Polarization dependence of the resonant Raman scattering from electrons in a spin-split subband of a III-V semiconductor quantum well

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We have shown that the resonance Raman spectrum of electron spin-flip excitations in a lowest spin-split conduction subband in a semiconductor quantum well depends on the directions of circular polarization of the incident and the scattered lights. In the case of resonance with a heavy-hole subband, this dependence can be detected only if this heavy-hole subband hybridizes with light-hole subbands. However, for resonance with a light-hole subband, the result is not so sensitive to the mixing of heavy- and light-hole subbands. We also found that under extreme resonant conditions, the amplitude of scattered light from charge-density excitations mixes with that from spin-density excitations because of the spin-orbit coupling associated to the spin splitting of conduction subbands. [S0163-1829(99)02140-2]

I. INTRODUCTION

In recent years there have been many studies on transport and optical phenomena associated to the spin-orbit splitting of the conduction band in III-V semiconductor quantum wells and nanostructures. The spin-orbit interaction which splits the conduction band consists of two terms. The Dresselhaus[1] term is due to the lack of inversion symmetry in the zinc-blende crystal Brillouin zone, and in addition the Rashba[2] term appears if the self-consistent potential within a quantum well is asymmetric along the growth direction. The spin-orbit interaction Hamiltonian derived from the $\mathbf{k}\cdot\mathbf{p}$ perturbation expansion has the form that the electronic spin $\mathbf{s}$ is coupled to a *magnetic* field $\mathbf{h}(\mathbf{k})$ which depends on the electron wave vector $\mathbf{k}$. The spin precession around the direction of $\mathbf{h}(\mathbf{k})$ leads to the splitting $|\mathbf{h}(\mathbf{k})|$ of the electron energy, which manifests itself in a split Raman band. Jusserand *et al.*[3] were the first to observe the spin-orbit splitting in the low-frequency spin-flip electronic Raman spectrum of an *n*-type modulation-doped GaAs/Al_xGa_{1-x}As quantum well, and deduced from it the spin-orbit splitting of about 0.4 meV. By analyzing the angular dependence of the spectrum, the contribution of the Dresselhaus term was separated from that of the Rashba term.[4]

The spin precession around $\mathbf{h}(\mathbf{k})$ also leads to various quantum interference phenomena which can be observed in transport and optical properties of two-dimensional (2D) degenerate electrons in quantum wells. For example, the spin-orbit interaction can induce an interference between light waves inelastically scattered from different vector components of spin-density fluctuations. As a consequence, a specific polarization dependence of the Raman spectrum was predicted,[5] which can be observed when the incident and/or scattered light waves are circularly polarized. For the case of nonresonant Raman scattering where the difference between the energy of an incident photon and the fundamental gap is much larger than the energy separation between adjacent hole subbands in a quantum well, calculation[5] has shown that when the circular polarization reverses its direction, there is a change of the relative intensities of Raman peaks corresponding to transitions between spin-split subbands. This theoretical prediction has been confirmed by a very recent experiment.[6] However, most experiments of electronic Raman scattering in quantum wells are performed at resonant conditions. In this case, components of the Raman scattering tensor depend strongly on which of the hole subbands is in resonance with the incident light. Consequently, we expect a different polarization behavior associated to the spin-orbit effects as compared to the nonresonant case.

In this paper we will study how the resonant conditions will modify the electronic Raman spectrum, which is asymmetric with respect to the direction of circular polarization. We consider a single quantum well in which a degenerate 2D electron gas occupies the lowest subband. Based on the symmetry of hole eigenstates, we have reached general conclusions regardless of the shape of the quantum well. In particular, the mixing of light- and heavy-hole subbands plays an important role when the resonance with the heavy-hole subband takes place. At extreme resonance conditions, our theory predicts in Raman spectra an unusual mixing of charge- and spin-density excitations due to the spin-orbit interaction.

II. RAMAN SCATTERING BY ELECTRONS IN A SPIN-SPLIT SUBBAND

In the presence of the spin-orbit interaction $\mathbf{h}(\mathbf{k})\cdot\mathbf{s}$, where $\mathbf{k}=(k_x,k_y)$ is the electron wave vector parallel to interfaces, the Hamiltonian of an electron in the lowest conduction subband $E(\mathbf{k})=k^2/2m^*$ has the form $H=E(\mathbf{k})+\mathbf{h}(\mathbf{k})\cdot\mathbf{s}$. Al-

though in general the spin-orbit interaction $\mathbf{h}(\mathbf{k})$ contains the Dresselhaus term[1] and the Rashba term,[2] its explicit expression depends on the crystallographic orientation of the well. If the growth direction z is along the [001] axis, then the Dresselhaus term can be written as[7]

$$h_x(\mathbf{k})=\alpha k_x(k_y^2-\kappa^2), \quad h_y(\mathbf{k})=\alpha k_y(\kappa^2-k_x^2), \tag{1}$$

where α is a constant and κ^2 is the average of the operator $-(\partial/\partial z)^2$ over the electronic state of the lowest subband. The value of κ is of the order of the inverse width of the well. The Rashba term[2] can be expressed as

$$h_x(\mathbf{k})=\gamma k_y, \quad h_y(\mathbf{k})=-\gamma k_x, \tag{2}$$

where γ is determined by the shape of the self-consistent potential in the quantum well along the growth direction. For a symmetric well, $\gamma=0$.

It is straightforward to solve the Schrödinger equation to obtain the two spin-split subbands for conduction electrons

$$E_{\pm}(\mathbf{k})=E(k)\pm|\mathbf{h}(\mathbf{k})|/2, \tag{3}$$

and the corresponding spin-dependent parts of the eigenfunctions

$$g_{\pm,\uparrow}(\mathbf{k})=\frac{1}{\sqrt{2}}e^{-i\phi_{\mathbf{k}}/2}, \quad g_{\pm,\downarrow}(\mathbf{k})=\pm\frac{1}{\sqrt{2}}e^{i\phi_{\mathbf{k}}/2}, \tag{4}$$

where $\phi_{\mathbf{k}}$ is the angle between the vector $\mathbf{h}(\mathbf{k})$ and the x axis.

We are interested in the low-frequency Raman scattering for which electronic transitions occur within a single spin-split subband or between the two subbands. We will label the frequency, the polarization vector, and the wave vector of the incident and scattered electromagnetic waves as, respectively, ω_L, $\mathbf{e}_L$, $\mathbf{q}_L$, and ω_S, $\mathbf{e}_S$, $\mathbf{q}_S$. We also define $\omega=\omega_L-\omega_S$ for the Stokes shift, and the 2D vector $\mathbf{q}$ for the projection of the vector $\mathbf{q}_L-\mathbf{q}_S$ onto the xy plane. In terms of these notations, the scattering cross section, weighted by the Fermi occupation function $F(E)$ of the initial and the final state, has the standard form

$$W(\omega,\mathbf{q})\propto\sum_{\mathbf{k}}\sum_{m,n=+,-}M_{m,n}(\mathbf{k},\mathbf{q})\{1-F[E_m(\mathbf{k}+\mathbf{q})]\}\times F[E_n(\mathbf{k})]\delta[E_m(\mathbf{k}+\mathbf{q})-E_n(\mathbf{k})-\omega]. \tag{5}$$

For a degenerate electron gas with high enough 2D density, which is the situation considered in this paper, we have $q\ll k_F$ and $|\mathbf{h}(\mathbf{k})|\ll E_F$. Then, the major contribution to Eq. (5) comes from the terms with k in the very near vicinity of the Fermi wave vector k_F. Under this condition, the transition probability from the state $\mathbf{k}$ in the nth spin-split subband to the state $\mathbf{k}+\mathbf{q}$ in the mth subband is given by

$$M_{m,n}(\mathbf{k},\mathbf{q})=\left|\sum_{\alpha,\beta=\uparrow,\downarrow}g^*_{m,\alpha}(\mathbf{k}_F)\gamma_{\alpha\beta}(\mathbf{k},\mathbf{q})g_{n,\beta}(\mathbf{k}_F)\right|^2, \tag{6}$$

where $\gamma_{\alpha\beta}(\mathbf{k},\mathbf{q})$ is the Raman scattering tensor. In the above expression, the eigenfunctions satisfy the relation $g_{m,\alpha}(\mathbf{k}_F+\mathbf{q})\simeq g_{m,\alpha}(\mathbf{k}_F)$.

For bulk semiconductors, the Raman scattering tensor $\gamma_{\alpha\beta}$ was derived by Hamilton and McWhorter.[8] It contains a part of light scattering from the charge-density fluctuations and a part from the spin-density fluctuations. This expression of $\gamma_{\alpha\beta}$ is valid for quantum-well systems[9] if the incident light is far from resonances with intermediate transitions from hole subbands to the final electron state above the Fermi level. The corresponding detuning of the resonances must be much larger than the energy separation $\Delta_{\mathbf{k}}$ between hole subbands. Under this condition, Eqs. (6) and (5) have been calculated in Ref. 5.

In this paper we will consider the opposite case of strong resonance with one of the hole subbands, say the νth subband $\varepsilon_\nu(\mathbf{k})$, when the detuning is much less than $\Delta_{\mathbf{k}}$. Here the main contribution to the scattering tensor comes from the intermediate states which lie very close to $\varepsilon_\nu(\mathbf{k}_F)$. We define $|S_\alpha\rangle$ as the zone-center Bloch states for electrons with spin α, and $|u_j\rangle$ as the zone-center Bloch states for holes. In terms of these states, the resonance Raman scattering tensor was derived as[10]

$$\gamma_{\alpha\beta}(\mathbf{k},\mathbf{q})=\sum_j\langle S_\alpha|\mathbf{pe}_S^*|u_j\rangle\langle u_j|\mathbf{pe}_L|S_\beta\rangle\sum_\nu A^{\pm}_{\nu j}, \tag{7}$$

with

$$A^{\pm}_{\nu j}=\frac{1}{\Delta^{\pm}_\nu}|\langle g(z)|f_{\nu,j}(z)\rangle|^2. \tag{8}$$

Here $\Delta^{\pm}_\nu=E_{\pm}(\mathbf{k}+\mathbf{q})-\varepsilon_\nu(\mathbf{k}-\mathbf{q}_S)-\omega_L$ is the resonance denominator and $g(z)$ is the z-dependent part of the electron envelope function. The z-component part of the hole envelope function is conventionally expressed in vector form $\mathbf{f}_\nu(z)$, which is the eigenvector of the Luttinger Hamiltonian.[12] For heavy-hole band and light-hole bands, which correspond to the total angular momentum $J=\frac{3}{2}$, we have $j=\pm\frac{3}{2}$ (heavy hole) and $\pm\frac{1}{2}$ (light hole). Hence $\mathbf{f}_\nu(z)=(f_{\nu,3/2}(z),f_{\nu,1/2}(z),f_{\nu,-1/2}(z),f_{\nu,-3/2}(z))$. However, for the split-off hole band with total angular momentum $J=\frac{1}{2}$, we have $j=\pm\frac{1}{2}$ and $\mathbf{f}_\nu(z)=(f_{\nu,1/2}(z),f_{\nu,-1/2}(z))$.

III. POLARIZATION DEPENDENCE

Positions of Raman peaks are determined by the extremal points in $\mathbf{k}$ space which satisfy the δ functions in Eq. (5). It has been shown[5] that the electron excitations within the + or − spin-split subband generate a peak (with inverse square-root singularity) in the Raman spectrum at $\omega=v_Fq$. At the same time, transitions between the + and − spin-split subbands produce two more peaks. The precise positions of these peaks are determined by the angular dependence of $|\mathbf{h}(\mathbf{k}_F)|$. For the simplest case that the cubic terms of $\mathbf{k}$ in Eq. (1) are ignored, $|\mathbf{h}(\mathbf{k}_F)|$ becomes isotropic and the peak positions are given by $\omega=v_Fq\pm|\mathbf{h}(\mathbf{k}_F)|$. The anisotropy of $|\mathbf{h}(\mathbf{k}_F)|$, caused by the cubic terms, is the origin of the complicated angular dependence of the peak positions, which has been discussed in detail in Ref. 4. In this paper we will analyze the dependence of the Raman peak intensities on the polarization vectors $\mathbf{e}_L$ and $\mathbf{e}_S$.

We need to calculate $M_{i,j}(\mathbf{k},\mathbf{q})$ at the extremal point $\mathbf{k}=\mathbf{k}_F^{\mathrm{ex}}$ on the Fermi line, which is determined from the δ function in Eq. (5) (the mathematical procedure to locate the extremal point was explained in detail in Refs. 4 and 5). For this purpose, the eigenfunctions of the Luttinger Hamiltonian

have to be solved in order to evaluate the expectation value $\langle g(z)|f_{\nu,j}(z)\rangle$ in the coefficient $A_{\nu j}^{\pm}$ given by Eq. (8). Solving the eigenvalue problem becomes increasingly difficult in the region of wave vectors where the light-heavy subband mixing is important. To tackle this problem, we will use the unitary transformation proposed by Broido and Sham.[13] If we choose the transformation to be independent of k_z and z, it will allow us to treat k_z as an operator $\hat{k}_z=-i(\partial/\partial z)$. Then, the 4×4 matrix representation of the original Luttinger Hamiltonian $\mathcal{H}$ is block diagonalized into two 2×2 matrices for the corresponding Hamiltonians $\mathcal{H}_1$ and $\mathcal{H}_2$. After deriving the eigenvectors $(f_{\nu 1h}(z),f_{\nu 1l}(z))$ and $(f_{\nu 2l}(z),f_{\nu 2h}(z))$ for $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, the eigenvectors of the full Luttinger Hamiltonian $\mathcal{H}$ are obtained with the inverse transformation

$$\begin{aligned}\mathbf{f}_{\nu 1}(z)&=(e^{i\varphi}f_{\nu 1h},e^{i\eta}f_{\nu 1l},e^{-i\eta}f_{\nu 1l},-e^{-i\varphi}f_{\nu 1h}),\\ \mathbf{f}_{\nu 2}(z)&=(e^{i\varphi}f_{\nu 2h},-e^{i\eta}f_{\nu 2l},e^{-i\eta}f_{\nu 2l},e^{-i\varphi}f_{\nu 2h}).\end{aligned}\tag{9}$$

Here the phase factors $e^{\varphi-\eta}=B/|B|$ and $e^{\varphi+\eta}=-iI/|I|$ are defined in terms of

$$B=-2\sqrt{3}\gamma_3(k_x-ik_y),$$

$$I=-\sqrt{3}\gamma_2(k_x^2-k_y^2)+i2\sqrt{3}\gamma_3k_xk_y,$$

with the Luttinger parameters[12] γ_1, γ_2, and γ_3. In order to demonstrate the essential physics with a compact presentation, we will use the spherical approximation $\gamma_2=\gamma_3=\overline{\gamma}$.

If the potential for holes is symmetric with respect to a reflection with respect to the xy plane, the eigenvectors given by Eq. (9) are then doubly degenerate with energies $\varepsilon_{\nu 1}(\mathbf{k})=\varepsilon_{\nu 2}(\mathbf{k})$. Such degeneracy at $k\neq0$ will be removed in asymmetric wells. However, these nondegenerate states can still be very close in energy, and hence must be included in the summation in Eq. (7) over the states with energies close to resonance.

Substituting the results of Eq. (9) into Eq. (8), the coefficients $A_{\nu j}$ can now be expressed in terms of the hole and electron envelope functions as

$$\begin{aligned}A_{h\nu}^{\pm}&=A_{\nu,3/2}^{\pm}=A_{\nu,-3/2}^{\pm}=\frac{1}{\Delta_{\nu 1}^{\pm}}|\langle g|f_{\nu 1h}\rangle|^2+\frac{1}{\Delta_{\nu 2}^{\pm}}|\langle g|f_{\nu 2h}\rangle|^2,\\ A_{l\nu}^{\pm}&=A_{\nu,1/2}^{\pm}=A_{\nu,-1/2}^{\pm}=\frac{1}{\Delta_{\nu 1}^{\pm}}|\langle g|f_{\nu 1l}\rangle|^2+\frac{1}{\Delta_{\nu 2}^{\pm}}|\langle g|f_{\nu 2l}\rangle|^2,\end{aligned}\tag{10}$$

where $\Delta_{\nu i}^{\pm}=E_{\pm}(\mathbf{k}+\mathbf{q})-\varepsilon_{\nu i}(\mathbf{k}-\mathbf{q}_S)-\omega_L$. By defining a 2D unit vector $\mathbf{n}\equiv\mathbf{h}(\mathbf{k})/|\mathbf{h}(\mathbf{k})|$ with components $n_x=\cos\phi_{\mathbf{k}}$ and $n_y=\sin\phi_{\mathbf{k}}$, from Eqs. (4), (6), (7), and (8), we obtain the transition probabilities $M_{\pm,\mp}$ for electrons between the two spin-split subbands

$$\begin{aligned}M_{\pm,\mp}=\frac{P_{cv}^4}{9}\Bigg(&\frac{(A_{l\nu}^{\pm}-3A_{h\nu}^{\pm})^2}{4}|P_z|^2+A_{l\nu}^{\pm 2}|\mathbf{P}_{||}\times\mathbf{n}|^2\\ &\mp i\frac{A_{l\nu}^{\pm}(A_{l\nu}^{\pm}-3A_{h\nu}^{\pm})}{2}\mathbf{P}\times\mathbf{P}^*\cdot\mathbf{n}\Bigg),\end{aligned}\tag{11}$$

where $\mathbf{P}=\mathbf{e}_S^*\times\mathbf{e}_L$, and P_{cv} is the Kane matrix element.[11]

The term $\mathbf{P}\times\mathbf{P}^*\cdot\mathbf{n}$ in Eq. (11) is due to the interference between the longitudinal and the transverse component of the spin-density fluctuations, and has different signs for the two transition probabilities. When $|\mathbf{h}(\mathbf{k})|\to0$, the spin-split bands merge and the two interference terms cancel each other in the Raman spectrum. Although the same form of interference also appears in nonresonant Raman scattering,[5] an important feature here is that $\mathbf{P}\times\mathbf{P}^*\cdot\mathbf{n}$ shows up together with the coefficients $A_{l\nu}$ and $A_{h\nu}$ which characterize the mixing of hole subbands. In a Raman scattering experiment with both the incident and the scattered light circularly polarized, it is easy to reverse simultaneously the polarization directions of both the incident and the scattered light. Such reversion will change $\mathbf{e}_L$ into $\mathbf{e}_L^*$ and $\mathbf{e}_S^*$ into $\mathbf{e}_S$. As a result, in Eq. (11), $\mathbf{P}$ becomes $\mathbf{P}^*$ and $\mathbf{P}\times\mathbf{P}^*$ changes its sign, while the other terms remain intact. Hence, the interference term can be derived by taking the difference of two Raman spectra with opposite circular polarizations in both the incident and the scattered light. The so-obtained *difference spectrum* can be calculated from the corresponding difference of transition probabilities

$$\Delta M_{\pm,\mp}=\mp i\frac{P_{cv}^4}{9}\mathbf{P}\times\mathbf{P}^*\cdot\mathbf{n}(A_{l\nu}^{\pm}-3A_{h\nu}^{\pm})A_{l\nu}^{\pm}.\tag{12}$$

When evaluating $\Delta M_{\pm,\mp}$ at Raman peak positions which are located at extremal points $\mathbf{k}=\mathbf{k}_F^{\mathrm{ex}}$, it will produce two peaks in the *difference spectrum*. Since $\Delta M_{+,-}$ and $\Delta M_{-,+}$ are of opposite signs, these two peaks are also of opposite signs and can be observed experimentally.[6]

Ivchenko and Pikus[10] have calculated the Raman scattering tensor $\gamma_{\alpha\beta}(\mathbf{k},\mathbf{q})$ assuming no mixing of the heavy- and light-hole subbands. Under this assumption, it is easy to show that $A_{l\nu}=0$ and $A_{h\nu}\neq0$ for scattering resonant with heavy-hole subbands, but $A_{h\nu}=0$ and $A_{l\nu}\neq0$ for scattering resonant with light-hole subbands. It then follows from Eq. (12) that $\Delta M_{\pm,\mp}=0$ if the resonant scattering is mediated by a heavy-hole subband, while $\Delta M_{\pm,\mp}\neq0$ if the resonant scattering is mediated by a light-hole subband. Therefore, the mixing of the heavy- and light-hole subbands must be taken into account in order to formulate a complete theory. For this purpose, we need to find the explicit expressions of $A_{h\nu}$ and $A_{l\nu}$.

In terms of the Pauli matrices σ_x, σ_y, and σ_z, the block diagonalized Hamiltonians $\mathcal{H}_1$ and $\mathcal{H}_1$ can be expressed in a suitable form

$$\begin{aligned}\mathcal{H}_1&=\gamma_1(k^2+k_z^2)+\overline{\gamma}(k^2-2k_z^2)\sigma_z+k_z|B|\sigma_x-|I|\sigma_y+u(z),\\ \mathcal{H}_2&=\gamma_1(k^2+k_z^2)-\overline{\gamma}(k^2-2k_z^2)\sigma_z-k_z|B|\sigma_x+|I|\sigma_y+u(z),\end{aligned}\tag{13}$$

where $k_z=-i(\partial/\partial z)$ and $u(z)$ is the potential energy of a hole in the quantum well. The terms proportional to $|B|$ and $|I|$ are responsible for the heavy- and light-hole subband mixing. The eigensolutions of H_1 and H_2 can be derived for any value of k, from which $A_{h\nu}$ and $A_{l\nu}$ can be readily calculated. However, since $|B|\sim k$ and $|I|\sim k^2$, in the region of small k, we can treat the $|B|$ and the $|I|$ term in $\mathcal{H}_{1,2}$ as perturbation. This will allow us to illustrate some characteristic features of subband mixing.

For the unperturbed Hamiltonians $H_i^{(0)}$ with $i=1$ and 2, the zeroth-order eigenfunctions $(f_{\nu ih}^{(0)},0)$ and $(0,f_{\mu il}^{(0)})$, as well as their corresponding eigenenergies $\varepsilon_{\nu ih}^{(0)}$ and $\varepsilon_{\mu il}^{(0)}$, can be easily obtained. They satisfy the relations $f_{\nu 1h}^{(0)}=f_{\nu 2h}^{(0)}$, $f_{\mu 1l}^{(0)}=f_{\mu 2l}^{(0)}$, $\varepsilon_{\nu 1h}^{(0)}=\varepsilon_{\nu 2h}^{(0)}$, and $\varepsilon_{\mu 1l}^{(0)}=\varepsilon_{\mu 2l}^{(0)}$. Since the transition probabilities $\Delta M_{\pm,\mp}$ are already finite at the level of unperturbed Hamiltonian if the scattering is resonant with a light-hole subband, here we will calculate the perturbation corrections to the νth heavy-hole subband. In the lowest order of its mixing with the light-hole subbands, the wave function correction is

$$f_{\nu il}^{(1)}=i\sqrt{3}\gamma\sum_{\mu}\frac{k^2\langle f_{\nu ih}^{(0)}|f_{\mu il}^{(0)}\rangle \mp 2k\left\langle f_{\nu ih}^{(0)}\left|\frac{\partial}{\partial z}f_{\mu il}^{(0)}\right.\right\rangle}{\varepsilon_{\nu ih}^{(0)}-\varepsilon_{\mu il}^{(0)}}f_{\mu il}^{(0)}, \tag{14}$$

where the $-$ sign is for $i=1$ and the $+$ sign is for $i=2$. From Eq. (10) the coefficient $A_{l\nu}$ is readily derived as

$$A_{l\nu}=\frac{|\langle g|f_{\nu 1l}^{(1)}\rangle|^2}{\Delta_{\nu 1}^{\pm}}+\frac{|\langle g|f_{\nu 2l}^{(1)}\rangle|^2}{\Delta_{\nu 2}^{\pm}}. \tag{15}$$

We should mention that in asymmetric quantum wells there will be different perturbation corrections to the degenerate energies $\varepsilon_{\nu 1h}^{(0)}$ and $\varepsilon_{\nu 2h}^{(0)}$. In this case the two resonance denominators $\Delta_{\nu 1}^{\pm}$ and $\Delta_{\nu 2}^{\pm}$ in Eq. (15) can be considerably different.

From Eqs. (15) and (14) we see that due to the mixing of heavy- and light-hole subbands, the coefficient $A_{l\nu}$ for resonant scattering from a heavy-hole subband is finite and increases with k. For an asymmetric quantum well, the right-hand side of Eq. (14) is dominated by the term linear in k because k is small. If d_ν is the range of hole confinement in a quantum well, then the expectation value of the operator $k_z=-i(\partial/\partial z)$ is proportional to d_ν^{-1}, and the energy denominator in Eq. (14) is proportional to d_ν^{-2}. Since the dynamics of the system is controlled by electrons with $k\simeq k_F$, the coefficient $A_{l\nu}$ given by Eq. (15) is proportional to $(k_f d_\nu^{-1}/d_\nu^{-2})^2=k_f^2 d_\nu^2$. On the other hand, the eigenfunctions in a symmetric quantum well are either even or odd with respect to reflection from the xy plane. For the lowest electron subband $g(z)$ is even. Hence, $f_{\nu il}^{(1)}$ in Eq. (15) is also even. Then, if ν in Eq. (14) corresponds to an even hole state, say the topmost heavy-hole subband, at the right-hand side of Eq. (14) the term linear in k vanishes. Consequently, $A_{l\nu}$ is proportional to $k_f^4 d_\nu^4$. These expressions are valid if the perturbation expansion parameter $k_f d_\nu$ is much less than 1. Beyond the perturbation region, $A_{l\nu}$ has to be derived numerically.

So far we have investigated the Raman spectra due to intersubband electron excitations between the spin-split subbands. By evaluating the coefficient $A_{l\nu}$, we conclude that, based on the characteristic properties of $\Delta M_{\pm,\mp}$ given by Eq. (12), our theory predicts an observable polarization effect in the resonance Raman spectrum. When resonant with a heavy-hole subband, this effect can be detected only if this heavy-hole subband hybridizes with light-hole subbands. However, for resonance with a light-hole subband, the effect is not so sensitive to the mixing of heavy- and light-hole subbands.

One additional prediction can be made for the third Raman peak at $\omega=qv_F$, which corresponds to intrasubband excitations within each spin-split subband. In an earlier work for nonresonant Raman scattering,[5] it has been shown that this peak does not depend on the direction of circular polarization. However, the situation will be different under resonant scattering. In this case, in Eq. (10) the energy denominator $\Delta_{\nu i}^{+}$ differs from the energy denominator $\Delta_{\nu i}^{-}$ because of the energy gap between the $+$ subband and the $-$ subband. As a result, this Raman peak is sensitive to the polarization of the light waves, as will be demonstrated below.

If the insignificant difference between the Fermi velocities in spin-split subbands is ignored, in Eq. (5) the transition probabilities $M_{+,+}$ and $M_{-,-}$ have equal contribution to the scattering cross section. Let us define

$$C_0^{\pm}=\frac{P_{cv}^2}{6}[(A_{l\nu}^{\pm}+3A_{h\nu}^{\pm})\mathbf{e}_S^*\cdot\mathbf{e}_L+3(A_{l\nu}^{\pm}-A_{h\nu}^{\pm})e_{Sz}^* e_{Lz}]$$

and $\mathbf{C}^{\pm}=-i(P_{cv}^2/3)A_{l\nu}^{\pm}\mathbf{P}$. The contribution of $M_{+,+}$ and $M_{-,-}$ to the scattering cross section is then proportional to

$$\begin{aligned}M_{+,+}+M_{-,-}=&|\mathbf{C}^+\cdot\mathbf{n}|^2+|C_0^+|^2+|\mathbf{C}^-\cdot\mathbf{n}|^2+|C_0^-|^2\\&+C_0^{+*}\mathbf{C}^+\cdot\mathbf{n}+C_0^+\mathbf{C}^{+*}\cdot\mathbf{n}-C_0^{-*}\mathbf{C}^-\cdot\mathbf{n}\\&-C_0^-\mathbf{C}^{-*}\cdot\mathbf{n}.\end{aligned} \tag{16}$$

Only when the incident and/or the scattered light is circularly polarized is the sum of the last four terms in the above Eq. (16) nonzero. It changes sign when circular polarizations reverse their directions. The so-produced effect can be seen when we take the difference of the two Raman spectra corresponding to opposite polarizations. This effect will manifest itself with increasing difference between $\Delta_{\nu i}^{+}$ and $\Delta_{\nu i}^{-}$, on which the values of $A_{l\nu}^{\pm}$ and $A_{h\nu}^{\pm}$ depend, as shown in Eq. (10).

An unusual feature of these last four terms in Eq. (16) is that the amplitudes $C_0^{\pm}$ and $\mathbf{C}^{\pm}$ appear in the forms of cross products. This means that amplitudes of inelastic light scattering from charge- and spin-density excitations interfere, and these four terms are just the corresponding interference terms. To clarify the physical picture, we notice that the coefficients C_0 and the components of the vector $\mathbf{C}\equiv(C_x,C_y,C_z)$ are expressed in terms of the scattering matrix γ as $C_0\propto\mathrm{Tr}[\gamma(\mathbf{k},\mathbf{q})]$ and $C_i\propto\mathrm{Tr}[\gamma(\mathbf{k},\mathbf{q})\sigma_i]$, where σ_i are Pauli matrices. Hence, C_0 is associated to light scattering by spin-independent charge-density excitations, while the coefficients C_i are connected to the scattered amplitudes of light by spin-density excitations. In the absence of the spin-orbit interaction, $C_0^+=C_0^-$ and $\mathbf{C}^+=\mathbf{C}^-$. Consequently, the interference terms in Eq. (16) vanish, and the remaining terms represent the independent contributions of spin- and charge-density excitations to the Raman cross section. However, for the problem investigated here, due to the spin-orbit coupling, these scattered amplitudes by spin-density excitations and charge-density excitations mix and the appropriate resonance conditions make it an observable effect.

It is well known that in a Coulomb gas the low-frequency charge-density excitations are screened, while Eqs. (5) and (16) are derived for a noninteracting electron gas. Furthermore, Coulomb interaction can create collective spin-density waves in the spin-split energy gap.[14] Because of all these complications, this topic requires a thorough study in the future.

IV. CONCLUSION

In summary, we have shown that the mixing of light- and heavy-hole subbands plays a very important role in the electron spin-flip resonance Raman spectrum with circularly polarized incident and scattered lights. An observable effect appears in the difference of two such Raman spectra taken with opposite circular polarizations. When the Raman scattering is resonant with a heavy-hole subband, this difference spectrum is enhanced with higher electron density, as well as with a weaker confinement of holes in the well. However, if the Raman scattering is resonant with a light-hole subband, there is no such strong dependence. We also have shown that interference of light scattering amplitudes from charge-density and spin-density excitations can be observed in the difference spectrum of spin conserving electron transitions. For that the resonance with incident photons must be strong enough to distinguish between contributions to the Raman tensor from the two spin-split electron states.

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