy range of interest $|E-h_0| \sim |r_N| E_{Th} \ll E_{Th}$.

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