Maximum magnetic moment in a field-cooled superconducting disk

Masahito Nakagawa, Shigeaki Utsumi, and Yasukage Oda
Department of Material Science, Faculty of Science, Himeji Institute of Technology, Ako-gun, Hyogo 678-1297, Japan

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We studied the maximum magnetic moments in a field-cooled superconducting disk for fields perpendicular to the disk surface. When the sample was field cooled, a part of the flux remained in the sample. We calculated the (positive) maximum magnetic moments only under the following two conditions. One is that the remaining flux not exceed the flux through the sample above T_c. The other is that the field component perpendicular to the sample surface be zero or positive. As a result, the maximum moment turned out to be about 50% of the full diamagnetic moment. If the observed paramagnetic Meissner effect was caused by flux compression under the two conditions, a positive magnetic moment over 50% of the full diamagnetic moment is impossible for a superconducting disk. For example, we calculated magnetic moments and others for a superconducting disk by assuming that the Bean state is realized around the center of the disk. [S0163-1829(99)11425-5]

I. INTRODUCTION

After some granular high-T_c superconductors were field cooled, positive magnetic moments were observed,[1–3] and this is referred to as the paramagnetic Meissner effect (PME) or Wohlleben effect. Afterwards many studies were made experimentally[4–8] and theoretically.[9–15] The PME is a very interesting phenomenon if it appears as a consequence of unconventional pairing as predicted theoretically. However, similar behaviors (positive magnetic moments) were observed in *s*-wave Nb disks.[19,20] In order to explain these experimental results consistently, several studies were made from the viewpoint of surface superconductivity.[16–18] In particular, Geim *et al.* discussed the origin of the PME on the basis of their observation of positive magnetic moments in mesoscopic superconductors.[18] On the other hand, Koshelev and Larkin pointed out that positive magnetic moments can be caused by flux compression into the sample due to inhomogeneous cooling; that is, the superconducting state extends from the edges of the sample.[21] Thus Rice and Sigrist proposed experimental methods to distinguish between the two different origins of paramagnetism, flux compression, and orbital magnetic moments (OMM's).[22] One of them is to measure the magnitude of the paramagnetic signal, because the paramagnetic signal due to OMM's may exceed the upper limit possible through flux compression. Indeed Knauf *et al.* reported a paramagnetic susceptibility larger than the full diamagnetic signal ($\chi=-1/4\pi$).[23] The maximum magnetic moment due to flux compression was calculated by Koshelev and Larkin for a thin superconducting strip assuming a Bean state; the value is about 27% of $1/4\pi$.[21] Therefore the results of Knauf *et al.* were difficult to be explained by flux compression. However, the results of Koshelev and Larkin may strongly depend on the sample geometry, or the Bean state assumed by Koshelev and Larkin may not be realized. Consequently, a much larger paramagnetic susceptibility may be caused by flux compression. Thus we calculated the maximum magnetic moment for a disk-shaped superconductor. In this geometry a demagnetizing field affects the magnetic moment largely; so we suppose that the maximum magnetic moment is the largest in all geometry. When obtaining the maximum magnetic moment we calculated it only under the following two conditions. One is that the flux which is larger than the flux through the disk at $T>T_c$ not remain in the disk at $T<T_c$. The other is that the field component perpendicular to the sample surface be zero or positive. Therefore, the (obtained) results are quite general.

This paper is organized as follows. In Sec. II we formulate the method to obtain the magnetic moment when an arbitrary distribution of field components perpendicular to the disk surface is given. In Sec. III we calculate the maximum magnetic moments. In Sec. IV we calculate magnetic moments and others by assuming that a Bean state is realized around the center of the disk. We will use the SI units from here on in.

II. FORMULATION FOR MAGNETIC MOMENT

In this section we formulate the method to obtain the magnetic moment when an arbitrary distribution of field components perpendicular to the disk surface is given.

We assume that the thickness of the disk is much smaller than the radius R and consequently we do not need to consider the z dependence of the current distribution, where z is the axis perpendicular to the disk surface. We integrate the current density with respect to z and we define $\boldsymbol{i}$ as the integrated value. When there is a stationary current $\boldsymbol{i}$ in a region of dS' at $\boldsymbol{r}'$, the vector potential $d\boldsymbol{A}$ at $\boldsymbol{r}''$ created by the current is

$$d\boldsymbol{A}=\frac{\mu_0}{4\pi|\boldsymbol{r}''-\boldsymbol{r}'|}\boldsymbol{i}(\boldsymbol{r}')dS'; \tag{2.1}$$

see Fig. 1. The flux $d\Phi$ which is created by the current $\boldsymbol{i}(\boldsymbol{r}')dS'$ inside a radius r from the center of the disk is

$$d\Phi=\oint d\boldsymbol{A}\cdot d\boldsymbol{s}'' \tag{2.2}$$

$$=\frac{\mu_0}{4\pi}dS'\oint\frac{\boldsymbol{i}(\boldsymbol{r}')\cdot d\boldsymbol{s}''}{|\boldsymbol{r}''-\boldsymbol{r}'|}. \tag{2.3}$$

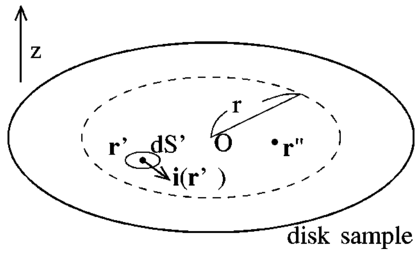

FIG. 1. Schematic illustration of the disk sample. The symbols are defined in the text.

Here, the integral path is a circumference of the radius r from the center of the disk. After this, we take the center of the disk as the origin of vectors and coordinates. In cylindrical coordinates,

$$d\Phi=\frac{\mu_0}{4\pi}r'\,dr'\,d\theta' \times\int_{-\pi}^{\pi}\frac{-i_r(r',\theta')\sin\alpha+i_\theta(r',\theta')\cos\alpha}{\sqrt{r'^2+r^2-2r'r\cos\alpha}}r\,d\alpha, \tag{2.4}$$

where i_r and i_θ are radial and circumferential components of $\boldsymbol{i}(\boldsymbol{r}')$, respectively. α is the angle between $\boldsymbol{r}'$ and $\boldsymbol{r}''$. Here $|\boldsymbol{r}''|=r$. The term of $\sin\alpha$ which is an odd function vanishes and Eq. (2.4) is expressed as

$$d\Phi=\frac{\mu_0}{4\pi}i_\theta(r',\theta')\frac{4}{k}F\left(\frac{r'}{r}\right)\left(\frac{r'}{r}\right)^{-1/2}r'\,dr'\,d\theta'. \tag{2.5}$$

Here,

$$k^2\equiv\frac{4r'/r}{(1+r'/r)^2}, \tag{2.6}$$

$$F\left(\frac{r'}{r}\right)\equiv\left(1-\frac{k^2}{2}\right)K(k^2)-E(k^2). \tag{2.7}$$

K and E are complete elliptic integrals of first and second kinds.

We consider a contribution of whole current. Using Eq. (2.5), the flux Φ through the disk inside the radius r is

$$\Phi(\tilde{r})=\frac{\mu_0H_{ext}}{\pi}R^2\int_0^1\tilde{I}_\theta(\tilde{r}')\frac{1}{k}F\left(\frac{\tilde{r}'}{\tilde{r}}\right)\left(\frac{\tilde{r}'}{\tilde{r}}\right)^{-1/2}\tilde{r}'\,d\tilde{r}', \tag{2.8}$$

where $\tilde{r}'\equiv r'/R$, $\tilde{r}\equiv r/R$, and

$$\tilde{I}_\theta(\tilde{r}')\equiv\frac{1}{H_{ext}}\int_0^{2\pi}i_\theta(R\tilde{r}',\theta')d\theta'. \tag{2.9}$$

Here H_{ext} is an external magnetic field of the z direction. For convenience, we show the expression of $\Phi(1)$,

$$\Phi(1)=\frac{\mu_0H_{ext}}{2\pi}R^2\int_0^1(1+\tilde{r}')F(\tilde{r}')\tilde{I}_\theta(\tilde{r}')d\tilde{r}'. \tag{2.10}$$

Using Eq. (2.8),

$$\phi(\tilde{r})\equiv\frac{1}{\Phi_{ext}}\frac{d\Phi}{d\tilde{r}}=\frac{1}{2\pi^2}\int_0^1\tilde{I}_\theta(\tilde{r}')\left(\frac{K(k^2)}{\tilde{r}'/\tilde{r}+1}+\frac{E(k^2)}{\tilde{r}'/\tilde{r}-1}\right)d\tilde{r}', \tag{2.11}$$

where $\Phi_{ext}=\pi R^2\mu_0H_{ext}$. Here Φ_{ext} corresponds to the flux which has penetrated through the disk at $T>T_c$, and $\Phi(\tilde{r})$ is related to B_z as follows:

$$\Phi(\tilde{r})=\int B_z dS-\pi R^2\tilde{r}^2\mu_0H_{ext}. \tag{2.12}$$

The integral region is inside a circle of radius r from the center of the disk. Note that B_z is a sum of the field created by the current and external magnetic field. From Eq. (2.12),

$$\phi(\tilde{r})=\frac{\tilde{r}}{\pi}\int_0^{2\pi}\left(\frac{B_z(\tilde{r},\theta)}{\mu_0H_{ext}}-1\right)d\theta. \tag{2.13}$$

Using Eq. (2.13), ϕ can be obtained from an arbitrary B_z distribution in the disk, and by solving the integral equation (2.11), $\tilde{I}_\theta$ can be obtained. The magnetic moment can be obtained as follows by using this $\tilde{I}_\theta$. The magnetic moment is obtained from

$$\boldsymbol{\mu}=\frac{1}{2}\int\boldsymbol{r}'\times\boldsymbol{i}(\boldsymbol{r}')dS'. \tag{2.14}$$

In cylindrical coordinates, μ_z is

$$\mu_z=\frac{1}{2}\int r'^2 i_\theta(r',\theta')d\theta'\,dr'. \tag{2.15}$$

The other components of $\boldsymbol{\mu}$ are zero. Equation (2.15) is reduced to

$$\tilde{\mu}_z\equiv\frac{\mu_z}{R^3H_{ext}}=\frac{1}{2}\int_0^1\tilde{r}'^2\tilde{I}_\theta(\tilde{r}')d\tilde{r}'. \tag{2.16}$$

Thus the magnetic moment can be obtained from an arbitrary B_z distribution inside the disk.

III. MAXIMUM MAGNETIC MOMENT

In this section we calculate the maximum magnetic moment under the condition of $\Phi(1)\leqslant 0$ and $B_z\geqslant 0$ in the disk region $(0\leqslant\tilde{r}\leqslant 1)$. Here $\Phi(1)$ means the flux through the disk created by current in the disk. Therefore, the former condition corresponds to the fact that the flux which is larger than the flux through the disk at $T>T_c$ does not remain in the disk at $T<T_c$. It is not obvious that the latter condition is satisfied. However, it is hard to consider that a region of $B_z<0$ appears (especially in the inhomogeneous cooling process). We should note that the maximum magnetic moment is infinite if the latter condition is absent. For information, in the Appendix we calculate the maximum magnetic moment under the condition that the critical current is finite instead of the latter condition. In this case a region of $B_z<0$ exists.

First, the condition $\Phi(1)\leqslant 0$ is written by

$$\int_0^1 \phi(\tilde{r})d\tilde{r} \leqslant 0. \tag{3.1}$$

From the condition $B_z \geqslant 0$ in the region $0 \leqslant \tilde{r} \leqslant 1$,

$$\int_0^{2\pi} B_z(\tilde{r},\theta)d\theta \geqslant 0 \tag{3.2}$$

is a necessary condition. By using Eq. (2.13) and from this condition,

$$\phi(\tilde{r}) \geqslant -2\tilde{r}. \tag{3.3}$$

Equation (3.3) is the only necessary condition for $B_z \geqslant 0$. However, there is no problem if the condition $B_z \geqslant 0$ is satisfied when we obtain the maximum $\tilde{\mu}_z$ under conditions (3.1) and (3.3).

$\tilde{\mu}_z$ can be obtained from ϕ as well as the B_z distribution in the disk; so we write $\tilde{\mu}_z(\phi)$. It is a linear functional with respect to ϕ. Thus, ϕ which gives the maximum $\tilde{\mu}_z$ under conditions (3.1) and (3.3) is obtained as

$$\phi(\tilde{r}) = \delta(\tilde{r}-\tilde{r}_0) - 2\tilde{r}, \tag{3.4}$$

where δ is a delta function and $\tilde{r}_0$ is some constant obtained later. Substituting this for Eq. (2.13),

$$\int_0^{2\pi} \frac{B_z(\tilde{r},\theta)}{\mu_0 H_{ext}} d\theta = \frac{\pi}{\tilde{r}} \delta(\tilde{r}-\tilde{r}_0). \tag{3.5}$$

Infinitely many B_z distributions which satisfy $B_z \geqslant 0$ and Eq. (3.5) exist. We give B_z independent of θ in them below:

$$B_z(\tilde{r},\theta) = \mu_0 H_{ext} \frac{1}{2\tilde{r}} \delta(\tilde{r}-\tilde{r}_0). \tag{3.6}$$

Using Eq. (3.4) the maximum $\tilde{\mu}_z$ (we define $\tilde{\mu}_{z,\max}$ as this value) is

$$\tilde{\mu}_{z,\max} = \tilde{\mu}_z[\delta(\tilde{r}-\tilde{r}_0)] + \tilde{\mu}_z(-2\tilde{r}). \tag{3.7}$$

In order to obtain $\tilde{r}_0$, we calculated $\tilde{\mu}_z[\delta(\tilde{r}-\tilde{\rho})] + \tilde{\mu}_z(-2\tilde{r})$ vs $\tilde{\rho}$ numerically with Eqs. (2.11) and (2.16), which is shown in Fig. 2. Each value is $\tilde{\mu}_z$ when the flux Φ_{ext} is concentrated only on the radius $R\tilde{\rho}$. Here a θ dependence of the flux distribution is arbitrary as long as $B_z > 0$. The magnetic moment changes from negative to positive as the flux approaches the center from the edge, and the maximum value is realized when $\tilde{\rho} \sim 0$. Consequently, $\tilde{r}_0 \sim 0$ is obtained. [$\tilde{\mu}_z(-2\tilde{r})$ is a constant $-8/3$. It results from the fact that a magnetic moment of the disk in the Meissner state is given by $-(8/3)R^3 H_{ext}$.] As a result, the maximum magnetic moment is realized when the whole Φ_{ext} is compressed around the center of the disk and the magnitude is about 4/3 which is 50% of that of Meissner state.

IV. CASE OF A CRITICAL BEAN STATE REALIZED AROUND THE CENTER OF THE DISK

In this section we obtain magnetic moments assuming constant finite critical current which is independent of magnetic field, i.e., the Bean model.

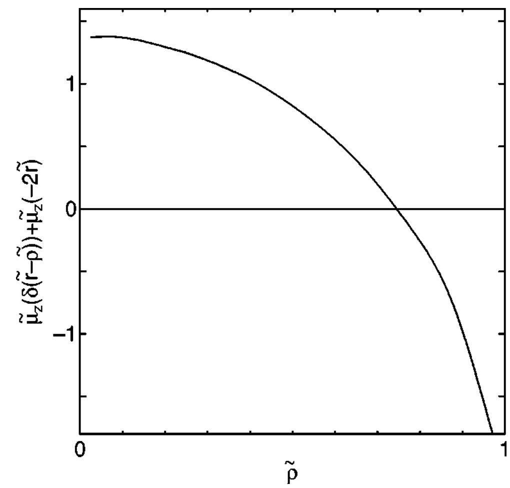

FIG. 2. Numerical result of $\tilde{\mu}_z$ when the flux Φ_{ext} is concentrated only on the radius $R\tilde{\rho}$.

We consider the situation that the flux through the disk exists only inside a radius r_b from the center of the disk and the critical current i_c flows here. f is defined as the ratio of flux compressed inside in field cooling (below T_c) to flux which has existed in the disk above T_c. This state is the same as that assumed by Koshelev and Larkin[21] except the sample geometry. After we fixed the values of r_b and f, we calculated magnetic moments numerically under the condition that the current inside a radius r_b (i_c is defined as the value) be constant. Thus the value of i_c is obtained after the calculation. The results are shown in Fig. 3. The $\times$'s correspond to the case that i_c turned out to be negative, but the possibility of the realization would be little. In $f=1$ magnetic moments are always positive and it becomes large as flux is compressed into the center. In calculations of thin strips by Koshelev and Larkin, the magnitude of the maximum magnetic moment is 27% of the Meissner state where $f=1$ and $r_b \to 0$. On the contrary, in a disk sample it is about

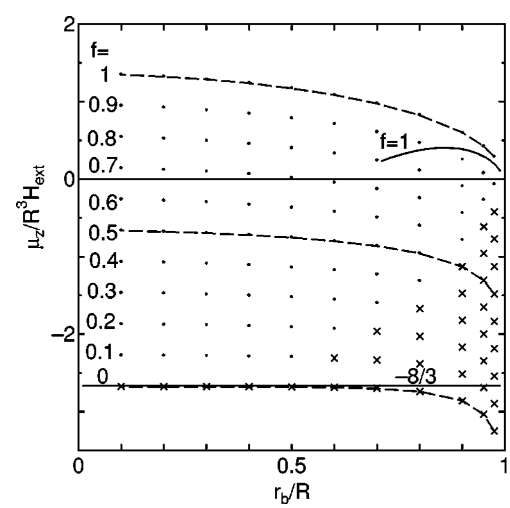

FIG. 3. Magnetic moments when $i_c=$const inside r_b and the Meissner state outside r_b. The $\times$'s correspond to the case that i_c flows in the negative direction. The line $(-8/3)$ shows the magnetic moment when the whole disk is in the Meissner state. The solid curve $(f=1)$ was predicted by Koshelev and Larkin (Ref. 21).

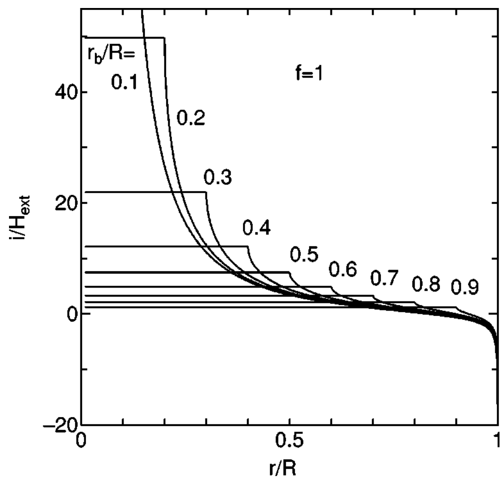

FIG. 4. The r/R dependence of current for $f=1$.

50% under the same conditions and it is larger than that of a thin strip. Koshelev and Larkin calculated the magnetic moment approximately in a disk sample only for $f\sim 1$ and $r_b \sim R$. The result is shown by the solid curve in $f=1$. However, it is not in agreement with our results. Their result is applicable only in $r_b\sim R$ and we may need to compare them where the conditions are satisfied more. The r/R dependence of the current is shown in Fig. 4 only for $f=1$. The current near $r\sim R$ flows in the direction in which the magnetic moment is negative, and the current near the center flows in the opposite direction. The contribution of the positive current exceeds that of the negative current; consequently a positive magnetic moment appears. The r_b/R dependence of i_c is shown in Fig. 5. Here i_c must be large in order to compress large flux into the center of the disk. The field dependence of the magnetic moment is shown in Fig. 6. If the model we assumed is realized, it would correspond to one of the points in the figure.

V. CONCLUSIONS

We calculated the maximum magnetic moment in the case of a disk sample under two conditions (see the text) and the

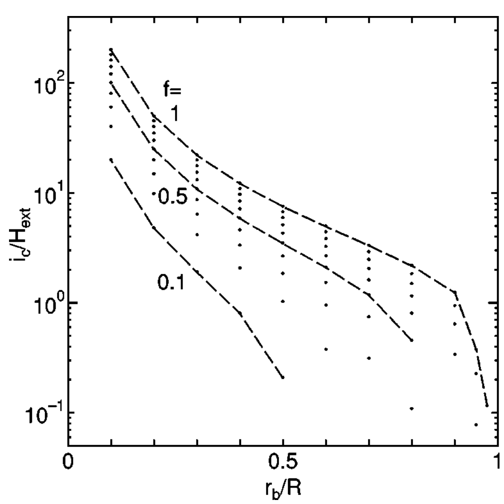

FIG. 5. The r_b/R dependence of i_c. The values of f are 1,0.9,0.8, . . . ,0 from top to bottom. The values of i_c which flow in the negative direction are not shown.

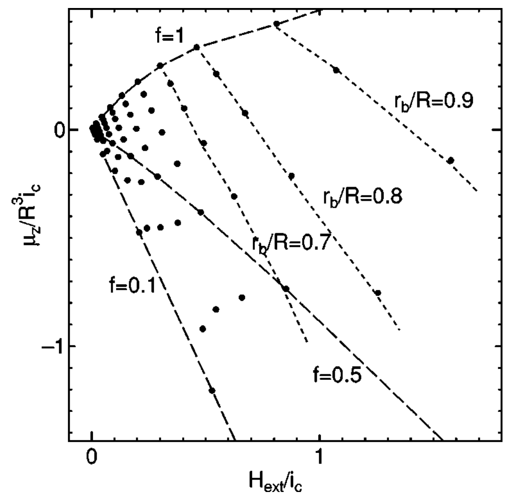

FIG. 6. The field dependence of magnetic moments. The dashed and dotted lines are guides to the eye. The values of f are 1,0.9,0.8, . . . ,0.1 from top to bottom and the values of r_b/R are 0.9,0.8, . . . ,0.1 from right to left. The values are not shown when i_c flows in the negative direction.

magnitude turned out to be 50% of that of the Meissner state. The conditions would be satisfied (especially in an inhomogeneous cooling process). In order to create a larger magnetic moment than 50%, we must consider the breakdown of either or both conditions. When OMM's exist, it is possible that the flux inside the sample in field cooling is larger than the flux through the disk at $T>T_c$ and then one of the conditions is not applicable.

We calculated magnetic moments and others in the case of the above-mentioned Bean state, and the maximum magnetic moment is about 50% of that of the Meissner state also in this case.

APPENDIX: MAXIMUM MAGNETIC MOMENT IN ANOTHER CASE

In this section we obtain maximum magnetic moments under the conditions of finite constant critical current density and $\Phi(1)=0$ without the condition of $B_z\geqslant 0$.

First, we consider the case without the former condition. For this purpose using an undetermined multiplier λ we obtain the current distribution $\tilde{I}_\theta(\tilde{r}')$ which causes $\tilde{\mu}_z -\pi^2\lambda\Phi(1)/\Phi_{ext}$ to be maximum. Using Eqs. (2.10) and (2.16),

$$\tilde{\mu}_z-\pi^2\lambda\frac{\Phi(1)}{\Phi_{ext}}=\frac{1}{2}\int_0^1 D(\lambda,\tilde{r}')\tilde{I}_\theta(\tilde{r}')d\tilde{r}', \qquad \text{(A1)}$$

where

$$D(\lambda,\tilde{r}')\equiv\tilde{r}'^2-\lambda(1+\tilde{r}')F(\tilde{r}'). \qquad \text{(A2)}$$

From the form of Eq. (A1) we can cause $\tilde{\mu}_z -\pi^2\lambda\Phi(1)/\Phi_{ext}$, i.e., $\tilde{\mu}_z$, to be large infinitely by taking an appropriate $\tilde{I}_\theta(\tilde{r}')$, i.e., $i_\theta(r',\theta')$.

Second, we calculate the current distribution that causes $\tilde{\mu}_z$ to be maximum under the condition of finite constant critical current density, namely,

$$\sqrt{\tilde{i}_\theta^2+\tilde{i}_r^2}\leqslant \tilde{i}_c, \tag{A3}$$

in an arbitrary region in the disk. The tilde refers to the value divided by H_{ext}. We calculate the current density which causes μ_z to be maximum under the looser condition of

$$|\tilde{I}_\theta(\tilde{r}')|\leqslant 2\pi\tilde{i}_c. \tag{A4}$$

Therefore, if the given current density satisfies Eq. (A3), then it is a solution. We calculate the condition that causes $\tilde{\mu}_z-\pi^2\lambda\Phi(1)/\Phi_{ext}$ to be maximum. As a result,

$$\tilde{I}_\theta(\tilde{r}')=\begin{cases} 2\pi\tilde{i}_c & \text{if } D(\lambda,\tilde{r}')>0, \\ -2\pi\tilde{i}_c & \text{if } D(\lambda,\tilde{r}')<0. \end{cases} \tag{A5}$$

Here $\tilde{i}_r$ and $\tilde{i}_\theta$ which satisfy Eqs. (A5) and (A3) exist and

$$\tilde{i}_r(r',\theta')=0, \tag{A6}$$

$$\tilde{i}_\theta(r',\theta')=\begin{cases} \tilde{i}_c & \text{if } D(\lambda,\tilde{r}')>0, \\ -\tilde{i}_c & \text{if } D(\lambda,\tilde{r}')<0. \end{cases} \tag{A7}$$

Next, we determine λ. Here $\Phi(1)$ is represented as follows using Eqs. (2.10) and (A5):

$$\Phi(1)=\mu_0 H_{ext}R^2\tilde{i}_c\int_0^1 \mathrm{sgn}[D(\lambda,\tilde{r}')](1+\tilde{r}')F(\tilde{r}')d\tilde{r}'. \tag{A8}$$

We determine λ so that $\Phi(1)$ is zero. We calculated λ numerically and we obtained $\lambda=0.406$ and

$$\begin{aligned} &D(\lambda,\tilde{r}')>0 \quad \text{if } 0.855>\tilde{r}'>0, \\ &D(\lambda,\tilde{r}')<0 \quad \text{if } 1>\tilde{r}'>0.855. \end{aligned} \tag{A9}$$

Therefore, Eq. (A7) is reduced to

$$\tilde{i}_\theta(r',\theta')=\begin{cases} \tilde{i}_c & \text{if } 0.855>\tilde{r}'>0, \\ -\tilde{i}_c & \text{if } 1>\tilde{r}'>0.855. \end{cases} \tag{A10}$$

Next, we calculate $\tilde{\mu}_z$ in this case. Substituting Eq. (A5) for Eq. (2.16),

$$\tilde{\mu}_z=\pi\tilde{i}_c\int_0^1 \tilde{r}'^2\ \mathrm{sgn}[D(\lambda,\tilde{r}')]d\tilde{r}'. \tag{A11}$$

Integrating it numerically,

$$\tilde{\mu}_z=0.260\tilde{i}_c, \tag{A12}$$

which is positive. We calculated H_z which is created by current in this case and the results are shown in Fig. 7.

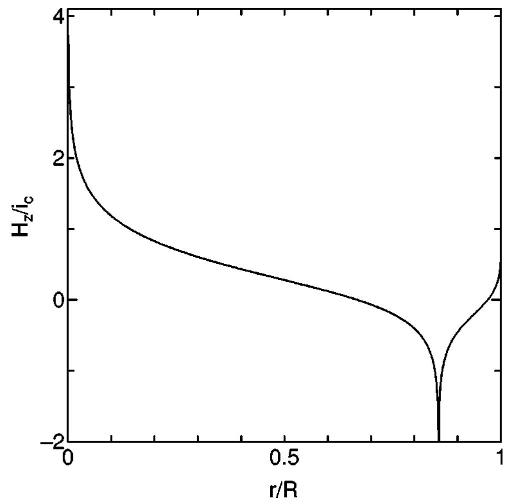

FIG. 7. The field distribution which gives the maximum magnetic moment under the condition that i_c be finite and $\Phi(1)=0$.

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