Resonant tunneling through linear arrays of quantum dots

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We theoretically investigate resonant tunneling through a linear array of quantum dots with subsequent tunnel coupling. We consider two limiting cases: (i) the strong Coulomb blockade, where only one extra electron can be present in the array; (ii) the limit of almost noninteracting electrons. We develop a density-matrix description that incorporates the coupling of the dots to reservoirs. We analyze in detail the dependence of the stationary current on the electron energies, tunnel matrix elements and rates, and on the number of dots. We describe interaction and localization effects on the resonant current. We analyze the applicability of the approximation of independent conduction channels. We find that this approximation is not valid when at least one of the tunnel rates to the leads is comparable to the energy splitting of the states in the array. In this case the interference of conduction processes through different channels suppresses the current. [S0163-1829(99)04040-0]

I. INTRODUCTION

In recent years, arrays of quantum dots have received an increasing amount of interest. With the progress of fabrication techniques, quantum dot arrays are coming within the reach of experimental investigation.[1] If the electron levels in the individual dots are aligned, we encounter here a situation of resonant tunneling. In this regime, the transport in the array becomes sensitive to precise matching of the electron levels in the dots that can be controlled by external gates. This opens up new possibilities to control the transport and perform sensitive measurements even in the simplest case of two dots.[2]

Resonant tunneling in arrays of quantum dots and layered semiconductor heterostructures exhibit some similarities. The latter situation has been intensively studied in the context of possible Bloch oscillations.[3] However, the Coulomb blockade dominates the properties of the arrays of quantum dots so that electron-electron interaction cannot be neglected as in the case of layered heterostructures.[4,5] A way to circumvent this difficulty is to perform an exact diagonalization of electron states in the array of coupled dots. Then one considers independent tunneling transitions between the resulting many-electron states.[6] This we call the independent channel approximation. This is approximate because it disregards the simultaneous tunneling of an electron through multiple conduction channels. Another approach is to restrict the basis to the resonant states of the uncoupled dots. Then the tunneling between the dots and the reservoirs is incorporated into a modified Liouville equation for the density matrix in this basis. For two quantum dots this has been done in Ref. 7 and here we extend this approach to the case of an array of an arbitrary number of dots.

In this paper we concentrate on an array of quantum dots where dots are connected in series and a tunnel coupling exists only between neighboring dots. This is the most interesting case because there is a unique path for the current and changes in any dot strongly affect the transport through the whole array. The first and last dot of the array are connected to leads. We assume that the voltage bias is sufficiently high so that the energy change during the tunneling of an electron between a reservoir and the array is much larger than the energy uncertainty due to this tunneling. We also assume that the resonant electronic energies in the array lie well between the Fermi levels of the leads. This enables us to use the density-matrix approach. We consider two limiting cases of the electron-electron interaction within the array. In the first case we assume that the long-range Coulomb repulsion between electrons in different dots of the array is so strong that only one or no extra electrons are present in the array (Coulomb blockade). This is to be contrasted with the case of ''free'' electrons. As we explain below (Sec. II), we do not disregard interactions completely in the latter case but rather account only for strong repulsion within each dot.

Using the density-matrix approach in the basis of localized states, we have obtained analytical results for the stationary current. Our results hold for arbitrary values of the parameters (within the applicability of our model) characterizing the arraylike dot energies and tunnel couplings: no assumption about homogeneity of the array has been made. This may facilitate the comparison with experiments and the design of resonant tunneling devices. We report the effects of localization and Coulomb repulsion on the resonant current when the energy level of the first and the last dot are independently varied. We have also considered another picture of the transport using the approximation of independent conduction channels in the array of dots. Using the density-matrix approach in the basis of delocalized states, we have calculated the occupations of the channels and their contributions to the current. We discuss in detail the range of validity of this approximation. In the limit of both weak *and* strong coupling to one or both of the leads, we obtain results in agreement with the former more general calculations. However, there can be substantial deviations from the predictions of this model when the tunnel rates and the coherent interdot couplings are comparable. To illustrate this, we study the dependence of the current on the transparencies of the tunnel barriers and find unusual features due to the inter-
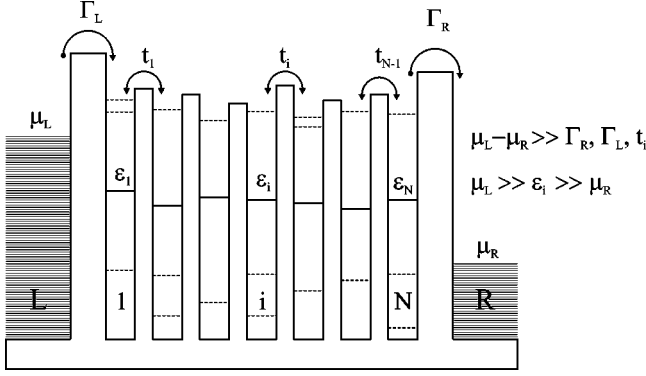
FIG. 1. Linear array of N quantum dots coupled to leads L and R. The energy levels of the uncoupled dots are given by full lines when relevant for resonant transport and dashed lines when irrelevant.

ference of electrons during tunneling.

The outline of the paper is as follows. In Sec. II we introduce the density-matrix description of a multidot system coupled to leads which we apply to the Coulomb blockade case in Sec. III and to the ''free'' electron case in Sec. IV. In Sec. V we compare the results with those obtained from the independent channel approximation and we discuss the deviations. We formulate our conclusions in Sec. VI.

II. ARRAY OF QUANTUM DOTS COUPLED TO RESERVOIRS

Let us first consider an array of N quantum dots (the ''device'') without any contacts (Fig. 1). We consider a quantum dot as some complex system with discrete many-electron states of which only two ground states participate in resonant transport. The one state is related to the other by the addition of an *extra* electron costing an addition energy which includes intradot charging, i.e., the Coulomb interactions between electrons in the dot. (For simplicity, we disregard the electron's spin degrees of freedom.[8]) We label the resonant states of dot $i = 1, \ldots, N$ by the number of extra electrons $n_i = 0,1$ and introduce fermionic operators $\hat{a}_i^\dagger, \hat{a}_i$ which create and annihilate, respectively, an extra electron in dot i. The many-electron eigenstates of the array of *uncoupled* dots will be denoted by $|n_1 \cdots n_N\rangle = |\{n_k\}\rangle = \Pi_{k=1}^N \hat{a}_k^{\dagger n_k}|\{0\}\rangle$. Let ε_i denote the energy for adding an electron to dot i and let $u_{|i-j|} \geq 0$ be the interdot charging energy due to the repulsive Coulomb interaction between the pair of extra electrons in dots $i \neq j$. A coherent interdot coupling with matrix element $t_i = t_i^*$ accounts for the tunneling of electrons between dots i and $i+1$. We obtain a Hubbard-type Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_u$ for the array of *coupled* dots:

$$\hat{H}_0 = \sum_{i=1}^N \varepsilon_i \hat{a}_i^\dagger \hat{a}_i + \sum_{i=1}^{N-1} t_i(\hat{a}_{i+1}^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_{i+1}), \quad (1a)$$

$$\hat{H}_u = \sum_{i<j=2}^N u_{|i-j|} \hat{a}_i^\dagger \hat{a}_i \hat{a}_j^\dagger \hat{a}_j. \quad (1b)$$

The density operator of the device $\hat{\sigma}$ evolves according to the Liouville equation $\partial_t \hat{\sigma} = -\mathrm{i}[\hat{H}, \hat{\sigma}]$ ($\hbar \equiv 1$) which con-

serves probability, i.e., $\mathrm{Tr}\,\hat{\sigma}(t) = 1$. By expanding $\hat{\sigma}$ in the many-electron eigenstates of the uncoupled dots, we obtain a $2^N \times 2^N$ density matrix $\boldsymbol{\sigma} = (\sigma_{\{n_k\},\{n_k'\}})$:

$$\hat{\sigma} = \sum_{\{n_k\},\{n_k'\}} \sigma_{\{n_k\},\{n_k'\}}|\{n_k\}\rangle\langle\{n_k'\}|.$$

We additionally introduce an $N \times N$ Hermitian matrix $\boldsymbol{\rho}$ with expectation values of single-electron operators ($i,j = 1, \ldots, N$):

$$\rho_{ij} = \langle \hat{a}_j^\dagger \hat{a}_i \rangle = \mathrm{Tr}\,\hat{a}_j^\dagger \hat{a}_i \hat{\sigma}.$$

Using the fermionic commutation relations, we find

$$\rho_{ii} = \sum_{\{n_{k \neq i}\}} \sigma_{n_1 \cdots \underset{i}{1} \cdots n_N, n_1 \cdots \underset{i}{1} \cdots n_N}, \quad (2a)$$

$$\rho_{ij} = \sum_{\{n_{k \neq i,j}\}} (-1)^{n_{i+1} + \cdots + n_{j-1}}$$

$$\times \sigma_{n_1 \cdots \underset{i}{1} \cdots \underset{j}{0} \cdots n_N, n_1 \cdots \underset{i}{0} \cdots \underset{j}{1} \cdots n_N}, \quad (2b)$$

where $\{n_{k \neq i,j}\}$ indicates that we sum over $n_k = 0,1$ for all dots $k = 1, \ldots, N$ except i,j. We will refer to $\boldsymbol{\rho}$ as the average occupation matrix with respect to electrons in individual dots whereas $\boldsymbol{\sigma}$ is the probability-density matrix with respect to many-electron states of the array.

Now we include leads L and R connected to the first and last dot of the device by a tunnel barrier, respectively (Fig. 1). The leads are considered here as electron reservoirs at zero temperature with a continuum of states filled up to their Fermi levels μ_L and μ_R, respectively. We assume that the energy levels of the device are located well between the chemical potentials of the leads, i.e., $\mu_L \gg \varepsilon_i, t_i \gg \mu_R$ (large bias), and that the level widths are much smaller than the bias, i.e., $\mu_L - \mu_R \gg \Gamma_{L,R}$ (discrete states). Under these assumptions an evolution equation for the density matrix $\boldsymbol{\sigma}$ of the device can be derived[9,10] by incorporating the details of the lead states into tunnel rates. Due to the high bias, electrons only tunnel through the barrier from lead L to dot 1 with a rate $\Gamma_L = 2\pi D_L |t_L|^2$ and from dot N to lead R with rate $\Gamma_R = 2\pi D_R |t_R|^2$. We assume that the density of states $D_{L,R}$ in leads is constant and that the tunneling matrix elements $t_{L,R}$ between lead and dot states depend only weakly on the energy. Due to the destructive interference of an electron tunneling between a discrete state in the device and the continuum of states in a reservoir, the rate for tunneling in one direction ($L \rightarrow 1, N \rightarrow R$) is constant in time whereas the rate of the reversed process ($L \leftarrow 1, N \leftarrow R$) is zero.[11] In general, transitions between discrete many-electron states a,b of a device with Hamiltonian $\hat{H}$ induced by tunneling to and from reservoirs can be included in a modification of the Liouville equation:[10]

$$\partial_t \sigma_{ab} = -\mathrm{i}[\hat{H}, \hat{\sigma}]_{ab} - \frac{1}{2}\left(\sum_{a'} \Gamma_{a \rightarrow a'} + \sum_{b'} \Gamma_{b \rightarrow b'}\right)\sigma_{ab}$$

$$+ \sum_{a'b'} \Gamma_{ab \leftarrow a'b'} \sigma_{a'b'}. \quad (3)$$

The first term of the modification describes the *separate* decay of states $a \to a'$ and $b \to b'$ due to (in general) *different* tunneling events between the device and the reservoir with rates $\Gamma_{a \to a'}$ and $\Gamma_{b \to b'}$, respectively. The second term describes the *joint* generation of states $a \leftarrow a'$ and $b \leftarrow b'$ due to a *single* tunneling event occurring with rate $\Gamma_{ab \leftarrow a'b'}$: the coherence lost by the simultaneous decay of states a', b' is transferred to states a, b. When we assume that there is a unique path for the current, then each state a is generated from a unique state a' by the tunneling of an electron to or from a reservoir: $\Gamma_{aa \leftarrow a'b'} = \Gamma_{a \leftarrow a'} \delta_{a'b'}$. The modified Liouville equation conserves probability: summing the equations for the diagonal elements gives $\partial_t \mathrm{Tr}\, \hat{\sigma}(t) = 0$, so

$$\sum_{\{n_k\}} \sigma_{\{n_k\},\{n_k\}} = 1. \tag{4}$$

We consider the following two cases for resonant transport through the array: (i) The "free"-electron (F) case where interdot Coulomb repulsion is negligible, i.e., $u_{|i-j|} \equiv 0$; up to N electrons can populate the array and all 2^N many-electron states lie between the chemical potentials $\mu_L \gg \mu_R$ and participate in resonant transport. (ii) The Coulomb blockade (CB) case where the interdot Coulomb repulsion is so strong that the smallest charging energy is large relative to the bias, i.e., $u_{N-1} \gg \mu_L - \mu_R$; many-electron states with more than one electron are highly improbable and can be neglected for resonant transport. We point out that in general the presence of electrons in the array modifies the rates for tunneling to or from the leads by Coulomb repulsion.[9] However, in the limiting cases considered here there is either no repulsion (F) or no other electron present in the array (CB) so two parameters Γ_L and Γ_R suffice to incorporate the details of the electronic states in the respective leads. The current flowing from dot N to reservoir R is determined by the average occupation and the tunnel rate:

$$\frac{I_N(t)}{e} = \Gamma_R \rho_{NN}(t). \tag{5}$$

With the leads included, the dynamics of the average occupation matrix $\boldsymbol{\rho}$ should be calculated from the full density matrix $\boldsymbol{\sigma}$ which evolves according to an equation of the type (3). However, in both cases considered here we can derive a dynamical equation for the matrix $\boldsymbol{\rho}$ which is solved more easily.

III. CURRENT IN THE COULOMB BLOCKADE CASE

When the long-range Coulomb repulsion is so strong that at most one electron can be present in the array of dots, we can restrict the set of many-electron basis states to

$$|0 \cdots 0\rangle, |0 \cdots \underset{i}{1} \cdots 0\rangle, \quad i = 1, \dots, N.$$

The average occupations of individual dots are simply equal to the nonzero probability densities of these states and

$$\rho_{ij} = \sigma_{0 \cdots \underset{i}{1} \cdots 0, 0 \cdots \underset{j}{1} \cdots 0}.$$

Conservation of probability (4) suggests that we additionally define an average occupation of the many-electron vacuum state:

$$\rho_{00} = \sigma_{0 \cdots 0, 0 \cdots 0}.$$

This quantity is positive $0 \leq \rho_{00} \leq 1$ and satisfies a conservation law:

$$\rho_{00} + \sum_{k=1}^{N} \rho_{kk} = 1. \tag{6}$$

In the restricted basis, the matrix elements of the Hamiltonian $\hat{H}$ which describe the coherent part of the evolution of the state only involve $\hat{H}_0$ [Eq. (1a)]. Modifying the Liouville equation according to Eq. (3), we obtain a dynamical equation for the average occupation matrix,

$$\partial_t \rho_{00} = -\Gamma_L \rho_{00} + \Gamma_R \rho_{NN}, \tag{7a}$$

$$\partial_t \rho_{ii} = \mathrm{i}(t_{i-1}\rho_{ii-1} + t_i \rho_{ii+1} - t_{i-1}\rho_{i-1i} - t_i \rho_{i+1i}) + \Gamma_L \rho_{00} \delta_{i1} - \Gamma_R \rho_{NN} \delta_{iN}, \tag{7b}$$

$$\partial_t \rho_{ij} = \mathrm{i}(\varepsilon_j - \varepsilon_i)\rho_{ij} + \mathrm{i}(t_{j-1}\rho_{ij-1} + t_j \rho_{ij+1} - t_{i-1}\rho_{i-1j} - t_i \rho_{i+1j}) - \frac{1}{2}\Gamma_R \rho_{iN} \delta_{jN}, \tag{7c}$$

where $j > i = 1, \dots, N$ and $t_0 = t_N \equiv 0$. On the right-hand side of Eq. (7a) the negative contribution describes the decay of the vacuum state due to the tunneling of an electron from lead L to dot 1 with rate Γ_L whereas the positive contribution describes the generation of this state due to the tunneling of the (only) electron in the device from dot N to lead R. In Eq. (7c) there is only a negative contribution due to the tunneling of an electron *out* of dot N to reservoir R with rate Γ_R. There is no negative contribution with Γ_L because we have incorporated the Coulomb blockade: we disregard the decay of many-electron states with one electron to a state with two electrons which occurs when an electron tunnels from lead L *into* dot 1. We can eliminate ρ_{00} from Eqs. (7) using Eq. (6) and obtain N^2 equations for the average occupations.

Equations (7) can be used to describe nonstationary transport with a typical relaxation time scale $\Gamma_{L,R}^{-1}$. Here we are interested in the stationary limit $\partial_t \boldsymbol{\rho} = 0$ only. In general, the solution of Eq. (7) can be obtained by inverting a matrix of dimension N^2. However, since the system under consideration is an array with subsequent tunnel coupling, most of the equations only relate matrix elements of $\boldsymbol{\rho}$ on neighboring dots. One can obtain the solution by iteratively expressing all matrix elements in terms of ρ_{00} and finally making use of the normalization constraint (6). The resulting stationary current in general reads

$$\frac{I_N^{\mathrm{CB}}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R}F_N + \dfrac{\Gamma_R}{4t_{N-1}^2}F_{N-1}}. \tag{8}$$

Here the dimensionless expression F_N depends only on $\varepsilon_1, \ldots, \varepsilon_N$ through the differences $\varepsilon_{ij} \equiv \varepsilon_i - \varepsilon_j, i < j$ and on $t_1, \ldots, t_{N-1}$. Note the curious property of Eq. (8): F_N enters the expressions for both I_N and I_{N+1}. This helps when deriving expressions for the current through an array with an increasing number of dots. For $N = 2$ we reproduce the result of Stoof and Nazarov:[12]

$$\frac{I_2^{CB}}{e} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}\left[2 + \left(\frac{\varepsilon_{12}}{t_1}\right)^2\right] + \frac{\Gamma_R}{4t_1^2}}. \qquad (9)$$

For $N = 3$ we obtain for the current through a triple dot system,

$$\frac{I_3^{CB}}{e} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}\left(3 + \frac{\varepsilon_{12}\varepsilon_{13} + 2t_1^2 - t_2^2}{t_1 t_2}\frac{\varepsilon_{12}\varepsilon_{13} + t_1^2 - t_2^2}{t_1 t_2} + \frac{\varepsilon_{12} + 2\varepsilon_{23}}{t_2}\frac{\varepsilon_{12} + \varepsilon_{23}}{t_2}\right) + \frac{\Gamma_R}{4t_2^2}\left(2 + \frac{\varepsilon_{12}}{t_1}\right)}. \qquad (10)$$

One can show $F_N(\varepsilon_1, \ldots, \varepsilon_N, t_1, \ldots, t_{N-1}) = N$ for $\varepsilon_{ij} \ll t_i = t$. This property will be used later on and it is derived in another way in Sec. V A.

We consider Eq. (8) in more detail for equal interdot couplings $t_i = t$ and several different configurations of the levels ε_i. At resonance $\varepsilon_{ij} = 0$ we find

$$\rho_{ii} = \frac{\frac{1}{\Gamma_R} + \frac{\Gamma_R}{4t^2}(1 - \delta_{iN})}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}N + \frac{\Gamma_R}{4t^2}(N-1)}$$

and the current is maximal:

$$\frac{I_{N,\max}^{CB}}{e} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}N + \frac{\Gamma_R}{4t^2}(N-1)}. \qquad (11)$$

Clearly the current is reduced when the array increases in size: the number of states participating in transport relative to the number of states of the array decreases $\propto 1/N$ due to the Coulomb blockade. In an actual system we expect a positive deviation from this decrease to occur when the spatial size of the array exceeds some range over which the Coulomb repulsion cannot exclude the occupation of a second dot in the array. Away from resonance, i.e., $\varepsilon_{ij} = O(\varepsilon) \gg t$, the conduction of the device decays exponentially with the size of the array N due to the localization of *the* electron in one of the individual dots: $I_N^{CB}/e \propto \Gamma_R(\varepsilon/t)^{-2(N-1)}$. To illustrate this we vary only the last level [Fig. 2(a)], i.e., we consider the solution of Eq. (7) for $\varepsilon_i = \varepsilon_N \delta_{iN}$:

$$\frac{I_N^{CB}}{e} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}\left[N + (N-1)\left(\frac{\varepsilon_N}{t}\right)^2\right] + \frac{\Gamma_R}{4t^2}(N-1)}.$$

As we increase the number of dots, the curve keeps its Lorentzian shape with respect to ε_N and always depends on both t and Γ_R since the electron is localized only just before tunneling out of the array:

$$\frac{I_N^{CB}}{I_{N,\max}^{CB}} \xrightarrow{N \to \infty} \frac{1 + \left(\frac{\Gamma_R}{2t}\right)^2}{1 + \left(\frac{\Gamma_R}{2t}\right)^2 + \left(\frac{\varepsilon_N}{t}\right)^2}.$$

Now we vary only the first level [Fig. 2(b)], i.e., we consider the solution of Eq. (7) for $\varepsilon_i = \varepsilon_1 \delta_{i1}$:

$$\frac{I_N^{CB}}{e} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R}\left\{N + \left[\left(\frac{\varepsilon_1}{t}\right)^2 + \left(\frac{\Gamma_R}{2t}\right)^2\right]\sum_{k=1}^{N-1} k\left(\frac{\varepsilon_1}{t}\right)^{2(N-1-k)}\right\}}.$$

For large N the normalized current vanishes when the detuning exceeds the tunnel coupling (since an electron is localized in the first dot) whereas near resonance the peak takes on a parabolic shape which is independent of the tunnel rate Γ_R:

$$\frac{I_N^{CB}}{I_{N,\max}^{CB}} \xrightarrow{N \to \infty} \begin{cases} 1 - \left(\frac{\varepsilon_1}{t}\right)^2, & \varepsilon_1/t < 1 \\ 0, & \varepsilon_1/t > 1. \end{cases}$$

For the case where the energies are configured as a "Stark ladder" of total width ε, we have plotted the current in Fig. 3(b). The localization of electrons clearly dominates the current since the tails of the current peak decrease rapidly with increasing N as in Fig. 2(a).

IV. CURRENT IN THE ''FREE''-ELECTRON CASE

When interdot Coulomb repulsion is altogether disregarded, all 2^N many-electron states $|\{n_k\}\rangle$ of the array of dots must be taken into account. Modifying the Liouville equation with $\hat{H} = \hat{H}_0$ according to the general prescription (3), we obtain the following set of 2^{2N} equations for the density matrix:

$$\partial_t \sigma_{n_1 \cdots n_N, n_1' \cdots n_N'} = -i[\hat{H}_0, \sigma]_{n_1 \cdots n_N, n_1' \cdots n_N'} \tag{12a}$$

$$-\frac{1}{2}\Gamma_L[(1-n_1)+(1-n_1')]\sigma_{n_1\cdots n_N, n_1'\cdots n_N'} - \frac{1}{2}\Gamma_R(n_N+n_N')\sigma_{n_1\cdots n_N, n_1'\cdots n_N'} \tag{12b}$$

$$+\Gamma_L n_1 n_1' \sigma_{0n_2\cdots n_N, 0n_2'\cdots n_N'} + \Gamma_R(1-n_N)(1-n_N')\sigma_{n_1\cdots n_{N-1}1, n_1'\cdots n_{N-1}'1}. \tag{12c}$$

Here Eq. (12a) describes transitions between the nonorthogonal states of the device with a fixed number of electrons due to the tunneling between neighboring dots. In contrast to the Coulomb blockade case, both tunneling into (Γ_L) *and* out of the array (Γ_R) give negative contributions (12b). Furthermore, there are tunnel processes which induce a transition between two pairs of many-electron states and give a positive contribution (12c) to the coherences. Only a subset of Eq. (12) forms a closed system of equations for the diagonal and some nondiagonal elements of $\boldsymbol{\sigma}$ (the remaining equations only couple a closed set of nondiagonal elements which are irrelevant). From this subset we can derive an even simpler dynamical equation for the average occupation matrix $\boldsymbol{\rho}$. Let us first derive the coherent part of this equation by considering ''free'' electrons in the array of dots without the leads. Because the Hamiltonian consists of only one-electron operators ($H_0 = \sum_{kl=1}^N H_{0kl} a_k^\dagger a_l$), the commutator in the

Heisenberg equation of motion for $\rho_{ij} = \langle a_j^\dagger a_i \rangle$ is readily expressed in other one-electron operators and the matrix elements of $\hat{H}_0$:

$$i\partial_t \rho_{ij} = \langle [a_j^\dagger a_i, \hat{H}_0] \rangle$$

$$= \sum_{l=1}^N H_{0il}\langle a_j^\dagger a_l \rangle - \sum_{k=1}^N \langle a_k^\dagger a_i \rangle H_{0kj}$$

$$= [\mathbf{H}_0, \boldsymbol{\rho}]_{ij}, \tag{13}$$

where $i,j = 1, \ldots, N$. This equation can also be derived by taking the average [defined in Eq. (2)] of the coherent part (12a). The contributions which describe the coupling to the leads are found by adding the average of the incoherent contributions (12b) and (12c) to the right-hand side of Eq. (13). We obtain the following closed system of only N^2 equations, which describes the dynamics of the average occupation number matrix:
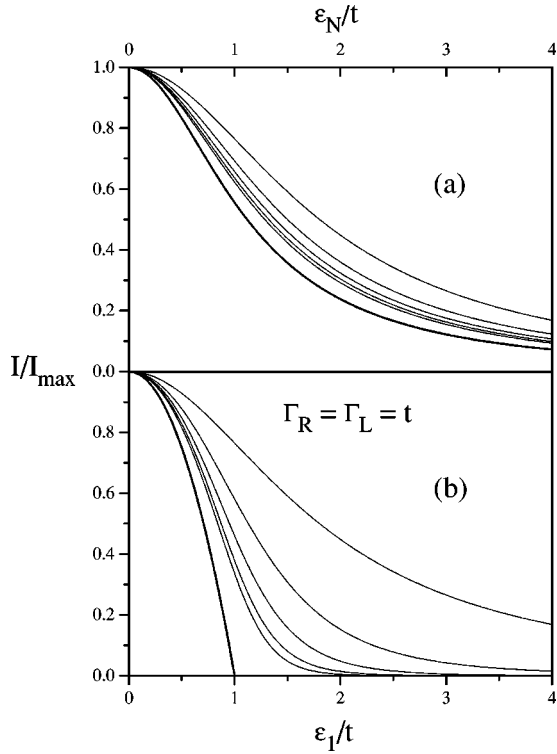
FIG. 2. Normalized resonant current for the Coulomb blockade case with $N=2,3,4,5,6$ (full curves downwards) and $N=\infty$ (thick full curve). (a) Variation of the last level, i.e., $\varepsilon_i = \varepsilon_N \delta_{iN}$. Increasing N does not alter the Lorentzian form of the curve. (b) Variation of the first level, i.e., $\varepsilon_i = \varepsilon_1 \delta_{i1}$. Electrons are localized in the first dot after tunneling through the left barrier resulting in the exponential decay of the current tails with N. For $N=2$, the curves in (a) and (b) coincide.
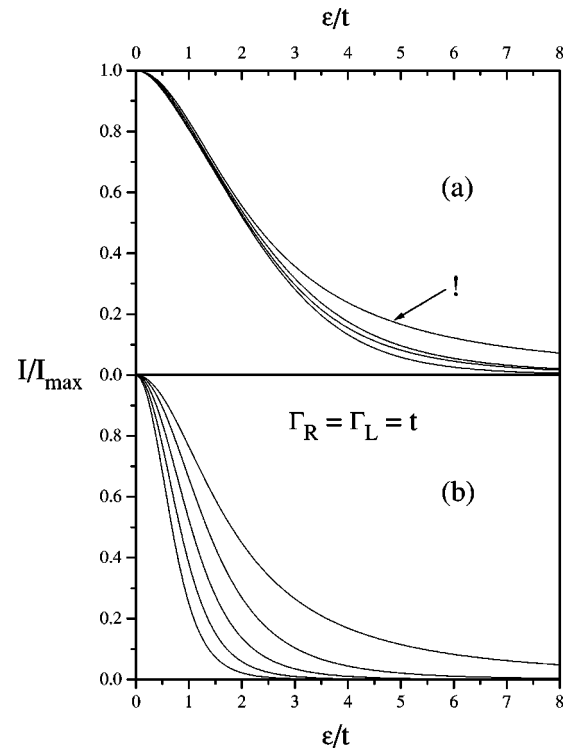
FIG. 3. Normalized current through an array of $N=2,3,4,5,6$ dots with energies configured as a Stark ladder $\varepsilon_i = i/(N-1)\varepsilon$ of varying width ε. (a) ''Free''-electron case: the curves for $N=2,3$ (indicated by the arrow) coincide for an isotropic array $t_i=t$ with $\Gamma_L=\Gamma_R$. (b) Coulomb blockade case.

$$\partial_t \rho_{ii} = i(t_{i-1}\rho_{ii-1} + t_i\rho_{ii+1} - t_i\rho_{i+1i} - t_{i-1}\rho_{i-1i})$$
$$+ \Gamma_L(1-\rho_{11})\delta_{i1} - \Gamma_R\rho_{NN}\delta_{iN}, \qquad (14a)$$

$$\partial_t \rho_{ij} = i(\varepsilon_j - \varepsilon_i)\rho_{ij} + i(t_{j-1}\rho_{ij-1}$$
$$+ t_j\rho_{ij+1} - t_{i-1}\rho_{i-1j} - t_i\rho_{i+1j}) - \frac{1}{2}\Gamma_L\rho_{1j}\delta_{i1}$$
$$- \frac{1}{2}\Gamma_R\rho_{iN}\delta_{jN}, \qquad (14b)$$

where $j > i = 1, \ldots, N$ and $t_0 \equiv t_N \equiv 0$. These equations closely resemble those for the Coulomb blockade case: the coherent contributions are exactly the same, but in contrast to Eq. (7b) the average occupations ρ_{11}, ρ_{NN} in Eq. (14a) are not coupled by incoherent transitions to some vacuum state. Furthermore, in Eq. (14b) there is a negative contribution due to the tunneling of an electron *into* dot 1 with rate Γ_L, which is absent in Eq. (7c) due to the Coulomb blockade.

Despite the close resemblance to Eq. (7) the (calculation of) stationary solution of Eqs. (14) for the general case is far more complicated. An analytical expression seems neither feasible nor instructive and we consider only a few representative cases here. Assuming equal couplings $t_i = t$, one finds that the resonant current peak ($\varepsilon_{ij} = 0$) is independent of N:

$$\frac{I_{N,\max}^F}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R} + \dfrac{\Gamma_L}{4t^2} + \dfrac{\Gamma_R}{4t^2}}. \qquad (15)$$

This result was previously obtained by Frishman and Gurvitz for tunneling through multiple-well heterostructures[5] using essentially the same approach as for the derivation of the modified Liouville equation[10] which we have used [Eq. (3)]. By solving Eqs. (14) for $N = 2$ we reproduce the result obtained by Gurvitz[13,10] from Eqs. (12),

$$\frac{I_2^F}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R} + \dfrac{1}{\Gamma}\left(\dfrac{\varepsilon_{12}}{t_1}\right)^2 + \dfrac{\Gamma_L}{4t_1^2} + \dfrac{\Gamma_R}{4t_1^2}}, \qquad (16)$$

where $\Gamma = \Gamma_L + \Gamma_R$. For $N = 3$, solving Eqs. (14) gives the current through a triple dot:

$$\frac{I_3^F}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R} + \dfrac{1}{\Gamma_L}\left(\dfrac{\varepsilon_{12}}{t_1}\right)^2 + \dfrac{1}{\Gamma_R}\left(\dfrac{\varepsilon_{23}}{t_2}\right)^2 + \dfrac{1}{\Gamma}\left(\dfrac{t_1}{t_2} - \dfrac{t_2}{t_1}\right)^2 + \dfrac{\Gamma_L}{4t_1^2} + \dfrac{\Gamma_R}{4t_2^2} - \dfrac{\left[\dfrac{1}{\Gamma_L}\dfrac{\varepsilon_{12}}{t_1} - \dfrac{1}{\Gamma_R}\dfrac{t_1}{t_2}\dfrac{\varepsilon_{23}}{t_2} - \dfrac{1}{\Gamma}\left(\dfrac{t_1}{t_2} - \dfrac{t_2}{t_1}\right)\dfrac{\varepsilon_{13}}{t_2}\right]^2}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R}\left(\dfrac{t_1}{t_2}\right)^2 + \dfrac{1}{\Gamma}\left(\dfrac{\varepsilon_{13}}{t_2}\right)^2 + \dfrac{\Gamma}{4t_2^2}}}. \qquad (17)$$

If one would first calculate $\boldsymbol{\sigma}$, then the subset of 20 relevant equations of Eqs. (12) (containing in total 64 equations) needs to be solved. For the case where the energies are configured as a ''Stark ladder'' of total width ε, we have plotted the current in Fig. 3(a). Comparison with Fig. 3(b) shows that the current for the ''free''-electron case is less sensitive to localization effects than in the Coulomb blockade case.

V. APPROXIMATION OF INDEPENDENT CONDUCTION CHANNELS

It is instructive to consider an approximate approach of independent conduction channels to our problem. In Sec. V A this approximation is introduced assuming that both reservoirs are *weakly* coupled to dots in the array. In this case the time Γ^{-1} needed for tunneling to or from a reservoir is much longer than typical time t^{-1} of the evolution of a coherent state in the array. Therefore, an electron completes many coherent oscillations in the array before tunneling. Somewhat surprisingly, this approach can also be used in the opposite limit of very *strong* coupling to the reservoirs as is shown in Sec. V B. In Sec. V C we compare the results of the independent channel approximation and discuss some peculiar features of the more general results obtained in Secs. III and IV.

A. Weak coupling to the leads

We first consider the array of dots without the leads. Let us denote the N localized (delocalized) eigenstates of a *single* electron in the array of uncoupled (coupled) dots by $|i\rangle$ ($|i'\rangle$), where $i = 1, \ldots, N$ and the single-electron vacuum by $|0\rangle$. By transforming to the basis of delocalized states that diagonalizes $\hat{H}_0$ [Eq. (1a)], we obtain new fermionic operators

$$\hat{a}_i'^\dagger = \sum_{k=1}^N \langle k|i'\rangle \hat{a}_k^\dagger, \quad \hat{a}_i' = \sum_{k=1}^N \langle 'i|k\rangle \hat{a}_k.$$

Since in a delocalized state there is a nonzero probability for finding an electron in both dot 1 and dot N, such a state can be regarded as a *conduction channel* which carries a current. The new operators add an electron to channel $|i'\rangle$ and remove an electron from channel $|i'\rangle$, respectively. By expanding the density operator of the array $\hat{\sigma}$ in the ''many-channel'' basis states $|n_1 \cdots n_N'\rangle = |\{n_k\}'\rangle = \Pi_{k=1}^N \hat{a}_k'^{\dagger n_k}|\{0\}\rangle$, we obtain a new density matrix $\boldsymbol{\sigma}'$. As in Sec. II, we define an average occupation matrix for the channels $\rho_{ij}' = \langle \hat{a}_j'^\dagger \hat{a}_i'\rangle$ which can be expressed in the density matrix $\boldsymbol{\sigma}'$. The advantage of the new basis is that the dynamical equations for the average occupations ρ_{ii}' are decoupled from the nondi-
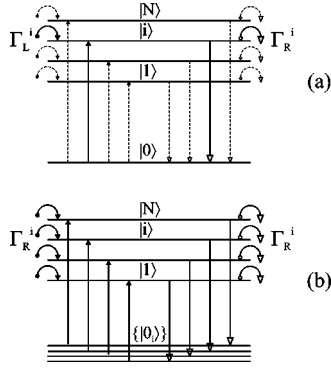
FIG. 4. Tunneling through N independent channels. (a) Coulomb blockade case: an electron tunneling through one of the channels blocks the remaining $N-1$ channels. The state must first decay to the vacuum before another electron can enter one of the channels. (b) ''Free''-electron case: N parallel channels for tunneling are available.

agonal elements when we only take $\hat{H}_0$ into account: $\partial_t \rho'_{ij} = -i(\varepsilon'_i - \varepsilon'_j)\rho'_{ij}$, where $\varepsilon'_{i,j}$ are the energies of the delocalized states i,j. In the Coulomb blockade case the effect of $\hat{H}_u$ is easily translated since the basis transformation preserves the trace of $\boldsymbol{\rho}$: the total occupancy of the channels is restricted to values ≤ 1 whereas in the ''free''-electron case all N channels can be occupied (Fig. 4).

Now we include the coupling to the reservoirs by a Golden Rule approach. Each channel $|i'\rangle$ is connected to lead L with modified matrix element $t^i_L = \langle i'|1\rangle t_L$ and to lead R with $t^i_R = \langle i'|N\rangle t_R$: an electron in lead L can tunnel through *any* channel to lead R. The rate for tunneling into channel $|i'\rangle$ is proportional to the probability to find the electron in dot 1 whereas the rate for tunneling out of channel $|i'\rangle$ is proportional to the probability to find the electron in N:

$$\Gamma^i_L = \Gamma_L |\langle i'|1\rangle|^2, \quad \Gamma^i_R = \Gamma_R |\langle i'|N\rangle|^2. \quad (18)$$

These probabilities depend on the energies ε_i of the localized states and the matrix elements t_i which couple them. When we assume that the uncertainty in the energy during the tunneling of an electron between leads and dots is much smaller than the level splitting, we can disregard correlations due to the simultaneous tunneling of one electron through multiple channels. Thus for weak coupling we can include the leads by only modifying the equations for the *diagonal* elements of $\boldsymbol{\rho}'$, i.e., the channel occupations. The channels can be treated as *independent* and their contributions to the current can simply be added.

In the Coulomb blockade case [Fig. 4(a)] the interdot repulsion couples the occupations of the channels and restricts their sum $\Sigma^N_{i=1} \rho'_{ii} < 1$. The tunneling of an electron into the empty device with rate Γ^i_L fills a channel $|i'\rangle$. This electron must tunnel out with rate Γ^i_R before another electron can tunnel into the device and occupy one channel. The equations for the occupations are

$$\partial_t \rho'_{00} = \sum_{i=1}^{N} \Gamma^i_R \rho'_{ii} - \left(\sum_{i=1}^{N} \Gamma^i_L \right) \rho'_{00},$$

$$\partial_t \rho'_{ii} = \Gamma^i_L \rho'_{00} - \Gamma^i_R \rho'_{ii}.$$

Using the implied conservation of probability $[\Sigma^N_{i=0} \rho'_{ii}(t) = 1]$, the stationary solution $[\lim_{t\to\infty} \partial_t \rho'_{ii}(t) = 0]$ is easily found. Summation of the contributions of the independent channels gives the stationary current:

$$\frac{I^{CB}_N}{e} = \sum_{i=1}^{N} \Gamma^i_L \rho'_{ii}(\infty) = \frac{\sum_{i=1}^{N} \Gamma^i_L}{1 + \sum_{i=1}^{N} \frac{\Gamma^i_L}{\Gamma^i_R}} = \frac{1}{\frac{1}{\Gamma_L} + \frac{1}{\Gamma_R} F_N}. \quad (19)$$

This is just the expression for the current through one double barrier: the tunneling time is just the sum of the tunneling times of the individual barriers where the time for tunneling through the right barrier is increased by a factor

$$F_N = \sum_{i=1}^{N} \frac{|\langle i'|1\rangle|^2}{|\langle i'|N\rangle|^2} > \frac{\sum_{i=1}^{N} |\langle i'|1\rangle|^2}{\sum_{i=1}^{N} |\langle i'|N\rangle|^2} = 1. \quad (20)$$

In the ''free''-electron case [Fig. 4(b)] the occupations of the channels are not coupled and their sum is restricted only by $\Sigma^N_{i=1} \rho'_{ii} < N$. The device is equivalent to N independent double barriers in parallel where the occupancy of channel i obeys

$$\partial_t \rho'_{ii} = \Gamma^i_L (1 - \rho'_{ii}) - \Gamma^i_R \rho'_{ii}.$$

From the stationary solution $[\lim_{t\to\infty} \partial_t \rho'_{ii}(t) = 0]$ the current is readily found:

$$\frac{I^F_N}{e} = \sum_{i=1}^{N} \Gamma^i_L \rho'_{ii}(\infty) = \sum_{i=1}^{N} \frac{1}{\frac{1}{\Gamma^i_L} + \frac{1}{\Gamma^i_R}}. \quad (21)$$

The tunneling time through each double barrier is the sum of the tunneling times $(\Gamma^i_{L,R})^{-1} > \Gamma^{-1}_{L,R}$ whereas the tunnel rate of the device is the sum of the tunnel rates of the N double barriers.

For a few representative cases we have explicitly worked out the approximate approach in the limit of weak coupling to both leads $\Gamma_{L,R} \ll \varepsilon_{ij}, t_i$. For the case of $N=2$ dots we have first calculated the two exact eigenstates of one electron in the array of coupled dots without the reservoirs. From these we obtained the tunnel rates to and from each of the delocalized states. Finally we summed the contributions of the independent channels to the total current: Eq. (19) precisely gives the result (9) without the term $(\Gamma_R/2t_1)^2$ whereas Eq. (21) gives the result (16) without the term $(\Gamma_L + \Gamma_R)/(2t_1)^2$ (the latter result was previously found in Ref. 4 for the case of a double quantum well). For the case of N dots with equal interdot couplings $t_i = t$ and aligned levels $\varepsilon_i = \varepsilon_j$, a simple result is also possible because in each delocalized state $|i'\rangle$ the electronic densities in the dots are

spatially symmetric. Then the modification factors of the rates are equal for each channel ($|\langle i'|1\rangle|^2 = |\langle i'|N\rangle|^2$) and they cancel in Eqs. (19) and (21). Thus for weak coupling to both reservoirs we obtain

$$\frac{I_{N,\max}^{\mathrm{CB}}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R}N}, \qquad (22)$$

$$\frac{I_{N,\max}^{\mathrm{F}}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\Gamma_R}}, \qquad (23)$$

which is just Eqs. (11) and (15) without the terms $(N-1) \times (\Gamma_R/2t)^2$ and $\Gamma_L\Gamma_R/(2t)^2$, respectively. Coulomb blockade increases the *effective* time for tunneling out of the device by a factor N with respect to the ''free''-electron case. The current increases *linearly* with each rate $\Gamma_L, \Gamma_R \ll t$ as expected from the enhanced tunneling.

B. Strong coupling to the leads

The terms which were found missing above become important when one of the tunnel rates to the reservoirs is comparable to the coherent interdot coupling. The correlations between the conduction channels can no longer be disregarded in this case: Eqs. (7) and (14) do not decouple into separate sets of equations for diagonal and nondiagonal elements on the basis of delocalized states. However, we will now show that in the limit of *strong* coupling to one or both of the reservoirs, we can use the independent channel approximation again. (Note that the energy uncertainties are assumed to be smaller than the bias $\Gamma_{L,R} \ll \mu_L - \mu_R$.)

First we consider the case where only the last dot N is strongly coupled to the right lead: $\Gamma_L \ll \varepsilon_{ij}, t_i \ll \Gamma_R$. The electronic state of the dot N and reservoir R form a continuum of states with a Lorentzian spectral density which is approximately constant over the energy range $t_{N-1} \ll \Gamma_R$:

$$D_{N+R}(\varepsilon) = \frac{1}{2\pi} \frac{\Gamma_R}{(\varepsilon - \varepsilon_N)^2 + (\Gamma_R/2)^2} \approx \frac{1}{2\pi} \frac{4}{\Gamma_R}.$$

The array of the remaining $N-1$ dots is *weakly* coupled to this continuum of states with matrix element t_{N-1}. Therefore we can apply the independent channel approach: the tunnel rate from dot $N-1$ to the continuum on the right is found with the Golden Rule,

$$\tilde{\Gamma}_R = 2\pi t_{N-1}^2 D_{N+R}(\varepsilon) = \frac{4t_{N-1}^2}{\Gamma_R}. \qquad (24)$$

For the case of aligned levels $\varepsilon_{ij}=0$ and equal couplings $t_i = t$, substitution of $\Gamma_R \to \tilde{\Gamma}_R$ and $N \to N-1$ in Eqs. (22) and (23), respectively, gives the maximum current in the limit of strong coupling to the right lead:

$$\frac{I_{N,\max}^{\mathrm{CB}}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{N-1}{\tilde{\Gamma}_R}} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{\Gamma_R}{4t^2}(N-1)}, \qquad (25)$$

$$\frac{I_{N,\max}^{\mathrm{F}}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{1}{\tilde{\Gamma}_R}} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{\Gamma_R}{4t^2}}, \qquad (26)$$

which is in agreement with our results (11) and (15), respectively. For strong coupling to the right lead, the current *decreases* as Γ_R^{-1}, which is somewhat surprising because tunneling is enhanced. The origin of this effect is the formation of linear combinations of the discrete state in dot N with reservoir states with energies roughly between $\varepsilon_N \pm \Gamma_R$. Because the tunnel processes from reservoir states back into the discrete state destructively interfere,[11] the discrete state irreversibly decays into the continuum. The resulting spectral density decreases with the energy uncertainty Γ_R. The eventual decrease of the current with the tunnel rate Γ_R can also be interpreted as a manifestation of the quantum Zeno effect as discussed in Ref. 10. More generally we have the following relation: for $\Gamma_L \ll \varepsilon_{ij}, t_i \ll \Gamma_R$,

$$I_N^{\mathrm{CB}}(\varepsilon_1 \cdots \varepsilon_N, t_1 \cdots t_{N-1}; \Gamma_L, \Gamma_R)$$

$$= I_{N-1}^{\mathrm{CB}}\left(\varepsilon_1 \cdots \varepsilon_{N-1}, t_1 \cdots t_{N-2}; \Gamma_L, \frac{4t_{N-1}^2}{\Gamma_R}\right),$$

$$I_N^{\mathrm{F}}(\varepsilon_1 \cdots \varepsilon_N, t_1 \cdots t_{N-1}; \Gamma_L, \Gamma_R)$$

$$= I_{N-1}^{\mathrm{F}}\left(\varepsilon_1 \cdots \varepsilon_{N-1}, t_1 \cdots t_{N-2}; \Gamma_L, \frac{4t_{N-1}^2}{\Gamma_R}\right).$$

This is clearly satisfied by Eqs. (10) and (17), respectively, and the general form (8) has this property.

Now consider the case where the array is coupled strongly to both leads, i.e., $\varepsilon_{ij}, t_i \ll \Gamma_{L,R}$. In the ''free''-electron case the reservoir L coupled to dot 1 gives a new continuum of states with a spectral density which is approximately constant over an energy range $t_1 \ll \Gamma_L$:

$$D_{1+L}(\varepsilon) = \frac{1}{2\pi} \frac{\Gamma_L}{(\varepsilon - \varepsilon_1)^2 + (\Gamma_L/2)^2} \approx \frac{1}{2\pi} \frac{4}{\Gamma_L}.$$

The tunnel rate from this continuum to dot 2 is

$$\tilde{\Gamma}_L = 2\pi t_1^2 D_{1+L}(\varepsilon) = \frac{4t_1^2}{\Gamma_L}.$$

The remaining $N-2$ dots are weakly coupled to a continuum on the left with matrix element t_1 and to the right with t_{N-1} and we can apply the independent channel approach to the $N-2$ conduction channels. For the case of aligned levels $\varepsilon_i = \varepsilon_j$ and equal couplings $t_i = t$, substitution of both $\Gamma_L \to \tilde{\Gamma}_L$ and $\Gamma_R \to \tilde{\Gamma}_R$ in Eq. (23) gives the maximum current in the limit of strong coupling to both leads,

$$\frac{I_{\max}^{\mathrm{F}}}{e} = \frac{1}{\dfrac{1}{\tilde{\Gamma}_L} + \dfrac{1}{\tilde{\Gamma}_R}} = \frac{1}{\dfrac{\Gamma_L + \Gamma_R}{4t^2}},$$

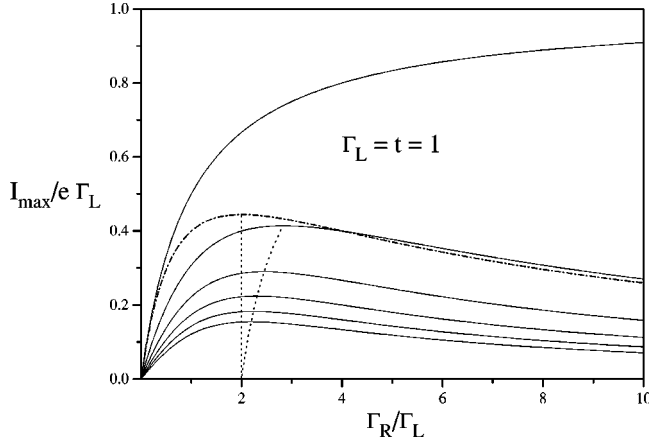in agreement with result (15). More generally, we have the relation for $\varepsilon_i, t_i \ll \Gamma_{L,R}$:

FIG. 5. Maximum resonant current as a function of the transparency of the right barrier: Coulomb blockade case for $N = 1,2,3,4,5,6$ (solid lines downwards) and ''free''-electron case (dot-dashed line for any $N>1$). The dotted lines show the position of the maxima.

$$I_N^F(\varepsilon_1 \cdots \varepsilon_N, t_1 \cdots t_{N-1}; \Gamma_L, \Gamma_R)$$

$$= I_{N-2}^F\left(\varepsilon_2 \cdots \varepsilon_{N-2}, t_2 \cdots t_{N-2}; \frac{4t_1^2}{\Gamma_L}, \frac{4t_{N-1}^2}{\Gamma_R}\right),$$

which is satisfied by Eq. (17). In the Coulomb blockade case, interdot repulsion prevents the discrete state in dot 1 from mixing with the reservoir: even if the energy uncertainty allows tunneling into dot 1 to occur on a small time scale Γ_L^{-1}, the next electron will have to wait for the previous one to tunnel out, which occurs on the much larger time scale $\max\{\varepsilon_{ij}^{-1}, t^{-1}, \Gamma_R^{-1}\}$. In the limit of strong coupling to both leads, the current in this case is *independent* of Γ_L and is correctly given by Eq. (25) in agreement with result (11).

Finally, we consider the case where only dot 1 is strongly coupled to the left lead, i.e., $\Gamma_R \ll \varepsilon_{ij}, t_i \ll \Gamma_L$. In the Coulomb blockade case, the weak-coupling result still applies as explained above. In the ''free''-electron case the discussion is completely analogous to the case of strong coupling of dot N to the right lead and the result is obtained by simply interchanging $L \leftrightarrow R$ and $1 \leftrightarrow N$.

C. Intermediate coupling to the leads: Maximum current

The competition between enhanced tunneling for weak coupling to the leads and destructive interference in the opposite limit implies that the current reaches a *maximum* value when the rate for tunneling into a dot is comparable to the coherent coupling to the neighboring dot. We can find the precise location and value of this maximum with the results obtained in Secs. III and IV, which also hold in this intermediate case.

First we consider the resonant current peak (11) and (15) as a function of the transparency Γ_R of the right tunnel barrier as plotted in Fig. 5. Starting from zero, the current initially increases linearly as expected from the enhanced tunneling to the right lead. Then a maximum is reached:

$$\frac{I_{max}^{CB}}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \sqrt{N(N-1)}\,\dfrac{1}{t}}, \quad \Gamma_R = \sqrt{\frac{N}{N-1}}2t, \quad (27a)$$

$$\frac{I_{max}^F}{e} = \frac{1}{\dfrac{1}{\Gamma_L} + \dfrac{\Gamma_L}{4t^2} + \dfrac{1}{t}}, \quad \Gamma_R = 2t. \quad (27b)$$

Increasing the transparency further will *reduce* the current as explained above. The maximum occurs when there is an optimal balance of the coherent tunneling into dot N and incoherent tunneling from this dot to reservoir R. At this point the *effective* time for tunneling out of the device is the same for weak [Eqs. (22) and (23), respectively], and strong coupling to the right lead [Eqs. (25) and (26), respectively], i.e., at these values of Γ_R we have

$$\frac{1}{\Gamma_R}N = \frac{1}{\tilde{\Gamma}_R}(N-1) \quad (CB),$$

$$\frac{1}{\Gamma_R} = \frac{1}{\tilde{\Gamma}_R} \quad (F),$$

where $\tilde{\Gamma}_R$ is given by Eq. (24). For the case of ''free'' electrons the effective time for tunneling out and therefore the position of the maximum is independent of the number of dots N. (For this case the Γ_R dependence of the resonant current peak has been discussed for a double dot system in Ref. 11.) In the Coulomb blockade case the nonmonotonic variation of the current with Γ_R persists for all N. The maximum occurs at a slightly higher value of Γ_R than for the ''free''-electron case: at $\Gamma_R = 2t = \tilde{\Gamma}_R$ the effective tunneling time for weak coupling is still larger than for strong coupling because the fraction of channels excluded by Coulomb repulsion is larger for N dots than for $N-1$ dots. The tunnel rate must be increased by a factor $\sqrt{N/(N-1)}$ to exactly balance the effective tunneling times. For large N this difference becomes negligible and the maximum occurs at the same position as for the ''free''-electron case but with a much smaller amplitude (Fig. 5).

Next we consider the resonant current peaks (11) and (15) as a function of the transparency Γ_L of the left tunnel barrier. In the ''free''-electron case the maximum current (15) remains unchanged when we interchange Γ_L and Γ_R. Therefore, the resonant current peak (15) also displays a maximum as a function of Γ_L at $\Gamma_L = 2t$. As a function of both rates the maximum current is

$$\frac{I_{N,max}^F}{e} = \frac{t}{2}, \quad \Gamma_L = \Gamma_R = 2t.$$

As discussed in Sec. V B, there is no such effect in the Coulomb blockade case: due to interdot repulsion, the resonant current peak (27a) will increase with Γ_L and saturate at a maximal value when $\Gamma_L \gg t$,

$$\frac{I_{N,max}^{CB}}{e} = \frac{t}{\sqrt{N(N-1)}}, \quad \Gamma_L \gg \Gamma_R = \sqrt{\frac{N}{N-1}}2t.$$

For $N=2$, the maximal current as a function of the rates is larger in the Coulomb blockade case whereas for $N>2$ it is larger in the ''free''-electron case.

VI. CONCLUSIONS

We have extended the density-matrix approach to resonant tunneling to the case of a linear array of quantum dots with strong, long-range electron-electron interaction. We have found exact analytical expressions for the stationary current in the array for an arbitrary set of parameters (within the applicability of our model) characterizing the array. Coulomb repulsion was found to reduce the resonant current by a factor of the order of the number of dots. The formation of a localized state in one of the dots when the energy level is displaced results in an exponential decay of the current with increasing size of the array. Our approach takes into account correlations between conduction channels in the array due to the coupling to the electron reservoirs. This makes our results also valid for relatively strong tunnel coupling to the reservoirs where the independent channel approximation does not work. These correlations manifest themselves in the eventual decrease of the resonant current when the rate for tunneling into the reservoir is increased.

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